A PROCEDURE FOR DETERMINING IF A SIGNIFICANT DIFFERENCE EXISTS BETWEEN TWO PERCENTAGES CALCULATED FROM INDEPENDENT SAMPLES

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Though noteworthy advances have been made in statistical procedures and in the utilization of data processing equipment, educators often encounter problems or delays when they are involved in a situation requiring the determination of whether two statistics differ significantly. Such situations occur because techniques in the field of statistics have been developed to the extent that alternative statistical tests which might be used to come to a decision about such a hypothesis are available for almost all research designs. Educators, either in their roles as teachers, students, administrators, researchers, or evaluators, must often make a choice between two or more possible methods of determining if a significant difference exists between two values, for example, two different percentages.

While many statistical procedures are available for making these determinations, the calculations involved often consume excessive amounts of time or require complex equipment which is not readily available. One possible solution to some of these problems was thought to be a table of statistically significant differences in observed percentages which was prepared by Cuthbert Daniel.[1] By using this table, Table 1, all calculations except the percentage of individuals in the groups who have some stated property could be eliminated. Once the percentages have been calculated, the proportion of individuals in one group having some stated property can be said to be greater than, less than, or equal to the proportion having this same property in another group. By using the figures in Table 1, one can determine if any existing difference is significant at the 0.05 level.

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TABLE 1

BY HOW MUCH MUST A PERCENT (A) OBSERVED IN ONE SAMPLE DIFFER FROM THAT OBSERVED (B) IN ANOTHER SAMPLE OF THE SAME SIZE TO BE STATISTICALLY SIGNIFICANT? (0.05 LEVEL OF SIGNIFICANCE)*

Lower per cent (B)	Size of Each Sample										
	20	25	30	35	40	45	50	60	70	80	90
10						15.8	14.7	13.3	12.2	11.2	10.5
20		26.0	23.6	21.7	20.1	18.8	17.8	16.1	14.8	13.8	13.0
30	30.9	27.4	25.0	23.1	21.5	20.2	19.2	17.4	16.0	15.0	14.1
40	30.8	27.6	25.3	23.4	21.9	20.6	19.6	17.9	16.6	15.5	14.6
50	29.6	26.7	24.5	22.8	21.4	20.2	19.2	17.6	16.3	15.3	14.5
60	27.3	24.8	22.8	21.3	20.1	19.0	18.1	16.7	15.5	14.6	13.8
70	23.8	21.7	20.2	18.9	17.8	17.0	16.2	15.0	13.9	13.1	12.4
80		17.5	16.4	15.4	14.6	13.9	13.3	12.4	11.6	10.9	10.4
90						9.3	9.0	8.4	7.9	7.5	7.2

	Size of Each Sample										
	100	120	140	160	180	200	250	300	400	500	1000
10	9.9	8.9	8.2	7.6	7.1	6.7	5.9	5.3	4.5	4.0	2.8
20	12.2	11.0	10.2	9.5	8.9	8.4	7.5	6.8	5.8	5.2	3.6
30	13.4	12.2	11.2	10.5	9.9	9.3	8.3	7.6	6.5	5.8	4.1
40	13.8	12.6	11.7	10.9	10.3	9.8	8.7	8.0	6.9	6.1	4.3
50	13.7	12.5	11.7	10.9	10.3	9.8	8.7	8.0	7.0	6.2	4.4
60	13.1	12.0	11.2	10.5	9.9	9.4	8.4	7.7	6.7	6.0	4.3
70	11.9	10.9	10.2	9.5	9.0	8.6	7.7	7.1	6.2	5.5	4.0
80	9.9	9.2	8.6	8.0	7.6	7.3	6.5	6.0	5.3	4.7	3.4
90	6.9	6.4	6.0	5.7	5.4	5.1	4.7	4.3	3.8	3.4	2.5

*Cuthbert Daniel, "Statistically Significant Differences in Observed Percents," The Journal of Applied Psychology, XXIV, No. 6 (December, 1940), p. 827.

Daniel reported that the values in this table could be used to determine the significance of differences between percentages from two independent samples or in a single sample. While the table values were reported as being applicable in both situations, plans were made to investigate the practical value of the table for determining whether a significant difference existed between two percentages from independent samples. The practical value of the table was to be assessed by comparing the results obtained through use of the table figures with those results obtained through the use of appropriate statistical procedures. It was anticipated that such comparisons would indicate that use of the table values would result in the same determination of the presence or lack of a significant difference as was obtained through the use of other statistical formulae and electronic data processing equipment.

Use of the Table

Table 1 gives the amount by which a percentage in one sample must exceed the percentage in a second sample to be significantly different at the 0.05 level of significance. Each time the table is used, Daniel states that the following conditions, requirements, or restrictions must be met.

1. A percent (A), having some property, must be found in one sample to be greater than a percent (b) which was found to have the same property in a second sample.

2. The two independent samples from which the percents were obtained must be roughly of the same size (within 40 percent in numerical size).

3. Both samples must be randomly selected. That is, every individual in the eligible population must stand an equal chance of being in the sample.

4. Interpolations, the use of values between entries in the table, are accurate throughout. Use of the table when the lower percent, (B), is below 10 percent or above 90 percent and when the sample size is below 20 or above 1000 is not permissible.

5. When percent (A) exceeds percent (B) by only the amount indicated in Table 1, the use of this table will result in an erroneous answer about one time in twenty comparisons.

If these standards are considered, use of the figures in Table 1 should give satisfactory results.

To use this table, one simply utilizes the procedure outlined below.

1. Select the lower of the two percentages, percent (B), from the left hand column of the table.

2. Move to the right along the row for percent (B) to the column which most closely represents the average size of the two samples involved. Interpolate to find the value if necessary.

3. Determine if the number in that cell or the interpolated value represents a larger difference than the absolute difference between the two sample percentages.

4. If it does, the two percentages do not differ significantly since the difference may be due to sampling errors. However, if it does not, the difference is statistically significant since it would occur no more than 5 percent of the time by chance.

By following this procedure, usable results should be obtained.

Comparison of Results

To check the results obtained through the use of Table 1 for determining if a significant difference exists between two percentages from independent samples, three different but generally recommended groups of formula were used to determine if the difference between two sample percentages was significant at the 0.05 level. The first of these formulae groups, formula group A.

$$n = n_1 + n_2 \tag{1}$$

$$p = \frac{n_1 p_1 + n_2 p_2}{n} \tag{2}$$

$$q = 1 - p \tag{3}$$

$$z = \frac{p_1 - p_2}{\sqrt{pqn/n_1 n_2}} \tag{4}$$

n - total number of sample units involved

n_1 - Size of sample 1

n_2 - size of sample 2

p - estimated proportion of population units possessing specified characteristic

p_1 - proportion of units in sample 1 possessing specified characteristic

p_2 - proportion of units in sample 2 possessing specified characteristic

q - estimated proportion of population units not possessing specified characteristic

z - z score

was used to compute a z score which could then be used to determine if the result was significant at the selected level of significance by referring to a table containing the probability of specific z-score values.[2] When 257 pairs of purposively selected percentages from samples meeting the previsously listed criteria were tested, the results, significant or not significant, obtained by using the table did not differ from those secured through use of the above formula group.

However, it was recognized that the above technique is generally considered to produce poor results when either n_1 or n_2 is less than 100 and either or both p_1 and p_2 are extreme, that is, less than 0.10 or greater than 0.90. In a situation of this type, formula group B should be used. To form this group, formulae (5) and (6) below should replace formula (4) in the previous sequence.

One should note that the use of formula (5)[3]

$$t = \frac{p_1 - p_2}{\sqrt{pq\ (1/n_1 + 1/n_2)}} \qquad (5)$$

t - t ratio

produces the same mathematical value as formula (4). The only difference lies in the interpretation of the results. When formula (5) is used, one must refer to a table which shows the distribution of t probability and involves the degrees of freedom.

$$df = (n_1 - 1) + (n_2 - 1) \qquad (6)$$

df - degrees of freedom

This technique can be used with large samples as well as small ones. Because of this, even though Table 1 cannot be used when p_1 or p_2 is less than 0.10 or when both are greater than 0.90, formula group B (formulae (1), (2), (3), (5), and (6) was used to determine whether a significant difference existed between the 257 pairs of purposively selected samples mentioned earlier. The results, significant or not significant, obtained by using Table 1 did not differ from those secured through use of this formula group.

While the results were very favorable, it was believed desirable to further determine if the table and formulae results would be congruent near the 0.05 level of significance. A specially prepared computer program was used to create 73 pairs of samples whose comparisons produced t scores in the range of 1.96 to about 2.30 with the degrees of freedom ranging from 40 to 120. When the results, significant or not significant, as determined by use of Table 1 were compared with those obtained by using formula group B and a t table, the results differed for six of the comparisons.

In order to apply statistical procedure to the results of this comparison, it was assumed that use of the values in Table 1 should result in no more than 5 percent of the comparisons producing results differing as to significance. Since 8.2 percent of the comparisons differed, it was decided to use the Poisson distribution to determine if this figure differed significantly from the expected population value.

$$H_O: p_d = 0.05 \qquad H_A: p_d > 0.05 \qquad \alpha = 0.01$$

The result of this latter statistical test indicated that the difference was not significant at the 0.01 level so the null hypothesis of no difference was not rejected. Thus, it is believed that use of Table 1 will not result in more than 5 percent of the determinations of significant or not significant differing from those that would be obtained through the use of formula group B.

For the third formula group, formula group C, which may be used to determine if a significant difference exists between two percentages from independent samples, formula (7)[5].

$$z = \frac{p_1 - l_2 \pm \frac{n}{2n_1 n_2}}{\sqrt{\frac{pq}{n_1} + \frac{pq}{n_2}}} \qquad (7)$$

replaces formula (4) in group A. It should be noted that this formula, formula (7), is equivalent to formulae (4) and (5) except that the term $n/2n_1n_2$, Yates's correction, is included as a correction for continuity. It may be used if neither n_1 or n_2 is very small. As a general rule, this formula can be used if n is greater than 40.

Formula group C, composed of formulae (1), (2), (3), and (7), was then used to determine whether a significant difference existed between 141 pairs of samples purposively selected from the 257 pairs mentioned earlier. The results, significant or not significant, obtained by using Table 1 differed from those obtained by using this formula group and a table of z-score probabilities for six of the comparisons.

The assumption was made that use of the values in Table 1 should result in no more than 5 percent of the comparisons producing results differing from those secured through the use of formula group C, but in reality only 4 percent of the actual results differed. Thus, it was unnecessary to test statistically whether the actual percentages of difference (4 percent) was greater than the assumed percent (5 percent). Because of this situation, the conclusion was made that use of Table 1 will probably not result in more than 5 percent of the determinations of significant or not significant differing from those that would be obtained through use of formula group C.

As a result of the comparisons described previously, it is evident that use of the values in Table 1 will produce results, significant or not significant, similar to those obtained through the three formulae groups described when the level of significance is 0.05 for a two-tailed test. Furthermore, it should be noted that the percentage of results differing increased as the comparisons moved from formula group A to formula group C. Thus, one would secure a higher degree of reliability if an earlier formula group would satisfy the educator's needs. But only the educator can decide which of these formulae groups will fit his data and meet his needs.

Examples

Though it has been shown that the results achieved through the use of the three procedures were similar, the question as how the information in Table 1 can be used must be considered. From an educator's point of view, it is well to consider certain instances in which such procedures are appropriate.

First, the figures in Table 1 may be used in determining if a significant difference exists between percentages from two independent samples in a single study. For example, a recent Ohio vocational agriculture study revealed the percentage of production agriculture students in the Southern and Western Regions of the State who reported they expected to enter a full-time agricultural occupation upon completing their education.[6] It was hypothesized that the percentages from the two areas would differ significantly.

$H_0: P_1 = P_2$ $\qquad$ $H_A: P_1 \neq P_2$ $\qquad$ $\alpha = 0.05$

P_1 - percentage of units in sample 1 possessing speified characteristic

P_2 - percentage of units in sample 2 possessing speified characteristic

Data collected during the conduct of the study produced the following results.

REGION	SOUTHERN	WESTERN
Sample Size (n)	297	310
Percent Planning to Enter an Agricultural Occupation	33.9	42.3

Referring to Table 1 in the row for 30 percent (B) and in column 300 $\frac{(297 + 310)}{2}$ for sample size, one finds the number 7.6. Since the difference between the two percentages, 8.4 percent, is greater than the 7.6 percent required for significance at 30 percent and greater than the 8.0 percent required at 40 percent, no interpolations are necessary. This means that a significant difference did exist. Thus, the null hypothesis of no difference is rejected and the alternate hypothesis is accepted.

The table values may also be used in a second way--to determine if a significant difference exists between results obtained through two separate studies involving independent samples. To demonstrate this use, let us consider separate studies which were conducted in two communities by the local teachers' organizations to determine the percentage of voters who favored an increase in property taxes to provide increased school funds. At the time these studies were announced, the county superintendent of schools decided to use the results to determine if a significant difference existed in voter support in the two communities. The superintendent hypothesized that there would be a difference in voter support.

H_0: $P_1 = P_2$ $\qquad$ H_A: $P_1 \neq P_2$ $\qquad$ $\alpha = 0.05$

When the results of these studies were compiled, the superintendent listed the following applicable information.

COMMUNITY	1	2
Sample Size (n)	50	60
Percent of Voters Supporting	25.0	43.0

Looking at the relevant percentages and sample sizes in Table 1, one finds the figure required for significance lies within the following limits.

Lower Percent (B)	Sample Size		
	50	[55]	60
20	17.8		16.1
[25]	[18.5]	[17.6]	[16.7]
30	19.2		17.4

[ ] - Interpolated Value

Since the observed difference in the percentages is 18.0 which is greater than the interpolated value of 17.6, the difference is statistically significant. Because of this, the null hypothesis of equal percentages is rejected and the alternate hypothesis accepted.

Conclusions

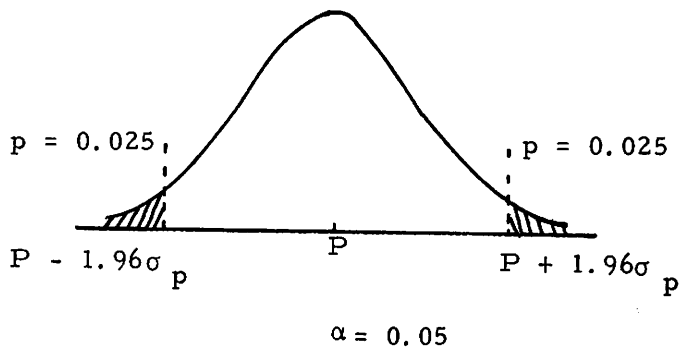
Results obtained in the comparisons described above indicated that use of Daniel's table will produce results which are consistent with those obtained through use of the formula groups under consideration when determining if a significant difference exists between two percentages from independent samples. Though the table may be used to determine if a significant difference exists between such percentages, one should consider certain conditions in interpreting the results. First, statistical significance is a statement about conditional probability. It does not guarantee that something important or meaningful has been found. All that a significant result implies is that one has observed something relatively unlikely given the hypothetical situation but relatively more likely given some alternative situation. The individual researcher must consider the circumstances contributing to the significance of the results and make a decision or a series of decisions in relation to the results. Merely finding a significant difference does not prove anything.

Second, virtually any study can be made to show significant results if one uses a large enough sample size, regardless of how nonsensical the content may be. This will occur even though large samples tend to produce more reliable statistics than those obtained from small samples. Refer to a good statistical reference on sampling if specific information is required for determining a suitable sample size.

Third, Daniel's table gives values for determining if a significant difference exists between two sample percentages at the 0.05 level of significance for a two-tailed test. The regions of rejection for such a test are shown in Figure 1. Consideration of this factor gives one the opportunity to express direction in the research hypothesis when this table is used. The table probability value ($\alpha = 0.05$) can be halved in such cases because a one-tailed test is called for while the table is designed for a two-tailed test.

FIGURE 1

REGIONS OF REJECTION FOR A TWO-TAILED TEST



For example, two samples, each composed of 100 individuals, were selected at random from the employees in two large corporations. As a part of a local school survey, these employees were asked whether sex education should be included as part of the school's curriculum. It was believed that a difference would exist between the percentages from the two groups who supported such education. In fact, since Corporation Number 1 had a more active public relations and community action program, it was hyopthesized that a greater percentage of its employees would favor such an educational program than would the employees from Corporation Number 2.

$H_0: P_1 = P_2 \qquad H_A: P_1 > P_2 \qquad \alpha = 0.025$

When the data were tabulated, the following results were obtained.

	Corporation 1	Corporation 2
Sample Size (n)	100	100
Percent Favoring	54.0	40.0

By using the value in Table 1, it was found that a difference of 13.8 percent was required for significance. Since the actual difference was 14.0 percent (54 - 40 = 14), the difference between the two percentages was significant at the 0.05 level. But since a direction was specified in the alternate hypothesis, the null hypothesis was rejected and the alternate accepted at the 0.025 level of significance. It was concluded that the percentage of employees favoring such an educational program from Corporation Number 1 was significantly greater than the percentage from Corporation Number 2.

If educators will follow the procedures outlined earlier and consider the three conditions discussed above when using Table 1, it is believed that accurate results will be obtained. However, the important factor in determining the value of any statistical test is the interpretation of the results by the individual. If the test result cannot be used in a worthy decision-making situation, application of that test is useless.

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