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MULTI-OBJECTIVE OPTIMIZATION OF THE BUFFER ALLOCATION PROBLEM CONSIDERING THE OVERALL EQUIPMENT EFFECTIVENESS INDICATOR

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This paper presents a formulation of the buffer allocation problem of a parallel serial production line of a footwear stitching process. This was analyzed under a multi-objective optimization approach that aims to maximize the value of the Overall Equipment Effectiveness indicator used in Lean Manufacturing, as well as to minimize the total cost of buffer allocation, this being a new proposal in the buffer allocation problem solution. The case study involves unreliable operating conditions. Process times, times between failures, and repair times are considered Normal distribution functions. The evaluation method used in the study involves the use of a simulation meta-model built from a design of experiments and simulations of the production line; also, the Evolutionary Solver algorithm is used to provide a solution to the mathematical model. The results report the allocation of buffers and their impact on the objectives, as well as a comparison between the optimization criteria.

Keywords: Buffer allocation problem, Multi-Objective, Meta-models, Simulation, Overall Equipment Effectiveness.

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1. INTRODUCTION

A significant number of companies in the world have opted for the implementation of the Lean Manufacturing philosophy, with the goal of continuous improvement of their production processes to be more competitive and generate greater profits, by eliminating or reducing everything that does not add value to products but does add cost and labor (Hernández-Vázquez *et al*., 2021; Socconini, 2019).

Part of the complexity of designing a production system is related to the buffer allocation problem (BAP); this involves analyzing and defining the optimal buffer distribution. The main reason for maintaining buffers is to allow workstations to operate independently from each other. Buffers have a significant impact on improving production line efficiency by eliminating detrimental effects due to failures or variations in processing times (Hernández-Vázquez *et al.*, 2022a; Kose & Kilincci, 2020). On the other hand, these can increase the cost of maintaining the system and decrease its profitability. Therefore, finding the optimal buffer capacities that result in a satisfactory process is a major problem in production systems research (Hernández-Vázquez *et al.*, 2019; Motlagh *et al.*, 2019).

This study presents as its main contribution a new proposal for a BAP solution, considering the multi-objective optimization of the Overall Equipment Effectiveness (OEE) indicator used in Lean Manufacturing and the minimization of the total cost in the allocation of buffers. This indicator represents the time that is actually worked, without downtimes, at the established capacity and without defects (Socconini, 2019). Seeking that buffer allocation within the production line improves OEE, and simultaneously considering the decrease in the total cost of buffer allocation is novel in the literature, for which the results achieved in this research are even more relevant.

The decrease in the number of buffers in the production line due to the cost they generate will have an impact on the value of the OEE indicator, a hypothesis that can be validated in the study described below.

It is important to mention that this work considers the analysis of a real case study from a company that is dedicated to the manufacture of footwear; therefore, the results obtained have a practical approach within a production line, whose operating conditions are considered unreliable, since there are downtimes due to machine breakdowns, repair time, as well as quality issues.

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Another aspect to highlight is the method of evaluation used. Like other works (Amiri & Mohtashami, 2012; Hernández-Vázquez J. I. *et al.*, 2024; Hernández-Vázquez J. O. *et al.*, 2022a; Hernández-Vázquez, J. O. *et al.*, 2022b; Mohtashami, 2014), meta-models built from experiment designs and simulations of the production line are used. The simulation software used in this work is PROMODEL; it was designed to analyze manufacturing processes of one or more products, assembly and transformation lines, among others (García-Dunna *et al.*, 2013). The use of this software in the analysis of BAP has been previously reported in other studies (Hernández-Vázquez J. I. *et al.*, 2024; Hernández-Vázquez J. O. *et al.*, 2022a; Hernández-Vázquez J. O. *et al.*, 2024a). Conversely, the Evolutionary Solver algorithm is used in the search for the solution as an optimization method.

The rest of this document is organized as follows. Section 2 explains the formulation of BAP, as well as mathematical modeling. Subsequently, section 3 describes the case study. Section 4 illustrates the meta-model developed. The optimization method used is described in Section 5 below. Section 6 details the numerical results obtained. Finally, a section of conclusions is presented where the scope of the results generated is addressed.

2. LITERATURE REVIEW

The buffer allocation problem is classified as an NP-hard combinatorial optimization problem in the design of production lines (Demir *et al.*, 2014; Li & Chen, 2025; Li *et al.*, 2025; Weiss *et al.*, 2019). This consists of defining the allocation of storage places (buffers) within a production line, in order to maximize the efficiency of the process.

Table 1 shows a review of BAP works with a multi-objective approach. Maximizing the average production or throughput rate (TH), minimizing total buffer size, and minimizing total cost of buffer allocation are some of the most popular criteria in the BAP study.

Recently, the impact of BAP on the OEE indicator has begun to be studied. In the work carried out by Hernández-Vázquez J. I. *et al*. (2024), the OEE indicator of an unreliable parallel serial line is optimized. On the other hand, Hernández-Vázquez J. O. *et al*. (2024) analyze the behavior of the OEE indicator of a serial line. In both cases, a single-objective OEE optimization is performed. Unlike these, in the present study, the BAP is analyzed with a multi-objective approach considering the OEE indicator and the buffer allocation costs in an unreliable parallel production line.

2.1 Formulation

In this study, BAP is analyzed under a multi-objective optimization approach, for which two criteria were considered. The first aims to maximize the value of the OEE indicator, while the second aims to minimize the total cost of buffer allocation. A suitable procedure for troubleshooting with such an approach is the LP metric method.

Equation (1) merges the objectives of the mathematical model by considering a weighting (or weight) for each of them and finds the minimum of the combined function (Amiri & Mohtashami, 2012; Hernández-Vázquez *et al.*, 2022a; Yu, 1973). The reasoning behind the formulation and how it contributes to the overall optimization process is set out by Amiri and Mohtashami (2012).

$$Min \ = \left[\sum_{j=1}^{m} W_j^p \left|\frac{f_j^* - f_j}{f_j^* - f_j^-}\right|^p\right]^{1/p}, \tag{1}$$

where

- $W_j =$ Objective weight j
- $m =$ Number of objectives
- $f_j^* =$ Optimal (ideal) value of the objective j
- $f_j^- =$ Target anti-optimal (anti-ideal) value of the objective j
- $f_j =$ Solution value of objective j
- $p > 1$

The first objective of the mathematical model will be to maximize the value of the OEE indicator for a given number of buffers.

Find $B = (B_1, B_2, \dots, B_n)$ to

$$Max\ Z_1 = f(B), \tag{2}$$

where

- B_i = Decision variable or buffer size in warehouse area i
- n = Number of warehouse areas
- $f(B)$ = Value of the OEE indicator of the production line, considering B

It is important to note that $f(B)$ is a regression meta-model that is generated through DOE and simulation. In the "Meta-model" section, the way in which this was obtained for the case study is described.

The second objective will be to minimize the total cost of buffer allocation.

Find $B = (B_1, B_2, \dots, B_n)$ to

$$Min\ Z_2 = C(B) = \sum_{i=1}^{n} C_i * B_i, \tag{3}$$

where

- B_i = Decision variable or buffer size in warehouse area i
- n = Number of warehouse areas
- C_i = Cost of Assigning a Buffer in Storage Area i
- $C(B)$ = Total cost of buffer allocation, considering B

Equation (4) is the function that combines the objectives of maximizing the value of the OEE indicator and minimizing the total cost of buffer allocation using the LP metric. This will be used to evaluate the quality of the solutions generated by the optimization method. The values of f_j^* and f_j^- are defined in the same way as other studies do (Amiri & Mohtashami, 2012; Hernández-Vázquez et al., 2022a). In section 3, the f_1^* f_2^* and f_1^- f_2^- values of equation (4) are established for the case study.

$$M(B) = \left[W_1^P \left| \frac{f_1^* - f(B)}{f_1^* - f_1^-} \right|^p + W_2^P \left| \frac{f_2^* - C(B)}{f_2^* - f_2^-} \right|^p \right]^{1/p} \tag{4}$$

The mathematical model contemplates constraints that are related to the buffer size of each storage area.

Due to production space constraints, warehouse areas cannot register an allocation greater than their capacity.

$$L_i \leq B_i \leq U_i \qquad \forall i = 1\ to\ n, \tag{5}$$

$$B_i \geq 0 \text{ and integer} \qquad \forall i = 1\ to\ n, \tag{6}$$

where

- L_i = Lower limit of B_i
- U_i = Upper limit or capacity of B_i

Table 1. Literature review of BAP with Multi-Objective Approach

Researcher(s)	Assumptions of production line	Solution Methodology: Evaluative	Solution Methodology: Optimization	Objectives
Chehade *et al.* (2010)	Unreliable	Simulation	Multi-objective ant colony optimization algorithm	Max throughput–Min total buffer size
Cruz *et al.* (2010)	Reliable	Generalized expansion method	Multi-objective genetic algorithm—NSGA-II	Max throughput–Min total buffer size
Abdul-Kader *et al.* (2011)	Unreliable	Non-linear mathematical programming	Lexicographic goal programming	Max throughput–Min total buffer size–Min makespan–Min total production time–Max maintenance time

Researcher(s)	Assumptions of production line	Solution Methodology Evaluative	Optimization	Objectives
Amiri and Mohtashami (2012)	Unreliable	Simulation metamodelling	Hybrid method (Genetic algorithm—line search)	Max throughput–Min total buffer size
Cruz *et al.* (2012)	Reliable	Generalized expansion method	Multi-objective evolutionary algorithm—NSGA-II	Max throughput–Min total buffer size–Min service rate
Bekker (2013)	Unreliable	Simulation	Cross-entropy method	Max throughput–Min total buffer size
Mohtashami (2014)	Unreliable	Simulation metamodelling	Hybrid method (genetic algorithm—line search)	Max production rate–Min total cost
Oesterle *et al.* (2016)	Unreliable	Simulation	Memetic NSGA-II	Min idle time–Min total unit costs–Max throughput
Wang *et al.* (2016)	Unreliable	Aggregation method	Gradient method	Max throughput–Min total cost
Dolgui *et al.* (2017)	Unreliable	Aggregation method	Evolutionary algorithms: SIBEA y SEMO	Max throughput–Min total cost
Su *et al.* (2017)	Unreliable	Decomposition extension-Markov approach	Hybrid method (Tabu search—NSGA-II)	Max throughput–Min total buffer size
Zandieh *et al.* (2017)	Unreliable	Simulation	Genetic algorithm–particle swarm optimization	Max production rate–Min total buffer size–Min total number of defective units
Kose and Kilincci (2018)	Unreliable	Simulation	Hybrid method (NSGA-II—MOBAP)	Max throughput–Min total buffer size
Bamporiki *et al.* (2019)	Unreliable	Simulation	MOOCEM	Max throughput–Min work in process
Motlagh *et al.* (2019)	Unreliable	Simulation	NSGA-II y NRGA	Max throughput–Min total cost – Min total buffer size
Renna (2019)	Unreliable	Simulation	Dynamic policy	Max throughput–Min total cost
Alaouchiche *et al.* (2021)	Unreliable	-----------	Integrated analytical method	Max throughput–Min energy consumption
Hernández-Vázquez *et al.* (2022a)	Unreliable	Simulation metamodelling	Hybrid method (genetic algorithm — simulated annealing)	Max throughput–Min total cost
Shi and Gao (2023)	Reliable	----------	Hybrid method (Tabu search — NSGA-II)	Max throughput–Min the average buffer level
Gao & Liu (2024)	Unreliable	-----------	The black widow optimizer and simulated annealing algorithm	Max throughput–Min energy consumption
Aghaei *et al*. (2025)	Unreliable	Simulation	NSGA-III	Max availability- Min overall system costs- Min buffer capacity

3. CASE STUDY

As a case study, a real process of a women's footwear stitching line was considered in the elaboration of a ballerina model. This has the behavior of an unreliable production line (with stoppages and repairs), whose process consists of 9 different workstations (from A to I) where the different operations are performed; in addition, there are 7 warehouse areas (buffers) within the process. Table 2 describes the operations carried out at each station, as well as the number of operators or machines.

Figure 1 shows the structure of the production line. The circles indicate the stations, the triangles represent the warehouse areas of the inventory in process, and the square points to the quality inspector. The raw material enters the production line at different stations and follows a marked flow in the process. Station D assembles the outputs of Stations A and C (Assembly 1); Station H assembles the outputs of Stations G and E (Assembly 2). Finally, Station I performs the last operation of the process to generate a finished product.

The production line includes a quality inspector, who can classify the inventory in process into one of the following categories:

• *Rejection*: The product has serious inconsistencies that cannot be reprocessed and therefore must be rejected.
• *Compliant*: The product does not present inconsistencies and must follow the sequence of the original process.

Table 2. Operations performed at each station

Station	Operation	Number of workers or machines
A	Assemble lining	1
B	Close slipper	1
C	Settling cut	1
D	Splice cut	1
E	Edging slipper	1
F	Cutting elastic	1
G	Sew bow	1
H	Sew elastic to slipper	1
I	Assemble lots	1

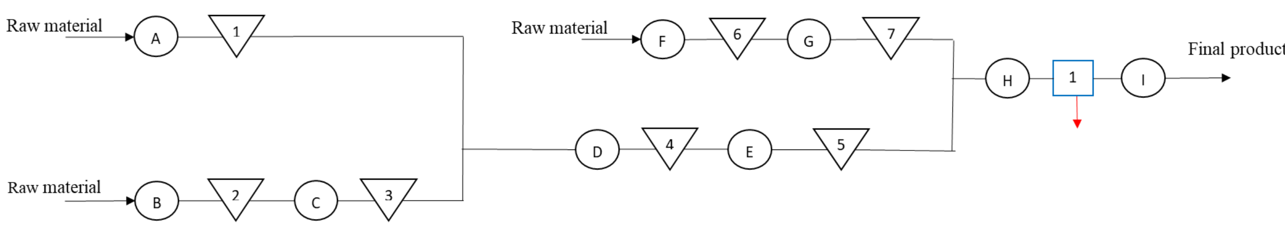

Figure 1. Production line

Table 3 presents the process times, times between failures, and repair times that exist in the case study, considering a Normal (N) distribution. Table 4 shows the inspection times and the probabilities of classifying the inventory in process in either of the categories mentioned above.

The upper limit of the warehouse areas (decision variables) was set to a value of 24. With regard to the lower limit, this study contemplates the value of a buffer, ensuring that a minimum capacity is maintained in any location analyzed. In addition, a cost of $100 MXN was considered for each buffer allocated in a warehouse area.

For the case study the values f_1^* f_2^* and f_1^- f_2^- of equation 4 are established as follows:

$f_1^* = 1$ Optimal OEE value (100%)
$f_1^- = 0$ Anti-optimal (anti-ideal) OEE target value (0%)
$f_2^* = 700$ Optimal buffer cost value (1 buffer in each of the warehouse areas)
$f_2^- = 16{,}800$ Anti-optimal (anti-ideal) value of the target cost of buffers (24 buffers in each of the warehouse areas).

Table 3. Process, failure-to-failure, and repair times for each station

Station	Processing time (Seconds)	Time between failures (minutes)	Time to repair (minutes)
A	N (34,5)	----------	----------
B	N(19.68,3)	N(150,10)	N (5,1)
C	N(15.78,3)	N(150,10)	N (7,1)
D	N(56.72,5)	----------	----------
E	N(62.54,5)	N(220,10)	N (5,1)
F	N(17,2)	----------	----------
G	N(39.25,4)	N(150,10)	N (5,1)
H	N(50,4)	N(180,10)	N (5,1)
I	N(50,4)	----------	----------

Table 4. Inspection times and classification probabilities

Inspector	Processing time (Seconds)	Probability	
		Rejection	Conform
1	N (10,2)	5%	95%

4. META-MODEL

A simulation model is a representation of a real-world system, while meta-models (as referred to in this paper) are a mathematical approximation of a simulation model (Kleijnen & Sargent, 2000). Meta-models are developed to obtain a better understanding of the relationship between the input and output variables of the system under study (Noguera & Watson, 2006).

In this research, a polynomial regression meta-model was developed; this category of meta-models has provided outstanding results in simulation work (Amiri & Mohtashami, 2012; Dengiz & Akbay, 2000; Durieux & Pierreval, 2004; Hernández-Vázquez J. I. *et al.*, 2024; Hernández-Vázquez J. O. *et al.*, 2022a). The methodology used for its elaboration is established by Amiri and Mohtashami (2012). It uses the design of experiments (DOE) and the simulation to fit a meta-model to the average rate of production as a response (y), considering warehouse areas as factors (x_i). It should be noted that, unlike what Amiri and Mohtashami (2012) did, the OEE indicator is considered a response (y). The calculation of the OEE indicator results from the multiplication of three factors: availability, efficiency, and quality, which is described in detail by Socconini (2019). Recently, Hernández-Vázquez *et al.* (2024b) presented an adaptation of the OEE indicator in the footwear industry in Mexico.

A complete factorial design 2^7, with which 128 combinations were generated. Each of the 128 combinations of the experiment was simulated in the PROMODEL software in order to analyze the response results (OEE). The study considered 10 replicates for each combination (i.e., 1,280 simulations). The simulation time was 8 hours for each replica with a warm-up time of 2 hours. The PC where these simulations were carried out includes an AMD Ryzen 5 4300U processor with Radeon Graphics 2.70 GHz and 8GB of RAM.

A regression meta-model was generated involving the main effects and their interactions between two, three, four, and five factors. The analysis of the ANOVA is presented in Table 5. Fisher's test demonstrates a high degree of significance; therefore, the meta-model is able to satisfactorily explain the variability in the response variable. The comparison of the results obtained with the meta-model and the simulation is another way to evaluate the validity of the latter; the approach suggested by Durieux and Pierreval (2004), also Amiri and Mohtashami (2012), was used. From the experimental design, ten combinations of values in the decision variables were randomly selected; the average absolute error turned out to be less than the 6% established by Durieux and Pierreval (2004). Therefore, it is considered to be sufficiently accurate (see Table 6).

Table 7 presents the meta-model developed for the case study, which estimates the value of the OEE indicator by evaluating the buffers assigned in the different warehouse areas (from B_1 to B_7).

Table 5. Anova análisis

OEE						
Source	df	SS	MS	F Value	P Value	
Model	63	1.26E-02	2.00E-04	4.4414	< 0.0001	Significant
Main effects	7	7.56E-03	7.56E-03	168.1154		
Interaction 2 factors	21	1.06E-03	1.06E-03	23.4779		
Interaction 3 factors	25	2.61E-03	2.61E-03	57.9966		
Interaction 4 factors	9	1.17E-03	1.17E-03	26.0608		
Interaction 5 factors	1	1.87E-04	1.87E-04	4.1556		
Residual	64	2.88E-03	4.50E-05			
Cor total	127					
R-Squared	81.38%					

Table 6. Meta-model validation

Combination	1	2	3	4	5	6	7	8	9	10	Average absolute error (%)
B_1	24	1	1	24	24	1	24	24	12	14	
B_2	1	24	24	24	1	24	24	24	10	7	
B_3	24	24	24	24	24	24	24	24	5	11	
B_4	1	1	24	24	24	24	1	1	8	18	
B_5	1	24	24	1	24	1	1	24	9	15	
B_6	1	1	1	24	24	1	24	24	20	13	
B_7	1	1	1	1	1	24	24	24	21	7	
Simulation	77.838%	79.320%	79.178%	77.616%	79.825%	78.737%	78.541%	78.624%	80.625%	79.889%	
Meta-model OEE	78.446%	79.905%	79.925%	78.472%	79.684%	78.599%	78.285%	79.461%	79.108%	79.067%	
Absolute error	0.7803%	0.7372%	0.9443%	1.1028%	0.1761%	0.1759%	0.3257%	1.0653%	1.8822%	1.0295%	0.8219%

5. OPTIMIZATION METHOD

To solve the mathematical models, Evolutionary Solver was used. The philosophy of the Evolutionary Solver is based on genetics, evolution, and survival of the fittest. In consequence, this type of algorithm is sometimes referred to as a genetic algorithm. The Evolutionary Solver has three crucial advantages over the standard Solver (or any other convex programming algorithm) for solving non-convex programming problems. First, the complexity of the objective function does not affect it. As long as the function can be evaluated for a given test solution, it does not matter if the function has discontinuities or many local optima. Second, the complexity of the given constraints (including non-convex constraints) does not have a substantial effect on the Evolutionary Solver either (although the number of constraints does). Third, because it evaluates all test solution populations, they are not necessarily in the same vicinity as the current best test solution, the Evolutionary Solver avoids being trapped in a local optimal (Hillier & Lieberman, 2014).

The parameters used by the Evolutionary Solver were: Convergence 0.0001, mutation rate 0.075, population size 100, random initialization value 0, maximum time without improvement 30 sec.

Table 7. Meta-model developed for the case study

Meta-model OEE $\boldsymbol{g(B)} =$					
0.782161171		2.94903E-05	$* B_3 * B_7$	-1.78013E-06	$* B_2 * B_5 * B_7$
0.000536841	$* B_1$	-1.50211E-05	$* B_4 * B_5$	-1.6695E-06	$* B_2 * B_6 * B_7$
-0.00051952	$* B_2$	-1.57741E-06	$* B_4 * B_6$	8.39652E-07	$* B_3 * B_4 * B_5$
-0.000284911	$* B_3$	3.46636E-06	$* B_4 * B_7$	-1.39838E-06	$* B_3 * B_4 * B_6$
0.000473297	$* B_4$	5.10914E-05	$* B_5 * B_6$	-7.87026E-07	$* B_3 * B_4 * B_7$
0.000117436	$* B_5$	2.91945E-05	$* B_5 * B_7$	-2.55977E-06	$* B_3 * B_5 * B_6$
-0.000422576	$* B_6$	3.75328E-05	$* B_6 * B_7$	-2.50535E-06	$* B_3 * B_5 * B_7$
-0.000366227	$* B_7$	9.60934E-07	$* B_1 * B_2 * B_4$	-2.1991E-06	$* B_3 * B_6 * B_7$
-4.45008E-06	$* B_1 * B_2$	-7.4708E-07	$* B_1 * B_2 * B_7$	-3.03073E-07	$* B_4 * B_6 * B_7$
-7.33048E-06	$* B_1 * B_3$	1.47208E-06	$* B_1 * B_3 * B_4$	-2.52802E-06	$* B_5 * B_6 * B_7$
-5.74242E-05	$* B_1 * B_4$	2.82913E-07	$* B_1 * B_3 * B_7$	-7.00764E-08	$* B_1 * B_3 * B_4 * B_7$
-2.10659E-05	$* B_1 * B_5$	1.40764E-06	$* B_1 * B_4 * B_6$	-7.31488E-08	$* B_1 * B_4 * B_6 * B_7$
-2.4071E-05	$* B_1 * B_6$	2.30946E-06	$* B_1 * B_4 * B_7$	6.46146E-08	$* B_2 * B_3 * B_4 * B_6$
-9.39619E-06	$* B_1 * B_7$	7.81511E-07	$* B_1 * B_5 * B_6$	1.3619E-07	$* B_2 * B_3 * B_5 * B_6$
2.57439E-05	$* B_2 * B_3$	8.35165E-07	$* B_1 * B_5 * B_7$	8.90882E-08	$* B_2 * B_3 * B_5 * B_7$
4.58642E-06	$* B_2 * B_4$	6.64362E-07	$* B_1 * B_6 * B_7$	6.53664E-08	$* B_2 * B_3 * B_6 * B_7$
4.86606E-05	$* B_2 * B_5$	-1.07335E-06	$* B_2 * B_3 * B_4$	8.82032E-08	$* B_2 * B_5 * B_6 * B_7$
4.57551E-05	$* B_2 * B_6$	-2.36993E-06	$* B_2 * B_3 * B_5$	8.27679E-08	$* B_3 * B_4 * B_6 * B_7$

Meta-model OEE					
$g(\boldsymbol{B}) =$					
2.6792E-05	$* B_2 * B_7$	-1.96989E-06	$* B_2 * B_3 * B_6$	1.41945E-07	$* B_3 * B_5 * B_6 * B_7$
-2.05914E-06	$* B_3 * B_4$	-3.88647E-07	$* B_2 * B_3 * B_7$	-6.00893E-09	$* B_2 * B_3 * B_5 * B_6 * B_7$
4.00136E-05	$* B_3 * B_5$	-8.86088E-07	$* B_2 * B_4 * B_6$		
3.08392E-05	$* B_3 * B_6$	-2.82986E-06	$* B_2 * B_5 * B_6$		

6. RESULTS

This section describes the results generated by solving the multi-objective mathematical model of Section 2, aimed at maximizing the value of the OEE and minimizing the total cost of buffer allocation.

The results consider different weights in the optimization objectives, as W_1 decreases its value, it means that the first objective (maximize the value of the OEE indicator) will have a lower degree of importance, therefore, the value of W_2, which represents the second objective (minimize the total cost of buffer allocation) will begin to increase and take on greater relevance in the search for solutions. Table 8 shows the nine solutions generated for the mathematical model with the different proposed weights of W_1 and W_2.

The highest value of the OEE indicator obtained in the solutions was 80.40%, with a total allocation of 53 buffers. Warehouse areas B_5 and B_6 presented the highest allocation of these, with 24 units respectively.

The lowest cost in the allocation of buffers achieved in the solutions was $700, considering one buffer in each warehouse area.

Figure 2 shows a comparison between the OEE indicator and the total cost of buffer allocation. As the total cost decreases, a decrease in the OEE value is also generated, so an acceptable budget for buffer allocation will have to be considered in order to maintain the OEE indicator at a required level in the production line.

Table 8. Solutions to the mathematical model

	W_1	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
	W_2	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
B^*	B_1	1	1	1	6	3	2	1	1	1
	B_2	1	1	1	1	1	1	1	1	1
	B_3	1	1	1	1	1	1	1	1	1
	B_4	1	1	13	1	1	1	1	1	1
	B_5	24	24	1	1	1	1	1	1	1
	B_6	24	24	1	1	1	1	1	1	1
	B_7	1	1	1	1	1	1	1	1	1
Total buffers		53	53	19	12	9	8	7	7	7
M(B)		0.17867	0.16686	0.15090	0.13019	0.10880	0.08715	0.06543	0.04362	0.02181
OEE		80.40%	80.40%	78.68%	78.40%	78.28%	78.23%	78.19%	78.19%	78.19%
Total cost		$5,300	$5,300	$1,900	$1,200	$900	$800	$700	$700	$700

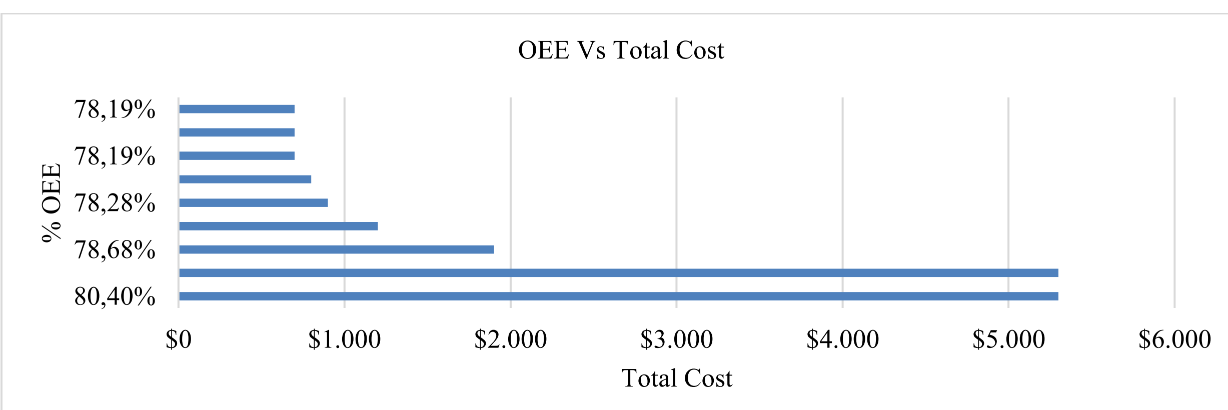

Figure 2. OEE Vs Total Cost

7. CONCLUSIONS

This work presented as its main contribution a new proposal for a BAP solution, considering a multi-objective optimization approach with the goal of maximizing the OEE indicator used in Lean Manufacturing, as well as minimizing the total cost of buffer allocation. A footwear stitching process was taken as a case study whose structure is that of a parallel production line in series, with unreliable operating conditions.

The evaluation method used considered a meta-model built from a design of experiments (DOE) and production line simulations through the use of PROMODEL software. This reflected the behavior of the OEE indicator in the production line of the case study.

Among the most relevant findings of the study is that when greater weight is assigned to the objective of minimizing the total costs of buffer allocation, the value of the OEE indicator is significantly affected in the solutions of the mathematical model, by decreasing the number of buffers. Over-inventory is considered a waste from the Lean Manufacturing approach; however, as shown in the analysis of the results, not considering an adequate investment in the number of buffers would significantly affect the value of the OEE indicator.

Unlike other studies where simulation is used as an evaluation method, the use of meta-models facilitated and accelerated the measurement of optimization objectives, allowing solutions to be achieved efficiently with the Evolutionary Solver algorithm. The implementation of meta-models helps in the analysis of complex production systems, taking advantage of one of the main virtues of simulation, and allows optimization methods to significantly improve their computational efficiency, since these behave as an analytical evaluation procedure (Hernández-Vázquez J. I. *et al.*, 20224; Hernández-Vázquez J.O. *et al.*, 2022a).

In future research, it would be interesting to use other multi-objective optimization methods different from the one presented in this paper and to compare the quality of the solutions generated. On the other hand, it would be possible to analyze production lines that are using AGVs and see their influence on the value of the OEE indicator. Another area of opportunity would be the use of the mathematical model presented in this paper for the analysis of other production lines with a different structure than the one analyzed.

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