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OPTIMIZED SUSTAINABLE OFF-ONLINE-HYBRID CHANNELS OF RECYCLING UNWANTED VEHICLES BY REUTILISING FORWARD SALE NETWORKS

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In line with continuously improved living standards, the amount of unwanted vehicles (UWVs) is increasing as the consumption of vehicles is being rapidly upgraded. This research aims to design a strategic UWV recycle system with resource-conservation and environmentally-friendly advantages. A nonlinear bi-level programming model is first constructed to optimize cooperation strategies between the manufacturer (leader) and the auto 4S shops (followers), including repurchasing prices and processing ways for distinct types of UWVs, the provided convenience and incentive policies to UWV-holders. Then, an efficient smoothing algorithm is developed to solve this complicated model in virtue of model reformulation and its property analysis. Numerical simulation is employed to reveal their practical implications. Main findings include: Reusing of existing vehicle sales network not only can reduce transportation cost, but also greatly improves sustainability of the UWV recycle system; Differential strategy of processing the UWVs with different damage-aging degrees can greatly improve their utilization rate and the total system profit; Governmental subsidy and differences of user groups both play critical roles in facilitating cooperation of recycle enterprises and efficiency of this system. It is seen that, unlike existing results, this study provides an off-online-hybrid-channel strategy for recycling the UWVs with the synergism of manufacturers and auto 4S shops in the decision-making framework of the Stackelberg game. The optimal strategies can be provided by the developed new model and algorithm, and the values of this research are further validated by numerical simulation.

Keywords: Numerical Simulation; Off-Online-Hybrid-Channel Recycle; Bi-Level Optimization; Smoothing Algorithm; System Modeling.

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1. INTRODUCTION

Unceasing innovation, together with higher life standards, has activated the potential market of used vehicles. According to the data in 2021, provided by the China Vehicle Dealers Association, the national transaction volume of the used vehicles is 17.585 million, increased by 22.6\% (Association, 2022) compared with that in 2020. Clearly, recycling and reusing the unwanted vehicles (UWVs) with different damaged and aged levels by advanced inspection and repair technology directly determine the quantities of UWVs either returning to the market or being directly dragged into a vehicle scrap center (Phuc, 2017), which are closely associated with saving of raw materials, reduction of energy consumption and greenhouse gas emissions (Sakai, 2014; Wang, 2020).

Unfortunately, in practice, it is seen that either many UWVs are idle owing to the inappropriately pricing by the recycle companies, or many badly aged vehicles are still used (Li, 2014). Although the widely used mode of 'who.sell.who.recycle' has been increasingly established in many regions or countries, the cost of transporting the collected aged vehicles is higher than that of processing them at the local collection sites. In addition, such a mode cannot reuse the idle vehicles still being well-condition. In other words, the existing mode in practice is not beneficial to the timely recycling of all the UWVs and to the promotion of the UWV transaction. Thus, by the principles of environmental protection and resource conservation, it is an urgent need to strategically explore more efficient recycling modes with less operational cost for recycling and reusing all the UWVs.

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In this paper, we intend to design an effective off-online-hybrid-channel system of recycling the UWVs with different damaged and aged levels, where the manufacturer online performs recycling of UWVs as the leader in the framework of the Stackelberg game, entrusting the well-developed forward network of the offline auto 4S shops (the follower) to refurbish or dismantle the UWVs. Then, the government pays proper subsidies to the manufacturer, proportional to the total recycled amount, rather than a one-time fiscal expenditure. In virtue of this operational mode, the willingness of all the recycling participants can be activated, and the UWV-holders' convenience of recycling may be improved. Besides these potential advantages, the sustainability of the recycle system can also be enhanced by the reutilization of the existing well-established vehicle sales networks, owing to the cost reduction of building new recycle facilities (Qu, 2022).

1.1.1 Third-Level Heading

2. LITERATURE REVIEW

2.1 Recycling modes of UWVs

Recently, many researchers have studied the issue of recycling UWVs from the perspective of recycling modes (Savaskan, 2004; Liu, 2017). Especially, designing a network of efficiently recycling UWVs is often considered to be the most important for such a reverse logistics system.

In the early stage of vehicle remanufacturing development, there are three types of pure modes to recycle the UWVs, as shown in Figure 1. By the first type, one recycles the UWVs directly by vehicle manufacturers from end-users (Mode M) (Zhang, 2022), where the vehicle manufacturers track the entire lifecycle of UWVs, and the users of UWVs directly hand them over to the corresponding manufacturers for reprocessing and disposal. However, due to the dispersed locations of the vehicle manufacturers in this mode, the collection and transportation costs of UWVs are often higher. By the second type, UWVs are collected by the offline auto 4S shops from end-users (Mode R) (Dey, 2021), who do not have the technology of reprocessing the used vehicles and need to transport the collected UWVs to manufacturers for the subsequent reprocessing and disposal, also resulting in higher transportation costs. By the third type, UWVs are recycled by an independent third-party recycler (enterprise) from end-users (Model T) (Shankar, 2018), who is responsible for the collection, disassembly, and transportation of the UWVs. Then, the disassembled parts are sold to the vehicle manufacturers for subsequent manufacturing. Clearly, this mode incurs a higher infrastructure construction cost during the establishment of the third-party recycler.

With the development of the industry of recycling UWVs, new hybrid modes are gradually forming. Gołębiewski *et al.* (2013) discussed three modes of recycling the UWVs: Extended producer responsibility recycling, third-party recycling (that is, the transportation of UWVs and dismantled materials is carried out through third-party logistics enterprises), and manufacturer alliance recycling, representing the joint recycling of multiple types of vehicle manufacturers. Driven by the introduction of the European End-of-Life Vehicle Directive, Mansour and Zarei (2008) proposed the cooperation between manufacturers and processing facilities to recycle end-of-life vehicles. According to the number, location, and capacity of collection centers and dismantling centers, as well as the flow of materials between different facilities, a multi-cycle reverse logistics optimization model was developed. In Mexico, Cruz-Rivera and Ertel (2009) put forward that vehicle manufacturers designed new collection facilities for a series of activities such as recycling, decontamination, and demolition. However, the main cost of the system is paid to the cost of establishing collection facilities and the cost of dragging old vehicles to collection facilities. Zhang *et al.* (2016) combined the comprehensive advantages of vehicle manufacturers, auto 4S shops, and third-party logistics and redesigned the comprehensive recovery model of the supply chain. In this model, auto 4S shops recycled the UWVs in each region. Then, the third-party logistics companies transported them in batches to the manufacturer for remanufacturing. However, this recycling mode requires a highly collaborative management mechanism. In order to increase overall profits by reducing warehouse costs, Liang and Ma (2015) set up an alliance of manufacturers and retailers for recycling the UWVs and storing them in a recycling warehouse, jointly established by the manufacturer and retailer.

It is noted that in the above-stated recycling modes, there is none to mention that UWVs are recycled with manufacturers cooperating with their subordinate auto 4S shops. It is clear that if the vehicle manufacturers become online recyclers of UWVs, while the auto 4S shops offline collect and directly process the UWVs at the ends of markets, then the convenience level of the provided recycle service can be improved for the UWV-holders, as well as reducing the cost of establishing new recycle facilities by making full use of existing well-developed sales networks. Both the increased amount of the recycled UWV and the cost savings are beneficial to improved sustainability of this recycle system.

In the existing recycle networks, governmental subsidies has been paid great attention, especially when the recycling willingness of UWV-holders is low (Wan, 2019; Zheng, 2011). Zhang *et al.* (2019) argued that the recycling price plays a key role in increasing the willingness of vehicle owners to participate in recycling activities. They suggested that the government should provide a subsidy to vehicle owners to increase their profits from vehicle recycling. Although the governmental subsidy is a hot topic in the existing research results, it is still a challenge for the government to provide an appropriate recycle subsidy in accordance with the contribution of recycling the UWVs to waste treatment and resource savings, rather than blindly implementing a subsidy so as to add a significant but unnecessary governmental expenditure burden. For example, the existent results did not consider how the governmental subsidy is provided in line with differences in the unit auto sale price and the aging degrees of the collected and processed UWVs by the recyclers. Owing to differential roles in protecting the environment and saving resources for the recycled UWVs of different types, a one-shot fixed governmental subsidy is not beneficial for its full use and alleviation of governmental burden.

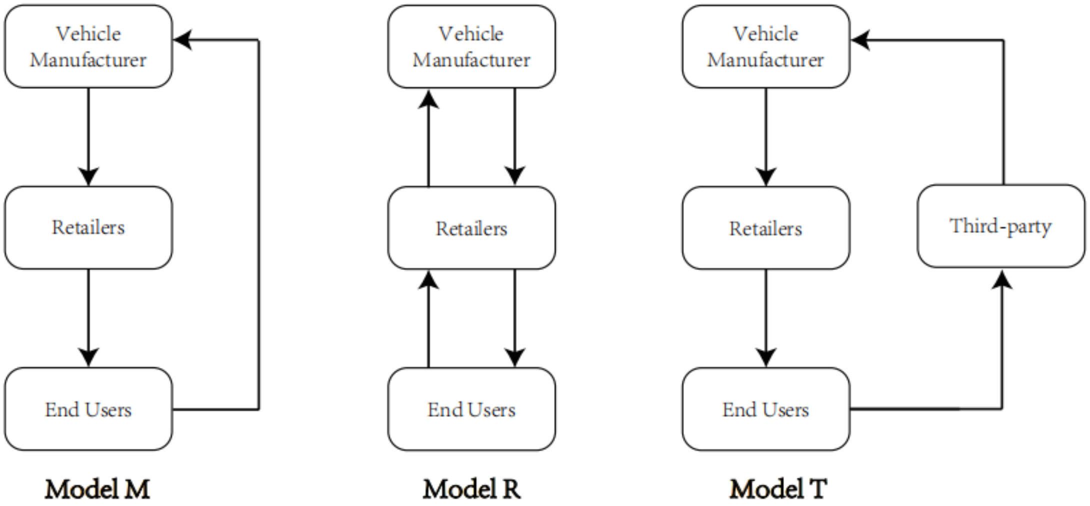


Figure 1. Pure modes of recycling UWVs

2.2 Optimization methods for recycling UWVs

Zarei *et al.* (2010) established a mixed integer goal programming model to optimize the generation and distribution of new vehicles and the recycling network of vehicles under an integrated system. An improved genetic algorithm was proposed to find an optimal solution to this model. Harraz and Galal (2011) also constructed a mixed integer goal programming model from the perspective of sustainable development, environmental improvement, and social benefits. The decision variables were associated with the location selection of collection and disassembly centers, and the material flows in UWVs. However, in this model, the repurchase price of UWVs was not considered as an endogenous variable. Sun *et al.* (2012) proposed a mixed integer linear programming model for collecting the used vehicles in Mexico, aiming at optimizing the number and the location of the sites for their collection, purification, and demolition operations. Demirel *et al.* (2016) studied a mixed integer linear programming model (MILP) for network design, where different participants were considered in this UWV-recycling system, and the manufacturers were responsible for retrieving the UWVs from end users, disassembling, shredding, and recycling them. Mahmoudzadeh *et al.* (2013) established a mixed integer programming model to find the optimal location of the scrap yard for a third-party recycling agency. It is noted that the models in these articles were solved by the off-the-shelf software packages, like IBM CPLEX (a popular optimization solver). Lin *et al.* (2018) established a mixed integer programming model to study the location-allocation problem of facilities for recycling the end-of-life vehicles such that the total operational cost was minimized, and an algorithm of Artificial Bee Colonies was used to solve this model. Zhang and Wang (2008) designed an MILP model to construct a closed-loop recycling network for end-of-life vehicles (ELVs). This network consisted of three echelons, namely, disassembly centers, remanufacturing factories, and redistribution warehouses, with the objective function of minimizing the total network cost. Ayvaz *et al.* (2021) studied an elastic recycling network for ELVs by constructing a multi-objective fuzzy mathematical programming model to address the optimal location and number of recycling facilities for ELVs in a fuzzy environment. Multiple indicators include the total cost of the network, the vehicle emissions, the number of jobs created, and the ecological tax profit. Summarily, the majority of the models in these results focus on location decisions in the design of recycling networks with the targeting of maximum efficiency or minimum cost. However, in real-world scenarios, vehicle manufacturer and auto 4S shops often seek their respective maximum profits when jointly recycling the UWVs. Thus, it seems more practicable to consider the respective optimal strategies for the vehicle manufacturer and the auto 4S shops in the framework of the Stackelberg game.

Notably, almost all of the models in the literature were solved by developing heuristic algorithms or by off-the-shelf optimization solvers. However, these solution methods cannot be directly applied to solve an optimization model with complementarity constraints (Luo, 2022). For the latter, it is necessary to develop new efficient algorithms to solve this type of complicated model, especially based on the property analysis of models. In order to further emphasize the diversity of this study, we here summarize the shortcomings of all the related works as follows.

- In the majority of the existing results, recycling modes of UWVs were designed by choosing new recycle facilities, rather than reutilizing the well-developed forward sales network of new vehicles.
- Little recycling models were designed on the basis of online and offline hybrid recycling channels in the literature.
- In all the optimization models of recycling UWVs, none of them considered the heterogeneity of the collected UWVs and their distinct strategies of processing them.
- In the majority of the existing models, decision variables were only associated with choices of location and distribution, not including the repurchase price. In practice, it is seen that the recycled amount of UWVs is often closely related to the repurchase price and public environmental awareness.
- In the available recycle systems, few models adopted a decentralized decision-making mode by taking into account the respective benefits of all the recyclers.
- For the complicated models, there were few efficient algorithms to be developed based on the analysis of model properties. Instead, they were often solved by either the off-the-shelf solvers or the heuristic algorithm, without the use of model properties.

2.3 Contribution of this research

Motivated by remedying the drawbacks of existing results on UWV recycle problems, we attempt to design a new recycle system in line with the following new ideas: (1) Instead of constructing a new recycle network, the recycle system offline collects and processes the UWVs by reutilising the existing well-built sales network of new vehicles, aiming at reduction of its total operational cost, such as the cost of infrastructure construction and the delivery cost. Indeed, it has been verified (Luo, 2024) that reutilization of the existent informative pharmaceutical sales network can greatly improve the efficiency of recycling the unwanted drugs. (2) In order to play the dominant role of manufacturers in remanufacturing, the manufacturer of vehicles is regarded to be the leader of this recycle game system, which commissions the recycling of UWVs to its subordinate auto 4S shops, and online performs the transaction with the UWV-holders. Intuitively, the structure of such a recycle system is depicted in Figure 2.

Mathematically, the above management system is formulated by a nonlinear bi-level programming model, where the buy-back price, the commission cost paid by the manufacturer to the auto 4S shops, and the proportion of refurbishing the collected UWVs are all the endogenous decision variables, so as to improve the efficiency of this recovery system. Compared with the related works in the literature, we state the novelties of this research as follows.

(1) In terms of damage/aging levels and reuse values in markets, the collected UWVs are classified into three types and are accordingly treated in distinct ways. Specifically, with regard to the first type, the collected UWVs are relatively new and are directly refurbished to be resold. With regard to the second, owing to the significant degree of damage/aging, one needs to optimize a proportion of them either to be refurbished or to be dismantled in terms of maximizing the total profit. With regard to the third, the collected UWVs are directly dismantled owing to their poor condition.

(2) The buy-back price, paid by the manufacturer to the UWV-holders, is the dynamic to drive the recycled amount of UWVs, being assumed to be a linear relationship between them, since the price is often determined to be smaller than its actual value in practice. For this reason, there is no need to consider the nonlinear relationship between the recycled amount and the price for simplicity of model building.

(3) Different from the existing model of recycling UWVs in practice, an innovative recycling mode is proposed, where cooperation between the auto manufacturer and the auto 4S shops is for the first time strategically incorporated into the system design of recycling the UWVs, and a bi-level programming model is built to provide optimal strategies for the operational problem of this recycle system. Indeed, for the pharmaceutical recycle problems, it has been shown by Luo and Wan (2024) that reutilizing the existent informative pharmaceutical sales network can greatly improve the efficiency of recycling the unwanted drugs.

(4) Due to the complexity of the built model, we also develop an efficient algorithm by employing the gradient information in the objective and constraint functions to generate better and better approximate solutions, rather than the direct solution methods or the heuristic algorithms in the literature (Wan and Zou, 2019). Remarkably, by scenario analysis and sensitivity analysis, a number of valuable findings can be revealed from the model in virtue of this algorithm.

The rest of this article is organized as follows. In Section 3, the studied problem and its formulation are presented. Section 4 is devoted to the property analysis of the model and the development of an efficient algorithm. In Section 5, scenario analysis is conducted. Sensitivity analysis is performed in Section 6. Preliminary real-life application of the developed model and algorithms presented in Section 7. Conclusions and suggestions for future research are given in Section 8.

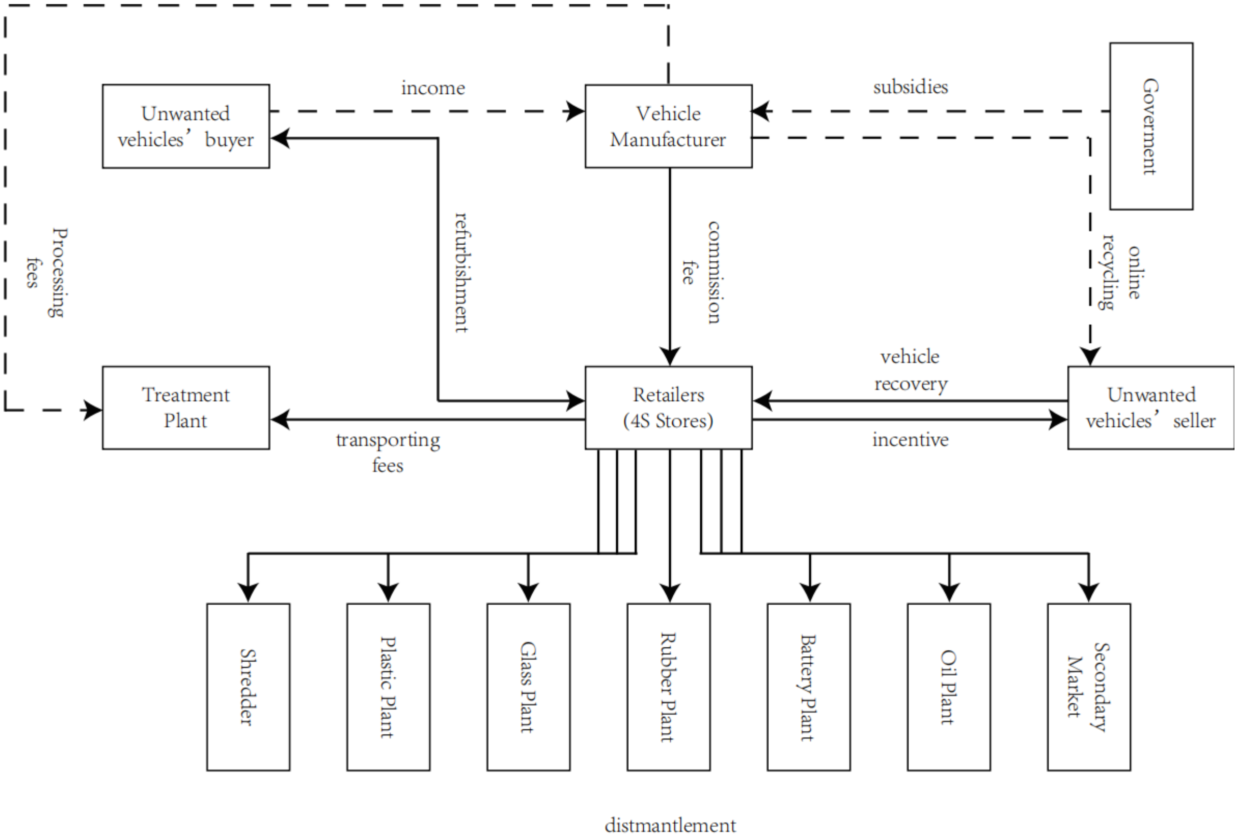


Figure 2. The system structure of recycling UWVs

Table 1. Comparison of this research with the closely related results

Authors	**Collected item**	**Collected by**	**decision type**		**Customer incentive**	**Government policy**	**Heterogeneous product**	**Establish recycle facilities**	**Built model**	**Solution method**
			Centred	**scattered**						
Zarei *et al.*, 2010	ELVs	New vehicles distributors	√						MILP	Genetic algorithms
Harraz *et al.*, 2011	ELVs	A 3PL	√		√		√	√	MILP	The branch and bound technique
Demirel *et al.*, 2016	ELVs	Manufacturers	√					√	MILP	Genetic algorithms
Sun *et al.*, 2024	ELVs	A collection centre	√					√	MILP	CPLEX
Mahmoudzadeh *et al.*, 2013	ELVs	Multi 3PLs	√		√		√	√	MILP	CPLEX

Authors	Collected item	Collected by	decision type		Customer incentive	Government policy	Heterogeneous product	Establish recycle facilities	Built model	Solution method
			Centred	scattered						
Lin *et al.*, 2018	ELVs	Recycling center	√					√	MILP	A metaheuristic algorithm
Zhang *et al.*, 2008	Automobile	Suppliers; 3PL Manufacturers	√		√			√	MILP	LINDO
Ayvaz *et al.*, 2021	ELVs	Multi 3PL	√					√	MFMM	Genetic algorithms
This Research	ELVs	4S shops		√	√	√	√		NBLP	Smoothing algorithm

3. THE STUDIED RECYCLE PROBLEM AND ITS BI-LEVEL PROGRAMMING MODEL

3.1 Problem description

In this paper, we consider a recycle system with one vehicle manufacturer and multiple auto 4S shops, which recycle the UMVs through offline and online hybrid channels, as shown in Figure 3.

Specifically, in order to make full use of irreplaceable technical remanufacturing advantages, the vehicle manufacturer in Figure 3 is the upper-level decision-maker (the leader), undertakes its dominant responsibility of recycling UWVs, and cooperates with multiple auto 4S shops (followers) to collect and treat the UWVs under the incentive of governmental subsidy. In this paper, with the offline detection of auto 4S shops, the UWVs are first classified into the following three types, mainly in line with the aging and damage degrees of UWVs:

The first type: The UWVs, which can be directly refurbished and resold online, often within four years old service life.

The second type: The UWVs, which can be either refurbished or dismantled, often with a service life between four and twelve years.

The Third type: The UWVs, which are directly dismantled for the reuse of their parts, often with a service life of more than twelve years.

Then, the auto 4S shops inform their detected reports to the manufacturer.

As the leader in this recycle system, the vehicle manufacturer first needs to online determine the unit commission cost paid to the auto 4S shops for each type of UWVs when the manufacturer online receives the filled information by the UWV users, such as the UWV brand, its driving mileage, and registration time. Basically, this unit commission cost is positively proportional to the unit repurchase price of each type of UWVs, paid offline by the auto 4S shops to the users of UWVs. In addition to the unit commission cost, the manufacturer also needs to decide the ratio of the second type of UWVs for further refurbishing or dismantling. Actually, benefiting from the recycling of all three types of UWVs, the manufacturer can earn income from selling the refurbished UWVs of the first and the second types, and also the subsidy from the government calculated by the recycled amount of all three types of UWVs. That is to say, the subsidy paid by the government is an intermediate variable that is calculated by the recycled amount of all three types of UWVs, rather than being a fixed model parameter as done in the literature.

With an appropriate incentive strategy from the vehicle manufacturer, the auto 4S shops in Figure 3 are willing to offline collect the UWVs from users with a recycle price as high as possible, so as to increase the total recycled amount of UWVs and get the commission revenue as much as possible. As lower-level decision-makers in this recycle system, the auto 4S shops need to optimize the unit repurchase price for the distinct types of UWVs, as the unit commission fees are paid by the vehicle manufacturer, so as to maximize their own profits. In this research, it is assumed that the unit repurchase price paid to the UWV-holders depends on the damage/aging degree of UWVs through offline detecting. Clearly, a higher repurchase price can increase the users' willingness to participate in recycling UWVs, but may increase the recycle cost of the vehicle manufacturer. Specifically, the auto 4S shops commissioned by the vehicle manufacturer provide higher quality service to the UWV users, who can choose to drive their UWVs to the nearest 4S shop for repairing, or wait at home for the 4S shops to collect. Then, the 4S shops conduct detection on the collected UWVs so as to determine their repurchase price.

Subsequently, the refurbished UWVs are sold through the manufacturer's online platform and are picked up at the corresponding offline 4S shops, while the dismantled parts are also sold to the suppliers of parts or are reused by the manufacturer.

Since the vehicle manufacturer and the 4S shops in this recycle system pursue their respective profit maximization, it is clearly a decentralized decision mode. Owing to their different roles in promoting the UMV recycle, we will construct a Stackelberg game model to formulate the decision processes of the manufacturers and the 4S shops. Indeed, since the manufacturer masters the key technology of vehicle recycling, it is reasonable for the manufacturer to be the leader of this system. Inspired by the provided incentives from the leader, the 4S shops participate in the recycling of UMVs, hence are regarded to be the followers of this system. Notably, different from existing recycle systems in the literature, the reutilization of the well-built forward sales network in this paper, i.e., the auto 4S shops, is clearly beneficial to the reduction of the total recycle cost and the consumed resources for building a new recycle network. Such a recycle system in Figure 3 will also be embedded into real-world scenarios in Section 5.

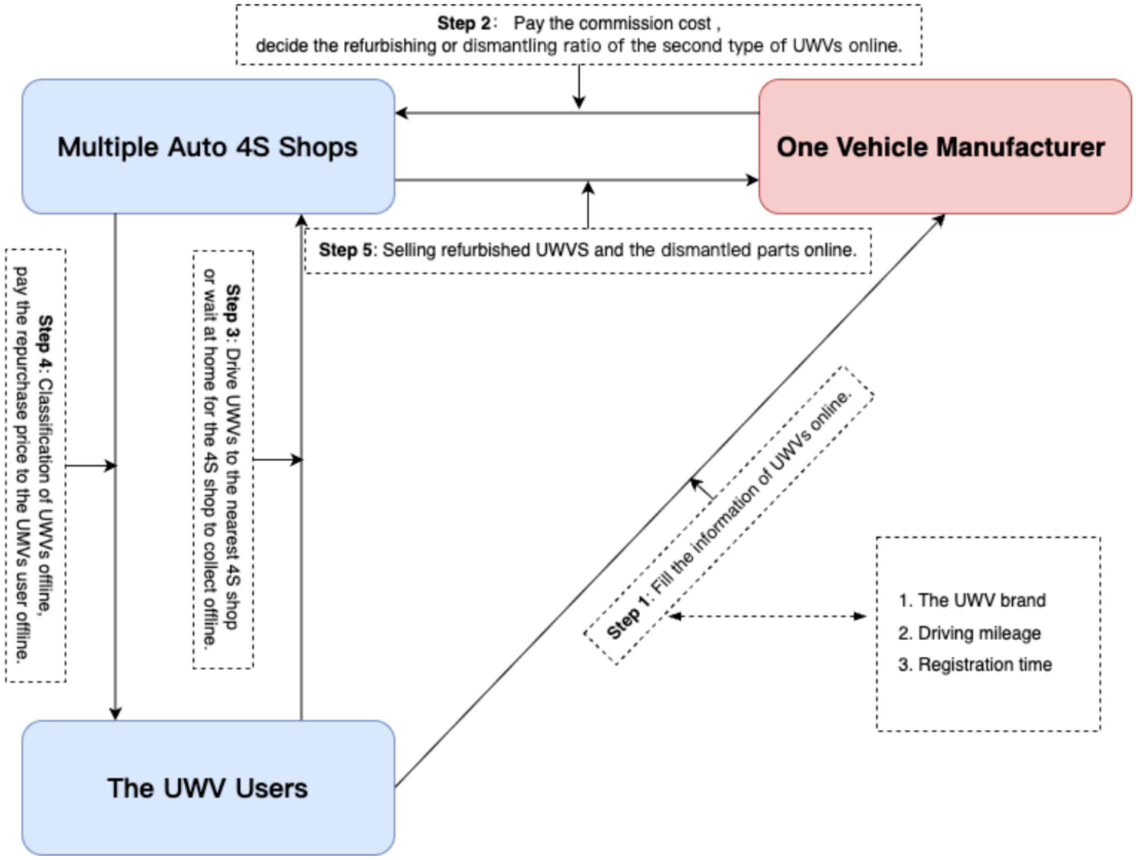


Figure 3. Off-online-hybrid-channels performed by one vehicle manufacturer and multiple auto 4S shops

In summary, compared with existing results in the literature, the studied system in this paper has the following novelties and advantages.

- Three different types of UWVs are considered, and their repurchase prices are optimized as endogenous decision variables of the system for UWVs. Particularly, whether the collected UWVs are refurbished or dismantled depends on their damage and aging degrees.
- By integrating the online and offline hybrid recycling channels, especially with reutilization of the well-built forward auto sales network, the service level of UMV recycle enterprises is improved, owing to the convenience of online transactions and Offline delivery and pickup for the users.
- By reutilizing the well-built forwards auto sales network, the total cost of the system can be greatly reduced, especially the saved resources to build a new recycle network.

- Instead of building facilities for processing and recycling batteries, tires, glass, and plastics, these recycled materials in this system will be sold separately to existing factories equipped with processing technology. This setting is closer to the practical situation of China, and is also helpful to realize the full utilization of resources and meet the requirements of sustainable development.

In order to show the values of doing so, we intend to mathematically address the following problems:

- How to formulate the management problem of this recycle system into a bi-level optimization model?
- As the leader in the built bi-level programming model, how does the vehicle manufacturer pay their partner 4S shops for the commission cost of UWVs at different levels of aging and damage in order to maximize its interests?
- As the followers in the built bi-level programming model, how does the retailer determine the repurchase price of UWVs to maximize profit?
- How does governmental subsidy affect the vehicle manufacturer's decisions and the recycled amount of UWVs?
- How do the auto 4S shops' inventory capacity and technology level affect the total system profit?

3.2 Notations

Before building a bi-level programming model for the designed recycle system, the notations used in this paper are given in Table 2.

Table 2. Nomenclature in models

Symbol	**Description**
Indices	
t	The type of UWVS, $t \in T = \{1,2,3\}$
i	The brand of UWVS, $i \in I$
j	The user of UWVS, $j \in J$
Vehicle manufacturer's parameters	
p_i	The unit sale price of the new vehicle with Brand i (CNY/unit)
q_i^A	The resale price of UWVS with Brand i and service life less than four years old (CNY/unit)
q_i^B	The resale price of UWVS with Brand i and service life between four-to-eight years old (CNY/unit)
q_i^C	The resale price of UWVS with Brand i and service life between eight-to-twelve years old (CNY/unit)
τ^A	The proportion of the refurbished UWVS with service life of less than four years old
τ^B	The proportion of the refurbished UWVS with service life between four-to-eight years old
τ^C	The proportion of the refurbished UWVS with service life between eight-to-twelve years old
4S shops' parameters	
D	The maximum inventory capacity of the auto 4S shops
f_i	The disposal cost for disposing a unit of hazardous materials of UWVS with Brand i (CNY/unit)
e_i^1	The unit cost of new parts for refurbishing UWVS with Brand i of the first type (CNY/unit)
e_i^2	The unit cost of new parts for refurbishing UWVS with Brand i of the second type (CNY/unit)
r_i	The unit price for refurbishing UWVS with Brand i (CNY/unit)
d_i	The unit price for dismantling UWVS with Brand i (CNY/unit)
c_i	The unit price of detecting and classifying UWVS with Brand i (CNY/unit)
TD_i	The unit price of transporting hazardous materials of UWVS with Brand i from the auto 4S shops to the disposal sites (CNY/unit)
s'_{i1}/s_{i1}	The resale of metal parts of UWVS with Brand i of the first/second type in the secondary market (CNY/unit)
s'_{i2}/s_{i2}	The resale of non-metal parts with Brand i of the first/second type in the secondary markets (CNY/unit)
s'_{i3}/s_{i3}	The resale of oil of UWVS with Brand i of the first/second type in the secondary market (CNY/unit)
s'_{i4}/s_{i4}	The resale of battery of UWVS with Brand i of the first/second type in the secondary market (CNY/unit)
s'_{i5}/s_{i5}	The resale of rubber of UWVS with Brand i of the first/second type in the secondary market (CNY/unit)
s'_{i6}/s_{i6}	The resale of glass of UWVS with Brand i of the first/second type in the secondary market (CNY/unit)
s'_{i7}/s_{i7}	The resale of plastic of UWVS with Brand i of the first/second type in the secondary market (CNY/unit) Secondary market (CNY/unit)
s'_{i8}/s_{i8}	The resale of ferrous/non-ferrous material of UWVS with Brand i of the first/second type in the secondary

Symbol	Description
	market (CNY/unit)
γ	The proportion of hazardous materials of UWVS
β_1	The proportion of available metal parts of UWVS
β_2	The proportion of available non-metal parts of UWVS
β_3	The proportion of available oil of UWVS
β_4	The proportion of available battery of UWVS
β_5	The proportion of available rubber of UWVS
β_6	The proportion of available glass of UWVS
β_7	The proportion of available plastic of UWVS
β_8	The proportion of the ferrous/non-ferrous material of UWVS
Government's parameters	
g_i^t	The unit subsidy given to the vehicle manufacturer by government of UWVS with Brand i of the t-th type (CNY/unit)
Customer' parameters	
A_{ij}	The largest recyclable amount of UWVS with Brand i of the first type at Collecting-site j
B_{ij}	The largest recyclable amount of UWVS with Brand i of the second type at Collection-site j
C_{ij}	The largest recyclable amount of UWVS with Brand i of the third type at Collection-site j
a_{ij}^t	The recyclable amount of UWVS with Brand i of the t-th type at Collection-site j when the repurchase price is 0
b_{ij}^t	The repurchase price coefficient of UWVS with Brand i of the t-th type at Collection-site j
Decision variables and intermediate variables	
ρ_i^t	The unit repurchase price that the auto 4S shops provide to users of UWVS with Brand i of the t-th type (CNY/unit)
θ_{ij}^t	The recycled amount of UWVS with Brand i of the t-th type by the auto 4S shops at Collection-site j
σ_t	The proportion of commission cost to the sale price of UWVS of the t-th type
η	The refurbishing percentage of the second type of UWVS

3.3 Optimization models of the auto 4S shops

The auto 4S shops, as the followers of the Stackelberg game in the system of recycling the UWVs, need to optimize the repurchase prices of all three types of UWVs so as to maximize their own profit, as the unit commission fees are paid by the vehicle manufacturer, the leader in this game.

Denote ρ_i^t the unit repurchase price of the t-type the UWVs with the i-th brand, $t = 1,2,3$, $i \in I$. Suppose that the recycled amount of each type of UWVs linearly depends on the unit repurchase price. That is to say, the recycled amount is defined by

$$\theta_{ij}^t = a_{ij}^t + b_{ij}^t \rho_i^t, \forall t = 1,2,3, \tag{3.1}$$

where a_{ij}^t is a given parameter, embodying the impacts of the public environmental awareness at the j-th collection site, i.e., the j-th group of UWV users, on the recycled amount of UWVs, even if no repurchase price is paid to the holders of UWVs, $b_{ij}^t$is a given model coefficient reflecting the price sensitivity on the recycled amount. Note that the linear model (3.1) is the simplest regression model to fit the practical data on the relation between the recycled amount and the take-back price. With this simplification, the objective in Model (3.11) is a quadratic function of the decision variables. It is possible that a nonlinear regression model more fits the practical data better than (3.1) if the unit repurchase price is very high. In this case, Model (3.11) will become more complicated. Taking into account that the paid repurchase price is much lower than its potential value in practice, the linear model (3.1) is reasonable.

Let σ_1, σ_2 and σ_3 be the determined percentages of the unit sale prices of the three types of UWVs by the vehicle manufacturer, respectively. Let p_i be the unit sale price of the i-th brand of UWVs. Then, the total income of the auto 4S shops, i.e., the total commission fees, is given by

$$CFI(\rho_i^1, \rho_i^2, \rho_i^3; \sigma_1, \sigma_2, \sigma_3) = \sigma_1 \sum_{i \in I} p_i \sum_{j \in J} \theta_{ij}^1 + \sigma_2 \sum_{i \in I} p_i \sum_{j \in J} \theta_{ij}^2 + \sigma_3 \sum_{i \in I} p_i \sum_{j \in J} \theta_{ij}^3. \tag{3.2}$$

The total repurchasing cost of the auto 4S shops is

$$RC(\rho_i^1,\rho_i^2,\rho_i^3)=\sum_{i\in I}\rho_i^1\sum_{j\in J}\theta_{ij}^1+\sum_{i\in I}\rho_i^2\sum_{j\in J}\theta_{ij}^2+\sum_{i\in I}\rho_i^3\sum_{j\in J}\theta_{ij}^3. \tag{3.3}$$

Since the auto 4S shops need to be responsible for offline classification and inspection of the recycled UWVs, the total cost of classification and inspection is calculated by

$$TCC(\rho_i^1,\rho_i^2,\rho_i^3)=\sum_{i\in I}c_i\sum_{j\in J}\left(\theta_{ij}^1+\theta_{ij}^2+\theta_{ij}^3\right). \tag{3.4}$$

Then, when the UWVs are required to be further refurbished or dismantled, such a part of operational cost is specified by

$$RDC(\rho_i^1,\rho_i^2,\rho_i^3)=\sum_{i\in I}r_i\sum_{j\in J}\left(\theta_{ij}^1+\eta\theta_{ij}^2\right)+\sum_{i\in I}d_i\sum_{j\in J}\left(\theta_{ij}^3+(1-\eta)\theta_{ij}^2\right), \tag{3.5}$$

where r_i and d_i are the unit refurbishment and dismantlement costs of the UWVs, respectively. In addition, some hazardous materials from dismantling the UWVs must be transported to the specified disposal sites. Thus, the auto 4S shops need to pay the following transportation costs:

$$TTC(\rho_i^2,\rho_i^3)=\sum_{i\in I}T\,D_i\gamma\sum_{j\in J}\left(\theta_{ij}^3+(1-\eta)\theta_{ij}^2\right), \tag{3.6}$$

where TD_i is the unit transportation cost from the auto 4S shops to the disposal sites. Consequently, the total profit of the auto 4S shops is

$$\Phi_r(\rho_i^1,\rho_i^2,\rho_i^3;\sigma_1,\sigma_2,\sigma_3,\eta)=CFI-RC-TCC-RDC-TTC \tag{3.7}$$

We next consider the practical constraints when the auto 4S shops seek for the maximal profit.

The first type of constraints is on the inventory capacity of the auto 4S shops. Let Dbe the maximum inventory capacity of the recycled UWVs. Then, it holds that:

$$\sum_{j\in J}(\theta_{ij}^1+\theta_{ij}^2+\theta_{ij}^3)\le D,\forall i\in I. \tag{3.8}$$

The second type of constraints is on the largest social stock of UWVs for all three types of UWVs. Let A_{ij}, B_{ij} and C_{ij} be the largest social stocks of all the three types of the i-th brand UWVs in the user group j.

$$\theta_{ij}^1\le A_{ij},\ \theta_{ij}^2\le B_{ij},\ \theta_{ij}^3\le C_{ij},\forall i\in I,\forall j\in J. \tag{3.9}$$

The last type of constraints is the nonnegativity of the decision variables, i.e.,

$$\rho_i^t\ge 0, t=1,2,3. \tag{3.10}$$

With the above analysis, we obtain an optimization model of the auto 4S shops as follows.

$$\begin{aligned}&\max\ \Phi_r(\rho_i^1,\rho_i^2,\rho_i^3;\sigma_1,\sigma_2,\sigma_3,\eta)=CFI-RC-TCC-RDC-TTC.\\&s.t.(3.8),(3.9),(3.10)\end{aligned} \tag{3.11}$$

Remark 3.1. Note that the linear model (3.1) is the simplest regression model to fit the actual data to capture the relationship between the incentive and the recycled amount of UWVs. With this simplification, the objective function in Model (3.11) is the quadratic function of the decision variables. In practice, it may be required to apply the nonlinear regression method to fit these data. In this case, the solution of Model (3.11) will become more complicated.

Remark 3.2. On the one hand, due to the reuse of existing developed retailer networks, there is no cost of infrastructure construction, and a part of the cost to deliver the collected UWVs in our model (3.11). Clearly, such a strategy is beneficial to the reduction of the total operational cost and carbon emission of recycling UWVs. On the other hand, the unit repurchase price of the UWVs is an endogenous variable in Model (3.11) to drive the total recycled amount of UWVs, rather than a fixed price. When the repurchase price is optimized in terms of the obtained subsidies from the government and the total system profit, the increased amount of the recycled UWVs is beneficial to green production.

3.4 Optimization models of the vehicle manufacturer

As an upper-level decision maker in the bi-level programming model, the vehicle manufacturer cooperates with the auto 4S shops, the followers in the Stackelberg game. For this reason, the vehicle manufacturer needs to decide how to pay the commission fee to the auto 4S shops and how to determine a proper percentage of refurbishing percentage of the second type of UWVs so as to maximize the total profit.

Specifically, the revenue of the vehicle manufacturer consists of two parts: The first part is the income from selling the refurbished UWVs and the reusable parts dismantled from the UWVS; The second part is the government subsidizes proportional to the total recycled amount of UWVs. More in details, the vehicle manufacturer online resells the refurbished UWVs of the first type and a part of the second type to the customers, who can pick them up offline at the nearest auto 4S shops. It's supposed that the resale prices of the refurbished UWVs are related with their service life. Denote q_i^A the resale price of those refurbished UWVs with a service life of less than four years, denote q_i^B the resale price of those refurbished UWVs with a service life between four years and eight years, and q_i^C the resale price of those refurbished UWVs with a service life between eight years and twelve years. Let τ_A, τ_B and τ_C be the proportions of the recycled UWVs with a service life of less than four years, those with a service life between four and eight years, and those with a service life between eight and twelve years, respectively. Then, for the vehicle manufacturer, the total income of selling the refurbished UWVs and the reusable parts is given by

$$\begin{aligned}
&SCI(\rho_i^1,\rho_i^2)+RPI(\rho_i^2,\rho_i^3)\\
&=\sum_{i\in I} q_i^A\,\tau_A\sum_{j\in J}(\theta_{ij}^1+\eta\theta_{ij}^2)+\sum_{i\in I} q_i^B\,\tau_B\sum_{j\in J}(\theta_{ij}^1+\eta\theta_{ij}^2)+\sum_{i\in I} q_i^C\,\tau_C\sum_{j\in J}(\theta_{ij}^1+\eta\theta_{ij}^2)\\
&+\beta_1(\sum_{i\in I} s_{i1}'\sum_{j\in J}(1-\eta)\theta_{ij}^2+\sum_{i\in I} s_{i1}\sum_{j\in J}\theta_{ij}^3)+\beta_2(\sum_{i\in I} s_{i2}'\sum_{j\in J}(1-\eta)\theta_{ij}^2+\sum_{i\in I} s_{i2}\sum_{j\in J}\theta_{ij}^3)\\
&+\beta_3(\sum_{i\in I} s_{i3}'\sum_{j\in J}(1-\eta)\theta_{ij}^2+\sum_{i\in I} s_{i3}\sum_{j\in J}\theta_{ij}^3)+\beta_4(\sum_{i\in I} s_{i4}'\sum_{j\in J}(1-\eta)\theta_{ij}^2+\sum_{i\in I} s_{i4}\sum_{j\in J}\theta_{ij}^3)\\
&+\beta_5(\sum_{i\in I} s_{i5}'\sum_{j\in J}(1-\eta)\theta_{ij}^2+\sum_{i\in I} s_{i5}\sum_{j\in J}\theta_{ij}^3)+\beta_6(\sum_{i\in I} s_{i6}'\sum_{j\in J}(1-\eta)\theta_{ij}^2+\sum_{i\in I} s_{i6}\sum_{j\in J}\theta_{ij}^3)\\
&+\beta_7(\sum_{i\in I} s_{i7}'\sum_{j\in J}(1-\eta)\theta_{ij}^2+\sum_{i\in I} s_{i7}\sum_{j\in J}\theta_{ij}^3)+\beta_8(\sum_{i\in I} s_{i8}'\sum_{j\in J}(1-\eta)\theta_{ij}^2+\sum_{i\in I} s_{i8}\sum_{j\in J}\theta_{ij}^3),
\end{aligned}\tag{3.12}$$

where η is the refurbishing percentage of the second type of UWVs.

Apart from the above income, the governmental subsidies are also the main income of the vehicle manufacturer. Instead of paying a fixed subsidy to encourage recycling of more UWVs in the current recycle practice, the governmental subsidies in this research are paid in line with the different types of recycled UWVs and their total recycled amounts. For the three types of UWVs, denote g_i^1, g_i^2 and g_i^3 the unit subsidies of the recycled UWVs of the i-th brand, respectively. Then, the total governmental subsidy is:

$$TSI(\rho_i^1,\rho_i^2,\rho_i^3)=\sum_{i\in I} g_i^1\sum_{j\in J}\theta_{ij}^1+\sum_{i\in I} g_i^2\sum_{j\in J}\theta_{ij}^2+\sum_{i\in I} g_i^3\sum_{j\in J}\theta_{ij}^3. \tag{3.13}$$

With regard to the operational cost, the vehicle manufacturer needs to pay a commission fee to the auto 4S shops, which is assumed to be a proportion of the unit sale price of the i-th brand autos, being associated with different damage and aging levels of the recycled UWVs. Let p_i be the unit sale price of the i-th brand autos, denote $\sigma_1 p_i$, $\sigma_2 p_i$ and $\sigma_3 p_i$ the paid commission fees of the three types of UWVs for each UWV of the i-th brand, respectively. Then, the total commission fee paid by the vehicle manufacturer is calculated by

$$CF(\sigma_1,\sigma_2,\sigma_3;\rho_i^1,\rho_i^2,\rho_i^3)=\sigma_1\sum_{i\in I} p_i\sum_{j\in J}\theta_{ij}^1+\sigma_2\sum_{i\in I} p_i\sum_{j\in J}\theta_{ij}^2+\sigma_3\sum_{i\in I} p_i\sum_{j\in J}\theta_{ij}^3. \tag{3.14}$$

Since the auto 4S shops are entrusted by the vehicle manufacturer to refurbish the UWVs of the first type and a part of the second type, it is necessary for the vehicle manufacturer to provide the auto 4S shops with the new parts used for the refurbishment. Denote e_i^1 the cost of the required new parts for refurbishing each unit UWVs of the first type, and denote e_i^2 The cost of the required new parts for refurbishing each unit UWVs of the second type. Then, the total cost of the used new parts for the refurbishment is defined by

$$NPF(\rho_i^1,\rho_i^2)=\sum_{i\in I}e_i^1\sum_{j\in J}\theta_{ij}^1+\sum_{i\in I}e_i^2\sum_{j\in J}\eta\,\theta_{ij}^2. \tag{3.15}$$

After dismantling, the auto 4S shops need to transport the hazardous materials to the disposal sites for treatment. This part of the disposal cost is also paid by the vehicle manufacturer, which is specified by

$$TF(\rho_i^2,\rho_i^3)=\sum_{i\in I}f_i\,\gamma\sum_{j\in J}\big((1-\eta)\theta_{ij}^2+\theta_{ij}^3\big), \tag{3.16}$$

where f_i is the unit disposal cost of hazardous materials. Consequently, for the vehicle manufacturer, the total profit is written as

$$\Phi_m(\sigma_1,\sigma_2,\sigma_3,\eta;\rho_i^1,\rho_i^2,\rho_i^3)=SCI+RPI+TSI-CF-NPF-TF. \tag{3.17}$$

Clearly, for the vehicle manufacturer, the decision variables must satisfy the following bound constraints:

$$\begin{cases}0\le\eta\le 1,\\ 0\le\sigma_t\le 1, \quad t=1,2,\ 3,\end{cases} \tag{3.18}$$

With the above analysis, an optimization model of the vehicle manufacturer is obtained as follows.

$$\begin{aligned}&\min\ -\Phi_m(\sigma_1,\sigma_2,\sigma_3,\eta;\rho_i^1,\rho_i^2,\rho_i^3)\\ &\quad=\mathrm{SCI}+\mathrm{RPI}+\mathrm{TSI}-\mathrm{CF}-\mathrm{NPF}-\mathrm{TF},\\ &\text{s.t.}\ \ (3.18),\\ &(\rho_i^1,\rho_i^2,\rho_i^3)\ \text{solves the following optimization model of the auto 4S shops:}\\ &\min-\Phi_r(\rho_i^1,\rho_i^2,\rho_i^3;\sigma_1,\sigma_2,\sigma_3,\eta)=CFI-RC-TCC-RDC-TTC\\ &\text{s.t. } (3.8),(3.9),(3.10).\end{aligned} \tag{3.19}$$

Remark 3.3. Theoretically, different from the existing models available in the literature, Model (3.19) proposed in this paper has a number of superiorities in terms of its practicality and resource reutilization, which are summarized as follows.

(1) The decentralized decision-making mode defined by Model (3.19) is closer to realistic situations than the centralized decision-making modes in the existing results.
(2) The recycled amount of UWVs in Model (3.19) is related with public environmental awareness and depends on the repurchase price (an endogenous decision variable of the model), instead of being supposed to be a fixed value in the majority of the results on the UWV recycle.
(3) Compared with the reverse recycling network in Demirel *et al.* (2016), Model (3.19) reuses a well-developed forward sales network of new vehicles, rather than establishing new recycling facilities. Thus, the recycle cost may be greatly reduced.
(4) In Model (3.19), the UWVs are recycled, and refurbished UWVs are resold through the "Internet+" online recycling channel, which can often improve the customer convenience of the UWV recycle.
(5) Unlike the existing models in the literature, Model (3.19) incorporates the heterogeneity of the recycled UWVs and their processing strategies, which may greatly improve their utilization rate and facilitate green production in practice.

Remark 3.4. It is seen that the proposed model (3.19) in this paper is a complicated bi-level programming problem, where the profit function of the vehicle manufacturer is associated with the auto 4S shops' decision variables ρ_i^1, ρ_i^2 and ρ_i^3, and the optimal policy of the auto 4S shops depends on the given strategy of the vehicle manufacturer. It is often difficult and valuable to address how to develop an efficient algorithm for finding the optimal solution of this game between the vehicle manufacturer (leader) and the auto 4S shops (followers).

4. MODEL PROPERTIES AND SOLUTION METHODS

4.1 Analysis of model properties

In order to develop an efficient algorithm to solve Model (3.19), its properties are analyzed as follows.

It is noted that the lower-level optimization problem (3.11) is a parametric quadratic programming model associated with the unknown parameters σ_1, σ_2, σ_3 and η.
Denote

$$x = (\rho_1^1, \dots, \rho_n^1, \rho_1^2, \dots, \rho_n^2, \rho_1^3, \dots, \rho_n^3)^T,$$

$$\omega = \begin{pmatrix} \sum_{j\in J}\left(a_{1j}^1 + (c_1 + r_1 - \sigma_1 p_1)b_{1j}^1\right) \\ \vdots \\ \sum_{j\in J}\left(a_{nj}^1 + (c_n + r_n - \sigma_1 p_n)b_{nj}^1\right) \\ \sum_{j\in J}\left(a_{1j}^2 + (c_1 + \eta r_1 + (1-\eta)(d_1 + \gamma TD_1) - \sigma_2 p_1)b_{1j}^2\right) \\ \vdots \\ \sum_{j\in J}\left(a_{nj}^2 + (c_n + \eta r_n + (1-\eta)(d_n + \gamma TD_n) - \sigma_2 p_n)b_{nj}^2\right) \\ \sum_{j\in J}\left(a_{1j}^3 + (c_1 + d_1 + \gamma TD_1 - \sigma_3 p_1)b_{1j}^3\right) \\ \vdots \\ \sum_{j\in J}\left(a_{nj}^3 + (c_n + d_n + \gamma TD_n - \sigma_3 p_n)b_{nj}^3\right) \end{pmatrix}^T$$

and

$$Q = \begin{pmatrix} Q_1 & & \\ & Q_2 & \\ & & Q_3 \end{pmatrix},$$

where

$$Q_t = \begin{pmatrix} 2\sum_{j\in J} b_{1j}^t & & \\ & \ddots & \\ & & 2\sum_{j\in J} b_{nj}^t \end{pmatrix}, \ t \in T = 1,2,3.$$

Then, Model (3.11) is rewritten as:

$$\begin{aligned} \min \quad & z = \frac{1}{2}x^T Q x + \omega^T x, \\ s.t. \quad & (3.8), (3.9), (3.10). \end{aligned} \tag{4.1}$$

Since it is true that $\sum_{j\in J} b_{ij}^t$ is greater than zero in practice for all $i \in I$ and $t = 1,2,3$, Q is positive definite. Thus, in Model (4.1), the objective function is strictly convex and all the constraints are linear. Consequently, it is easy to see that the optimal solution of Model (4.1) has the following property.

Theorem 4.1. For any given value of the upper-level decision variable $(\sigma_1, \sigma_2, \sigma_3, \eta)$, let $\rho^* = (\rho_i^1, \rho_i^2, \rho_i^3)$ be a feasible point of the lower-level optimization model (3.11), which satisfies the $Karush-Kuhn-Tucker$ (KKT) conditions. Then, ρ^* is a global minimizer of Model (4.1) or Model (3.11).

4.2 Reformulation of models and development of algorithms

The objective function in the upper-level optimization model is linear with respect to the decision variables of the vehicle manufacturer, and apart from the constraints defined by the lower-level optimization problem, all the other constraints are also linear. By Theorem 4.1, the bi-level programming model (3.19) is reformulated into an ordinary constrained optimization problem.

Denote

$$\begin{cases} D^{'} = D - \sum_{i\in I}\sum_{j\in J}(\theta_{ij}^{1} + \theta_{ij}^{2} + \theta_{ij}^{3}), \\ m_{ij}^{'} = A_{ij} - \theta_{ij}^{1}, \\ d_{ij}^{'} = B_{ij} - \theta_{ij}^{2}, \\ e_{ij}^{'} = C_{ij} - \theta_{ij}^{3}. \end{cases} \tag{4.2}$$

Then, the constraints in Model (4.1) are simplified as:

$$\begin{cases} D^{'} \geq 0, m_{ij}^{'} \geq 0, d_{ij}^{'} \geq 0, e_{ij}^{'} \geq 0, \quad \forall i \in I, \forall j \in J, \\ \rho_{i}^{t} \geq 0, t = 1,2,3, \quad \forall i \in I. \end{cases} \tag{4.3}$$

Let $\lambda, \mu_{ij}^{1}, \mu_{ij}^{2}, \mu_{ij}^{3}, \zeta_{i}^{1}, \zeta_{i}^{2}$ and ζ_{i}^{3} be the Lagrangian multipliers corresponding to these constraints, respectively. Then, the Lagrangian function corresponding to Model (4.1) reads

$$\begin{aligned} & L(\rho_{i}^{1}, \rho_{i}^{2}, \rho_{i}^{3}, \lambda, \mu_{ij}^{1}, \mu_{ij}^{2}, \mu_{ij}^{3}, \xi_{i}^{1}, \xi_{i}^{2}, \xi_{i}^{3}) \\ = \quad & -\Phi_{r}(\rho_{i}^{1}, \rho_{i}^{2}, \rho_{i}^{3}; \sigma_{1}, \sigma_{2}, \sigma_{3}, \eta) - \lambda D^{'} - \sum_{i\in I}\sum_{j\in J}(\mu_{ij}^{1} m_{ij}^{'} + \mu_{ij}^{1} d_{ij}^{'} + \mu_{ij}^{1} e_{ij}^{'}) \\ & -\sum_{i\in I}(\zeta_{i}^{1}\rho_{i}^{1} + \zeta_{i}^{2}\rho_{i}^{2} + \zeta_{i}^{3}\rho_{i}^{3}). \end{aligned} \tag{4.4}$$

By the first-order optimality conditions, the unique optimal solution of the lower-level optimization model (3.11) satisfies the following equalities:

$$\begin{aligned} & \textstyle\sum_{j\in J}\big((-\sigma_{1}p_{i} + c_{i} + r_{i} + \rho_{i}^{1}\mu_{ij}^{1}b_{ij}^{1} + \lambda)b_{ij}^{1} + \theta_{ij}^{1}\big) - \zeta_{i}^{1} = 0, i \in I \\ & \textstyle\sum_{j\in J}\Big((-\sigma_{2}p_{i} + c_{i} + \eta r_{i} + (1-\eta)(d_{i} + TD_{i}\gamma) + \rho_{i}^{2}\mu_{ij}^{2}b_{ij}^{2} + \lambda\big)b_{ij}^{2} + \theta_{ij}^{2}\Big) - \zeta_{i}^{2} = 0, i \in I \\ & \textstyle\sum_{j\in J}\big((-\sigma_{3}p_{i} + c_{i} + d_{i} + TD_{i}\gamma + \rho_{i}^{3} + \mu_{ij}^{3} + \lambda)b_{ij}^{3} + \theta_{ij}^{3}\big) - \zeta_{i}^{3} = 0, i \in I, \end{aligned} \tag{4.5}$$

and the complementarity constraints:

$$\begin{cases} \lambda \geq 0,\ D^{'} \geq 0,\ \lambda D^{'} = 0, \\ \mu_{ij}^{1} \geq 0,\ m_{ij}^{'} \geq 0,\ \mu_{ij}^{1} m_{ij}^{'} = 0,\ i \in I,\ k \in K, \\ \mu_{ij}^{2} \geq 0,\ d_{ij}^{'} \geq 0,\ \mu_{ij}^{2} d_{ij}^{'} = 0,\ i \in I,\ k \in K, \\ \mu_{ij}^{3} \geq 0,\ e_{ij}^{'} \geq 0,\ \mu_{ij}^{3} e_{ij}^{'} = 0,\ i \in I,\ k \in K, \\ \zeta_{ij}^{1} \geq 0,\ \rho_{i}^{1} \geq 0,\ \zeta_{ij}^{1}\rho_{i}^{1} = 0,\ i \in I, \\ \zeta_{ij}^{2} \geq 0,\ \rho_{i}^{2} \geq 0,\ \zeta_{ij}^{2}\rho_{i}^{2} = 0,\ i \in I, \\ \zeta_{ij}^{3} \geq 0,\ \rho_{i}^{3} \geq 0,\ \zeta_{ij}^{3}\rho_{i}^{3} = 0,\ i \in I. \end{cases} \tag{4.6}$$

From (4.5), it follows that:

$$\begin{cases}\zeta_i^1 = \sum_{j\in J}\big((-\sigma_1 p_i + c_i + r_i + \rho_i^1 \mu_{ij}^1 b_{ij}^1 + \lambda) b_{ij}^1 + \theta_{ij}^1\big), i \in I \\ \zeta_i^2 = \sum_{j\in J}\Big(\big(-\sigma_2 p_i + c_i + \eta r_i + (1-\eta)(d_i + TD_i\gamma) + \rho_i^2 \mu_{ij}^2 b_{ij}^2 + \lambda\big) b_{ij}^2 + \theta_{ij}^2\Big), i \in I \\ \zeta_i^3 = \sum_{j\in J}\big((-\sigma_3 p_i + c_i + d_i + TD_i\gamma + \rho_i^3 + \mu_{ij}^3 + \lambda) b_{ij}^3 + \theta_{ij}^3\big), i \in I.\end{cases} \tag{4.7}$$

Denote $Y = (\lambda,\ \mu_{ij}^1,\ \mu_{ij}^2,\ \mu_{ij}^3,\ \rho_i^1,\ \rho_i^2,\ \rho_i^3)$ and $F(Y) = (D',\ m'_{ij},\ d'_{ij},\ e'_{ij},\ \zeta_{ij}^1,\ \zeta_{ij}^2,\ \zeta_{ij}^3)$. Then, the complementarity conditions (4.6) are formulated as the following parametric complementarity problem:

$$Y \geq 0,\ F(Y) \geq 0,\ Y^T F(Y) = 0. \tag{4.8}$$

Owing to the existence of the complementarity constraints (4.8), Model (3.19) is the so-called mathematical programs with equilibrium constraints (MPECs), and many standard powerful optimization algorithms cannot be directly used to solve such a model (3.19) owing to the damaged constraint qualification and its non-smoothness, which are basic conditions often required in the theory of standard optimization. For this reason, we now use the partially smoothing method proposed in Chen and Wan (2015) to approximate the complementarity constraints (4.18). Specifically, the research defines.

$$\Phi_\varepsilon^{ij}(Y) = (\phi_{\varepsilon,1}(Y),\ \phi_{\varepsilon,2}^{ij}(Y),\ \phi_{\varepsilon,3}^{ij}(Y),\ \phi_{\varepsilon,4}^{ij}(Y),\ \phi_{\varepsilon,5}^{i}(Y),\ \phi_{\varepsilon,6}^{i}(Y),\ \phi_{\varepsilon,7}^{i}(Y))^T, \tag{4.9}$$

where

$$\begin{cases}\Psi_\varepsilon(t) = \frac{2t}{\pi}\arctan(\frac{t}{\varepsilon}), \\ \phi_{\varepsilon,1}(Y) = \frac{1}{2}(\lambda + D' - \Psi_\varepsilon(\lambda - D')), \\ \phi_{\varepsilon,2}^{ij}(Y) = \frac{1}{2}(\mu_{ij}^1 + m'_{ij} - \Psi_\varepsilon(\mu_{ij}^1 - m'_{ij})), \\ \phi_{\varepsilon,3}^{ij}(Y) = \frac{1}{2}(\mu_{ij}^2 + d'_{ij} - \Psi_\varepsilon(\mu_{ij}^2 - d'_{ij})), \\ \phi_{\varepsilon,4}^{ij}(Y) = \frac{1}{2}(\mu_{ij}^4 + e'_{ij} - \Psi_\varepsilon(\mu_{ij}^3 - e'_{ij})), \\ \phi_{\varepsilon,5}^{i}(Y) = \frac{1}{2}(\rho_i^1 + \zeta_i^1 - \Psi_\varepsilon(\rho_i^1 - \zeta_{ij}^1)), \\ \phi_{\varepsilon,6}^{i}(Y) = \frac{1}{2}(\rho_i^2 + \zeta_i^2 - \Psi_\varepsilon(\rho_i^2 - \zeta_{ij}^2)), \\ \phi_{\varepsilon,7}^{i}(Y) = \frac{1}{2}(\rho_i^3 + \zeta_i^3 - \Psi_\varepsilon(\rho_i^3 - \zeta_{ij}^3)).\end{cases} \tag{4.10}$$

Then, (4.8) is approximated by the following inequalities:

$$Y \geq 0,\ F(Y) \geq 0,\ \Phi_\varepsilon^{ij} \leq 0, \tag{4.11}$$

and the bi-level programming model (3.19) is reformulated as the following standard smooth optimization problem:

$$\begin{aligned} \max\ & G(u) \\ s.t.\ & 0 \leq \sigma_1 \leq 1,\ 0 \leq \sigma_2 \leq 1,\ 0 \leq \sigma_3 \leq 1,\ 0 \leq \eta \leq 1, \\ & Y \geq 0,\ F(Y) \geq 0,\ \Phi_\varepsilon^{ij}(Y) \leq 0,\ i \in I,\ j \in J, \end{aligned} \tag{4.12}$$

where ε is called the smoothing parameter and

$$\begin{cases} Y = (\lambda,\ \mu_{ij}^1,\ \mu_{ij}^2,\ \mu_{ij}^3,\ \rho_i^1,\ \rho_i^2,\ \rho_i^3), \\ u = (\sigma_1, \sigma_2, \sigma_3, \eta, \rho_i^1, \rho_i^2, \rho_i^3, \lambda, \mu_{ij}^1, \mu_{ij}^2,\ \mu_{ij}^3), \\ F(Y) = (D',\ m'_{ij},\ d'_{ij},\ e'_{ij},\ \zeta_{ij}^1,\ \zeta_{ij}^2,\ \zeta_{ij}^3), \\ G(u) = -\phi_m(\sigma_1, \sigma_2, \sigma_3, \eta; \rho_i^1, \rho_i^2, \rho_i^3). \end{cases} \tag{4.13}$$

With the above preparation, we are now in a position to present an efficient algorithm to find an equilibrium solution of (3.19) by solving a series of approximate smooth problems defined by (4.12).

Algorithm 3.1. (Model Reformulating and Smoothing Methods(MRSM))

Step 1. Given an initial point u_1, Take $\varepsilon_1 > 0$, $\epsilon_{stop} > 0$ small enough, and take $\beta \in (0,1)$. Set $l := 1$.

Step 2. Let ε_l be the current parameter. In the case of $\varepsilon = \varepsilon_l$, solve the subproblem (4.12) by a solver for the smooth nonlinear programming problems. The optimal solution is referred to as $\tilde{u}_l$, and the corresponding optimal value of the objective function is $G(\tilde{u}_l)$.

Step 3. If $\|G(\tilde{u}_l) - G(\tilde{u}_{l-1})\| < \epsilon_{stop}$, set $u_l := \tilde{u}_l$. Output the solution u_{l} and the optimal profits of the vehicle manufacturer and the auto 4S shops. The algorithm stops. Otherwise, set $\varepsilon_{l+1} := \beta\varepsilon_l$, $u_{l+1} = \tilde{u}_l$, $l := l + 1$. Return to Step 2.

Remark 4.1. In Chen and Wan (2015), it has been proved that if the termination condition:

$$\| \min\{Y, F(Y)\}\| \leq \varepsilon_{stop}$$

in Step 3 of Algorithm 3.1 holds, then the sequence $\{u_l\}$ generated by Algorithm 3.1 globally converges to a stationary point of Model (3.19) under some mild conditions.

5. SCENARIO ANALYSIS

In this section, scenario analysis will be conducted to validate the developed models and algorithm, and a number of valuable practical managerial implications will be revealed through numerical simulation.

5.1 Setting of distinct scenarios

The model established in this paper uses the existing sales network to recycle UWVs. The economic development levels of different regions are different, and the corresponding users' environmental protection awareness and price sensitivity have significant differences. In addition, the proposed model is used to recycle different brands of UWVs. So, it is interesting to study how the brands of UWVs affect the recycling strategy and efficiency of the recycling network. In particular, four different situations are considered for the sale price and resale price of different brands of UWVs, corresponding to lower or higher sale prices and resale prices. In summary, this section explores the efficiency of UWV recycling strategies in different recycling networks under four scenarios in regions with different levels of development. Clearly, the above three regions represent the following typical scenarios:

Table 3. Public environmental protection awareness and price sensitivity in different regions

Region	a_1	a_2	a_3	b_1	b_2	b_3
Developed region	0	2	30	0.005	0.02	0.2
Developing region	0	1	10	0.007	0.03	0.5
Underdeveloped region	0	0	5	0.009	0.04	0.8

Developed region: Higher environmental awareness with lower price sensitivity.
Developing region: Medium environmental awareness with medium price sensitivity.
Underdeveloped region: Lower environmental awareness with higher price sensitivity.

Taking into account the different brands and types of UWVs in practice, the following four scenarios with different sale and resale prices can be set up.

Scenario I (High sale price, high resale price): the sale price of UWVs and the resale prices of refurbished vehicles with different service life are $p = 50000$, $q^A = 45000$, $q^B = 40000$, and $q^C = 35000$ respectively. This means that refurbished UWVs on the secondary market are still popular and have a high rate of value preservation. Of course, vehicles with a high sale price also have a high cost for new parts during refurbishment. Set $e_1 = 3000$ and $e_2 = 30000$. In order to recycle these UWVs with a high rate of value preservation, the government's subsidy are also high. Therefore, take $g^1 = 3000$, $g^2 = 5000$, and $g^3 = 12000$.

Scenario II (High sale price, low resale price): in this case, the resale prices of UWVs on the secondary market drop to $q^A = 25000$, $q^B = 20000$, and $q^C = 15000$. Other parameters are consistent with **Scenario I**. This is a brand of refurbished UWVs with less demand in the secondary market and a lower rate of value preservation.

Scenario III (Low sale price, high resale price): the sale price of UWVs and the resale prices of refurbished UWVs with different service life are $p = 30000$, $q^A = 25000$, $q^B = 20000$, and $q^C = 15000$ respectively. This brand of UWVs has a low sale price, and its value remains basically unchanged after returning to the market with new parts replaced. It has an ultra-high cost performance. Since its sale price is not high, the cost of replacing new parts during refurbishment is relatively low. Set it to $e_1 = 2000$ and $e_2 = 10000$. The government's subsidies are: $g^1 = 2000$, $g^2 = 3000$, and $g^3 = 5000$.

Scenario IV (Low sale price, low resale price): the resale price of refurbished UWVs on the secondary market drops to $q^A = 15000$, $q^B = 10000$, and $q^C = 8000$. Other parameters are consistent with **Scenario III**. This is a product with low value and less demand in the secondary market.

For all the above four scenarios, we collect the realistic data of model parameters, such as the sale price and the resale price of UWVs, from the 2022 National Used Vehicle Market Depth Analysis Report released by the China Vehicle Circulation Association. Another part of the technical data, including the unit transportation cost, the unit processing cost, and the material weight percentages of UWVs, comes from the article Demirel *et al.* (2016), where an optimization model was built to solve the practical management problems of recycling the end-of-life vehicles in Turkey. For the model parameters related with the UWV market and the performance of users, we estimate their values through local inquiries from the auto 4S shops, the customers, and the distributors in Changsha, China. We list the collected or estimated data as follows.

$$\begin{cases} \tau_A = 0.2, \tau_B = 0.2, \tau_C = 0.6, \beta_1 = 0.06, \beta_2 = 0.04, \beta_3 = 0.005, \beta_4 = 0.001, \beta_5 = 0.01, \\ \beta_6 = 0.01, \beta_7 = 0.01, \beta_8 = 0.66, \gamma = 0.204, ss_1 = 1200, s_1 = 500, ss_2 = 6000, \\ s_2 = 2000, ss_3 = 6250, s_3 = 2500, ss_4 = 3100, s_4 = 1500, ss_5 = 3000, s_5 = 1000, \\ ss_6 = 1000, s_6 = 300, ss_7 = 800, s_7 = 200, ss_8 = 600, s_8 = 200, ff = 500, \\ c = 400, r = 600, d = 1600, TD = 200, A = 100, B = 200, C = 900, D = 950. \end{cases} \tag{5.1}$$

5.2 Optimal strategies in different scenarios

All the computer codes of Algorithm 3.1 are written and implemented on the MATLAB R2020a platform. By implementing the algorithm in the four scenarios in different development regions to solve Model (3.19), the corresponding optimal decisions and profits can be obtained, as shown in Tables 4 and 5.

The numerical results in Tables 4 and 5 indicate that:

(1) In the developed regions, the commission fee of vehicle manufacturers entrusting the auto 4S shops is higher than that in the underdeveloped regions, resulting in lower profit for vehicle manufacturers in developed regions (see the values of σ_1, σ_2, σ_3, ϕ_m in Table 4). In developed regions, the cost of land resources is higher, which directly leads to higher establishment costs for the auto 4S shops. If the vehicle manufacturer wants to attract the auto 4S shops to participate in recycling, he must inevitably increase their commission fee. When the resale price is low, it seems difficult for the vehicle manufacturer to benefit from the recycle system. From this perspective, it is beneficial for environmental protection and sustainable development if the manufacturer insists on recycling UWVs and refurbishing or dismantling them, owing to its enhancing their brand image and user loyalty, a long-term benefit.

(2) In most cases, the profit of the auto 4S shops in the recycle system is higher than that of the vehicle manufacturer. Especially in the developed regions, the public's high environmental awareness can bring the maximum profit to the auto 4S shops (see the values of Φ_r in Table 5). It can be explained that the public's high level of environmental awareness has greatly enhanced the willingness of UWV-holders to participate in recycling.

(3) From the comparison between the results in Tables 4 and 5, it follows that the differences of the sale price and resale price of the UWVs greatly affect the recovery strategy of the vehicle manufacturer and the auto 4S shops. In **Scenario I** and **Scenario II**, the resale prices of UWVs are different. The manufacturer processes the second type of UWVs based on the way of cost minimization and profit maximization (see the different values of η in Table 4). For example, the vehicle manufacturer chooses to dismantle all of the UWVs when the sale price of UWVs is high, and the resale price is low. The main reason for vehicle manufacturers to participate in the recycling lies in that the government provides more subsidies.

(4) Higher price sensitivity and higher environmental awareness both have positive impacts on the recycle of UWVs.

However, comparing the scenarios in the regions with different levels of development, it is seen that it is more profitable to select the regions with higher price sensitivity to recycle UWVs. That is to say, the recycling model of UWVs proposed in this article is more suitable for these regions with high price sensitivity.

Table 4. Comparison of optimal solutions of vehicle manufacturers in different regions

Region	Scenario	σ_1	σ_2	σ_3	η	ϕ_m	$cputime(/s)$
Developed region	I	0.8200	0.4179	0.1406	1	7.6103e+05	0.32
	II	0.8200	0.4388	0.1406	0	-2.9094e+06	0.50
	III	0.3801	0.8306	0.1412	1	-1.5307e+06	0.74
	IV	0.0400	0.8306	0.1412	0	-3.3513e+06	0.49
Developing region	I	0.5914	0.2859	0.0924	1	5.9892e+06	0.31
	II	0.5914	0.3068	0.0924	0	1.8187e+06	0.22
	III	0.3757	0.5720	0.1438	1	2.1228e+05	0.37
	IV	0.2051	0.6136	0.1438	0	-1.6522e+06	2.56
Underdeveloped region	I	0.4644	0.2199	0.0732	1	7.4089e+06	0.24
	II	0.4644	0.2408	0.0732	0	3.7384e+06	0.21
	III	0.9289	0.4400	0.1464	1	4.5892e+05	0.23
	IV	0.9289	0.4400	0.1464	0	-2.0011e+06	0.27

Table 5. Comparison of optimal solutions of the auto 4S shops in different regions

Region	Scenario	ρ_i^1	ρ_i^2	ρ_i^3	Φ_r	$\phi_m + \Phi_r$
Developed region	I	20000	9900	2420	5.3205e+06	6.0815e+06
	II	20000	9900	2420	5.3205e+06	2.4111e+06
	III	4251	9900	670	2.2247e+06	6.9398e+05
	IV	0	9900	670	2.1343e+06	-1.2170e+06
Developing region	I	14286	6633	1280	3.6069e+06	9.0961e+06
	II	14286	6633	1280	3.6069e+06	5.4256e+06
	III	4197	6633	767	1.7664e+06	1.9787e+06
	IV	2063	6633	767	1.6729e+06	2.0656e+04
Underdeveloped region	I	11111	5000	806	2.6392e+06	1.0048e+07
	II	11111	5000	806	2.6392e+06	6.3777e+06
	III	11111	5000	806	2.6392e+06	3.0982e+06
	IV	11111	5000	806	2.6392e+06	6.3816e+05

Table 6. The cost structure of vehicle recycle system

model	Total cost	Fixed cost	Transportation cost	Repurchasing cost	The cost of new parts
M_{us}	$1.1580e+07$	0	$2.6520e+04$	$3.5872e+06$	$6.3000e+06$
$M_{Demirel}$	$9.9643e+05$	$5.8337e+05$	$2.0609e+04$	$8.7926e+03$	0
Percentage of total cost (\%) ($M_{us}/M_{Demirel}$)	–	0/58.55	0.23/2.07	30.98/0.88	54.40/0

Table 7. The income structure of vehicle recycle system

model	Total income (no subsidy)	Total profit (no subsidy)	Income from refurbished vehicle	Income from dismantling	subsidy
M_{us}	$2.0676e+07$ $(1.1576e+07)$	$9.0961e+07$ (-3908)	$1.1399e+07$	$1.7615e+05$	$9.1e+06$
$M_{Demirel}$	$2.4231e+05$	$-7.5431e+05$	0	$2.4231e+05$	0
Percentage of total income (%) ($M_{us}/M_{Demirel}$)	–	–	55.13/0	0.86/100	44.01/0

Remark 5.1 In the Model (3.19), if the auto 4S shops do not make a profit during the recycling process, it will no longer cooperate with the vehicle manufacturer, which is in line with the real-world profit-seeking behavior of decision makers. Therefore, the value of the objective function of the auto 4S shops in (3.11) should be guaranteed to be nonnegative when implement Algorithm 3.1 to solve Model (3.19).

5.3 Advantages of the developed model

In order to further highlight the advantages of our recycle network, it is compared with the recycling strategy proposed in Demirel *et al.* (2016), where the recycled amount of UWVs is set to be fixed number (950), rather than a variable depending on the decision variables in the studied recycle system. With the same setting of other model parameters, we apply the two different models to solve the problems of recycling the UWVs. The numerical results are shown in Tables 6 and 7.

By comparing the results in Tables 6 and 7, the following conclusions can be drawn.

(1) Compared with the model $M_{Demirel}$, although the cost of our model is relatively large, the corresponding benefit is also enormous. The main reason for the high cost is that the cost of new parts for refurbished UWVs and the cost of recycling UWVs account for a large proportion of $54.4\%$and 30.98%, respectively. This is conducive to the better performance of refurbished UWVs and improving the willingness of UWV-holders to sell the UWVs.

(2) The total system profit of our model is 9.0961×10^6, whereas that of $M_{Demirel}$ is negative (see the third column in Table 7, which is closely related to the government's support for recycle system of the UWVs. Clearly, in practice, the government provides subsidies to the enterprises involved in recycling for the following two reasons: (i) Encouraging them to improve the level of refurbishment on UWVs; (ii) Increasing the repurchase price to increase the recycled amount, thereby reducing environmental pollution. Without governmental subsidy, the total profit from the system is -3098, which depends on the different treatment methods for UWVs with different levels of damage and aging.

(3) In the cost composition of $M_{Demirel}$, the maximum share is achieved by 58.55% of the fixed cost, and the minimum share is achieved by 0.88% of the repurchase cost. However, in our model, the fixed cost of the model is 0, which is the advantage of the recycling network established by us. Instead of establishing new recycling facilities, we utilize existing developed vehicle sales networks to recycle UWVs. In addition, in our model, the repurchase cost of UWVs accounts for 30.98%, which means that through the model proposed in this article, the recycle system is more willing to pay more money to end-users to recycle UWVs, rather than paying for the high cost of facility construction like the model $M_{Demirel}$. In addition, it is seen that in the model $M_{Demirel}$, the proportion of transportation cost is 2.07%, while in our model, the lowest proportion of transportation cost is 0.23%, almost 10 times that of our model. Thanks to the reuse of existing sales networks, the expenditure of transportation costs in the total cost has been greatly reduced. With this analysis, it is concluded that our model is more conducive to enhancing the willingness of UWV-holders to participate in recycling, thereby recycling more UWVs, which is crucial for sustainable development.

(4) In the profit composition, the model $M_{Demirel}$ derives its profit entirely from the sale of reusable parts and materials after dismantling. In our model, the main profit is composed of the sales of refurbished UWVs and the governmental subsidy. The profit from dismantling accounts for only 0.86%. Compared with the $M_{Demirel}$, the solution of M_{us} that incorporates the heterogeneity of UWVs into the model is more suitable for the vehicle recycle system, and can greatly improve their utilization rate and system profit.

(5) In terms of resource conservation, cost allocation, and the enthusiasm of recycling enterprises to participate in recycling, reusing existing developed vehicle sales networks and governmental subsidy can both greatly improve the sustainability of the vehicle recycle system.

6. SENSITIVITY ANALYSIS

In this section, owing to the practical difficulties in exactly evaluate the model parameters, we conduct sensitivity analysis so as to explore the impacts of the fluctuation of some critical model parameters on the optimal strategy. Particularly, by this analysis, the following issues can be addressed:

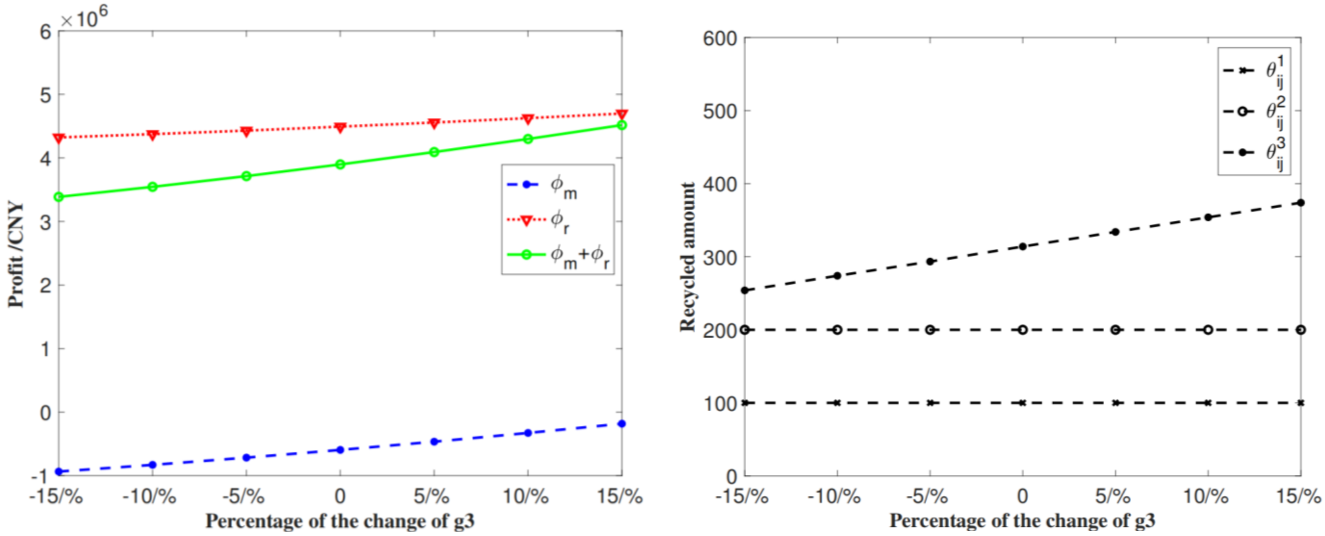
1. With regard to the governmental strategy, i.e., the provided subsidy to the third type of the recycled UWVs, how does it affect the optimal recycled amount? And how does it affect the decisions of vehicle manufacturers and the auto 4S shops?
2. How does the fluctuation of inventory capacity in the auto 4S shops affect the profit of decision makers and the recycled amount of UWVs?
3. How does the disassembly cost in the auto 4S shops affect the profits of decision makers and the recycled amount of UWVs?

For the three different types of UWVs, the parameters of the public environmental awareness and price sensitivity coefficient are taken as $a_1 = 0,\ b_1 = 0.005$; $a_2 = 2,\ b_2 = 0.02$; $a_3 = 30,\ b_3 = 0.2$, respectively. The other model parameters are chosen to be the same as in (5.1), except for the analyzed parameter. Clearly, the efficiency and stability of Algorithm 3.1 ensure the feasibility of conducting such an analysis.

6.1 Impacts of governmental subsidy

The literature Zhang *et al.* (2016) shows that government subsidy is very important to establish an effective recycling system. A larger subsidy will undoubtedly attract more enterprises to recycle the UWVs. However, this will certainly bring huge financial burden to the government. Therefore, for different development regions, it is meaningful and necessary to find an optimal government subsidy. Meanwhile, numerical results in 5.2 indicate that the first and the second types of UWVs are all recycled, but only a part of the third type is recycled. It's worth looking into how the governmental subsidy (g_3) affect the recycled quantity of this type of UWVs.

We first change the value of g_3 with a maximal fluctuation of 15% by a step size of 5%. Then, we implement Algorithm 3.1 to solve the models (3.19) corresponding to these values of parameters. The impacts of this fluctuation are illustrated in Figure 4.



(a) On profit (b) On the recycled amount

Figure 4. Impacts of governmental subsidy g_3

Looking at Figure 4 finds the following practical insights are found:

(1) An increasing governmental subsidy provided to the third type of UWVs can improve the total profit of the recycle system (see the middle green curve in Figure 4(a)), despite the negative profit for the vehicle manufacturer. Importantly, it can increase the recycled amount of UWVs (see Figure 4(b)). Thus, it is beneficial to the reduction of environmental pollution and the reuse of waste resources.

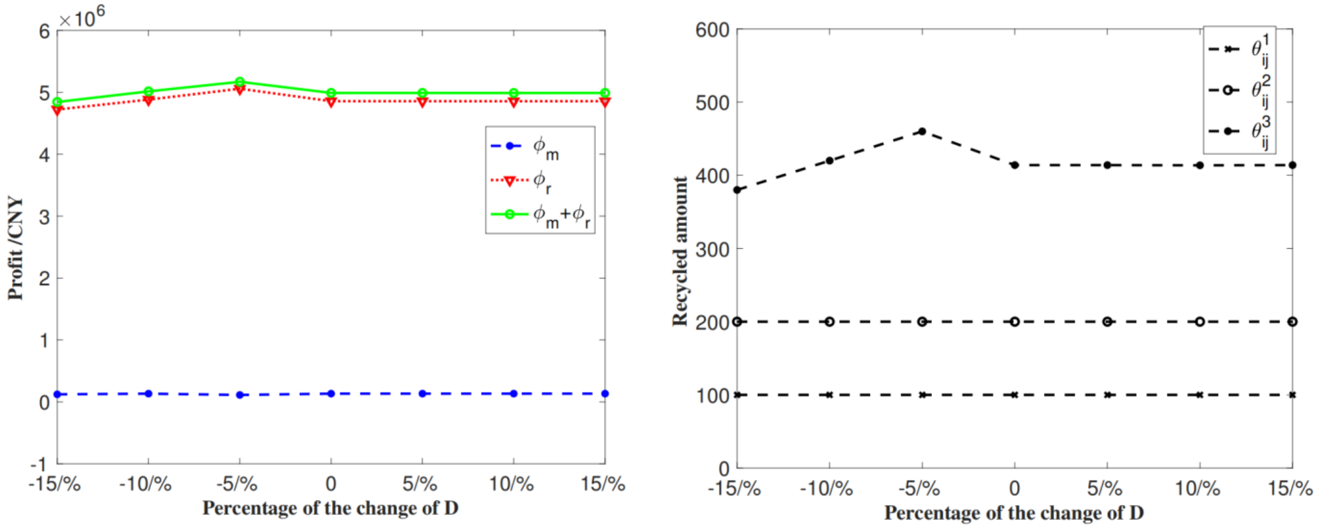
(2) Governmental subsidy and resale of refurbished UWVs are the two main return sources for the vehicle manufacturer, and the increase of governmental subsidy can reduce the loss of the manufacturer (see the bottommost blue curve in Figure 4(a)). It is concluded that the governmental subsidy plays a key role in promoting the effective recovery of UWVs.

(3) For the vehicle manufacturer, participating in the recycle of UWVs is mainly driven by social responsibility and the public brand image, even if the earned profit purely from the recycling of UWVs is negative.

6.2 Impacts of the auto 4S shops' inventory capacity

Owing to the larger volume of UWVs, it is valuable to study the impacts of the inventory capacity in the auto 4S shops on the optimal strategy. In particular, it is better to find the optimal inventory capacity.

We first change the value of the capacity D with a maximal fluctuation of 15% by a step size of 5%. Then, we implement Algorithm 3.1 to solve the models (3.19) corresponding to these values of parameters. The impacts of this fluctuation are illustrated in Figure 5.



(a) On profit (b) On recycled amount

Figure 5. Impacts of inventory capacity D

From the numerical results in Figure 5, it follows that:

(1) Inventory capacity has a greater impact on the profit of the auto 4S shops and the total profit of the recycle system within the range $(700{,}800)$(see the top two curves in Figure 5(a)), compared with the more stable profit for the vehicle manufacturer.

(2) No matter how the inventory capacity of the auto 4S shops changes, the first and the second types of UWVs are all recycled, but it affects the recycled quantity of the third type (see the topmost curve in Figure 5(b)). More in detail, the recycled amount of the third type of UWVs rises up when the inventory capacity increases at the beginning, but it decreases and tends to be stable for a greater inventory capacity (see the topmost curve in Figure 5(b)).

(3) From the perspective of full utilization of facilities and waste resources, there exists an optimal inventory capacity to be suggested for the auto 4S shops, which is about 700 in this numerical analysis. With this inventory capacity, the recycled amount of the third type of UWVs can be maximized, and the construction cost of facilities can be reduced.

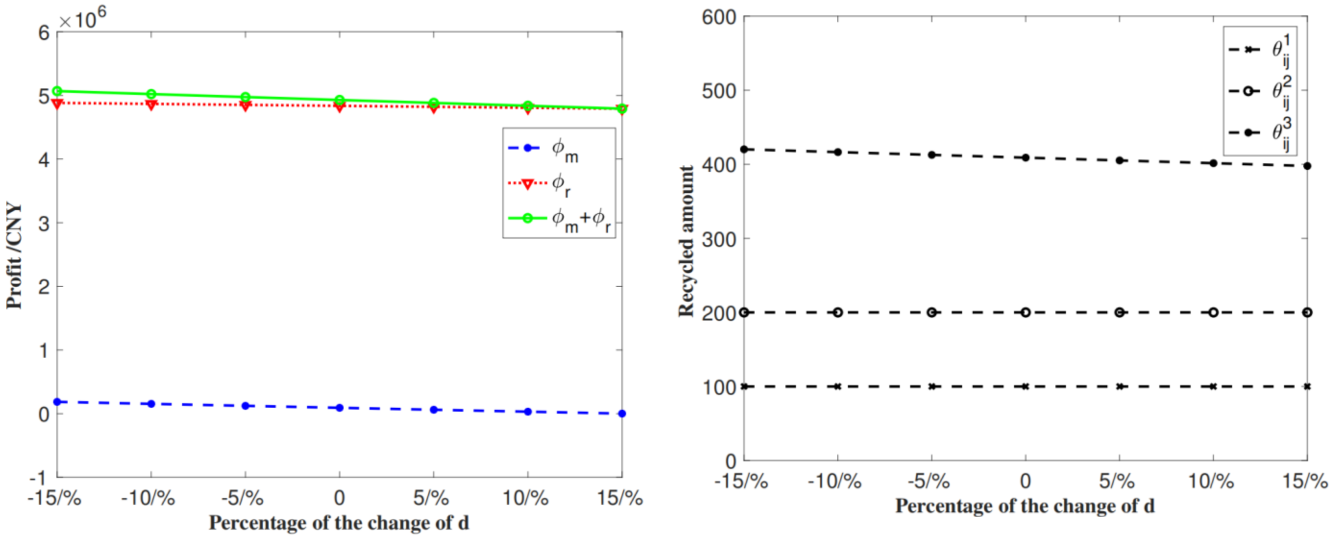
6.3 Impacts of unit disassembly cost

Lower operational cost often enables enterprises to gain a more competitive edge. Disassembly cost is the dominant part of the total cost in the recycle system. So, it is better to explore how the change of unit disassembly cost affects the optimal strategy.

We first change the value of the unit disassembly cost d with a maximal fluctuation of 15% by a step size of 5%. Then, we implement Algorithm 3.1 to solve the models (3.19) corresponding to these values of parameters. The impacts of this fluctuation are illustrated in Figure 6. From the numerical results in Figure 6, the following insights are obtained.

(1) For all three types of UWVs, their recycled amounts are closely related to the unit disassembly cost, and all of them reduce when the unit disassembly cost rises up (see Figure 6(b)). Especially, the downward trend is sharper for the types of UWVs (see the topmost curve in Figure 6(b)).

(2) The increase in the unit disassembly cost causes a decline of all the profits in the recycle system (see the topmost curve in Figure 6(a)). Therefore, it is important for the auto 4S shops to improve its disassembly technical level to enhance their competitive edge to gain the opportunity to cooperate with vehicle manufacturers.



(a) On profit (b) On recycled amount

Figure 6. Impacts of unit disassembly cost d

7. PRELIMINARY REAL-LIFE APPLICATIONS

In order to further verify the values of the developed model and algorithm in this paper, we now conduct a real-life case study by optimizing a recycle system with competing interests between an auto manufacturer and its subordinate 4S sales shops in Hunan Province, China. As shown in Figure 3, the manufacturer and the 4S sales shops collaborate by the mode of off-online dual channels to recycle the UWVs in this system, where the manufacturer as a leader operates an online platform for the UWV transactions and strategic decision-making, while its subordinate 4S sales shops are responsible for the offline collection, detection, and processing (refurbishment/dismantling) of UWVs.

All the data related with this case study are directly collected or estimated by consulting the manufacturer, the 4S sales shops, the governmental agencies, and the UWV holders involved with the studied recycle system. In detail, the market size $a = 950$ units; The unit sale price $p_i = 50{,}000$ (CNY); The resale price $q^A = 45{,}000$ (CNY) for the autos within 0-4 years old; The resale price $q^B = 40{,}00$ (CNY) for the autos within 4-8 years old; The resale price $q^C = 35{,}000$ (CNY) for the autos within 8-12 years old; For the recycled UWVs of the first type, the governmental subsidy $g^1 = 3{,}000$ (CNY); For those of the second type, the governmental subsidy $g^2 = 5{,}000$ (CNY); For those of the third type, the governmental subsidy $g^3 = 12{,}000$ (CNY); The unit detection cost $c_i = 400$ (CNY); The unit refurbishment cost $r_i = 600$ (CNY); The unit dismantling cost $d_i = 1{,}600$; The unit costs of purchasing new parts $(e_i^1, e_i^2) = (3{,}000,\ 30{,}000)$ (CNY); The coefficients of public awareness and price sensitivity in the model are estimated to be: $a_1 = 0$, $a_2 = 2$, $a_3 = 30$, $b_1 = 0.005$, $b_2 = 0.02$, and $b_3 = 0.2$.

Then, we apply Algorithm 3.1 to solve the reformulated model (4.12), and get the optimal strategies and the maximized profits for the manufacturer and the 4S sales shops. In Table 8, we list all the numerical results.

Table 8. The optimal strategies and the maximized profits in case studies

Decision Maker	Optimal Decisions	Profit (CNY)
Manufacturer	$\sigma_1 = 0.82, \sigma_2 = 0.41, \sigma_3 = 0.14, \eta = 1$	7.61×10^5
4S Shop	$\rho^1 = 20000, \rho^2 = 9900, \rho^3 = 2420$	5.32×10^6
Total System	−	6.08×10^6

From the numerical results in Table 8, it is found that:

(1) Owing to the 4S Shop's reuse of existing facilities, channel synergy plays a positive role in reducing the transportation costs, compared with building new recycling centers (also see the listed fixed costs in Table 6). The hybrid channel can increase the recycled amount of UWVs due to convenient accessibility, especially compared with the adopted pure offline recycle models (also see the results from the model $M_{Demirel}$ in Table 7).

(2) For the system of recycling UWVs, the profit of manufacturers (leaders) depends heavily on the governmental subsidies and on the high-value refurbished sales (q^A, q^B). Indeed, for the manufacturer, the governmental subsidy is 44% of the total income (also see Table 7). Without g^3 (12,000/unit for Type-3 UWVs). The manufacturer's profit drops by 14%, which further indicates that governments play critical roles in recycling low-value UWVS. In contrast, the profit of 4S shops (followers) dominates the total benefits from forming this cooperation system, sharing 87.5% of the total income (see the ratio of profits from the 4S shop to the total System in Table 8), especially owing to the optimized ρ^t. That is to say, the proposed recycle system in this paper can promote the enthusiasm of the auto sales shops for the recycle of UWVs, not just selling new vehicles.

(3) Clearly, processing all the UWVs of Type-2 (full refurbishment) can maximize resource utilization. Even if differential strategies for the UWVs of different types can be optimized in this paper, it follows from **Scenario I** in Table 4 that all the UWVs of Type 2 are suggested to be refurbished in the proposed recycle system. Indeed, refurbishing all the UWVs of Type-2 can yield a higher profit/unit than dismantling them (the optimized $\eta = 1$ in Table 8). For the UWVs of Type-3 (the aged years are over 12), their recycle contributes only 7% to the total profit in the studied system, but clearly is beneficial to environmental protection.

8. CONCLUSIONS AND FUTURE RESEARCH

In the context of the national encouragement of ``Internet + recycling" in China, we have established an off-online-hybrid-channel reverse recycling system with online recycling by the manufacturer (leader) and offline processing by the auto 4S shops (followers). Then, a bi-level programming model has been built to optimize managerial strategies of this system, where the repurchase price of UWVs, the commission cost of the manufacturer, and the ratio of UWVs refurbished are all the endogenous variables. Notably, different from all the existing models of recycling UWVs in the literature, our recycle model has addressed how to reutilize the well-developed forward vehicle sales network, i.e., the auto 4S shops. In addition, our recycle model also takes into account the damaged and aged levels of UWVs for the subsequent processing of refurbishment or dismantling, which is clearly beneficial to the improvement of resource utilization and the system profit.

Since no any algorithm in the literature can be directly applied to solve the complicated bi-level programming model, we have reformulated the bi-level programming model into a series of standard smooth optimization subproblems, and developed an efficient algorithm, instead of heuristic algorithms, to find the optimal strategy of the recycle system.

Through scenario analysis and sensitivity analysis, we have revealed a number of important managerial insights. Specifically, for UWVs with different levels of damaged and aged, differentially processing the recycled vehicles with different degrees of damage and aging can improve the utilization rate of UWVs. Indeed, the optimal refurbishment ratio of the second type of UWVs has been provided in this study, which sufficiently shows the practical value of the proposed recycle model. Besides, it has been validated that the reuse of the well-developed forward sales network can enhance the willingness of UWV-holders to participate in recycling, increase the recycled amount of UWVs, as well as reduce the construction costs and optimize the operational cost composition. Thus, it is beneficial for sustainable development.

For the auto 4S shops, it is important to improve its disassembly technical level to enhance their competitive edge to gain the opportunity to cooperate with vehicle manufacturers. The technical improvement for refurbishing and dismantling of the UWVs also plays a critical role in improving the performance of the total recycle system. And, for the vehicle manufacturer, participating in the recycle of UWVs may be mainly driven by the social responsibility and the public brand image since the earned profit purely from the recycling of UWVs may be negative. Governmental subsidy plays a key role in promoting the effective recovery of UWVs since it is the main return source of the manufacturer.

Regional differences of developing levels and public awareness of environmental protection seriously affect the profit distribution between the auto manufacturer and the 4S shops, as well as the operational cost composition of the recycle system.

In future research, the obtained results in this paper can be modified and extended in the following three aspects.

(1) This paper uses the repurchase price as an endogenous variable in the model to influence the recycled amount of UWVs, while in reality, the recycled amount of UWVs is closely related to the measures taken by the auto 4S shops. So, other incentives to be provided by the auto 4S shops can be considered to promote the recycling of UWVs, such as the service level and advertising.

(2) The UWV recycle system designed in this paper takes into account the fact that the vehicle manufacturer and the auto 4S shops both make their own decisions to maximize interests, while in practice, the government has been playing a leading role in dealing with environmental issues. For example, the governmental strategies to support UWV recycling are necessary and beneficial to the reduction of greenhouse gases, energy consumption, and sustainable development. In this case, one needs to consider the government as the leader and establish a three-level Stackelberg game model.

(3) Basically, the proposed offline-online hybrid recycle channel in this paper is a class of prospective modes for recycling UWVs, although the built model and algorithm have been applied in solving the UWV recycle problems under

different scenarios. It is interesting to explore how this mode and the corresponding model can improve the economic and environmental benefits of the existent ones in practice. For example, since no one dares to be in the first cabins of the Destiny, it may be difficult to persuade manufactures to replace their current strategies with this new mode.

(4) In practice, uncertainty of model parameters and multi-objective optimization should also be taken into account in the recycling of UWVs (Xia *et al.*, 2024). For example, the unit transportation cost and the market base of UWVs in each region are often vague and stochastic. In this case, when the uncertainty is considered, uncertainty optimization techniques are needed to find new 'optimal' strategies. Furthermore, when the model proposed in this paper is applied to the recycling management problem in a given region, it is necessary to investigate an appropriate model to estimate the parameters, such as public awareness of environmental protection.

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