Drilling Optimization through Rig Hydraulics Using a Mathematical Model

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Abstract

Hydraulics play a crucial role in the advancement and enhancement of drilling operations. The utilization of the Burgoyne and Young models for predicting the rate of penetration (ROP) and for the refinement of hydraulic parameters underscores the critical importance of jet velocity, jet impact force (JIF), and equivalent circulation density (ECD). It is of utmost importance to optimize flow rates, as they have a direct and significant impact on both the impact force and the equivalent circulating density of the jets. Consequently, this paper aims to thoroughly examine the fundamental interrelations among various parameters, including ECD, weight on bit (WOB), revolutions per minute (RPM), and flow rate, as well as to assess and evaluate how hydraulic optimization contributes to the improvement of drilling efficiency. The initial rate of penetration has experienced an increase of tenfold, with the smallest recorded increase being 1.65 times, indicating that ROP has been effectively optimized as a hydraulic parameter. The findings of this study suggest a substantial enhancement in the rate of penetration due to hydraulic optimization, underscoring the significance of flow rate in achieving optimal drilling outcomes. This research highlights the necessity of a well-planned and executed hydraulic strategy to maximize drilling efficiency and minimize operational challenges. By understanding and controlling the interplay between hydraulic parameters and drilling performance, operators can achieve significant improvements in drilling speed and overall operational effectiveness.

Introduction

The drilling operation represents a significant financial investment in the process of hydrocarbon extraction from beneath the surface. Therefore, enhancing drilling efficiency by optimizing parameters such as weight on bit is crucial, as it affects the rate of penetration. However, it is important to note that increasing the weight on bit does not always correlate with improved penetration rates. In softer formations, additional weight can lead to higher torque, while in harder formations, it may adversely affect the bit life (Al-Mahasneh 2017). Increasing weight on bit (WOB) sometimes resulted in the reduction of rate of penetration because of the bit foundering effect and that means increasing WOB breaks the rock cuttings and enhances rate of penetration, rate of penetration (ROP) up to the founder point and WOB causes an excess damage to the bit and lowers the ROP (Islam and Hossain 2021). Whereas hydraulic optimization mainly discussed the optimizing of pump pressure while maintaining high enough flow rate to carry the rock cuttings to the surface (Mustafa et al. 2021; Song et al. 2022). One way to optimize the rig hydraulic is to improve the bit design, which means optimizing the nozzle diameter while minimizing the frictional pressure drop to obtain an optimum jet impact force (JIF) (Ramsey 2019). In addition, it was suggested by different scholars (Longwa Milia 2008; Gulraiz and Gray 2020; Yavari 2023) that the optimum pressure drop across a polycrystalline diamond compact (PDC) bit should be 46% of the pump pressure, therefore higher ROP is achieved when the pressure drop is higher than 50% of the total pump pressure, and hydraulic horsepower is higher than 65% of the standpipe pressure, another property should be considered is

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the effect of thixotropy on the drilling hydraulics, where the changes in shear rate results in the fluctuations in the annular pressure losses. Additionally, geo-mechanical properties of the formation play a significant role such as torque and drag of the drill string, drilling vibrations, hole cleaning, to achieve more accurate ROP from the estimation models (Li 2023). A comprehensive understanding of the performance and wear characteristics associated with several types of drilling bits is crucial for enhancing drilling efficiency and minimizing operational costs. PDC bits are subjected to both axial and torsional impacts during the drilling process (Huang and Li 2018). Research indicated that as the duration of impact increases, the average torque tends to decrease, and higher impact speeds further enhance rock-breaking efficiency. Additionally, roller cone bits are susceptible to damage and wear, especially at the shoulder of the rubber ring and the sealing face of the metal ring, primarily due to elevated shear stress encountered during drilling activities. Consequently, the wear of roller cone bits has a direct effect on the overall efficiency of the drilling operation (Darwesh et al. 2020.). The rate of penetration is influenced by multiple factors, including WOB, rotary speed, depth, formation strength, and hydraulic conditions, leading to the development of various mathematical models aimed at predicting the rate of penetration (Tanko 2020). Among these, the Burgoyne and Young model stands out as one of the most comprehensive for forecasting the rate of penetration for roller cone bits (Li et al. 2023; Huang and Li 2018).

Darwesh et al. (2020) have suggested normalization factor values of 11200, 9, 4, and 1000 for the true vertical depth (TVD), equivalent pore pressure, weight on bit per inch of bit diameter, and jet impact force, respectively. Additionally, the concept of mechanical specific energy (MSE), initially was introduced (Tanko 2020) and has gained significant traction in enhancing drilling efficiency and has proven effective in real-time estimation of optimal WOB while varying revolutions per minute (RPM) to penetrate specific formation intervals using positive displacement motors (PDM). This paper aims to enhance drilling performance by optimizing hydraulic parameters, including flow rate, jet impact force, and equivalent circulating density, through a mathematical modeling approach.

Methodology

Drilling data was extracted from wells drilled in South-West Queensland Australia. Different wells were drilled with roller cone bit and PDC bit. Key parameters are as follows: ROP, drill string design, TVD, WOB, rotary speed, bit record, flow rate (Q), bit diameter (d_b) and mud weight (Mw) are recorded during drilling operations. **Table 1** presents the drilling data extracted from wells drilled in South-West Queensland Australia. This paper analyzed the data to improve the drilling efficiency by optimizing the hydraulic system while considering the effect of drilling parameters on ROP, including pore pressure gradient (pg), equivalent circulating density (ECD), bit wear, and jet impact force (Fj).

Table 1—Drilling data from South-West, Queensland, Australia.

ROP, ft/hr	TVD, ft	WOB, klb	Rotary, RPM	Q, gpm	db, in	ρg, ppg	ECD, ppg	Bit Tooth wear.	JIF, lbf
71.8	718.5	12.9	115	358	10.625	9.1	9.1	0.25	205.83
166.7	4416.0	6.3	132	327	7.875	9.1	9.3	0.125	365.96
147.6	720.1	13.5	118	367	10.625	9.1	9.1	0.125	273.12
195.2	4586.6	9.5	130	389	7.875	9.0	9.2	0.25	512.19
350.4	2103.0	6.0	120	370	12.25	9.0	9.0	0.125	347.73
537.7	18986.0	6.6	134	397	8.50	9.0	9.2	0.125	355.65
206.7	2519.7	7.9	132	499	10.625	9.0	9.1	0.25	619.84
74.1	7637.7	5.7	84	368	7.875	9.0	9.2	1	337.11

ROP Prediction. The Burgoyne and Young model were employed in this study to forecast and enhance the hydraulic parameters on ROP. The coefficients a_j and x_j were determined through the application of multiple regression analysis. Subsequently, the data was utilized to calculate the parameters xj.

$$\ln(ROP) = a_1 + a_2x_2 + a_3x_3 + a_4x_4 + a_5x_5 + a_6x_6 + a_7x_7 + a_8x_8 \text{ ,} \tag{1}$$

or

$$\mathrm{x}_5 = \ln\left(\frac{\mathrm{C_r}\frac{1000\mathrm{WOB}}{\mathrm{d_b}} - 0.02\Delta\mathrm{P_b}\left(\frac{\mathrm{d_b}-1}{\mathrm{d_b}}\right)}{\left(\frac{1000\mathrm{WOB}}{\mathrm{d_b}}\right)_{\mathrm{c}}}\right) \; for\ PDC\ bits, \tag{2}$$

whereby TVD is the true vertical depth, ft; p_g is the pore pressure gradient, ppg; and p_c is equivalent circulating density, ppg; C_r represents the dimensionless weight distribution between the 12¼″ drill bit and 13½″ under-reamer. In the absence of the under-reamer, C_r is equal to 1. Initially, the C_r was 0.942, but it was later decreased to 0.02 due to being excessively high (Tanko 2020). ΔPb is a bit pressure drop, psi; db denotes the bit diameter; WOB indicates the weight, klb; N represents the rotary speed.

$$\mathrm{x}_6 = \ln\left(\frac{\mathrm{N}}{100}\right) \text{ for roller cone bits,} \tag{3}$$

or

$$\mathrm{x}_6 = \ln\left(\frac{\mathrm{N}}{160}\right) \text{ for PDC bits,} \tag{4}$$

Then, parameters for bit wear and jet impact force are:

$$\mathrm{x}_7 = -\mathrm{h}, \tag{5}$$

$$\mathrm{x}_8 = \ln\left(\frac{\mathrm{F_j}}{1000}\right), \tag{6}$$

where N represents the rotational speed in revolutions per minute, and F_j denotes the jet impact force in pounds-force.

Hydraulics and ROP Optimization. The hydraulic parameters in the Burgoyne and Young model, **Eqs. 7** and **8** include the equivalent circulating density (p_c) and the jet impact force (F_j),

$$\mathrm{F_j} = \frac{\rho \mathrm{Q} \mathrm{v_n}}{1930} = \frac{\rho \mathrm{Q}^2}{6016\mathrm{A_n}}, \tag{7}$$

$$\rho_\mathrm{c} = \rho + \frac{\mathrm{P_a}}{0.052 \times \mathrm{TVD}}, \tag{8}$$

The calculation of annular pressure, P_a, involves the utilization of the Darcy-Weisbach Equation, considering the Reynolds' number and Chen's correlation friction factor for the flow within the annulus. It is crucial to ensure that the carrying capacity of the cutting surpasses the slip-velocity, while optimizing the flow rate.

$$v_{slip} = 1.54\sqrt{\frac{d_c(p_c - M_w)}{M_w}}, \tag{9}$$

$$v_{min} = 2.5 \times v_{slip}, \tag{10}$$

$$Q_{min} = \frac{v_{min}}{2.448(d_2^2 - d_1^2)} \tag{11}$$

The slip velocity (vslip), minimum annular velocity (vmin) and minimum flow rate (Qmin) are determined based on **Eqs. 9** to **11**, accordingly. It is essential for the flow rate in the annulus to exceed the minimum flow rate to effectively transport the cuttings to the surface. The ROP is calculated using the Burgoyne and Young model, and the flow rate is optimized through the Qoptimum model. Subsequently, the optimized total nozzle area (An, min) is established based on the jet impact force. Finally, the flow rate is cross-checked with the minimum flow rate to ensure proper hole cleaning.

Results and Discussion

The data related to Pearson's correlation analysis is presented in **Figure 1**. By examining the correlation matrix, relationships among various drilling parameters were identified. The ROP shows a significant correlation with TVD, RPM, WOB, and bit wear (h). While the Burgoyne and Young model initially assumed these parameters to be independent (Li et al. 2023), it is important to note that TVD, RPM, WOB, bit diameter (db), mud weight (Mw), pore pressure gradient (pg), and flow rate (Q) are all considered independent variables. The jet impact force is influenced by flow rate, mud weight, and pressure drop. Given the low to moderate correlation among the model parameters, it is reasonable to conclude that these parameters are independent of one another.

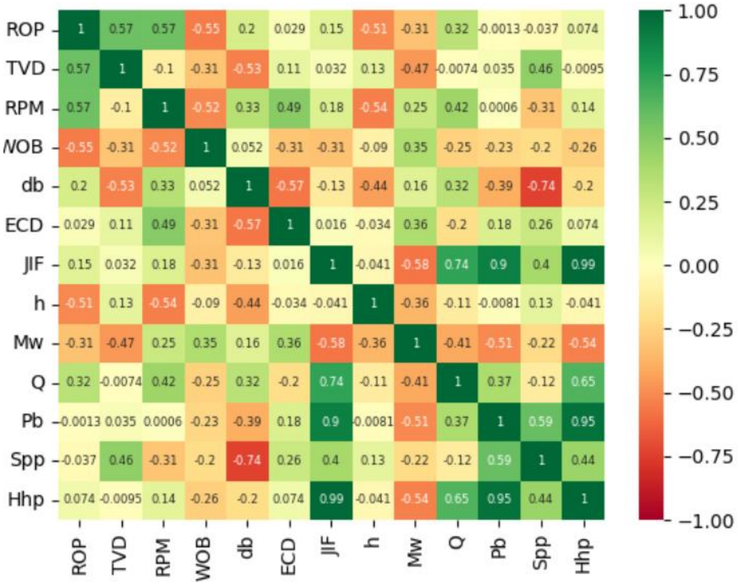

Figure 1—Pearson correlation matrix heatmap for the drilling data.

As shown in **Figure 2**, the ROP generally decreases with increasing depth due to the increasing formation strength. However, one data point deviates from this trend, likely due to other factors that have a greater influence on ROP, as previously discussed Furthermore, there is a clear relationship between ROP and revolutions per minute, with higher rotational speeds leading to faster drilling rates as illustrated in **Figure 3** illustrate that ROP initially increases with an increase in WOB, peaking at approximately seven kilo pounds. However, beyond this point, ROP decreases due to the founder effect. This effect occurs when the bit encounters deeper layers with reduced cutting efficiency, likely due to damage, as supported by reference (Islam and Hossain 2021).

The flow rate follows a similar trend, rising with ROP before declining. In terms of hydraulic parameters, both jet impact force and hydraulic horsepower contribute to increased ROP. However, jet impact force begins to decline at around 450 pounds per foot, whereas hydraulic horsepower continues to rise alongside ROP.

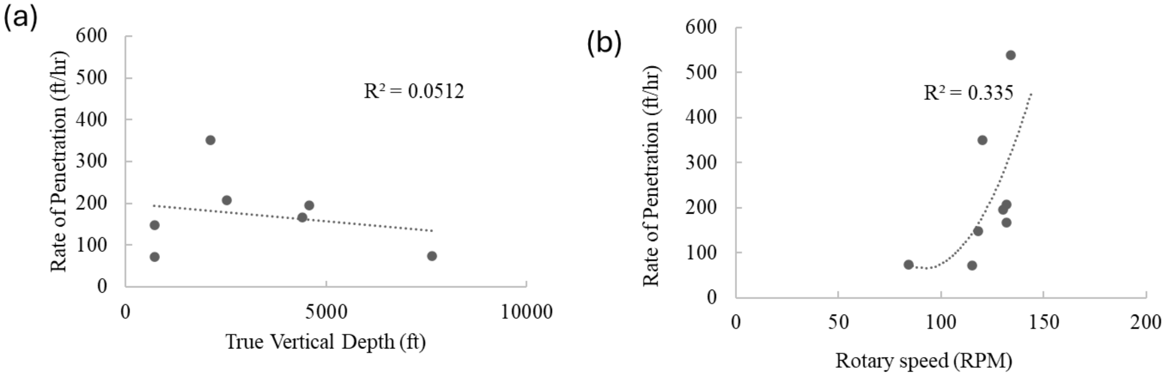

Figure 2—(a) Rate of penetration against true vertical depth; (b) Rate of penetration against rotary speed.

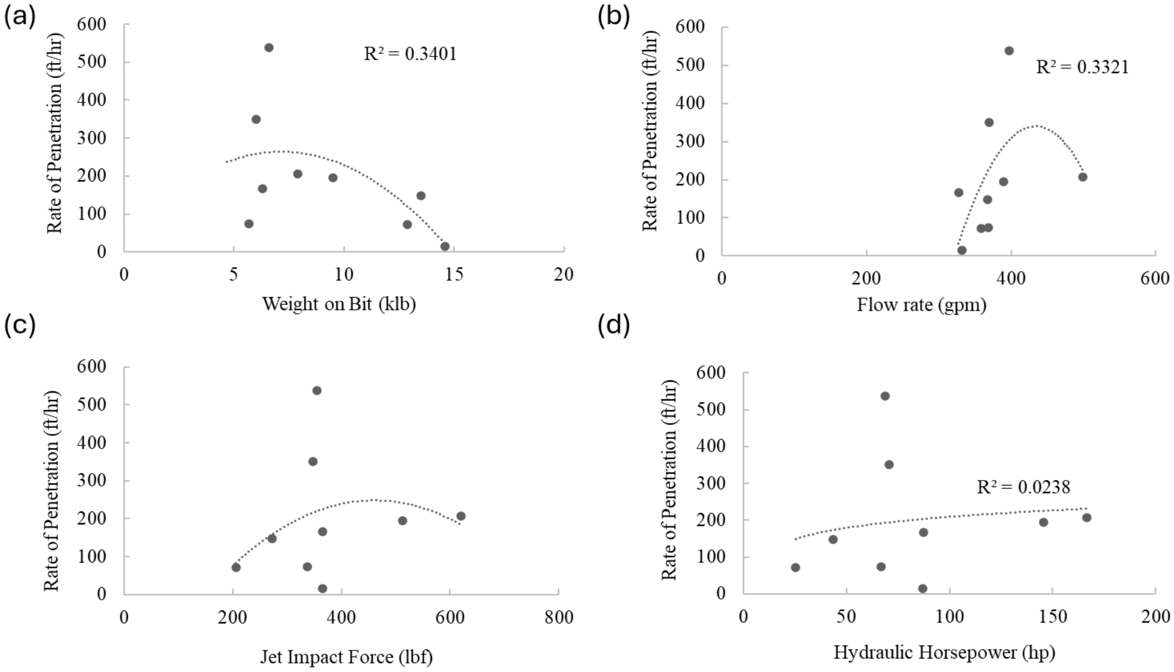

Figure 3—(a) Rate of penetration against weight on bit; (b) Rate of penetration against flow rate; (c) Rate of penetration against jet impact force; (d) Rate of penetration against hydraulic horsepower.

The calculations for optimal nozzle area, ECD, and jet impact force were performed using the equations outlined in the methodology section. The results are presented in **Table 2**. Additionally, minimum flow rates were analyzed to ensure that the mud flow rates are sufficient for transporting rock cuttings to the surface.

Table 2—Optimized hydraulic parameters.

Optimum Bit Pressure Drop	Optimum Flow Rate	Optimum Nozzle Area	Optimum ECD	Optimum Jet Impact Force	Minimum Flow Rate
2400	1259	0.744	9.35	3223.82	189
2400	1214	0.717	9.35	3106.32	189
2400	1010	0.593	9.09	2569.92	264
2400	573	0.337	9.07	1458.40	191

The anticipated penetration rate was calculated through multiple regression analysis, with the evaluation of the Burgoyne and Young model coefficients presented in **Table 3**. The findings revealed that RPM and bit wear exerted the greatest influence on ROP, followed by hydraulics and weight on bit. Furthermore, the results indicated a weak formation strength based on the coefficient range of a1, while coefficients a6, a7, and a8 highlighted the importance of focusing on rotary speed, bit wear, and jet impact force. At higher depth, normal compaction, under-compaction, and differential pressure are worth optimizing.

Table 3—Coefficient of Burgoyne & Young model.

Factors	Coefficients	Values
Formation Strength	a1	24.34
Normal Compaction	a2	-0.0013
Under-compaction	a3	-0.0620
Differential Pressure	a4	0.0085
Weight on Bit	a5	0.7686
Rotary Speed	a6	-9.67
Bit Tooth Wear	a7	3.91
Jet Impact Force	a8	1.53

The impact of flow rate on ROP is illustrated in **Figure 4**. As the flow rate rises, the parameters x4 and x8, representing the equivalent circulating density and jet impact force, are impacted. Specifically, an increase in jet impact leads to a higher ROP, whereas a rise in equivalent circulating density results in a lower rate of penetration. Despite this, ROP experiences an increase with flow rate due to the heightened jet impact force. However, it decreases because of the differential pressure between equivalent circulating density and the high pore pressure gradient.

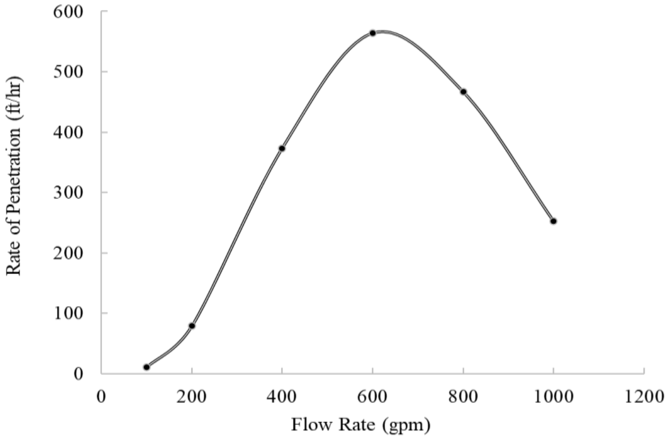

Figure 4—Rate of penetration versus flow rate.

Figure 5 depicts the correlation between the ROP and the jet impact force. Elevating the jet impact force results in an increase in the ROP. Nevertheless, boosting the jet impact force necessitates a rise in the flow rate, which could potentially lead to a decrease in the ROP once it reaches the optimal level. Therefore, it is crucial to optimize the jet impact force by reducing the total nozzle area to the maximum bit pressure drop. Furthermore, the enhancement in the ROP is attributed to the reduction in the total nozzle area and becomes more pronounced at higher jet impact forces.

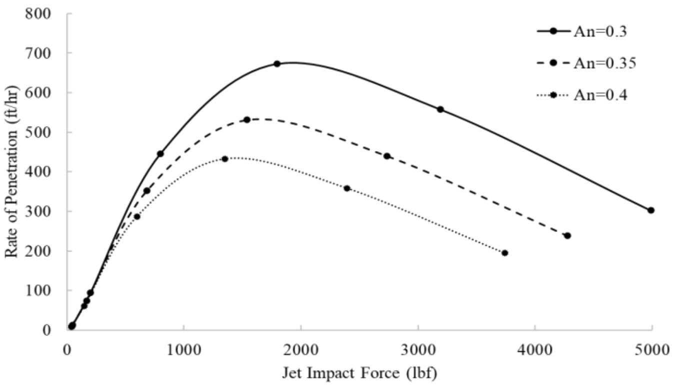

Figure 5—Rate of penetration against jet impact force at different total nozzle areas (An).

The ROP refers to how quickly the drill bit advances through the formation, while ECD represents the effective mud density inside the wellbore, accounting for both the static mud weight and the dynamic effects of fluid circulation. Therefore, **Figure 6** presents the relationship between rate of penetration and equivalent circulating density, where ROP increases with ECD, is primarily due to optimized differential pressure, improved jet impact force, and more efficient cuttings removal. However, controlling ECD within optimal limits is crucial to maintaining drilling efficiency without causing wellbore stability issues.

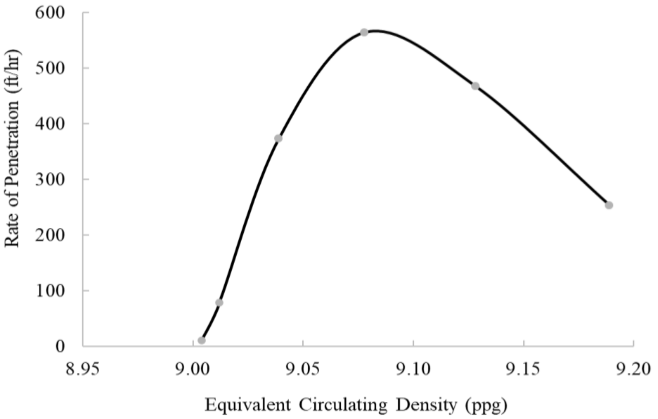

Figure 6—Rate of penetration against equivalent circulating density.

Table 4 demonstrates that the optimized Jet impact force has a considerable influence on the ROP. On average, there is an increase of ten times the original ROP and a minimum of 1.65 times the original ROP. However, the actual ROP value could be lower than expected due to other factors that may affect the ROP when increasing the jet impact force or the accuracy of the model.

Table 4—The actual ROP and optimized ROP.

Actual ROP	Optimized ROP	Percentage Increased (%)
71.85	1236.04	1620%
147.64	1709.97	1058%
350.39	2301.25	557%
206.69	548.30	165%

Conclusions

The research has demonstrated the efficacy of hydraulic optimization in enhancing drilling efficiency by concentrating on the improvement of the rate of penetration through the optimization of jet impact force and equivalent circulating density. Additionally, the study noted an increase in rate of penetration when applying the Burgoyne and Young model. The findings suggest that factors, including depth, weight on bit, revolutions per minute, flow rate, equivalent circulating density, and jet impact force, require optimization. The effect of flow rate on both equivalent circulating density and jet impact force is vital for boosting drilling efficiency. On average, rate of penetration experienced a tenfold increase compared to the baseline, with a minimum enhancement of 1.65 times the original rate of penetration. In summary, this paper highlights the critical role of flow rate in influencing rate of penetration, emphasizing the necessity for further investigation and optimization of flow rate management strategies to attain optimal drilling performance.

Nomenclature

ANN =Artificial Neural Network
ECD =Equivalent Circulating Density
GA =Genetic Algorithm
GPM =Gallon per Minute
Hhp =Hydraulic Horsepower
JIF =Jet Impact Force
MLR =Multiple Linear Regression
MSE =Mechanical Specific Energy
PDC =Polycrystalline Diamond Compact
RF =Random Forest
ROP =Rate of Penetration
RPM =Rotary per Minute
SPP =Standpipe Pressure
TVD =True Vertical Depth
WOB =Weight on Bit
A_b =Bit Area
a_j =model coefficients
A_n =Nozzle Area
$A_{n\ min}$ =Minimum Nozzle Area
Cd =Dimensionless Nozzle Discharge Coefficient
Cr =Dimensionless Weight Split
d_1 =Drill Pipe Outer Diameter
d_2 =Hole Inner Diameter
d_b =Bit Diameter
d_c =Cutting Diameter
d_e =Equivalent diameter
ϵ =Relative Roughness
f =Friction factor
F_j =Jet Impact Force
h =Fractional Bit Tooth Wear
H_{hp} =Hydraulic Horsepower
m =Flow Rate Exponent

M_w =Mud Density
N =Rotary per Minute
ρ =Mud Density
P_a =Annulus Pressure
P_b =Bit Pressure Drop
ΔP_b =Bit Pressure Drop
ρ_c =Cutting Density
ρ_g =Pore pressure gradient
P_{LOT} =Leak-off Test Pressure
P_p =Pump Pressure
Q =Flow Rate
Q_{min} =Minimum Flow Rate
$Q_{optimum}$ =Optimum Flow Rate
T =Bit Torque
v_a =Annular velocity
v_{min} =Minimum Velocity
v_n =Nozzle Velocity
v_{slip} =Slip Velocity
$(WOB/d)t$ =Threshold Weight on Bit per inch bit diameter
x_j =Model parameters

Conflict of Interest

The authors declare that there is no conflict of interest.

References

Al-Mahasneh, M. A. 2017. Optimization of Weight on Bit During Drilling Operation Based on Rate of Penetration. *Int. J. Oil, Gas Coal Eng.* **5**(2): 13-27.

Bani Mustafa, A., Abbas, A. K., Alsaba, M., et al. 2021. Improving Drilling Performance Through Optimizing Controllable Drilling Parameters. *J. Pet. Explor. Prod. Technol.* **11**(3): 1223-1232.

Darwesh, A. K., Rasmussen, T. M., and Al-Ansari, N. 2020. Controllable Drilling Parameter Optimization for Roller Cone and Polycrystalline Diamond Bits. *J. Pet. Explor. Prod. Technol.* **10**(4): 1657-1674.

Gulraiz, S. and Gray, K. E. 2020. Thixotropy Effects on Drilling Hydraulics. *J. Nat. Gas Sci. Eng.* **84**(1): 103653.

Huang, Z. and Li, G. 2018. Optimization of Cone Bit Bearing Seal Based on Failure Analysis. *Adv. Mech. Eng.* **10**(3):168-179.

Islam, M. R. and Hossain, M. E. 2021. *Drilling Engineering*. Amsterdam, Netherlands: Elsevier.

Li, L. 2023.Intelligent Optimization for a Full-Sized PDC Bit with Composite Percussive Rock Breaking Drilling. *J. Intell. Constr.* **1**(4): 9180021.

Li, L. L., Liu, X., Zhou, Y., et al. 2023. Intelligent Optimization for a Full-sized PDC Bit with Composite Percussive Rock Breaking Drilling. *J. Intell. Constr.* **1**(4): 9180021.

Longwa Milia, R. L. 2008. Optimization of Drilling Hydraulics in Vertical Holes. PhD dissertation, Bandar Seri IskandarUniversiti Teknologi PETRONAS.

Ramsey, M. S. 2019. *Practical Wellbore Hydraulics and Hole Cleaning.* Amsterdam, Netherlands: Elsevier.

Song, X., Li, G., Xu, Z., et al. 2022. *Fundamentals of Horizontal Wellbore Cleanout*. Amsterdam, Netherlands: Elsevier.

Tanko, A. 2020. Rate of Penetration Optimization Using Burgoyne and Young Model (a Case Study of Niger Delta Formation). *Int. J. Petrochemical Sci. Eng.* **5**(2):1-12.

Yavari, H. 2023. An Approach for Optimization of Controllable Drilling Parameters for Motorized Bottom Hole Assembly in a Specific Formation. *Results Eng.* **20**(1): 101548.

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