

DYNAMIC COMPUTATIONAL APPROACH FOR ROUTING AND SCHEDULING LAST-MILE DISTRIBUTION

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This paper proposes an efficient dynamic mathematical programming approach to route and schedule downstream last-mile distributions of goods and assign a set of delivery points to delivery persons for minimizing the number of delivery persons required over multiple time periods while satisfying predetermined time restrictions. This approach aims to develop a decomposition MIP-based procedure that dynamically changes the members of the delivery point sets. This procedure allows for easily solvable optimization models, reducing computational efforts and minimizing memory usage and execution times. To demonstrate the effectiveness of the proposed approaches, the paper presents two datasets of 25 last-mile case-problems. The key findings include significant computational improvements in terms of reduced numbers of constraints and variables, which result in reduced computational effort, time, and memory usage required for optimizing the routes, schedules, and assignments of last-mile distributions.

Keywords: Dynamic Modeling; Hybrid Optimization; Travelling Salesman; Last-mile delivery; Scheduling and Routing; Mixed Integer programming.

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1. INTRODUCTION

Given the expansive network of distribution and delivery in supply chains, the term "miles delivery" can be used to describe the distances covered within the supply chain network, comprising various entities and individuals involved in transporting goods to end consumers through a designed physical distribution process. The concept of the "first mile" refers to the distance traveled to transport raw materials and partially completed products from upstream suppliers, while the "last mile" pertains to the distance covered to deliver goods to downstream customers. In this study, however, our focus is specifically on the latter. We define last-mile delivery as the final distance traveled in the distribution and delivery network, culminating in the customer receiving their goods. Accordingly, last-mile delivery service can be defined as the service that delivers a product to its ultimate destination from a depot, warehouse, or distribution center.

Last-mile distribution has drawn considerable interest from distributors, given its increasing demand in multi-channel distribution networks, which involve serving numerous delivery points with the least number of delivery personnel across various time periods. The final delivery destination in this study typically comprises a sales or delivery point, such as a personal residence. Delivery personnel play a vital role in delivering goods to end-users and customers, serving as the primary agents for distributing products and achieving strategic objectives set forth by distributors. The task of assigning, routing, and scheduling delivery personnel holds significant importance in managing last-mile delivery efficiently.

Distributors handling multiple delivery points must carefully assess distribution routes and allocate delivery personnel to maintain an effective delivery team and reduce overall delivery costs. Routing delivery personnel is a critical aspect of last-mile distribution management, involving determining the most optimal geographical path for each delivery person to follow when transporting goods to a specific set of delivery points. In the context of last-mile delivery management, planners often face the challenge of routing and scheduling individual delivery personnel. They must devise an efficient strategy to route and schedule deliveries to minimize the number of delivery personnel required to serve a given set of locations.

In real-life scenarios, distributors must allocate multiple delivery personnel to cater to specific delivery points, while also sequencing the order in which customers are served within each route across multiple time periods, typically spanning several days.

Our main focus in last-mile service is to efficiently assign delivery routes, schedules, and tasks to the fewest possible deliverymen in order to deliver all items to various delivery points over multiple time periods. By concentrating on optimizing last-mile delivery assignments and routing, distributors can ensure satisfactory service levels to their delivery points while

minimizing delivery costs. The challenge of managing multiple deliverymen in last-mile delivery is similar to the multiple traveling salesmen problem (mTSP), a more complex version of the traditional traveling salesman problem (TSP) that involves more than one salesman (Bektas, 2006).

This study proposes and implements effective methods for last-mile delivery tailored specifically to extended versions of the classical traveling salesmen problem. These extended versions are particularly relevant to the multiple salesmen problem, as they involve salesmen spending longer service times at delivery points compared to deliverymen. Our primary goal is to develop a comprehensive approach to address the multi-deliveryman problem in last-mile delivery, similar to the mTSP. This involves optimizing delivery schedules, routes, and assignments for individual deliverymen while adhering to time constraints. Each route must start and end at the main distribution center, with each delivery point visited and served multiple times within a specified time period. The time required to provide delivery service at each delivery point is predetermined based on customer type. Similar to the mTSP, the significance of addressing this problem lies in the substantial cost savings achieved by minimizing the number of deliverymen employed and optimizing their routes and schedules.

In this setting, computer-aided systems that support optimization methods have become effective tools for analyzing and enhancing real-world issues such as mTSP and last-mile delivery service problems. These systems enable users to explore operations in a safe and flexible way, helping to determine delivery routes and schedules, allocate delivery points to delivery personnel, and enhance last-mile services through optimized planning and routing processes. Despite the numerous beneficial problems that these tools can address, mTSP has been identified as an NP-hard issue, meaning it cannot be solved efficiently as the problem size increases. The existing research has not yet found a successful algorithm or method to solve these problems in a reasonable time frame (Reus, 2016), which is where our research comes in. This paper introduces a unique discrete algorithmic modeling that utilizes a computer-aided system to support mathematical optimization models, enabling the efficient solution of mTSP-like problems within a reasonable timeframe. This is demonstrated through the solution of 25 case problems. Our research addresses a gap in the literature by proposing a method to optimize the number of delivery personnel in a hybridized way, considering their work time constraints, service, and travel times.

The structure of this paper is as follows: Section 2 provides a brief overview of existing research on the multiple traveling salesmen problem. Section 3 defines the problem addressed in this paper. Section 4 details the development of our proposed approaches. Section 5 showcases the application of these approaches to 25 case problems and presents the results, and finally, Section 5 summarizes the paper and draws conclusions.

2. RELEVANT LITERATURE REVIEW

Comprehensive reviews are available on several variations and approaches proposed for solving the traveling salesman problems (TSP) and multiple traveling salesmen problems (mTSP). The approaches have been proposed primarily based on the application areas.

2.1 TSP and mTSP Applications

The TSP arises in main real-world applications, including the drilling problem of printed circuit boards in an actual production environment, overhauling gas turbine engines of aircraft, analysis of the structure of crystals presented by Bland and Shallcross (1989), the connection of components on a computer board, order-picking and material handling in warehouses proposed by Ratliff & Rosenthal (1983). Moreover, TSP has evolved from a theoretical benchmark to an essential model for automated smart manufacturing and logistics (Bello *et al.*, 2021; Bengio *et al.*, 2021; Wu *et al.*, 2023) and real-world manufacturing and logistics routing (Li *et al.*, 2024)

Whereas mTSP has numerous real-life applications. Basu *et al.* (2000), Wang & Regan (2002), and Macharis & Bontekoning (2004) reported a comprehensive review on various applications of mTSP. The main applications of mTSP include production scheduling presented by Tang *et al.* (2000), Carter & and Ragsdale (2002), school bus routing reported by Angel *et al.* (1972), crew scheduling described by Svestka & Huckfeldt (1973), mission planning presented by Brummit & Stentz (1998), designing system networks suggested by Saleh & Chelouah (2004), security service investigated by Calvo & Cordone (2003) and Kim & Park (2004), vehicle routing (VRP) discussed by Ralphs, (2003), Mitrović-Minić *et al.* (2004), and Granado *et al.* (2021). Nekovář *et al.* (2023) focused on multi-robot path planning to coordinate exploration and inspection. Onder *et al.* (2017) applied mTSP in manufacturing and maintenance, while Mo *et al.* (2023) used it in healthcare logistics for ambulance or home care worker routing.

2.2 TSP and mTSP Variations

A number of novel TSP variations have surfaced in recent years to address practical industrial issues. The Asymmetric TSP models direct pathways for warehouse material flow (Zhang *et al.*, 2023; Nekovář *et al.*, 2023). The Time-Dependent TSP takes into consideration variable travel times in smart-city logistics, and the Green or Energy TSP focuses on reducing emissions in electric vehicle routing (Qin *et al.*, 2022; Cai *et al.*, 2023). TSP in industry for robotic arm scheduling, CNC toolpath optimization, and healthcare logistics, including ambulance routing (Chen *et al.*, 2023; Mo *et al.*, 2023). These variations demonstrate how TSP is still crucial for logistics, manufacturing, and intelligent infrastructure design.

Furthermore, Bektas (2006) listed several variations on the mTSP, instead of one depot, the multi-depot mTSP has multiple depots, with multiple salesmen at every depot. In the first version, salesmen return to their source terminal/depot from which they began; In the second version, salesmen are not required to get back to their source depots, but depots must receive their exact number of salesmen; In the third variation the number of salesmen is a constant bounded by an upper limit; The fourth variation based on inconstant number of salesmen with a fixed cost for initiating a salesman. Zhang *et al.* (2023) presented an asymmetric traveling salesman problem as a branch of the traveling salesman problem.

In mTSP version with a fixed charge, the fixed charges and tour costs for the salesmen are minimized. There is one more mTSP variation based on a time window for visiting different nodes. Bektaş and Laporte (2011) used time-window constraints (mTSP_{TW}), where the customer must be served within the time boundary. Groba *et al.* (2018) discussed the mTSP to prove that integrating forecasting within a metaheuristic evolutionary-based method, such as genetic algorithms, can yield better results in a dynamic scenario than their simple non-predictive version. Honglu *et al.* (2018) proposed genetic algorithms to solve the minsum mTSP with multiple depots, a closed path, and the requirement for minimizing the number of locations served by individual salesmen. Saeed Osman (2025) explored a multi-period distributed approach for last-mile delivery services. Lei *et al.* (2018) proposed an indefinite period but for a single TSP, in which customer nodes need to be visited an unknown multiple or an infinite number of times, but cannot be visited more than once on the same trip. Cheikhrouhou and Khoufi (2021) researched the min-sum vs. min-max objectives, which help minimize the total travel cost and ensure balanced workloads among salesmen. Ćbroška *et al.* (2023) studied drone-integrated systems, where mTSP has been extended to truck and drone operations. Jiang *et al.* (2019) tackled the environmental aspect by solving a TSP variant that minimizes carbon emissions using mathematical modeling to provide a foundation for green logistics.

2.3 TSP and mTSP Formulations

With regard to TSP and mTSP mathematical formulations, many approaches are reported in the literature. Orman & Williams (2006) and Öncan *et al.* (2009) have provided surveys on several formulations of the problem. Among these, the formulations proposed by Applegate *et al.* (2003) and Zhang *et al.* (2024). Bagagiolo and Benetton (2012) applied the dynamic programming technique in order to characterize the value function of the TSP as the unique viscosity solution of a suitable Zhang *et al.* (2020) proposed a logistics collaboration model to address the collaborative vehicle routing problem that involves shared carriers and a depot to decrease transportation distances and improve capacity utilization. Hamilton–Jacobi equation. Liu & Chen (2021) proposed parallelized GPU-based solvers for the TSP problem, while Zhong *et al.* (2024) suggested refinements to enhance edge elimination and reduce the computation time.

Hassanpour *et al.* (2021) proposed a robust optimization model integrated with a vehicle routing problem for solving hazmat location-routing problems. Kuo *et al.* (2022) presented a mixed-integer programming model to minimize the total travelling costs and explored the cooperation of the vehicle routing problem with drones. Applegate *et al.* (2023) developed flow-based models that represent cities as nodes in a flow network for routing with demand. The most popular and successful approach for solving the TSP problem is still the branch-and-cut approach (Applegate *et al.*, 2023). Chen & Huang (2023) proposed dynamic constraint learning techniques, which employ machine learning to forecast useful cuts and bounds, for accelerating the process of solving TSP.

2.4 TSP and mTSP Algorithms and Heuristics

Several authors have proposed heuristic methods for solving TSP and mTSP problems. Fischetti *et al.* (2002) have presented surveys of algorithms for the problem. Several well-known heuristic approaches have been developed to solve these problems, which include the algorithms presented by Dell'Amico & Toth (2000), and Vairaktarakis (2003) reconsidered the version of the TSP and found applications in machine scheduling and workforce planning. Kalczynski and Kamburowski (2005) showed that the problem of minimizing the expected makespan in a two-machine flow shop is equivalent to TSP. However, the best available algorithm for the symmetric TSP was developed by Applegate *et al.* (2006). Honglu *et al.* (2018) adopted the particle swarm optimization algorithm to solve mTSP. They found out that the suggested algorithms for solving the mTSP are feasible with room for improving their efficiency. Guilherme *et al.* (2018) considered a genetic local search procedure

that incorporates a variable neighborhood search method for solving $mTSP$. Their outputs demonstrated that the distributed approach provided better results. Lei *et al.* (2018) proposed exact and heuristic approaches for solving the TSP problem with certain sizes. The authors concluded that better algorithms are to be developed in the future for a very large number of nodes, like of 100 nodes or more. Zhang *et al.* (2023) proposed a discrete mayfly algorithm for solving the asymmetric traveling salesman problem, where a salesman travels a group of cities at the minimum cost and returns to the starting city. Saeed Osman (2025) developed a distributed algorithm for last-mile deliveries.

Among metaheuristics, Tabu Search (Xie *et al.*, 2022), Ant Colony Optimization (Nekovář *et al.*, 2023), Genetic Algorithms (Mahmoudinazlou and Kwon, 2023; Wang *et al.*, 2023), Simulated Annealing (Li *et al.*, 2024), and Variable Neighborhood Search (Han *et al.*, 2025) have demonstrated good performance in solving TSP and $mTSP$ problems. Cunha *et al.* (2024) used Adaptive Large Neighborhood Search, which constantly changes its search strategy to handle large or complex problems more efficiently. Li *et al.* (2024) combined global search and local refinement in hybrid models.

2.5 Last-mile Applications

Macioszek (2018) conducted a conceptual review of last-mile delivery to provide a holistic view of the logistical challenges at both ends of the supply chain, and a comprehensive overview of last-mile delivery modes to assess their efficiency, sustainability, and trends, using a literature review and comparative analysis to create a structured understanding of delivery options.

Bruni *et al.* (2023) developed a tactical-operational model for Unmanned Aerial Vehicle delivery for optimizing routing while accounting for non-linear energy consumption and shared depot usage through an MIP model. Yang *et al.* (2021) addressed cold chain logistics with the objective of designing a resilient network for temperature-sensitive goods by applying robust optimization for network design to mitigate uncertainties. Kuo *et al.* (2022) presented a mixed-integer programming model to minimize the total travelling costs and explored the cooperation of the vehicle routing problem with drones. Hassanpour *et al.* (2021) proposed a robust optimization model integrated with a vehicle routing problem for solving hazmat location-routing problems. Zhang *et al.* (2020) proposed a logistics collaboration model to address the collaborative vehicle routing problem that involves shared carriers and a depot to decrease transportation distances and improve capacity utilization. Yilmaz and Demirel (2023) applied the Hesitant Fuzzy Linguistic Term Sets approach for the evaluation of six different out-of-home delivery methods for LMD.

For complex, NP-hard routing problems in last-mile delivery, heuristic and metaheuristic approaches provide good-quality, feasible solutions within reasonable computation time, as explored by (Aggarwal *et al.*, 2024) and (Chen and Chen, 2022), who both developed bio-inspired algorithms for efficient route finding based on Ant Colony Optimization. Ky Phuc and Anh (2024) addressed the complex 2-Echelon Vehicle Routing Problem by designing an efficient Adaptive Large Neighborhood Search heuristic for systems involving satellite hubs, though their work identifies. Zhou *et al.* (2025) investigated the joint optimization of truck-drone routes using a combination of mathematical optimization and heuristics to coordinate these vehicles in tandem, but a critical gap remains in handling complex real-world constraints such as dynamic no-fly zones and varying weather conditions.

There are still a number of important research gaps in TSP, $mTSP$, and LMD investigations. Since neural and metaheuristic solvers have trouble handling a large number of nodes, scalability remains a problem (Luo *et al.*, 2023). There is no common framework for the hybrid integration of operations research and heuristics/metaheuristics/machine learning (Chen & Huang, 2023). Nonetheless, as far as we know, there is no published work that presents the problem addressed in this paper or suggests a method to optimize the number of deliverymen in a hybrid and dynamic manner, subject to their work time limit and service and traveling times.

3. PROBLEM STATEMENT

The last-mile delivery problem (LMDP) explored in this paper is particularly challenging, as it resembles a multi-period $mTSP$ problem and requires dynamically assigning, scheduling, and routing last-mile deliveries in an optimal way that minimizes the number of deliverymen. What differentiates between $mTSP$ and LMDP is that salesmen take longer service times at sales-points compared to the time spent by deliverymen at delivery points. In our LMDP, every delivery point can be visited and served multiple times, but cannot be visited more than once in the same period (day). To date, no published research has addressed routing and scheduling large-scale last-mile deliveries involving over 100 delivery points with the goal of minimizing the number of deliverymen. This study tackles that gap using a well-structured dynamic computational approach. The main contribution of this work is the development of a dynamic MIP-based model and an algorithm supported by a computer-aided system. This system integrates optimization models to effectively solve large-scale multi-period $mTSP$ -like problems within a reasonable computation time. The approach is validated through the resolution of 25 case problems.

4. DYNAMIC MODELING FOR MULTI-TRAVELING DELIVERYMEN MULTI-PERIOD PROBLEM

4.1 MIP-based Multi-Traveling Deliverymen Multi-Period Model

The proposed MIP-based formulation of the Multi-Traveling Deliverymen Multi-Period (*mTD-mP*) model is based on a graph $G = (I, R)$, where I is the set of $|I|$ delivery points, and R is the set of $\frac{1}{2} * |I| * |I - I|$ bidirectional arcs. Traversing arc $i-j$ requires traveling time T_{ij} , where $i \& j \in I$, while serving delivery point i requires service time S_j . It is assumed that node 1 is the main distribution center (depot), and the main distribution center operates with m deliverymen. Before we introduce the formulation of *mTD-mP* in detail, we introduce the notation shown in Table 1.

Table 1. Notations used in *mTD-mP* Model

	Description	Type
B	An arbitrarily big number	IP
d	Period/day index; $d \in D$	IN
D	The set of all working days	SE
$ D $	The cardinality of set D .	IP
e_{dk}	Equal to 1 if deliveryman k is assigned to visit any delivery point on day d , and 0 otherwise	DV
f_k	The working time available for each deliverymen k per period (<i>day</i>)	IP
F	The working time available for a given deliveryman in set K per period	IP
i/j	The delivery point index; $i \& j \in I$ or I'	IN
I	The set of all points visited by deliverymen including delivery points and the main depot	SE
I'	A subset of delivery points visited by deliverymen, excluding the distribution center	SE
$ I $	The cardinality of set I .	IP
G_d	The relative position of a period d in set D	IP
K	The set of all deliverymen	SE
$ K $	The cardinality of set K .	IP
k	Deliverymen index; $k \in K$	IN
N_i	The number of scheduled visits to the delivery point i	DP
p_{dk}	The total time spent by deliveryman k on day d to visit the assigned delivery points	DV
P	The total time spent by individual deliveryman to visit the assigned delivery points on a given period	DV
q_i	The required number of deliveries/visits by deliverymen to delivery point i	IP
R_i	The relative position of a delivery point i in set I	IP
S_j	The average service/delivery time at delivery point j	IP
T_{ij}	The traveling time between delivery point i and j	IP
V_k	Equal to 1 if deliveryman k is hired, and 0 otherwise	DV
w_{dkij}	Equal to 1 if deliveryman k travels from delivery point i to j on day d , and 0 otherwise	DV
W_{ij}	Equal to 1 if individual deliveryman travels from delivery point i to j on a given period, and 0 otherwise	DV
<i>SE: Set</i>	<i>IN: Index</i>	<i>IP: Input Parameter</i>
		<i>DV: Decision Variable</i>

The objective function and functional constraints of the proposed *mTD-mP* model are detailed as follows.

Objective Function:

$$\text{Min } \sum_{k=1}^{|K|} B * V_k + \sum_{d=1}^{|D|} \sum_{k=1}^{|K|} (f_k - \sum_{i=1}^{|I|} \sum_{j=1}^{|I|} (T_{ij} + S_j) w_{dkij}) \tag{1}$$

Subject to:

$$\sum_{i=1}^{|I|} \sum_{j=1}^{|I|} w_{dkij} \leq B * e_{dk} \quad \forall k \in K \text{ and } d \in D \tag{2}$$

$$\sum_{d=1}^{|D|} \sum_{i=1}^{|I|} \sum_{j=1}^{|I|} w_{dkij} \leq B * V_k \quad \forall k \in K \tag{3}$$

$$\sum_{i=1}^{|I|} \sum_{k=1}^{|K|} w_{dkij} = 1 \quad \forall d \in D \text{ and } j \in I' \quad (4)$$

$$\sum_{j=1}^{|I|} \sum_{k=1}^{|K|} w_{dkij} = 1 \quad \forall d \in D \text{ and } i \in I' \quad (5)$$

$$\sum_{d=1}^{|D|} \sum_{i=1}^{|I|} \sum_{k=1}^{|K|} w_{dkij} = q_j \quad \forall j \in I' \quad (6)$$

$$\sum_{d=1}^{|D|} \sum_{j=1}^{|I|} \sum_{k=1}^{|K|} w_{dkij} = q_i \quad \forall i \in I' \quad (7)$$

$$\sum_{i=1}^{|I|} w_{dkij} = e_{dk} \quad \forall j=1, k \in K, \text{ and } d \in D \quad (8)$$

$$\sum_{j=1}^{|I|} w_{dkji} = \sum_{j=1}^{|I|} w_{dkij} \quad \forall i \in I, k \in K, \text{ and } d \in D \quad (9)$$

$$\sum_{i=1}^{|I|} \sum_{j=1}^{|I|} (T_{ij} + S_j) w_{dkij} \leq f_k \quad \forall k \in K \text{ and } d \in D \quad (10)$$

$$R_i - R_j + |I| \sum_{k=1}^{|K|} w_{dkij} \leq |I| - 1 \quad \forall i \neq j \in I' \text{ and } d \in D \quad (11)$$

$$w_{dkij} = 0 \text{ or } 1 \quad \forall i \& j \in I, k \in K, \text{ and } d \in D \quad (12)$$

$$e_{dk} = 0 \text{ or } 1 \quad \forall k \in K \text{ and } d \in D \quad (13)$$

$$V_k = 0 \text{ or } 1 \quad \forall k \in K \quad (14)$$

Objective function (1) is to minimize the number of employed deliverymen to serve different delivery points over multiple periods. It also maximizes the utilization of individual deliverymen by minimizing their unutilized time. Constraints (2) and (3) ensure that if a deliveryman is hired on a given period, he must have at least one delivery point to visit. Constraints (4) and (5) assume that each delivery point is to be visited no more than once per period by a deliveryman coming from the main distribution center or another delivery point. Constraints (6) and (7) ensure that each delivery point is to be visited as many as required over the given multiple periods. Constraint (8) ensures that if a deliveryman is hired on a given period must return back to the distribution center from the delivery point lastly served. Constraint (9) executes the rule of net flow that the deliveryman inflowing each delivery point must outflow from it on any given period. This constraint also ensures that all deliverymen who leave the main distribution center must return to it on any given day. Constraint (10) guarantees that the total traveling and service time for individual deliverymen on multiple periods each deliveryman does not exceed the allowable daily working time (time limit) to visit and travel between delivery points on a given day. We also adopted the elimination of sub-tours (*Miller-Tucker-Zemlin, 1960*) in Constraint (11). The declarations of w_{dkij} , e_{dk} , and V_k as binary decision variables are given in Constraints (12), (13), and (14).

For the $mTD-mP$ model, we compute the number of constraints is $[2*|I-1| + 2*|K| + 6*|K|*|D| + 2*|D|*|I-1| + |I-1|^2*|D| + |K|*|I-1|^2*|D| + |K|*|I|*|D|]$, and the number of variables is $[|K| + |K|*|D| + |K|*|I|*|I-1|*|D|]$, where the cardinality of sets is denoted by $|\bullet|$. For example, for $|D|=5$ periods, and $|I'| = |I-1| = 40, 80, \text{ and } 100$ delivery points, the number of variables and constraints, and solutions including the optimal number of salesmen, CPU time, and memory usage are summarized in Table 2.

Table 2. Size and Outputs of $mP-mTD$ Model

Set Cardinalities			Constraints and Variables		Solution			
$ I' $	$ K $	$ D $	No of Constraints	No of Variables	Relative Gap	Optimal No. of Salesmen	CPU Time	Memory Usage
15	2	5	1,510	2,585	0	1	9.094	3 MB
30	3	5	5,391	14,451	0.000121	2	16.891	4 MB
60	4	5	20,027	74,467	0.000098	3	67.781	11 MB
90	6	5	43,963	207,055	*****Resource limit exceeded*****			

The results in Table 2 show that $mTD-mP$ model is capable of providing optimal solutions within reasonable memory and CPU constraints for a small number of delivery points. However, it may not be able to return a solution for a larger

number of delivery points as it exceeds resource limits, causing the solver to terminate. Additionally, the model requires an upper bound on the number of available deliverymen to be specified before implementation, which can be challenging to determine accurately using simple methods.

To address these limitations, we propose a unique dynamic optimization modeling. This approach utilizes dynamic sets of delivery points and partitions tasks among deliverymen and multiple time periods to optimize routes and schedules and minimize the number of deliverymen needed while maximizing their utilization.

4.2 Dynamic Optimization Modeling

For solving the multi-traveling deliverymen multi-period problem addressed in this paper, we propose a dynamic optimization (DO-2mTDP) approach. The DO-2mTDP iteratively solves optimization models over sets of deliverymen and periods. The dynamic optimization nature of the DO-2mTDP approach is based on the use of dynamic sets of delivery points to perform partial computations as required for minimizing the model size, including the number of variables and constraints, computational times, and memory storage requirements.

All the sets, including the delivery points set used in *mTD-mP* model, have fixed/static members. The DO-2mTDP approach will dynamically change the members of the delivery points set. Generally, a set whose members can change is called a dynamic set to stand in contrast of the static delivery points set whose members will never change, resulting in unneeded computations and, in turn, long computational times and more memory consumption. The distinction here is important and will make an important computational improvement in terms of reduced numbers of constraints and variables, and that will result in reduced computational time and memory usage required to obtain an optimal delivery points assignment to one-deliveryman-one-period in every iteration. We outline the primary steps of the DO-2mTDP algorithm in Figure 1.

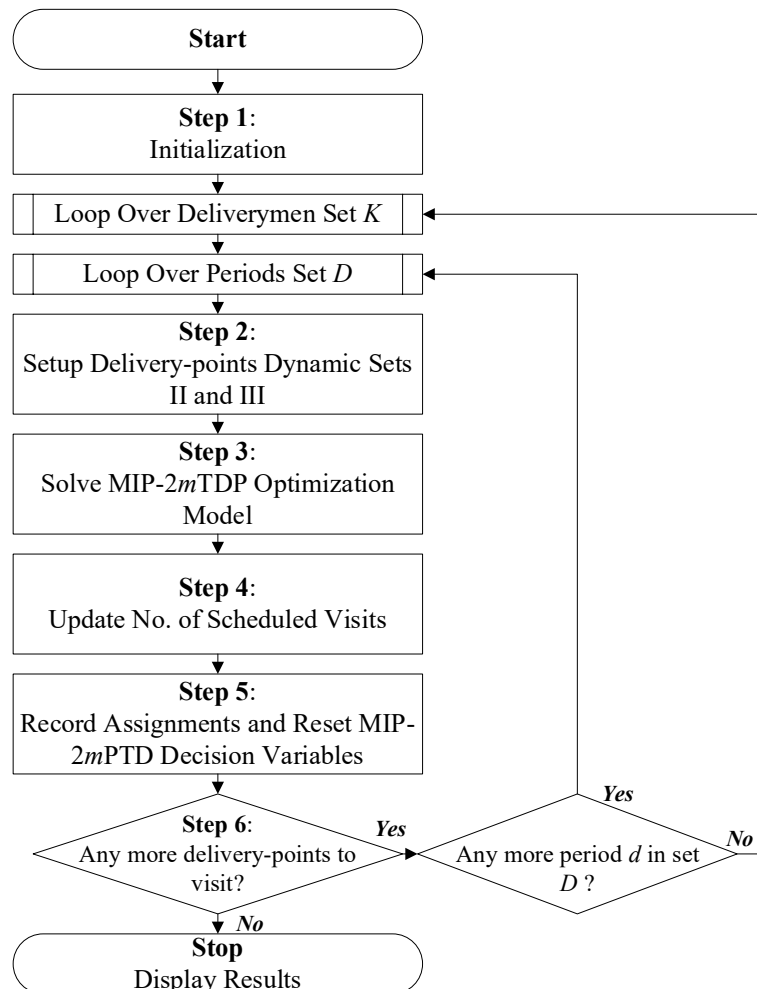


Figure 1. Overall Steps of DO-2mTDP Procedure

DO-2mTDP solution procedure begins by Step 1, which is an initiation phase for declaring input parameters. This step includes setting the number of scheduled visits to individual delivery points to zero.

Then DO-2mTDP loops over two sets, the deliverymen set and periods set. Each loop includes repetitive steps for each iteration. While looping over deliverymen set, the dynamic procedure assigns the available time of individual deliverymen, h_k , to a un-superscripted parameter H .

Hence, Step 3 of DO-2mTDP improves the solution quality of the $mTD-mP$ model. The MIP-based optimization model (MIP-2DmTDP) is solved for dynamic delivery points sets rather than static set I and subset I' . However, prior to solving the optimization model in Step 3, Step 2 involves setting up two dynamic sets, II and III , of delivery points for each iteration, given static set I . Only the delivery points that are still required to be scheduled for delivery/service will be declared as members of the first dynamic set II , while the main distribution center and all delivery points that are yet to be scheduled for delivery will be declared as members of the second dynamic set III .

In Step 4, DO-2mTDP updates the number of scheduled visits to delivery points and traveling times as obtained in Step 3. If a delivery point has been served with the required number of visits, then an arbitrary long traveling time to/from that given delivery point would be assigned to avoid reconsidering it in the next iterations. Step 5 of DO-2mTDP records the assignment of delivery points to deliverymen by giving x_{ij} in values from 2DmTP to x_{dkij} in DO-2mTDP, which are superscripted with the indexes k and d , and then resetting x_{ij} values to zero. In Step 6, DO-2mTDP compares the number of scheduled versus required visits to all delivery points; if all delivery points were assigned for visits as required, then DO-2mTDP stops. Otherwise, the procedure continues to loop over periods set and deliverymen set until the terminating condition has been satisfied.

4.2.1 DO-2mTDP Pseudo Code

The code of the DO-2mTDP procedure is detailed as follows:

Given S_j, q_i, f_k , and T_{ij} for all $k \in K$ and i and $j \in I$

Step 1: Initialization

- ⊖ For all delivery points $i \in I'$,
→ set $N_i = 0$
- ⊖ For all $k \in K$,
→ set $F = f_k$
- ⊖ For all $d \in D$,

Step 2: Setup Dynamic Sets II and III

- ⊖ For all locations/points including the main distribution center and delivery points $i \in I$,
→ set $i \in II$ if $R_i > 1$ and $N_i < q_i$
→ set $i \in III$ if $R_i = 1$ or $N_i < q_i$

Step 3: Solve the MIP-2DmTDP optimization model

Step 4: Update the number of scheduled visits to delivery points and traveling times

- ⊖ For all $i \in I'$,
→ $N_i = N_i + \sum_{j=1}^{|I'|} W_{ij}$
→ If $N_i = q_i$ then
⊖ For all $j \in I$,
– $T_{ij} = B$ and $T_{ji} = B$

Step 5: Record the assignment and reset MIP-2DmTDP decision variable

- ⊖ For all $i \in I$ and $j \in I$
→ $w_{dkij} = W_{ij}$ and $W_{ij} = 0$ (required to allow multiple visits to a delivery point)

Step 6: Check the terminating condition,

- ⊖ If $\sum_{i=1}^{|I'|} N_i = \sum_{i=1}^{|I'|} q_i$ then STOP
- ⊖ Else
→ If $G_d < |D|$ Go to Next d
→ Go to Next k

4.2.2 MIP-2DmTDP Optimization Model

Step 3 in the DO-2mTDP solves the multiple traveling-deliverymen multi-periods optimization model (2DmTP). This model is solved iteratively for dynamic sets of individual deliverymen over multiple periods. The 2DmTP assumes that all input parameters are deterministically identified.

Objective Function:

$$\text{Min } F - \sum_{i=1}^{|III|} \sum_{j=1}^{|III|} (T_{ij} + S_j) W_{ij} \quad (15)$$

Subject to:

$$\sum_{j=1}^{|III|} W_{ij} \leq 1 \quad \forall i \in II \quad (16)$$

$$\sum_{i=1}^{|III|} W_{ij} \leq 1 \quad \forall j \in II \quad (17)$$

$$\sum_{i=1}^{|III|} W_{ij} = 1 \quad \forall j=1 \quad (18)$$

$$\sum_{j=1}^{|III|} W_{ij} = 1 \quad \forall i=1 \quad (19)$$

$$\sum_{j=1}^{|III|} W_{ji} = \sum_{j=1}^{|III|} W_{ij} \quad \forall i \in II \quad (20)$$

$$\sum_{i=1}^{|III|} \sum_{j=1}^{|III|} (T_{ij} + S_j) W_{ij} \leq F \quad (21)$$

$$R_i - R_j + |I| \sum_{k=1}^{|K|} W_{ij} \leq |I| - 1 \quad \forall i \neq j \in II \quad (22)$$

$$W_{ij} = 0 \text{ or } 1 \quad \forall i \& j \in I \quad (23)$$

The MIP-2DmTDP optimization model is to be solved for dynamic sets II and III instead of the static set I and subset I' . Therefore, Objective Function (15) minimizes the unutilized time for individual deliverymen on a given day. Constraints (16) and (17) assume that each delivery point is to be visited no more than once on that iterated day. Constraint (18) ensures that the assigned deliveryman must return to the main distribution center on any iterated period. In contrast, Constraint (19) confirms that the assigned deliveryman must begin from the main distribution center. Constraint (20) imposes the net flow rule. Constraints (20), (21), (22), and (23) are a modified version of Constraints (9), (10), (11), and (12), respectively, after removing index k and d .

5. COMPUTATIONAL CASE PROBLEMS

We conducted computational work for 25 scaled-up case problems, each with a different number of delivery points to be visited and served over five business days. Given the number of delivery points for every case-problem, we created random integers numbers for the service/delivery times between 5 and 45 minutes, number of required visits between 1 and 3, and travel times between delivery points and between the main distribution center and delivery points between 10 and 40 minutes. The raw data is not presented here; however, the descriptive summaries for the 25 case problems are reported in Table 3.

We solved the 25 case-problems using DO-2mTDP and reported the computational outputs used for comparative analysis. DO-2mTDP procedure was coded and executed in General Algebraic Modeling System (GAMS 24.8.5) with CPLEX solver. The computer used for solving all case-problems is a Dell with a 64-bit OS, Intel(R) Xeon(R), x64-based processor @ 3.60GHz, a 700GB hard drive, and 16.0 GB of RAM.

To report the solution quality of the proposed approach, three primary metrics are used; the optimal number of salesmen, memory usage, and total CPU time. We considered CPU time as a metric for demonstrating the reduced running times for solving the multiple traveling-deliverymen multi-periods case-problems using the DO-2mTDP. The solution metrics obtained for solving the 25 case-problems are reported in Table 4.

Table 3. Number of Delivery points, Required Visits, and Service and Traveling Times (*in min.*)

Case No.	No. of Delivery points	Average Service Time	Average No. of Required Visits	Average Traveling Time	Case No.	No. of Delivery points	Average Service Time	Average No. of Required Visits	Average Traveling Time
1	40	24.18	2.15	24.61	14	450	24.89	1.99	25.00
2	60	24.10	2.17	24.91	15	500	25.11	1.99	25.00
3	80	24.46	2.09	24.90	16	550	25.14	1.98	25.00
4	100	25.75	2.05	24.89	17	600	24.87	2.00	25.00
5	120	25.83	2.02	24.99	18	650	24.83	2.02	24.99
6	140	25.95	2.03	25.03	19	700	24.74	2.02	24.99
7	160	25.45	2.02	25.03	20	750	24.83	2.01	24.98
8	180	25.49	1.98	24.99	21	800	24.76	2.01	24.99
9	200	25.46	1.99	25.01	22	850	24.86	2.02	24.99
10	250	24.96	2.00	25.00	23	900	24.86	2.01	24.99
11	300	25.14	1.99	25.01	24	950	24.95	2.02	24.99
12	350	25.18	1.99	25.01	25	1000	24.99	2.02	24.99
13	400	25.06	1.99	25.01					

Table 4. Solution Metrics for DO-2mTDP Approaches

Case No.	No. of Delivery points	Optimal No. of Salesmen	CPU Time (sec)	Memory Usage (MB)	Case No.	No. of Delivery points	Optimal No. of Salesmen	CPU Time (sec)	Memory Usage (MB)
1	40	2	1.3	3	14	450	18	117.1	35
2	60	4	1.5	3	15	500	21	178.6	43
3	80	4	2.1	4	16	550	22	234.4	52
4	100	5	5.1	4	17	600	24	297.9	61
5	120	6	4.3	5	18	650	26	459.1	71
6	140	7	5.4	6	19	700	28	503.3	82
7	160	7	7.3	7	20	750	31	679.0	94
8	180	8	8.7	8	21	800	32	793.2	106
9	200	9	19.3	9	22	850	33	1076.1	119
10	250	11	24.9	13	23	900	35	3898.8	133
11	300	13	35.3	18	24	950	38	1580.8	148
12	350	15	54.7	23	25	1000	39	1921.0	164
13	400	17	79.4	29					

The inclusion of the dynamic sets of delivery points eased the complexity of computational works. It reduced the number of constraints and variables in the dynamic optimization approach. It also minimized the computational resources needed for obtaining optimal assignments of delivery points to the minimum number of deliverymen over multi-periods. This optimal assignment is accomplished while satisfying the deliverymen time-limit constraints. The summary of results shown in Table 4 provided evidence that DO-2mTDP is a promising method that offers competitive solutions to large-scale multiple traveling-deliverymen multi-periods problems defined in this research, and enables the efficient distribution of goods.

6. CONCLUSIONS

This paper studies the challenge of coordinating multiple delivery personnel over several time periods for last-mile delivery operations. The problem involves a set of delivery points characterized by service times, required visit frequencies, travel times between locations, and individual time constraints for each delivery person. We proposed an optimization-based approach utilizing embedded mixed-integer programming (MIP) models, which dynamically update the set of active delivery points.

Our method simultaneously determines optimal routes, schedules, and assignments of delivery points to each delivery person across multiple periods, with the objective of minimizing total delivery costs. This is achieved by reducing the number of delivery personnel required and optimizing both routing and scheduling decisions.

The core contribution of our work lies in introducing a novel computational framework for solving the multi-period multiple traveling deliverymen problem under time constraints. The proposed Dynamic Optimization for the Multi-period Multiple Traveling Deliverymen Problem (DO-2mTDP) incorporates dynamically adjusted subsets of delivery points. This enables partial computations, effectively reducing memory usage and computational complexity.

By adopting this dynamic subset strategy, DO-2mTDP is capable of efficiently solving large-scale instances involving up to 1000 delivery points across multiple periods within reasonable computation times. The approach significantly reduces the computational effort required to optimize the objectives.

Overall, this research offers promising avenues for addressing a wide range of real-world logistics problems, such as adapting the proposed dynamic approach to variations of the multi-period multiple traveling salesman problem (mTSP). Conducting comparative and sensitivity analysis of parameters could be considered as a future work.

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