FRACTALS

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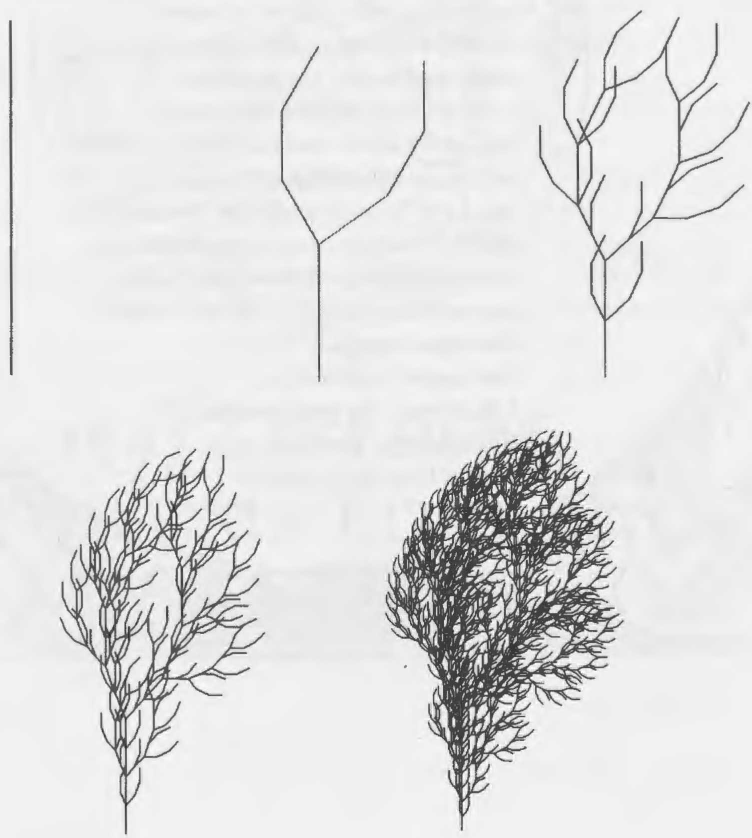
Fractals are everywhere—in the flowing of a stream; in the intertwining branches of a tree; in the rise and fall of voices in conversation. The language of fractal geometry describes the underlying patterns that we intuitively sense in each of these. Fractals are complex designs which are created by repeating a simple mathematical rule many times. They allow us to examine the subtle structures of our world and show us how the repetition of simple patterns results in the infinite variety of objects that we see around us. Fractals are exciting because they offer us a better visual description of the way natural objects change and grow than do simple shapes like circles and squares. For this reason, scientists often use them as models to describe natural processes.

The best way to understand fractals is to see how one develops.



Although the formal study of fractals began in the late 1970s, philosophers and artists have been using the ideas behind fractal geometry for centuries. Since the time of the ancient Greeks, philosophers have suggested that we can train our minds to think abstractly by learning to see increasingly subtle patterns in the world. The study of fractal geometry teaches us to see the subtlest of patterns in the chaos of our experience.

The human mind has an innate tendency to search for patterns. Many artists use this tendency in their visual portrayal of structure. Natural objects are so complex that it is not possible to communicate every detail in a picture. A cluttered image results from the attempt to draw everything we see. Artists often solve this problem by outlining the form of an object and allowing the viewer's mind to infer the rest of the structure. Ansel Adams discussed such a technique in his Basic Photo Series Book, *Natural Light Photography*:

> Consider photographing rocky landscapes, another instance of textural rendition. Beyond a certain distance, a great field of granite boulders will appear as perfectly smooth stones, the natural textures being beyond the resolving power of the lens and/or the emulsion. In order to suggest the substance of these stones it is necessary to include in the very near foreground a boulder in which the texture is adequately revealed. You can then say that the photograph 'reads well.' While you cannot see the texture in the distant boulders, you can see it in the near boulder, and you assume that all the boulders are the same material.

Adams reminds us that in art, as in everyday life, we need structure AND detail to form a coherent and meaningful view of the world.

Our minds crave complexity. If we lived in a completely ordered environment, everything would be identical and our senses would not be useful. Everywhere we turned, we would see and hear exactly the same thing. After a few minutes of uniformity, our minds would cease to respond to sensory messages. Rigid order satisfies us aesthetically only when our senses are overloaded and need a break.

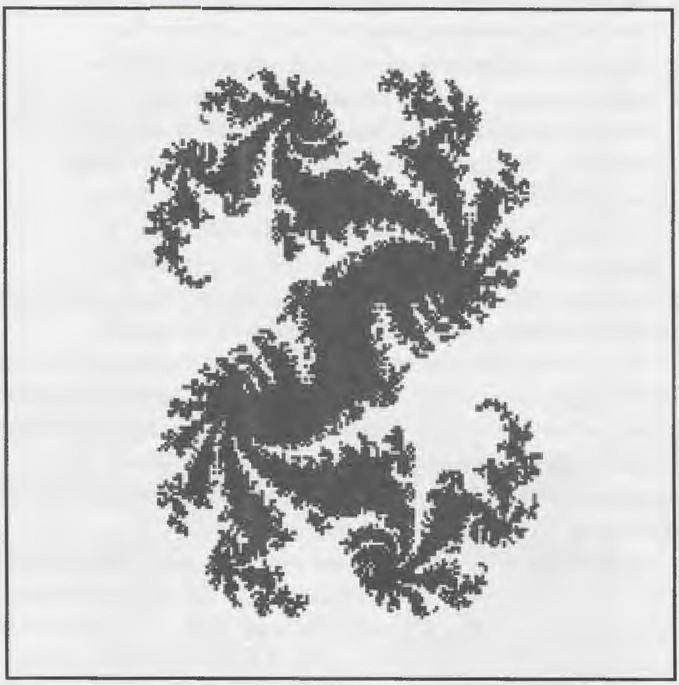
Complete randomness does not meet our aesthetic needs, either. If we flip a coin a large number of times, we will not find any pattern in the resulting string of heads and tails. Because there is no structure in random occurrences, they are not complex. Our minds are not able to make any more sense out of completely random experiences than completely ordered ones.

Great music is neither monotone nor noise. A great painting is neither a blank canvas nor completely random splotches of color. Artists incorporate the random and the ordered into their work, but it is the way in which they do this, the structure they impose, which is beautiful and meaningful. Those objects and processes which are most complex and beautiful involve both an underlying form and the potential for infinite variation. In other words, they are fractals.

To generate a fractal, we choose a simple "replacement rule" and then repeat it many times. Computers are usually used to create pictures of fractals because it is difficult to perform the rule enough times by hand to get a feel for the end result. The replacement rule might be geometric, such as the one used to create the bush that we saw earlier. In this fractal, each line segment is divided into five pieces, two of which remain aligned and three of which are duplicated and angled. Each of the new line segments is replaced using the same procedure.

Another type of replacement rule involves algebraic equations. Classic Julia sets, for example, are created by using the equation (New Z)=Z^2+C. Here is a picture of a Julia set.

The original Z in the equation is a point in the complex plane



and C is a complex number which stays constant. To generate this image, the computer starts with the point at the upper left corner of the monitor.

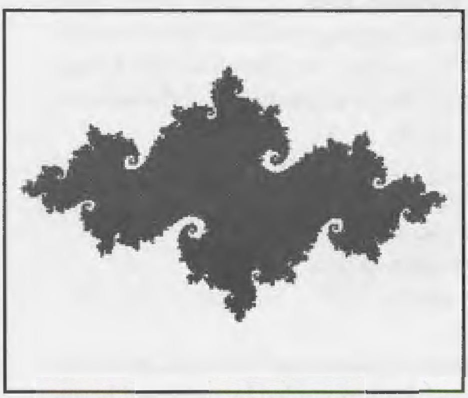
It takes the complex number Z corresponding to that point, multiplies Z by itself, and adds C to it. It then replaces the original Z with the result, squaring it and adding C to get the next iterate. The computer repeats this process until the iterates become "large," which means they are "escaping" to infinity, or until the operation has been performed a certain predetermined number of times. If the iterates do not escape to infinity, we say that the original point remained "bounded" under iteration, and the computer colors that point. Otherwise, it leaves the point uncolored. The computer then proceeds to the next point on the screen and repeats the entire process.

Sometimes, instead of leaving the background blank, the computer assigns color to a point to indicate how many iterations it took for the results to become "large." Those points which escape first form the background color for the fractal. Those points which escape after one iteration form the outside color band. Each successive iteration produces a new, smaller escape band. Those points which never escape are colored black. The fractal on plate 31 was colored in this way. Because this image is a zoom from deep within the fractal, we cannot see the background or the outside color bands, but we can see which points escaped during each of the last few iterations.

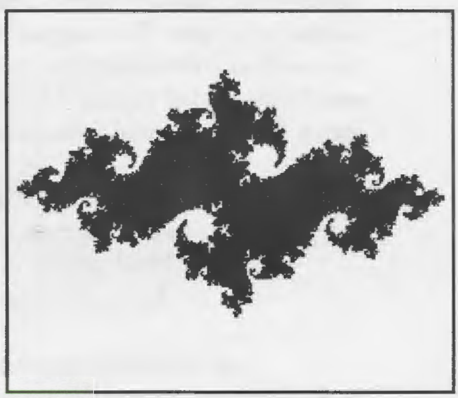
Using different values for C creates different pictures. The following images demonstrate how very small changes in C affect the resulting fractal.

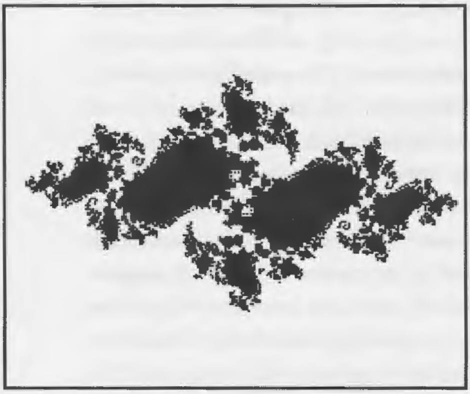
It is interesting to notice the similarities and differences between

C=-0.736 + 0.097i

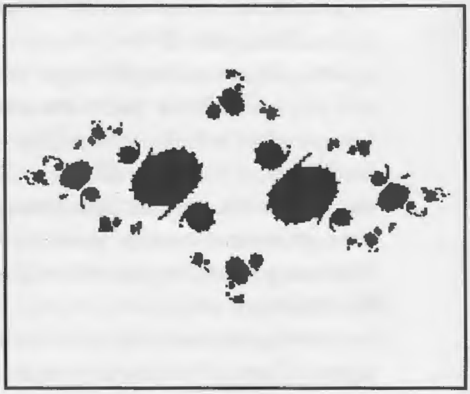


C=-0.743 + 0.097i





C=-0.744 + 0.097i



C=-0.756 + 0.097i

fractals created with different replacement rules. The first Julia set is more filled in than the others. What causes this? Some of these fractals "blow away" as we increase the number of iterations. There are other values for C which do not produce an image at all. Which values for C produce images that will not blow away? We notice that these four images are all symmetric around the center point. Is that true of all Julia sets? Both the Julia sets and the bush fractal have features which are repeated on different scales throughout the image. Do all fractals have this property? These are the types of questions that fractal geometry explores.

The study of mathematics has long been used to develop our intuitive ability to recognize patterns and structure. Math students first learn to recognize simple patterns, and gradually learn to think about more abstract structures. Plato argued that this training would ultimately enable the mind to contemplate the subtlest concepts: the Beautiful, the Good, and the other ideal Forms. Fractal geometry is an extension of this ancient search for the essential forms in our world.

> To see a world in a grain of sand
> And a heaven in a wild flower
> Hold infinity in the palm of your hand
> And eternity in an hour

Blake described what it means to understand the fractal structure in the world around us. We are everywhere surrounded by profound beauty. By learning to see subtle patterns, we develop a deeper understanding and appreciation of the wonders around us.

FOR MORE INFORMATION

An Eye For Fractals, Michael McGuire, Addison-Wesley Publishing Company, Redwood City, California, 1991. This beautiful book is an excellent introduction to fractal geometry and its use in art.

Nick's Fractal Page, http://www.bush.edu/~nick/nick.html, gives more detail on how Mandelbrot and Julia sets are generated. Nick offers copies of the freeware program *Fractint*, which is one of the most user-friendly fractal programs around. This page also has links to fractal galleries and other related sites.