Principles for Assumptions Generation in Enthymeme-Based Dialogue

DIEGO S. ORBE LEIVA*, Institute for Computer Science and Engineering (CONICET-UNS), Department of Computer Science & Engineering, Universidad Nacional del Sur, Argentina
ALEJANDRO J. GARCÍA, Institute for Computer Science and Engineering (CONICET-UNS), Department of Computer Science & Engineering, Universidad Nacional del Sur, Argentina
SEBASTIAN GOTTIFREDI, Institute for Computer Science and Engineering (CONICET-UNS), Department of Computer Science & Engineering,Universidad Nacional del Sur, Argentina

In enthymeme-based dialogues, involved participants create assumptions in order to decode arguments from the exchanged enthymemes. This work introduces the concept of *assumptions operator*, which formalizes the mechanism for generating these assumptions, and proposes a set of principles to guide the construction of these operators. Said principles are inspired by Grice's Maxims of Conversation, as well as Govier's ARG conditions for cogent arguments. Then, in order to analyze how the used operator influences the dialogue and how that dialogue differs from the one in which the original argument is sent, we propose a framework to compare both scenarios, the former being the *enthymemic* one and the latter the *complete* one. Finally, we formally show that if the used assumptions operator complies with a set of the aforementioned principles, then most arguments in the *complete* dialogue have their counterpart in the *enthymemic* one. Furthermore, we show that under certain conditions, the *enthymemic* dialogue preserves some semantic properties from the *complete* one, specifically: *conflict-freeness, acceptability and admissibility.*

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1 Introduction

Argumentation-based dialogue representation, which formalizes dialogues between agents that exchange arguments, is a very active field of research [5, 4, 18]. Even though there are several works which explore argumentative dialogues and their applications [1, 7, 21, 2], most approaches involve a dialogue in which agents exchange information through logical arguments. These arguments typically contain all the necessary information to entail a claim. This is far from how dialogues are handled in real-world scenarios. Usually, humans provide arguments via enthymemes, which are described in [24]: "An argument is said to be an enthymeme if there are premises needed to make the argument valid that are only tacitly, but not explicitly stated or advanced as part of the argument."

*Corresponding Author.

Authors' Contact Information: Diego S. Orbe Leiva, ORCID: 0000-0001-6980-083X, diego.orbe@cs.uns.edu.ar, Institute for Computer Science and Engineering (CONICET-UNS), Department of Computer Science & Engineering, Universidad Nacional del Sur, Bahía Blanca, Buenos Aires, Argentina; Alejandro J. García, ORCID: 0000-0002-2680-9903, ajg@cs.uns.edu.ar, Institute for Computer Science and Engineering (CONICET-UNS), Department of Computer Science & Engineering, Universidad Nacional del Sur, Bahía Blanca, Buenos Aires, Argentina; Sebastian Gottifredi, ORCID: 0009-0009-8031-2576, sg@cs.uns.edu.ar, Institute for Computer Science and Engineering (CONICET-UNS), Department of Computer Science & Engineering,Universidad Nacional del Sur, Bahía Blanca, Buenos Aires, Argentina.

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Humans exchange enthymemes for several reasons: First, they could assume that the omitted premises are already known by the recipient of the message. Secondly, they could assume that those premises can be found in an external source (*e.g.,* a common-knowledge set). Finally, in certain types of dialogues, omitting premises could be used as a strategical move (*e.g.*, in persuasion dialogues, agents could prefer to omit a premise that the recipient of the message could use against them). Note, however, that dealing with enthymemic-based dialogues is not simple. Consider the following example.

Example 1.1 (Informal dialogue)**.** In this example, Matias is trying to persuade Federico to meet up at a bar after work.

Matias: *Hey Federico, Let's go to a bar!*
Federico: *I can't go today. I have an early meeting tomorrow and I can't drink.*
Matias: *We could go only to get something to eat.*
Federico: *Okay, see you there!*

Even in this brief dialogue, the nature of enthymemes is apparent. Before we illustrate some of the issues that arise when dealing with enthymemes, we provide a framework for structuring the exchanged messages. Each message is defined as a tuple $\langle \mathtt{claim}, \mathsf{R}, \mathsf{F} \rangle$, where R is a set of rules and F is a set of premises. Then, a message is an argument if the claim is entailed from R and F. Otherwise, that message is an enthymeme. Observe the second line of the dialogue, where Federico exchanges the following enthymeme: $\langle \neg\mathtt{go\_to\_bar}, \emptyset, \{\mathtt{cant\_drink}, \mathtt{early\_meeting}\} \rangle$. Note that the connection between the claim and its premises is not explicitly stated. By receiving this enthymeme, Matias could decode the following arguments:

(1) *I can't go today. I cannot drink given that I have an early meeting tomorrow, and I won't go to a bar if I cannot drink.*

$$\left\langle \neg\mathtt{go\_to\_bar}, \left\{ \begin{matrix} \mathtt{cant\_drink} \rightarrow \neg\mathtt{go\_to\_bar}, \\ \mathtt{early\_meeting} \rightarrow \mathtt{cant\_drink} \end{matrix} \right\}, \{\mathtt{early\_meeting}\} \right\rangle$$

(2) *I can't go to a bar today, given that I cannot drink and I have an early meeting tomorrow.*

$$\left\langle \neg\mathtt{go\_to\_bar}, \{\mathtt{cant\_drink}, \mathtt{early\_meeting} \rightarrow \neg\mathtt{go\_to\_bar}\}, \left\{ \begin{matrix} \mathtt{cant\_drink}, \\ \mathtt{early\_meeting} \end{matrix} \right\} \right\rangle$$

(3) *I can't go to a bar today because I have other plans tonight.*

$$\langle \neg\mathtt{go\_to\_bar}, \{\mathtt{have\_plans} \rightarrow \neg\mathtt{go\_to\_bar}\}, \{\mathtt{have\_plans}\} \rangle$$

Both arguments (1) and (2) use the premises provided by the enthymeme, but they differ in how the premises are connected to the enthymeme's claim. On the other hand, argument (3) discards the premises of the original enthymeme and provides new ones instead. Nevertheless, in every instance an argument is decoded (even though they may not be suitable) by creating *assumptions*.

The example above shows that, given an enthymeme, different assumptions could be created in order to decode an argument from it. Furthermore, the assumptions created will dictate if the resulting argument is suitable or not. A very appealing challenge for AI should be to design agents that can receive an enthymeme and then create the assumptions necessary to decode a suitable argument from them. Most works in the literature provide a fixed mechanism for assumption creation (*i.e.,* there is only one way in which said assumptions are created). For instance, some of these mechanisms obtain assumptions using external information, such as common-knowledge sets, argumentation schemes [20, 25], or the participants' mental model [6]. Notable exceptions include [27] and [16]. In the former, no assumptions are created from enthymemes. Instead, their dialogue system provides different kinds of messages to explicitly complete enthymemes. In the latter, the mechanism that creates assumptions does not use external information.

Our proposal differs from the works described above by defining the concept of assumptions operators, which provide an abstract framework capable of characterizing different assumptions generation methods. Throughout this paper, we will present concrete operators to illustrate how assumptions are created. Then, we propose a set of principles that serve as guidelines for the construction of said operators. These principles draw inspiration from Grice's Maxims of Conversation [12], as well as Govier's ARG conditions for cogent arguments [11]. Furthermore, we show that if a concrete operator complies with a subset of these principles, the result of its application will always yield a suitable argument. To the best of our knowledge, our proposal is the first to define principles as guidelines for assumption creation. Furthermore, we believe that our approach is a step forward to build systems for logical analysis of real-world arguments, and how the assumptions created can impact the quality of the resulting arguments.

To introduce the aforementioned principles, and explore how operators could adhere to them, we will propose a formalization of enthymemes using ASPIC+. We opted to use ASPIC+ because it is the most widely adopted structured argumentation formalism. In addition, we propose a simple dialogue system in which agents exchange enthymemes. This dialogue system is equipped with an assumptions operator, which modifies the exchanged enthymemes in order to decode a complete ASPIC+ argument from them. It is important to note that the dialogue system itself is not a contribution of this work, but rather a framework from which we analyze how the assumptions operator influences the dialogue. Nevertheless, the dialogue system is general enough to accommodate different types of dialogues, such as *persuasion, inquiry, information seeking and deliberation* dialogues.

Another challenge of dealing with enthymemic dialogues is the fact that, given an enthymeme, it is not always possible to obtain the original argument from which said enthymeme was created. Works like [6] explore how to obtain the most relevant argument from an enthymeme (*i.e.,* the argument that most resembles the original intended one from which the enthymeme is created), but this is not possible in every scenario. Given this situation, we instead explore how assumptions could yield a suitable argument in the context of the dialogue. In order to do so, we define a framework to contrast dialogues in which enthymemes are exchanged with the one in which agents send complete arguments. This framework allows us to show that most arguments from the latter are either part of the former, or have an enthymemic counterpart in the former. Finally, we show that if a concrete operator adheres to some of our proposed principles, the former dialogue retains some semantic properties from the latter. This is of utmost importance because it ensures that, under some conditions, the exchange of arguments or enthymemes can result in similar outcomes.

To summarize, our contributions in this work are the following:

(1) We propose a formalization of enthymemes which are built from ASPIC+ arguments. In comparison to other works that explore enthymeme representation using the ASPIC+ formalism [13, 27], we define enthymemes based on their contents (rules and premises), instead of their structure.
(2) We introduce the concept of assumptions operator, which serves as an abstract framework for defining different assumptions creation methods, such as [16, 20, 25, 6], among others. Furthermore, we define a dialogue system for enthymeme-based dialogues, which is equipped with an assumptions operator.
(3) We propose a set of principles as guidelines for the construction of suitable assumptions operators. We show that if an operator complies with these principles, its application will yield a *cogent argument* as defined in [11].
(4) We define a framework to contrast the dialogues in which enthymemes are sent with the one in which complete arguments are sent instead. We then show that most arguments from the latter are either part of the former, or have an enthymemic counterpart in the former.
(5) We show that under certain restrictions, some semantic properties are retained by the arguments in the latter dialogue that have their counterpart in the former one. More specifically: *conflict-freeness*, *acceptability* and *admissibility*.

The rest of this paper is structured as follows: In Section 2, we explore some preliminary concepts, including the ASPIC+ argumentation framework, as well as Grice's Maxims of Conversation and Govier's ARG Conditions.

Section 3 then presents the representation of enthymemes using ASPIC+. Then, in Section 4 the concept of *assumptions operator* is introduced and a dialogue system in which agents exchange enthymemes and arguments is presented. Subsequently, in Section 5, a set of principles is proposed based on Grice's Maxims [12] and Govier's ARG conditions [11], to which the aforementioned operators should adhere. In Section 6, we explore some desirable properties that arise when the equipped operator complies with some of the presented principles. Finally, in Section 7, related work is discussed, and in Section 8, the conclusions of this work are given.

2 Preliminaries

In this work, we introduce the concept of *assumptions operator*, which is an abstract framework to formalize the process by which an enthymeme is modified in order to decode a complete argument from it. Since our aim is to define principles that serve as guidelines for the construction of suitable operators, we define these principles based on criteria discussed in philosophy, argumentation, and informal logic studies. More specifically, our principles are inspired by Grice's Maxims of Conversation [12] and Govier's *ARG conditions* of cogent arguments [11], which will be briefly introduced next. Then, in the following subsection, we will provide a summary of the ASPIC+ argumentation formalism [19], which we will use to provide a representation of arguments and enthymemes.

2.1 Grice's Maxims and Govier's ARG Conditions

In [12], conversations are described as cooperative interactions to some extent. That is, even if the participants are part of some kind of *competitive* dialogue (*e.g.,* persuasion, negotiation), it is assumed that they pursue a common goal of exchanging information, and as such, they will comply with a set of implicit rules during said interactions. Given this assumption, Grice defines a general principle, denoted as *cooperative principle* that the participants of a dialogue should comply with, or at the very least, be aware of. He then distinguishes a set of maxims and sub-maxims which describe said principle. The aforementioned maxims are defined as follows:

(1) **Quantity**
- **(Gr1A)** *Make your contribution as informative as is required (for the current purposes of the exchange).*
- **(Gr1B)** *Do not make your contribution more informative than is required.*

(2) **Quality**
- **(Gr2A)** *Do not say what you believe is false.*
- **(Gr2B)** *Do not say that for which you lack adequate evidence.*

(3) **Relation**
- **(Gr3A)** *Be relevant.*

(4) **Manner**
- **(Gr4A)** *Avoid obscurity of expression.*
- **(Gr4B)** *Avoid ambiguity.*
- **(Gr4C)** *Be brief (avoid unnecessary prolixity).*
- **(Gr4D)** *Be orderly.*

In [11] a *cogent argument* is defined as an argument whose premises are rationally acceptable and are ordered so as to provide rational support for the argument's conclusion. A set of conditions, denoted as the **ARG conditions**, is then presented in order to assess whether an argument is cogent or not.[1]

- **(GoA) - Acceptability:** Are the argument's premises acceptable?
- **(GoR) - Relevance:** Are the premises relevant to the argument's conclusion?
- **(GoG) - Good grounds:** Do the premises provide enough grounds for the argument's conclusion?

[1]In [15], the ARS criteria are presented to assess the quality of an argument. Said criteria are similar to the ARG conditions presented above.

2.2 A Brief Summary of the ASPIC+ Argumentation Formalism

In this work, we examine the process by which assumptions are created from enthymemes in order to decode complete arguments. Therefore, it is necessary to use a structured argumentation formalism. Although several alternatives exist in the literature, including *ABA* [23], *ASPIC+* [19], *Deductive Argumentation* [3], and *DeLP* [10], we opted to use ASPIC+ [19, 9] because it is the most widely adopted structured argumentation formalism. Furthermore, as we will discuss in following sections, there are a number of works exploring enthymemes' formalization that use ASPIC+ [13, 27, 28, 30].

In ASPIC+, a logical language $\mathcal{L}$ closed under negation "¬" (*e.g.,* $a, b, \neg a, \neg c$) is used, where $x = \overline{y}$ denotes that either $x = \neg y$, or $y = \neg x$. Then, the following elements of the framework are defined: First, an *argumentation system* is a triple $AS = \langle \mathcal{L}, \mathcal{R}, n \rangle$ where $\mathcal{L}$ is the aforementioned logical language, $\mathcal{R} = \mathcal{R}_s \cup \mathcal{R}_d$ is a set of strict ($\mathcal{R}_s$) inference rules of the form $\varphi \leftarrow \{\varphi_1, \ldots, \varphi_n\}$ and defeasible ($\mathcal{R}_d$) inference rules of the form $\varphi \Leftarrow \{\varphi_1, \ldots, \varphi_n\}$, where φ_i, φ are meta-variables ranging over wff in $\mathcal{L}$. Finally, n is a rule naming function. Then, a *knowledge base* is a set $\mathcal{K} \subseteq \mathcal{L}$ consisting of two disjoint subsets $\mathcal{K}_n$ (*axiomatic premises*, which depict premises that are certain and thus, cannot be attacked), and $\mathcal{K}_p$ (*ordinary premises*, which depict premises that can be attacked). Finally, an ASPIC+ *argumentation theory* is a tuple $AT = \langle AS, \mathcal{K} \rangle$ where AS is an argumentation system and $\mathcal{K}$ is a knowledge base.

Then, given an argumentation theory AT, an argument $\mathcal{A}$ is a structure obtainable by applying one or more of the following steps finitely many times:

(1) *φ is an argument if $\varphi \in \mathcal{K}$, where:*
- $\text{conc}(\mathcal{A}) = \varphi,\ \text{sub}(\mathcal{A}) = \{\varphi\}$.
- $\text{defRules}(\mathcal{A}) = \text{strRules}(\mathcal{A}) = \emptyset$.
- $\text{axPremises}(\mathcal{A}) = \{\varphi\} \cap \mathcal{K}_n,\ \text{orPremises}(\mathcal{A}) = \{\varphi\} \cap \mathcal{K}_p$.

(2) *$\varphi \Leftarrow \mathcal{A}_1, \ldots, \mathcal{A}_n$ is an argument if $\mathcal{A}_1, \ldots, \mathcal{A}_n$ are arguments such that $\varphi \Leftarrow conc(\mathcal{A}_1), \ldots, conc(\mathcal{A}_n) \in \mathcal{R}_d$, where:*
- $\text{conc}(\mathcal{A}) = \varphi,\ \text{sub}(\mathcal{A}) = \{\mathcal{A}\} \cup \text{sub}(\mathcal{A}_1) \cup \ldots \cup \text{sub}(\mathcal{A}_n)$.
- $\text{defRules}(\mathcal{A}) = \{\varphi \Leftarrow \text{conc}(\mathcal{A}_1), \ldots, \text{conc}(\mathcal{A}_n)\} \cup \text{defRules}(\mathcal{A}_1) \cup \ldots \cup \text{defRules}(\mathcal{A}_n),\ \text{strRules}(\mathcal{A}) = \text{strRules}(\mathcal{A}_1) \cup \ldots \cup \text{strRules}(\mathcal{A}_n)$
- $\text{axPremises}(\mathcal{A}) = \text{axPremises}(\mathcal{A}_1) \cup \ldots \cup \text{axPremises}(\mathcal{A}_n)$, $\text{orPremises}(\mathcal{A}) = \text{orPremises}(\mathcal{A}_1) \cup \ldots \cup \text{orPremises}(\mathcal{A}_n)$

(3) *$\varphi \leftarrow \mathcal{A}_1, \ldots, \mathcal{A}_n$ is an argument if $\mathcal{A}_1, \ldots, \mathcal{A}_n$ are arguments such that $\varphi \leftarrow conc(\mathcal{A}_1), \ldots, conc(\mathcal{A}_n) \in \mathcal{R}_s$, where:*
- $\text{conc}(\mathcal{A}) = \varphi, \text{sub}(\mathcal{A}) = \{\mathcal{A}\} \cup \text{sub}(\mathcal{A}_1) \cup \ldots \cup \text{sub}(\mathcal{A}_n)$
- $\text{defRules}(\mathcal{A}) = \text{defRules}(\mathcal{A}_1) \cup \ldots \cup \text{defRules}(\mathcal{A}_n)$, $\text{strRules}(\mathcal{A}) = \{\varphi \leftarrow \text{conc}(\mathcal{A}_1), \ldots, \text{conc}(\mathcal{A}_n)\} \cup \text{strRules}(\mathcal{A}_1) \cup \ldots \cup \text{strRules}(\mathcal{A}_n)$
- $\text{axPremises}(\mathcal{A}) = \text{axPremises}(\mathcal{A}_1) \cup \ldots \cup \text{axPremises}(\mathcal{A}_n)$, $\text{orPremises}(\mathcal{A}) = \text{orPremises}(\mathcal{A}_1) \cup \ldots \cup \text{orPremises}(\mathcal{A}_n)$

Given an argument $\mathcal{A}$, we denote its knowledge as:

$$\text{knowledge}(\mathcal{A}) = \text{defRules}(\mathcal{A}) \cup \text{strRules}(\mathcal{A}) \cup \text{axPremises}(\mathcal{A}) \cup \text{orPremises}(\mathcal{A}).$$

Furthermore, we denote the set of all arguments created on the basis of AT as $\mathbb{A}_{AT}$. When no ambiguity arises, we will denote said set simply as $\mathbb{A}$.

For instance, consider an argumentation theory $AT = \left\langle AS, \mathcal{K}_n \cup \mathcal{K}_p \right\rangle, AS=\langle \mathcal{L}, \mathcal{R}, n \rangle$ such that its rules are $\mathcal{R} = \{x \Leftarrow y, b,\ y \leftarrow a\}$, and its premises are $\mathcal{K}_n=\{\mathbf{b}\}, \mathcal{K}_p = \{a\}$ (from now on, we will use boldface literals to depict axiomatic premises). Figure 1 depicts the graphical representation of argument $\mathcal{A}$ created on the basis of AT, as well as its sub-arguments $\mathcal{A}, \mathcal{S}_1, \mathcal{S}_2$, and $\mathcal{S}_3$. Note that $\text{knowledge}(\mathcal{A}) = \mathcal{K}_n \cup \mathcal{K}_p \cup \mathcal{R}$.

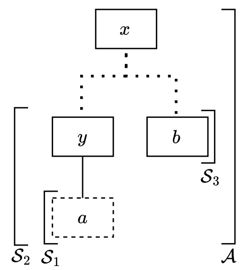

Fig. 1. Graphical representation of argument $\mathcal{A}$ and its sub-arguments $\mathcal{A}, \mathcal{S}_1, \mathcal{S}_2$ and $\mathcal{S}_3$. Dotted edges denote defeasible rules, solid edges strict rules, and dashed nodes depict ordinary premises.

ASPIC+ generates abstract argumentation frameworks by relating arguments via binary defeats. Briefly, an argument $\mathcal{A}$ *attacks* an argument $\mathcal{B}$ if: $\text{conc}(\mathcal{A}) = \overline{\text{conc}(\mathcal{S})}$ for some $\mathcal{S} \in \text{sub}(\mathcal{B})$ such that the top rule of $\mathcal{S}$ is defeasible (*rebuttal*), $\text{conc}(\mathcal{A}) = \overline{h}$ where $h \in \text{orPremises}(\mathcal{B})$ (*undermining*), or $\text{conc}(\mathcal{A})$ is in conflict with some defeasible rule in $\mathcal{B}$ (*undercutting*). Then, given a *preference ordering* $\preceq$ over $\mathbb{A}$ (*i.e.,* given $\mathcal{A}, \mathcal{B} \in \mathbb{A}$, $\mathcal{A} \prec \mathcal{B}$ denotes that $\mathcal{B}$ is preferred over $\mathcal{A}$), an argument $\mathcal{A}$ *defeats* $\mathcal{B}$ if $\mathcal{A}$ attacks $\mathcal{B}$, and $\mathcal{B} \not\prec \mathcal{A}$ (*i.e.,* $\mathcal{B}$ is not preferred over $\mathcal{A}$). Given an argumentation theory AT and the preference ordering $\preceq$, an *abstract argumentation framework* is a pair $AF = \langle A, D \rangle$ such that $A = \mathbb{A}_{AT}$ and $D \subseteq A \times A$ is the set of all defeats with respect to $\preceq$. The justified arguments of $AF = \langle A, D \rangle$ are defined under various semantics [8] as follows:

(1) $S \subseteq A$ is conflict free *iff* $\forall \mathcal{X}, \mathcal{Y} \in S : (\mathcal{X}, \mathcal{Y}) \notin D$.
(2) $\mathcal{X} \in A$ is acceptable with respect to $S \subseteq A$ *iff* $\forall \mathcal{Y} \in A$ *such that* $(\mathcal{Y}, \mathcal{X}) \in D : \exists \mathcal{Z} \in S$ *such that* $(\mathcal{Z}, \mathcal{Y}) \in D$.
(3) $S \subseteq A$ is an admissible set *iff* S *is conflict free and* $\mathcal{X} \in S$ *implies* $\mathcal{X}$ *is acceptable w.r.t.* S.
(4) **complete** (co): $S \subseteq A$ *is a complete extension iff* S *is admissible and if* $\mathcal{X} \in A$ *is acceptable w.r.t.* S *then* $\mathcal{X} \in S$.
(5) **preferred** (pr): $S \subseteq A$ *is a preferred extension iff it is a set inclusion maximal complete extension.*
(6) **grounded** (gr): $S \subseteq A$ *is the grounded extension iff it is the set inclusion minimal complete extension;*
(7) **stable** (st): $S \subseteq A$ *is a stable iff* S *is conflict free and* $\forall \mathcal{Y} \notin S, \exists \mathcal{X} \in S$ *s.t.* $(\mathcal{X}, \mathcal{Y}) \in D$

Finally, given a semantics $\sigma \in \{\texttt{co}, \texttt{pr}, \texttt{gr}, \texttt{st}\}$, $\mathcal{A}$ is denoted as *credulously justified* on the basis of AF under semantics σ if $\mathcal{A}$ belongs to at least one σ-extension of AF. Likewise, argument $\mathcal{A}$ is *skeptically justified* on the basis of AF under semantics σ if $\mathcal{A}$ belongs to all σ-extensions of AF. Then, a formula $\varphi \in \mathcal{L}$ is *skeptically / credulously* justified on the basis of an AF under semantics $\sigma \in \{\texttt{co}, \texttt{pr}, \texttt{gr}, \texttt{st}\}$ if φ is the conclusion of a *skeptically / credulously* justified argument under semantics σ. From now on, if we do not explicitly specify whether a formula is skeptically or credulously justified, then the distinction is inconsequential.

3 Enthymemes

In most real-world argumentative dialogues, the exchanged arguments do not necessarily align with the definition of *logical arguments*, where the set of premises is sufficient to entail the argument's claim. Instead, most of these exchanged arguments are enthymemes. In [24], enthymemes are described by the following definition: "An argument is said to be an enthymeme if there are premises needed to make the argument valid that are only tacitly, but not explicitly stated or advanced as part of the argument." In other words, enthymemes could be conceived as partially completed arguments.

In recent years, a number of works have explored enthymemes and their formalization, adopting different approaches. Works such as [7, 14, 22] use deductive argumentation and logic-based argumentation as the underlying framework for enthymemic representation. In comparison, works like [13, 27, 28, 30] adapt the ASPIC+ argumentation formalism to accommodate enthymemic arguments. In [13], enthymemes are defined as Argument-trees in which some of the nodes representing rules lack some of their child nodes (*i.e.,* some premise is unstated), or some edges between nodes are *illegitimate* (*i.e.,* a premise is connected to a rule to which it is not relevant). In contrast, works like [27, 28, 30] define enthymemes as a forest of trees, which is the result of pruning some of the argument's nodes and edges. Finally, [16] use DeLP as the underlying argumentation formalism to represent both the agents involved, as well as the enthymemes exchanged. In this work, we opted to represent arguments and enthymemes by encapsulating their contents (claim, rules and premises) in a tuple. A more in-depth discussion of the representation in comparison to other works in the area will be discussed in Section 7. For now, given an ASPIC+ argument, we define our representation of arguments as follows:

Definition 1 (Argument)**.** Let $\mathcal{X}$ be an ASPIC+ argument on the basis of the ASPIC+ argumentation theory AT. The argument $\mathcal{A}$ that comes from $\mathcal{X}$ is a tuple $\langle h, R, F\rangle$ such that, $h = \mathsf{conc}(\mathcal{X})$, $R = \mathsf{defRules}(\mathcal{X}) \cup \mathsf{strRules}(\mathcal{X})$, $F = \mathsf{axPremises}(\mathcal{X}) \cup \mathsf{orPremises}(\mathcal{X})$. We denote the set of all possible *arguments* as $\mathbb{A}$.

Example 3.1. Given ASPIC+ arguments $\mathcal{A}, \mathcal{S}_1, \mathcal{S}_2, \mathcal{S}_3$ shown in Figure 1, their corresponding arguments under our representation are as follows:

$$\mathcal{S}_1 = \langle a, \emptyset, \{a\}\rangle \qquad \mathcal{S}_2 = \langle y, \{y \leftarrow a\}, \{a\}\rangle \qquad \mathcal{S}_3 = \langle b, \emptyset, \{\mathbf{b}\}\rangle$$

$$\mathcal{A} = \left\langle x, \begin{Bmatrix} x \Leftarrow y, b, \\ y \leftarrow a \end{Bmatrix}, \begin{Bmatrix} a, \\ \mathbf{b} \end{Bmatrix} \right\rangle$$

Note that, given an ASPIC+ argumentation theory AT, if there is an ASPIC+ argument $\mathcal{X}$ on the basis of AT, there will exist an argument in $\mathcal{A} \in \mathbb{A}$ that shares $\mathcal{X}$'s contents. The converse also holds true, if $\mathcal{A}$'s contents are part of some AT, then there will be an ASPIC+ argument on the basis of AT with its contents and its conclusion.

Remark 1. Given an argument $\mathcal{A} \in \mathbb{A}$, $\mathcal{A} = \langle h, R, F\rangle$ and an ASPIC+ argumentation theory $AT = \langle AS, \mathcal{K}\rangle$, $AS = \langle \mathcal{L}, \mathcal{R}, n\rangle$ such that $R \subseteq \mathcal{R}$, $F \subseteq \mathcal{K}$, then there exists an ASPIC+ argument $\mathcal{X}$ on the basis of AT such that $\mathsf{conc}(\mathcal{X}) = h$, $\mathsf{knowledge}(\mathcal{X}) = R \cup F$.

Then, following the notion of enthymeme defined in [24], given an argument $\mathcal{A}$, we denote that an enthymeme contains a subset of $\mathcal{A}$'s premises. The formal definition of *enthymemes* is introduced as follows:

Definition 2 (Enthymeme)**.** Let $\mathcal{A} = \langle h, R, F\rangle$ be an argument (Definition 1), we denote that $\mathcal{E} = \langle h, R', F'\rangle$ is an *enthymeme* from $\mathcal{A}$ iff $R' \cup F' \subset R \cup F$, and $\mathcal{E} \notin \mathbb{A}$.

We denote the set of all possible *enthymemes* as $\mathbb{E}$.

Notation 3.1. Given an ASPIC+ argument $\mathcal{X}$, such that argument $\mathcal{A}$ comes from $\mathcal{X}$, and let $\mathcal{E}$ be an enthymeme from $\mathcal{A}$. Then, $\mathcal{E}$ is an enthymeme that comes from ASPIC+ argument $\mathcal{X}$.

As the reader may have noticed, arguments and enthymemes share the same structure. We have defined them in such a way in order to use them interchangeably as messages during the dialogue defined in Section 4. As such, given an enthymeme or an argument $\mathcal{M} \in \mathbb{A} \cup \mathbb{E}$, $\mathcal{M} = \langle h, R, F\rangle$, we define the following sets:

- $\mathsf{conc}(\mathcal{M}) = h$, $\mathsf{rules}(\mathcal{M}) = R$, $\mathsf{premises}(\mathcal{M}) = F$
- $\mathsf{heads}(\mathcal{M}) = \{x \mid x \text{ is the head of a rule in } R\}$
- $\mathsf{bodies}(\mathcal{M}) = \{x \mid x \text{ is part of the body of a rule in } R\}$
- $\mathsf{literals}(\mathcal{M}) = \mathsf{heads}(\mathcal{M}) \cup \mathsf{bodies}(\mathcal{M}) \cup F \cup \{h\}$

Notation 3.2. Given a tuple $\mathcal{M} = \langle h, R, F \rangle$, $\mathcal{M} \in \mathbb{A} \cup \mathbb{E}$, when no ambiguity arises, we will refer to it as the union of its components, that is: $\mathcal{M} = R \cup F$. This extends to set-operations: For instance, given a tuple $\mathcal{M} = \langle h, R, F \rangle$ and a set R' of defeasible and strict rules, then $\mathcal{M} \cup R' = \langle h, R \cup R', F \rangle$. Furthermore, we denote the set $\mathbb{M} = \mathbb{A} \cup \mathbb{E}$.

Given the incomplete nature of enthymemes, we will introduce the concept of *baseless, disconnected* and *relevant* literals. First, given a tuple $\mathcal{M} \in \mathbb{M}$, we denote that a literal is *baseless* if it lacks support from a rule or premise in $\mathcal{M}$ (*i.e.,* it is not the head of any rule and it is not a premise in $\mathcal{M}$). On the other hand, a literal is *disconnected* if it does not provide support to another literal nor $\mathcal{M}$'s claim (*i.e.,* it is not part of the body of any rule and it is not $\mathcal{M}$'s conclusion). The sets of *baseless* and *disconnected literals* are defined as follows:

- $\text{baseless}(\mathcal{M}) = \{l \mid l \in \text{literals}(\mathcal{M}) \setminus (\text{heads}(\mathcal{M}) \cup \text{premises}(\mathcal{M}))\}$
- $\text{disconnected}(\mathcal{M}) = \{l \mid l \in \text{literals}(\mathcal{M}) \setminus (\{\text{conc}(\mathcal{M})\} \cup \text{bodies}(\mathcal{M}))\}$.

Note that, if an enthymeme $\mathcal{E}$ comes from argument $\mathcal{A}$, then at least one rule or premise will be part of $\mathcal{A}$ but not of $\mathcal{E}$. As such, said *missing gap* of information will result in, at least, one literal being part of baseless($\mathcal{E}$). This is addressed by the following remark.

Remark 2. If $\mathcal{E} \in \mathbb{E}$, then $\text{baseless}(\mathcal{E}) \neq \emptyset$.

Finally, we will also denote which literals are explicitly *relevant* to a claim. Given a tuple $\mathcal{M} = \langle h, R, F \rangle$, we denote that a literal $l \in \text{literals}(\mathcal{M})$ is *relevant* if either: $l = h$, or there exists a rule $l' \Leftarrow Body \in R$ or $l' \leftarrow Body \in R$ such that $l \in Body$ and l' is relevant. We denote the set of relevant literals of $\mathcal{M}$ as relevant($\mathcal{M}$).

Example 3.2. Consider an argumentation theory $\mathbb{AT}$ such that $\mathcal{A}$ is an argument obtainable from it, where:

$$\text{conc}(\mathcal{A}) = a,\ \text{knowledge}(\mathcal{A}) = \begin{Bmatrix} a \Leftarrow b,\ c & c \Leftarrow e \\ b \leftarrow d & d \leftarrow f \\ \mathbf{e} & f \end{Bmatrix}$$

Then, $\mathcal{E}_1,\ \mathcal{E}_2$ are two enthymemes from $\mathcal{A}$ defined as follows:

$$\mathcal{E}_1 = \left\langle a, \{b \leftarrow d\}, \begin{Bmatrix} \mathbf{e}, \\ f \end{Bmatrix} \right\rangle\ \mathcal{E}_2 = \left\langle a, \begin{Bmatrix} a \Leftarrow b,\ c \\ c \Leftarrow e \\ b \leftarrow d \end{Bmatrix}, \emptyset \right\rangle$$

Table 1 describes $\mathcal{E}_1$ and $\mathcal{E}_2$ information. Observe that two very structurally different enthymemes can be obtained from the same argument. In $\mathcal{E}_1$, the enthymeme's claim is neither a premise nor the head of any rule in said enthymeme, as such, $a \in \text{baseless}(\mathcal{E}_1)$. Furthermore, some premises such as e and f are not part of the body of any rule, and so, they are part of disconnected($\mathcal{E}_1$). In contrast, observe that every literal in $\mathcal{E}_2$ is relevant to its claim, and therefore, there are no disconnected literals: $\text{relevant}(\mathcal{E}_2) = \text{literals}(\mathcal{E}_2),\ \text{disconnected}(\mathcal{E}_2) = \emptyset$.

Table 1. Information of enthymemes $\mathcal{E}_1, \mathcal{E}_2$, which were obtainable from argument $\mathcal{A}$.

-	$\mathcal{E}_1$	$\mathcal{E}_2$
conc	a	a
heads	$\{b\}$	$\{a, b, c\}$
bodies	$\{d\}$	$\{b, c, d, e\}$
premises	$\{e, f\}$	$\emptyset$
literals	$\{a, b, d, e, f\}$	$\{a, b, c, d, e\}$
baseless	$\{a, d\}$	$\{e, d\}$
disconnected	$\{b, e, f\}$	$\emptyset$
relevant	$\{a\}$	$\{a, b, c, d, e\}$

In the following section, we will show how enthymemes are employed in argumentative dialogues, and outline the necessary mechanisms for agents to reason with them.

4 Assumptions Operators in Enthymemic Dialogues

This section introduces the concept of *assumptions operator*, which formalizes the process of completing an enthymeme in order to obtain a complete argument from it. Also, we will present a dialogue system in which two agents exchange enthymemes, where said system is equipped with an assumptions operator. Since our aim in this work is to explore how these operators are used in argumentative dialogues, the dialogue system presented serves merely as a framework within which the operator can function, and it is a simplified version of works such as [27, 16].

As we briefly explained in Section 1, and more specifically in Example 1.1, there are several ways in which an argument can be derived from an enthymeme. Given an enthymeme, an agent might add assumptions to "connect" its premises in order to entail its claim. Another agent could instead discard all premises and create new assumptions to support the enthymeme's claim. Hence, we will define an *assumptions operator* as a function that receives an enthymeme and creates a tuple by adding, removing or modifying its contents.

Definition 3 (Assumptions operator)**.** An assumptions operator is a function $\oplus : \mathbb{E} \mapsto \mathbb{M}$. We denote the application of operator $\oplus$ on enthymeme $\mathcal{E}$ as $\oplus\mathcal{E}$.

Next, we will introduce two concrete operators that will be used throughout the remainder of this work. Later in this section, we will discuss how the usage of each of them could lead to different semantic results in terms of the justified arguments. As we will show in the following examples, these two assumptions operators behave very differently, being the former a simple one, and the latter a more complex one. We have decided to do so because in Section 5, we will use them to exemplify the principles that will be used to characterize this kind of operators. The first one is called the *simple operator* and is denoted $\oplus_s$. Intuitively, given an enthymeme $\mathcal{E}$, $\oplus_s$ creates an ordinary premise to support $\mathcal{E}$'s conclusion if it is not supported already, otherwise it returns the original enthymeme $\mathcal{E}$.

Definition 4 (Simple Operator)**.** Let $\mathcal{E}$ be an enthymeme, the application of the *simple operator* $\oplus_s$ on $\mathcal{E}$ is denoted as $\oplus_s\mathcal{E} = \mathcal{E} \cup (\{\mathsf{conc}(\mathcal{E})\} \cap \mathsf{baseless}(\mathcal{E}))$

Example 4.1. Consider again the two enthymemes $\mathcal{E}_1$ and $\mathcal{E}_2$ introduced in Example 3.2:

$$\mathcal{E}_1 = \left\langle a, \{b \leftarrow d\}, \begin{Bmatrix} \mathbf{e}, \\ f \end{Bmatrix} \right\rangle \quad \mathcal{E}_2 = \left\langle a, \begin{Bmatrix} a \Leftarrow b,\ c \\ c \Leftarrow e \\ b \leftarrow d \end{Bmatrix}, \emptyset \right\rangle$$

Then, the application of $\oplus_s$ on $\mathcal{E}_1$ and $\mathcal{E}_2$ results in:

$$\oplus_s\mathcal{E}_1 = \left\langle a, \{b \leftarrow d\}, \begin{Bmatrix} \mathbf{e}, & f, \\ \underline{a} & \end{Bmatrix} \right\rangle \qquad \oplus_s \mathcal{E}_2 = \mathcal{E}_2$$

Observe that in $\oplus_s\mathcal{E}_1$, a is added to the set of premises, since $\mathsf{conc}(\mathcal{E}_1) \in \mathsf{baseless}(\mathcal{E}_1)$. In future examples, the rules and premises created by $\oplus$ will be underlined in order to highlight the new elements added by the assumptions operator. In contrast, we will now define the *needed assumptions operator*, denoted as $\oplus_{\updownarrow}$, which will behave in a more complex manner: Given an enthymeme $\mathcal{E}$, $\oplus_{\updownarrow}$ creates defeasible rules that "connect" $\mathcal{E}$'s conclusion with the rest of its contents. In addition, $\oplus_{\updownarrow}$ creates ordinary premises for each baseless literal that is not relevant for $\mathcal{E}$'s conclusion.

Definition 5 (Needed Assumptions Operator)**.** Let $\mathcal{E} = \langle h, R, F\rangle$ be an enthymeme, the application of the *needed assumptions operator* $\oplus_{\updownarrow}$ on $\mathcal{E}$ results in $\oplus_{\updownarrow}\mathcal{E} = \langle h, R \cup R', F \cup F'\rangle$ such that:

If disconnected($\mathcal{E}$) = $\emptyset$ then, $R' = \emptyset, F' = \{x \mid x \in \text{baseless}(\mathcal{E}) \cap \text{relevant}(\mathcal{E})\}$.
Otherwise, $R' = \{x \Leftarrow Y \mid x \in \text{baseless}(\mathcal{E}) \cap \text{relevant}(\mathcal{E}),\ Y = \text{disconnected}(\mathcal{E})\}$, $F' = \{x \mid x \in \text{baseless}(\mathcal{E}) \setminus \text{relevant}(\mathcal{E})\}$

Example 4.2. Recall enthymemes $\mathcal{E}_1, \mathcal{E}_2$ from example 3.2, the application of $\oplus_{\updownarrow}$ on those results in:

$$\oplus_{\updownarrow}\mathcal{E}_1 = \left\langle a, \left\{ \begin{matrix} b \leftarrow d, \\ \underline{a \Leftarrow b, e, f} \end{matrix} \right\}, \left\{ \begin{matrix} \mathbf{e}, & f \\ \underline{d} & \end{matrix} \right\} \right\rangle \qquad \oplus_{\updownarrow}\mathcal{E}_2 = \left\langle a, \left\{ \begin{matrix} a \Leftarrow b, c \\ c \Leftarrow e \\ b \leftarrow d \end{matrix} \right\}, \left\{ \begin{matrix} \underline{d}, \\ \underline{e} \end{matrix} \right\} \right\rangle$$

Recall that rules and premises created by the operator are underlined. On the one hand, in $\oplus_{\updownarrow}\mathcal{E}_1$ the operator adds $R' = \{a \Leftarrow b, e, f\}$, that is a rule that connects the conclusion with the rest of the information included in $\mathcal{E}_1$ and also adds an ordinary premise, since $F' = \{d\}$. On the other hand, in $\oplus_{\updownarrow}\mathcal{E}_2$, since its conclusion and all the other literals are connected through rules, then $R' = \emptyset$ and the operator only adds the ordinary premises $F' = \{d, e\}$ because both literals are 'baseless'.

As shown by the two examples above, the assumptions operator definition allows for the characterization of a wide range of operators, from very simple to more complex ones. In the next section, we will introduce different principles with the aim of characterizing and analyzing these kinds of operators when used in dialogues that contain enthymemes. Before that, for the rest of this section, we will introduce a dialogue system for dialogues in which agents exchange both arguments and enthymemes. It is important to note that the dialogue system itself is not a contribution of this paper, rather, it serves as a means of illustrating how the equipped *assumptions operator* could yield different semantic results. Our proposed dialogue system is general enough to accommodate different kinds of dialogues, such as *persuasion, inquiry, information seeking and deliberation* ones.

In our proposed dialogue system, two agents prop and opp exchange arguments and enthymemes. Said agents are defined by an argumentation theory $\mathbb{AT}_{\mathsf{ag}}$, which represents their knowledge. In addition, the dialogue system is equipped with an assumptions operator $\oplus$ which will create assumptions for every exchanged enthymeme. The formal definition of a dialogue is as follows:

Definition 6 (Dialogue)**.** Let $\mathsf{A} = \{\mathsf{prop}, \mathsf{opp}\}$ be a set of two agents and $\oplus$ be an assumptions operator. A *dialogue* $\mathbb{D}^{\oplus}_{\mathsf{A}}$ is a sequence of messages $[m_1, \ldots, m_n]$ where for all $i \in \{1, \ldots, n\}$, $m_i = \langle \mathsf{x}, M \rangle$, $\mathsf{x} \in \mathsf{A}$, $M \in \mathbb{M} \cup \{\mathsf{pass}\}$, and if $i \mod 2 = 1$, then $m_i = \langle \mathsf{prop}, M \rangle$, otherwise $m_i = \langle \mathsf{opp}, M \rangle$.

Notation 4.1. From now on, we denote that $\overline{\mathsf{prop}} = \mathsf{opp}$ and $\overline{\mathsf{prop}} = \mathsf{opp}$. Furthermore, when no ambiguity arises, we will denote the dialogue $\mathbb{D}^{\oplus}_{\mathsf{A}}$ as simply $\mathbb{D}$.

The definition above dictates that a dialogue involving prop and opp is a sequence of messages which could be either an argument (Definition 1), an enthymeme (Definition 2), or pass, which denotes that the agent does not want to exchange anything into the dialogue.

Given a dialogue $\mathbb{D}^{\oplus}_{\mathsf{A}}$, we define the *argumentation theory from* $\mathbb{D}^{\oplus}_{\mathsf{A}}$ as the ASPIC+ argumentation theory which contains the information obtainable from the application of the *assumptions operator* on each of the exchanged enthymemes, as well as the contents from the exchanged arguments.

Definition 7 (Argumentation Theory from a Dialogue)**.** Given a dialogue $\mathbb{D}^{\oplus}_{\mathsf{A}}$, its argumentation theory is a tuple $\mathbb{AT}_{\mathbb{D}} = \langle \mathbb{AS}, \mathcal{K} \rangle$, $\mathbb{AS} = \langle \mathcal{L}, R, nom \rangle$, where $\mathcal{L}$ is a logical language, *nom* is a rule naming function and:

$$R = \left\{\text{rules}(\oplus\mathcal{E}) \mid \langle \_, \mathcal{E} \rangle \in \mathbb{D}^{\oplus}_{\mathsf{A}},\ \mathcal{E} \in \mathbb{E}\right\} \cup \left\{\text{rules}(\mathcal{A}) \mid \langle \_, \mathcal{A} \rangle \in \mathbb{D}^{\oplus}_{\mathsf{A}},\ \mathcal{A} \in \mathbb{A}\right\}$$
$$\mathcal{K} = \left\{\text{premises}(\oplus\mathcal{E}) \mid \langle \_, \mathcal{E} \rangle \in \mathbb{D}^{\oplus}_{\mathsf{A}},\ \mathcal{E} \in \mathbb{E}\right\} \cup \left\{\text{premises}(\mathcal{A}) \mid \langle \_, \mathcal{A} \rangle \in \mathbb{D}^{\oplus}_{\mathsf{A}},\ \mathcal{A} \in \mathbb{A}\right\}$$

For simplicity sake, we denote from now on that knowledge($\mathbb{D}^{\oplus}_{\mathsf{A}}$) = $R \cup \mathcal{K}$.

Note that the argumentation theory from $\mathbb{D}^{\oplus}_{\mathsf{A}}$ will be defined not only by the information exchanged in the messages, but also the assumptions created by the equipped operator. This will be illustrated in the following example:

Example 4.3. Let $\mathsf{A} = \{\mathsf{prop}, \mathsf{opp}\}$ be two agents and recall operators $\oplus_s$ and $\oplus_{\updownarrow}$ (Definition 4, 5 respectively). Then, we define dialogues $\mathbb{D}_{\mathsf{A}}^{\oplus_s} = [m_1, m_2]$ using $\oplus_s$ and $\mathbb{D}_{\mathsf{A}}^{\oplus_{\updownarrow}} = [m_1, m_2]$ using $\oplus_{\updownarrow}$ where m_1, m_2 are defined as follows:

$$m_1 = \left\langle \mathsf{prop}, \left\langle x, \left\{ x \Leftarrow y, b \right\}, \{y,\ b\} \right\rangle \right\rangle \qquad m_2 = \left\langle \mathsf{opp}, \left\langle \neg x, \left\{ \neg y \leftarrow z, a \right\}, \emptyset \right\rangle \right\rangle$$

$$\mathsf{knowledge}(\mathbb{D}_{\mathsf{A}}^{\oplus_s}) = \left\{ \begin{array}{c} x \Leftarrow y, b, \\ \neg y \leftarrow z, a, \\ \\ y,\ b,\ \underline{\neg x} \end{array} \right\} \mathsf{knowledge}(\mathbb{D}_{\mathsf{A}}^{\oplus_{\updownarrow}}) = \left\{ \begin{array}{c} x \Leftarrow y, b, \\ \neg y \leftarrow z, a, \\ \underline{\neg x \Leftarrow \neg y,} \\ \\ y,\ b,\ \underline{z},\ \underline{a} \end{array} \right\}$$

Observe that both dialogues share the same messages and agents, but do not share the same assumption operator. Despite the exchange of identical enthymemes in both dialogues, the resulting argumentation theories differ on the assumptions created by their respective operators. Furthermore, we are interested in showing how the equipped assumptions operator could lead to different status of the arguments instantiated by $\mathbb{AT}_{\mathbb{D}}$. As such, we explore the status of the argumentation framework from $\mathbb{AT}_{\mathbb{D}}$, which we denote as $\mathbb{AF}_{\mathbb{D}}$, in the following example.

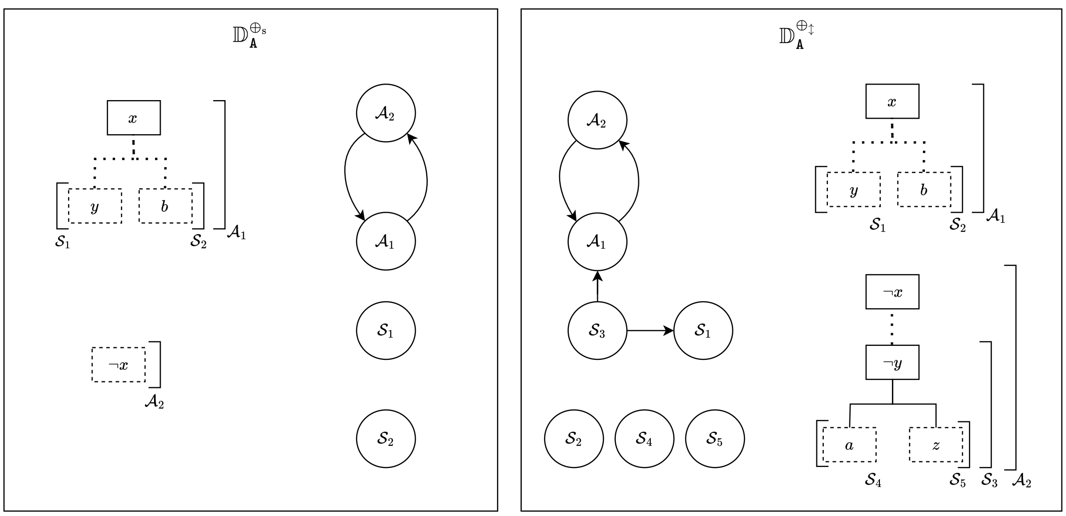

Fig. 2. Arguments obtained from dialogues $\mathbb{D}_{\mathsf{A}}^{\oplus_s}$ and $\mathbb{D}_{\mathsf{A}}^{\oplus_{\updownarrow}}$ with their resulting argumentation frameworks: circle nodes represent arguments (with their labels inside) and solid arrows denote the attack relation (*e.g.,* $\mathcal{S}_3$ attacks $\mathcal{A}_1$).

Example 4.4. Consider dialogues $\mathbb{D}_{\mathsf{A}}^{\oplus_s}, \mathbb{D}_{\mathsf{A}}^{\oplus_{\updownarrow}}$ introduced in Example 4.3, Figure 2 shows the arguments created from $\mathbb{AT}_{\mathbb{D}_{\mathsf{A}}^{\oplus_s}}$ and $\mathbb{AT}_{\mathbb{D}_{\mathsf{A}}^{\oplus_{\updownarrow}}}$, as well as their corresponding ASPIC+ argumentation framework. Then, the extensions under the *preferred* semantics for each of those argumentation frameworks is as follows:

$$\text{For } \mathbb{D}_{\mathsf{A}}^{\oplus_s} :\ \{\mathcal{S}_1, \mathcal{S}_2, \mathcal{A}_1\}, \{\mathcal{S}_1, \mathcal{S}_2, \mathcal{A}_2\}.$$

$$\text{For } \mathbb{D}_{\mathsf{A}}^{\oplus_{\updownarrow}} :\ \{\mathcal{S}_2, \mathcal{S}_3, \mathcal{S}_4, \mathcal{S}_5, \mathcal{A}_2\}$$

It is important to note that, by using different operators, some arguments differ in their status. For instance, note that argument $\mathcal{A}_1$ is credulously accepted under the preferred semantics in $\mathbb{D}_{\mathsf{A}}^{\oplus_s}$ but not in $\mathbb{D}_{\mathsf{A}}^{\oplus_{\updownarrow}}$.

As we mentioned above, during a dialogue agents exchange arguments or enthymemes. In order to avoid "foul play" during the dialogue, we assume that agents comply with some restrictions on the kinds of messages they send. Given that our dialogue system is general enough to accommodate different kinds of dialogues (*e.g.,* inquiry, persuasion, deliberation), the defined restrictions are general too. First, every message must come from an ASPIC+ argument that does not contain two sub-arguments with complementary conclusions. Furthermore, we enforce that said argument should be minimal, that is, it should not contain two different sub-arguments with the same conclusion. Finally, if part of said argument is present in the dialogue's knowledge, then it should be part of the exchanged message. These restrictions are introduced as follows:

Definition 8 (Valid message)**.** Let $\mathbb{D}^{\oplus}_{\mathsf{A}} = [m_1, \ldots, m_n]$ be a dialogue, and let $m = \langle \mathsf{ag}, \mathcal{M} \rangle$ such that $\mathcal{M} \in \mathbb{M}$ comes from the ASPIC+ argument $\mathcal{X}$. Then, m is a valid message if the following conditions hold.

(1) there are no $\mathcal{B}$ and $\mathcal{C}$ in $\mathrm{sub}(\mathcal{X})$ such that $\mathrm{conc}(\mathcal{B}) = \overline{\mathrm{conc}(\mathcal{C})}$,
(2) there are no $\mathcal{B}$ and $\mathcal{C}$ in $\mathrm{sub}(\mathcal{X})$ such that $\mathcal{B} \neq \mathcal{C}$ and $\mathrm{conc}(\mathcal{B}) = \mathrm{conc}(\mathcal{C})$,
(3) if $x \in \mathsf{knowledge}(\mathbb{D}^{\oplus}_{\mathsf{A}}) \cap \mathsf{knowledge}(\mathcal{X})$, then $x \in \mathcal{M}$.

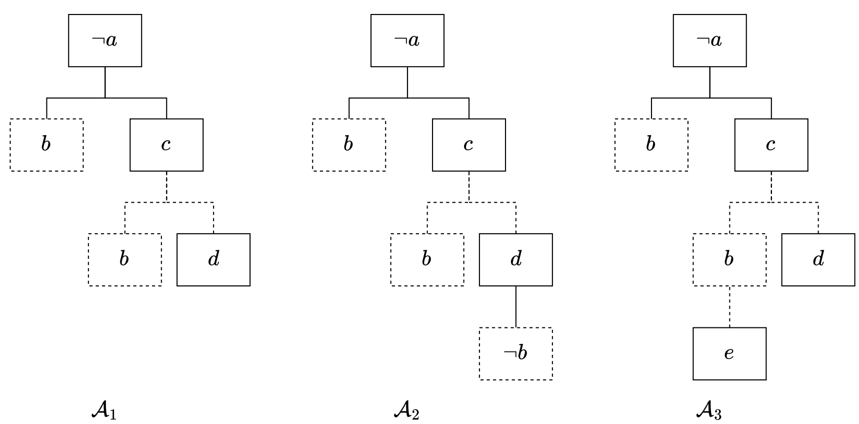

Fig. 3. Arguments mentioned in Example 4.5, from which m_a, m_b, m_c and m_d are obtained.

Example 4.5. Consider the dialogue $\mathbb{D}^{\oplus\updownarrow}_{\mathsf{A}}$ introduced in Example 4.3, such that prop wants to exchange a message, so it considers the following ones:

$$m_a = \langle \mathsf{prop}, \langle \neg a, \{\neg a \leftarrow b, c\}, \{b\}\rangle\rangle \qquad m_b = \langle \mathsf{prop}, \langle \neg a, \{\neg a \leftarrow b, c\}, \emptyset\rangle\rangle$$
$$m_c = \langle \mathsf{prop}, \langle \neg a, \{\neg a \leftarrow b, c\}, \{\neg b\}\rangle\rangle \qquad m_d = \langle \mathsf{prop}, \langle \neg a, \{b \Leftarrow e\}, \{b\}\rangle\rangle$$

Enthymemes m_a and m_b both come from $\mathcal{A}_1$ (see Figure 3, where all the arguments are shown), m_c comes from $\mathcal{A}_2$ and m_d from $\mathcal{A}_3$. Observe that m_a is the only valid message according to Definition 8. Note that m_b does not comply with item 3, given that b is part of $\mathsf{knowledge}(\mathbb{D}^{\oplus\updownarrow}_{\mathsf{A}})$, but not of m_b. Enthymeme m_c does not comply with item 1 since $\mathcal{A}_2$ has two sub-arguments with contradictory claims. Finally, observe that m_d does not comply with item 2, because its original argument, $\mathcal{A}_3$, has two distinct sub-arguments for the same claim.

5 Principles for Assumptions Generation

In the previous section, we have introduced the concept of assumptions operator, which given an enthymeme, creates a new tuple (enthymeme or argument) by modifying its input enthymeme. As shown in Example 1.1, there are several ways to define them, including some that yield undesirable arguments, such as the third one decoded by Matias in said example. As such, we need tools to evaluate how operators should behave in order to obtain a desirable argument from their application.

In this section, we will propose a set of principles which address some criteria that the assumptions operators should comply with. These principles are inspired by the *ARG conditions for arguments* introduced in [11], as well as the *Maxims of Conversation* proposed in [12]. As briefly introduced in Section 2, Grice's Maxims were proposed to outline a general principle for conversational exchanges, whereas Govier's ARG conditions were proposed to dictate whether an argument is cogent or not.

Since we aim to define some rationale for the assumptions operator behavior, we adopted the following approach. First, we examine how the maxims and conditions could be interpreted in order to define the principles presented. Then, we show that, if a concrete operator complies with a subset of the principles defined, the result of its application is a *cogent argument*, under our interpretation of the ARG conditions.

The remainder of this section will be devoted to the definition of the aforementioned principles, which will be categorized into two distinct groups: *basic* and *argumentative*. The former being a group that formalizes simple principles which every operator should comply with (*e.g.,* the preservation of the enthymeme's contents), and the latter being a set of principles that is strongly inspired by the ARG conditions. Then, we will demonstrate that if an operator complies with the latter, the result of its application is a cogent argument, in accordance with our interpretation of the ARG conditions.

5.1 Basic Principles

In this first subsection, we will propose a group of three basic principles for assumptions operators. To motivate the first principle, consider for example the enthymeme $\mathcal{E} = \langle x, \{y \Leftarrow z\}, \{w\}\rangle$ and an assumptions operator $\oplus_1$, which is defined in such a way that it receives $\mathcal{E}$ and returns $\oplus_1\mathcal{E} = \langle x, \{x \Leftarrow a\}, \{a\}\rangle$. Note that, although it returns an argument, some information that was part of the received enthymeme ($y \Leftarrow z$ and w) was removed. As seen in Example 1.1, this trait is undesirable for assumptions operators. An operator should take into consideration the fact that the sender had a reason for exchanging said information in the first place. Furthermore, if we assume that the exchanged enthymeme comes from an argument which follows Grice's maxims **(Gr1a)**, **(Gr3A)** and also Govier's condition **(GoR)**. As such, by removing premises from the exchanged enthymeme, an operator may be perceived as violating the above mentioned maxims and conditions. Hence, we consider that an assumptions operator should preserve the contents from the original enthymeme. We formalize this intuition by the definition of *lossless operators*.

Principle 1 (Lossless operator)**.** Let $\mathcal{E} \in \mathbb{E}$ and $\oplus$ an assumptions operator. Then, $\oplus$ is a *lossless operator* if and only if $(\mathsf{rules}(\mathcal{E}) \cup \mathsf{premises}(\mathcal{E})) \subseteq (\mathsf{rules}(\oplus\mathcal{E}) \cup \mathsf{premises}(\oplus\mathcal{E}))$.

That is, an assumptions operator that does not remove any premise from the original enthymeme is a lossless operator. Clearly, the operator $\oplus_1$ shown in the example above does not satisfy Principle 1. Note that the assumptions operators $\oplus_s$ and $\oplus_\updownarrow$ introduced in the previous section are lossless operators. We will show this at the end of the subsection.

To motivate the following principle, consider again the enthymeme $\mathcal{E} = \langle x, \{y \Leftarrow z\}, \{w\}\rangle$, and two new assumptions operators $\oplus_2$ and $\oplus_3$, defined in such a way that $\oplus_2\mathcal{E} = \mathcal{E}$, and $\oplus_3\mathcal{E} = \langle x, \{x \leftarrow y, w;\ y \Leftarrow z\}, \{w, \mathbf{z}\}\rangle$. Although both operators are lossless, some flaws can be identified in them. First, note that $\oplus_2$ returns the same enthymeme, hence, the argument that originates the enthymeme could never be decoded from its application. Furthermore, the resulting enthymeme from $\oplus_2$'s application would never comply with Govier's condition **(GoG)**. Second, note that $\oplus_3$ creates only strict knowledge, which we consider to be excessively bold, and furthermore, the resulting enthymeme would not comply with Grice's maxim **(Gr2B)**. In order to address these issues, we propose the concept of *additive operator*. An operator is considered *additive* if it always adds knowledge to the exchanged message, and the added knowledge is not strict (that is, the operator adds neither strict rules nor axiomatic premises).

Principle 2 (Additive operator)**.** Let $\mathcal{E} \in \mathbb{E}$ and $\oplus$ an assumptions operator. Then, $\oplus$ is an *additive operator* if and only if $(\text{rules}(\oplus\mathcal{E}) \cup \text{premises}(\oplus\mathcal{E})) \setminus (\text{rules}(\mathcal{E}) \cup \text{premises}(\mathcal{E}))$ is a non-empty set of defeasible rules and/or ordinary premises.

Note that Principle 2 does not state that the operator should preserve the contents from the original enthymeme. That is, an *assumptions operator* could be *additive* but not *lossless* and vice versa.

To motivate the third principle, recall that Definition 3 imposes no restriction on the returned enthymeme. For instance, consider again the enthymeme $\mathcal{E} = \langle x, \{y \Leftarrow z\}, \{w\}\rangle$, and the operator $\oplus_4$ such that, $\oplus_4\mathcal{E} = \langle o, \{y \Leftarrow z\}, \{w, z\}\rangle$. That is, an operator could return an enthymeme with a different conclusion. In this work, we consider that said modification is undesirable, a sensible operator should preserve the conclusion of the original enthymeme.

Principle 3 (Coherent operator)**.** Let $\mathcal{E} \in \mathbb{E}$ and $\oplus$ an assumptions operator. Then, $\oplus$ is a *coherent operator* if and only if $\text{conc}(\oplus\mathcal{E}) = \text{conc}(\mathcal{E})$.

Now, we will define a class of operators to depict the ones that comply with the *lossless, additive* and *coherent* principles described above. We will denote this class of operators as *basic operators*.

Definition 9 (Basic Operator)**.** Let $\oplus$ be an assumptions operator. Then, $\oplus$ is a *basic operator* if it is *lossless, additive* and *coherent*.

Example 5.1. Recall the *simple operator* $\oplus_s$ defined in Definition 4:

$$\oplus_s\mathcal{E} = \mathcal{E} \cup (\{\text{conc}(\mathcal{E})\} \cap \text{baseless}(\mathcal{E}))$$

- Since $\oplus_s$ does not remove any premise from the original enthymeme, $\oplus_s$ is *lossless.*
- Furthermore, $\oplus_s$ is *coherent*, given that it does not modify the enthymeme's claim.
- However, $\oplus_s$ is not *additive*: consider enthymeme $\mathcal{E} = \langle x, \{x \Leftarrow a, b\}, \emptyset\rangle$, given that $x \notin \text{baseless}(\mathcal{E})$ then $\oplus_s\mathcal{E} = \mathcal{E}$. As such, $\oplus\mathcal{E} \setminus \mathcal{E} = \emptyset$.

Thus, we can conclude that $\oplus_s$ is not a *basic operator.*

In contrast to $\oplus_s$, we will now show that the *needed assumptions operator* introduced in Definition 5 complies with all the basic principles, and thus, is a *basic operator.*

Proposition 1. The needed assumptions operator $\oplus_{\updownarrow}$ is a *basic operator.*

Proof. See Appendix A. □

5.2 Argumentative Principles

In this subsection, we will introduce the second group of principles that we call *argumentative principles*, and are based on the ARG conditions of *cogent* arguments introduced in [11] and described in Section 2. As explained in [11], a premise cannot be deemed acceptable if it is inconsistent with another in the argument, and thus, said argument would neither comply with Govier's **(GoA)** condition nor Grice's maxim **(Gr2A)**. Consider the following example: Let $\mathcal{E} = \langle x, \{y \Leftarrow z\}, \{w\}\rangle$ be an enthymeme, and $\oplus_5$ an operator such that $\oplus_5\mathcal{E} = \mathcal{E} \cup \{x \Leftarrow y,\ z \Leftarrow \neg w,\ \neg w\}$. Observe that, even though an ASPIC+ argument can be obtained from $\oplus\mathcal{E}$, an inconsistency arises from w and $\neg w$, which is an undesirable trait for the operator. Hence, we propose the following principle, to ensure that the set of created assumptions by the operator does not contradict itself or the original enthymeme's knowledge.

Principle 4 (Consistent operator)**.** Let $\oplus$ be a *basic operator* and $\mathcal{E} \in \mathbb{E}$. Then, $\oplus$ is a *consistent operator* if and only if $(\mathsf{literals}(\oplus\mathcal{E}) \setminus \mathsf{literals}(\mathcal{E})) \cap \overline{\mathsf{literals}(\oplus\mathcal{E})} = \emptyset$.

Principle 4 only considers the newly created premises instead of the whole resulting enthymeme. That is, given a *consistent* operator $\oplus$, if the original enthymeme $\mathcal{E}$ has contradictory literals, then $\oplus\mathcal{E}$ will also be inconsistent. Note however that, if $\oplus$ is consistent and $m = \langle \_, \mathcal{E}\rangle$ is a *valid message* under Definition 8, then the resulting enthymeme $\oplus\mathcal{E}$ will be a consistent one.

The following two principles are inspired both by Govier's condition **(GoR)** and Grice's maxims **(Gr1B)** and **(Gr3A)**. Consider for example the enthymeme $\mathcal{E} = \langle x, \{y \Leftarrow z\}, \{w\}\rangle$ and the operator $\oplus_6$ such that $\oplus_6\mathcal{E} = \langle x, \{y \Leftarrow z,\ x \Leftarrow y, w\}, \{w, z, v, u\}\rangle$. Observe that some literals in $\oplus_6\mathcal{E}$ are irrelevant for its conclusion (v and u), and thus, it does not comply with **(GoR)** stated above. In particular, said set of literals corresponds to $\mathsf{disconnected}(\oplus_6\mathcal{E})$. We can conclude then that, if the application of an operator yields an enthymeme with no disconnected literals, then every one of them will be relevant for its conclusion.

Principle 5 (Connective operator)**.** Let $\oplus$ be a *basic operator* and $\mathcal{E} \in \mathbb{E}$ such that $\mathsf{disconnected}(\mathcal{E}) \neq \emptyset$. Then, $\oplus$ is a *connective operator* if and only if $\mathsf{disconnected}(\oplus\mathcal{E}) = \emptyset$.

Consider $\mathcal{E} = \langle x, \{y \Leftarrow z\}, \{w\}\rangle$ and an operator $\oplus_7$ such that its application on $\mathcal{E}_7$ results in $\oplus_7\mathcal{E} = \mathcal{E} \cup \{x \Leftarrow v,\ v \Leftarrow y, w\}$. Observe that $\mathsf{disconnected}(\oplus_7\mathcal{E}) = \emptyset$, which means that $\oplus_7$ is *connective*. Note however that, by adding v into the resulting enthymeme, it creates an unnecessary link between x and the rest of disconnected literals, which could be perceived as not complying with Grice's maxim **(Gr4C)**. To address this, we introduce the concept of *strongly connective* operators as an strengthened version of the above definition. Intuitively, we consider that an operator is *strongly connective* if it is connective and if it only creates assumptions using the existing literals from the original enthymeme.

Principle 6 (Strongly connective operator)**.** Let $\oplus$ be a *connective operator* and $\mathcal{E} \in \mathbb{E}$. Then, $\oplus$ is a *strongly connective operator* if and only if $\mathsf{literals}(\mathcal{E}) = \mathsf{literals}(\oplus\mathcal{E})$.

Then, our following argumentative principle is based on Grice's Maxims **(Gr1B)**, **(Gr4C)** and ARG Condition **(GoR)**. Consider an enthymeme $\mathcal{E} = \langle x, \{x \Leftarrow a, b;\ a \Leftarrow b\}, \emptyset\rangle$, such that the application of operator $\oplus_8$ results in $\oplus_8\mathcal{E} = \mathcal{E} \cup \{b \Leftarrow c, b \Leftarrow d\} \cup \{c, d\}$. Notice that the result of its application could yield an argument (the claim is entailed by the premises in $\oplus_8\mathcal{E}$). In fact, multiple arguments could be decoded from it, for instance: $\mathcal{E} \cup \{b \Leftarrow c,\ c\}$ and $\mathcal{E} \cup \{b \Leftarrow d,\ d\}$. Note however that, in both arguments, some premises created by $\oplus_8$ are irrelevant: for example, in the former the rule $b \Leftarrow d$ is not used, and in the latter, $b \Leftarrow c$. This situation shows that the operator does not comply with **(Gr1B)**,**(Gr4C)** and **(GoR)**. To address this, we define the concept of *concise operators*. Intuitively, we denote that an operator is *concise* if every piece of information in $\oplus\mathcal{E}$ entails a different literal, that is, there are not two pieces of information (rules or premises) that entail the same literal. Before we introduce the formal definition of *concise operators*, we define a function $\mathsf{h}(X)$ such that: $\mathsf{h}(X) = X$ if X is an axiomatic or ordinary premise, otherwise $\mathsf{h}(x \Leftarrow Y) = x$ and $\mathsf{h}(x \leftarrow Y) = x$.

Principle 7 (Concise Operator)**.** Let $\oplus$ be a *basic operator*, $\mathcal{E} \in \mathbb{E}$ holding that $\mathsf{h}(x) \neq \mathsf{h}(y)$ where $x, y \in \mathcal{E}$ and $x \neq y$. We say that $\oplus$ is a *concise operator* if and only if for every $x, y \in \oplus\mathcal{E}$ with $x \neq y$ it holds that $\mathsf{h}(x) \neq \mathsf{h}(y)$.

Finally, our last argumentative principle is based on both the condition **(GoG)** and Grice's maxim **(Gr1A)**. For instance, consider $\mathcal{E} = \langle x, \{y \Leftarrow z\}, \{w\}\rangle$ and the operator $\oplus_9$ such that $\oplus_9\mathcal{E} = \mathcal{E} \cup \{x \Leftarrow v,\ v \Leftarrow y, w\}$, observe that the resulting enthymeme does not have the necessary premises in order to entail $\mathcal{E}$'s claim (in particular, z is not supported by any premise). In contrast, if $\oplus_{10}\mathcal{E} = \mathcal{E} \cup \{x \Leftarrow w\}$ then the resulting enthymeme would have enough grounds to support x (in fact, it would have some irrelevant premises). Hence, our last argumentative principle states that an operator is *conclusive* if its application results in an enthymeme with enough information to decode an ASPIC+ argument from it, and said argument has the same conclusion as the original enthymeme.

Principle 8 (Conclusive operator)**.** Let $\oplus$ be a *basic operator* and $\mathcal{E} \in \mathbb{E}$.Then, given an argumentation theory $\mathbb{AT}$ such that $\mathsf{knowledge}(\mathbb{AT}) = \oplus\mathcal{E}$, we say that $\oplus$ is a *conclusive operator* if there exists an ASPIC+ argument $\mathcal{A}$ from $\mathbb{AT}$ such that $\mathsf{conc}(\mathcal{A}) = \mathsf{conc}(\mathcal{E})$.

Recall that Principle 5 states that if an operator is *connective* then the resulting tuple will have no disconnected literals. Note that this does not imply that its claim will be fully supported (*e.g.,* if $\oplus\mathcal{E} = \langle x, \{x \Leftarrow y, z\}, \{y\}\rangle$, then its premises will not entail x). On the other hand, Principle 8 states that an ASPIC+ argument can be decoded from the resulting enthymeme, but it does not guarantee that every premise in $\oplus\mathcal{E}$ will be part of it (*e.g.,* if $\oplus\mathcal{E} = \langle x, \{x \Leftarrow w,\ y \Leftarrow z\}, \{w, z\}\rangle$, even though an argument can be decoded from $\oplus\mathcal{E}$, the rule $y \Leftarrow z$ will not be a premise in said argument). Finally, if an operator is *concise*, we can guarantee that its application will yield a tuple in which every piece of information entails a different literal. Of course, this does not imply that the claim will be fully supported nor that the result will have no disconnected literals. Nevertheless, if the three principles are complied simultaneously and if the exchanged message is valid, then we can guarantee that there is one and only one ASPIC+ argument $\mathcal{X}$ that can be built from $\mathbb{AT}_{\mathbb{D}}$ such that $\mathsf{knowledge}(\mathcal{X}) = \oplus\mathcal{E}$. This result is shown in the next theorem.

Theorem 1. Let $\oplus$ be a *connective, conclusive* and *concise* operator, $m = \langle \mathsf{ag}, \mathcal{E}\rangle$ a *valid message* such that $\mathbb{D}^{\oplus}_{\mathsf{A}} = [m_1, \ldots, m]$, $\mathcal{E} \in \mathbb{E}$. Then, given $\mathbb{AF}_{\mathbb{D}} = \langle A, D\rangle$, there is one and only one ASPIC+ argument $\mathcal{X} \in A$ such that $\mathsf{conc}(\mathcal{X}) = \mathsf{conc}(\oplus\mathcal{E})$, $\mathsf{knowledge}(\mathcal{X}) = \oplus\mathcal{E}$.

Proof. See Appendix A. □

Then, we define a class of operators that complies with the argumentative principles defined in this section. We define an operator as *(strongly) argumentative* if it is *consistent, (strongly) connective, concise* and *conclusive.*

Definition 10 ((Strongly) Argumentative Operator)**.** Let $\oplus$ be a *basic operator.* $\oplus$ is an *(strongly) argumentative operator* if it is *consistent*, *concise*, *(strongly) connective* and *conclusive.*

We will show that if an assumptions operator $\oplus$ complies with all our proposed argumentative principles, then the resulting enthymeme yields a *cogent argument*, as defined by Govier in [11]. Recall from Section 2 that a *cogent argument* is one that complies with Govier's **ARG conditions**.

- If $\oplus$ is a *consistent operator*, then every newly added premise does not contradict any other premise in the message (neither from the original one nor newly created), and thus, they are acceptable under our interpretation of condition **(GoA)**.
- If $\oplus$ is a *connective operator* then $\mathsf{disconnected}(\oplus\mathcal{E}) = \emptyset$, which means that every premise (both original and newly added) is relevant to the message's conclusion, and thus the resulting argument complies with condition **(GoR)**.
- If $\oplus$ is a *concise operator*, then every piece of information in $\oplus\mathcal{E}$ will entail only one literal. As such, for every literal, there will be only one rule or premise that addresses it, thus avoiding the inclusion of irrelevant pieces of knowledge. In conclusion, this principle yields arguments that comply with condition **(GoR)**.
- Finally, if $\oplus$ is a *conclusive operator*, then there is enough information in $\oplus\mathcal{E}$ to decode an ASPIC+ argument with $\mathsf{conc}(\mathcal{E})$ as its conclusion. As such, the resulting argument also complies with condition **(GoG)**.

Another desirable property that arises from using assumptions operators that comply with all the argumentative principles is that every premise uttered in the dialogue should be relevant. This is shown in Proposition 2. Intuitively, if an operator complies with every argumentative principle and the messages exchanged are valid, then every piece of knowledge in $\mathbb{AT}_{\mathbb{D}}$ will be a premise for an argument in $\mathbb{AF}_{\mathbb{D}}$.

Proposition 2. Let $\mathbb{D}^{\oplus}_{\mathsf{A}} = [m_1, m_2, \ldots, m_n]$ be a dialogue where every message m_i ($i \in [1, n]$) is *valid*, and $\oplus$ is an *argumentative operator*. Then, given $\mathbb{AF}_{\mathbb{D}} = \langle A, D \rangle$ it holds that $(\forall x \in \mathsf{knowledge}(\mathbb{D}^{\oplus}_{\mathsf{A}}))(\exists \mathcal{X} \in A)\ x \in \mathsf{knowledge}(\mathcal{X})$.

Proof. See Appendix A. □

In this section, we have introduced a set of principles to characterize and evaluate the behavior of assumptions operators. Table 2 shows a summary of them. We have shown several examples of operators that do not comply with these principles, and how their application yields undesirable results (*e.g.*, arguments with irrelevant premises, arguments with contradictory premises, and so on). Now we will show in Proposition 3 that the *needed assumptions operator* $\oplus_{\updownarrow}$, introduced in Definition 5, is not only a *basic operator* (see Proposition 1), but also a *strongly argumentative* one.

Proposition 3. The *needed assumptions operator* $\oplus_{\updownarrow}$ is a *strongly argumentative operator*.

Proof. See Appendix A. □

Table 2. Brief summary of the proposed principles.

Category	Principle	Intuition
Basic	*lossless*	The operator preserves the enthymeme's contents.
	additive	The operator always adds defeasible knowledge.
	coherent	The operator preserves the enthymeme's claim
Argumentative	*consistent*	The operator creates knowledge consistent with both the original knowledge and the newly created one.
	connective	The resulting tuple does not have disconnected literals.
	strongly connective	The operator is connective and does not add new literals to the resulting enthymeme.
	concise	The resulting tuple does not have two pieces of information entailing the same literal.
	conclusive	The resulting tuple has enough information to decode an ASPIC+ argument which entail its original conclusion.

6 Semantic Properties of Enthymemic Dialogues

In Section 4, we introduced a dialogue system equipped with an assumptions operator. Then, in Section 5, we proposed a set of principles that serve as guidelines for the construction of suitable operators. In this section, we will introduce a framework to contrast dialogues in which enthymemes are exchanged to the one in which agents send complete arguments. This framework allows us to show that most arguments from the latter are either part of the former, or have an enthymemic counterpart in the former. We will show that if a concrete operator adheres to some of our proposed principles, the former dialogue retains some semantic properties from the latter. This is of utmost importance because it ensures that under some conditions, by exchanging enthymemes instead of complete arguments, a similar outcome can be achieved.

In order to analyze how an assumptions operator influences the dialogue's outcome, and if the outcome corresponds to the one in which arguments were exchanged instead, we next introduce a framework to contrast them. Consider a dialogue $\mathbb{D}^{\oplus}_{\mathsf{A}}$ and a message $m = \langle \mathsf{ag}, \mathcal{E} \rangle$, such that the enthymeme $\mathcal{E}$ comes from argument $\mathcal{A}$.

Then, we can distinguish three distinct dialogue instances. First, the one in which m has not been exchanged yet. Secondly, the instance in which m is introduced into the dialogue. Finally, an instance in which $\mathcal{A}$ is exchanged instead of $\mathcal{E}$. Following this intuition, we introduce the concepts of *base, enthymemic and complete theories and frameworks* as follows.

Definition 11 (Base, enthymemic and complete theories)**.** Let $\mathbb{D}^{\oplus}_{\mathsf{A}} = [m_1, \ldots, m_n]$ be a dialogue and let $m = \langle \mathsf{ag}, \mathcal{E} \rangle$ be a message such that the enthymeme $\mathcal{E} \in \mathbb{E}$, $\mathcal{A} \in \mathbb{A}$ and $\mathcal{E}$ comes from $\mathcal{A}$. Then, we denote the base, enthymemic and complete theories (together with their respective frameworks) as follows:

- The *base theory* $\mathbb{T}_{\mathbb{B}} = \mathbb{AT}_{\mathbb{D}}$, the *base framework* $\mathbb{F}_{\mathbb{B}} = \mathbb{AF}_{\mathbb{D}}$.
- The *enthymemic theory* $\mathbb{T}_{\mathbb{E}} = \mathbb{AT}_{\mathbb{D}'}$ and *enthymemic framework* $\mathbb{F}_{\mathbb{E}} = \mathbb{AF}_{\mathbb{D}'}$, where $\mathbb{D}' = [m_1, \ldots, m_n, m]$.
- The *complete theory* $\mathbb{T}_{\mathbb{C}} = \mathbb{AT}_{\mathbb{D}''}$ and *complete framework* $\mathbb{F}_{\mathbb{C}} = \mathbb{AF}_{\mathbb{D}''}$ where $\mathbb{D}'' = [m_1, \ldots, m_n, m']$, and $m' = \langle \mathsf{ag}, \mathcal{A} \rangle$.[2]

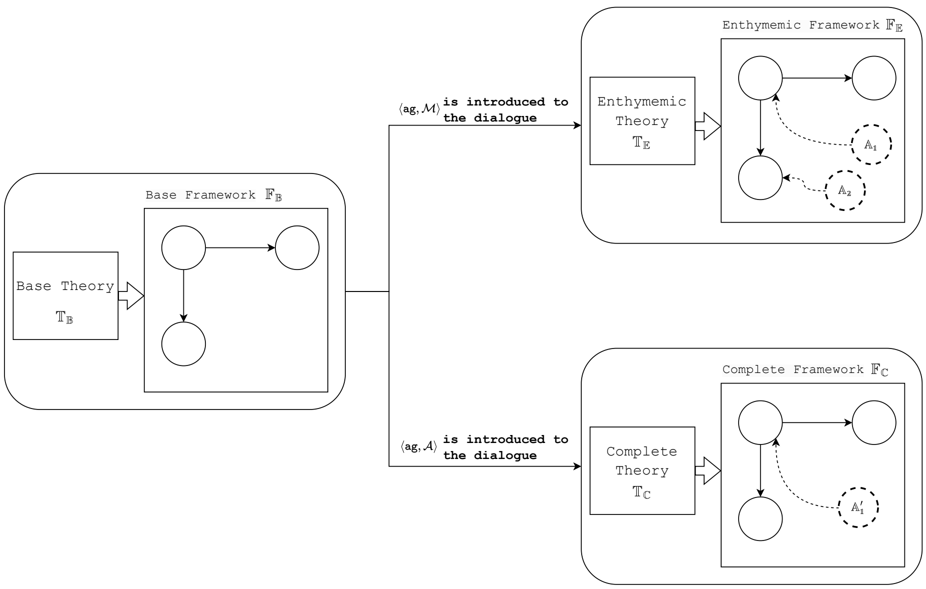

Fig. 4. Base, enthymemic and complete theories together with their corresponding frameworks

Figure 4 provides a visual depiction of the *Base, enthymemic, and complete theories*, as well as their corresponding frameworks. On the leftmost side of the figure, the *base theory* is shown, in which neither $\mathcal{E}$ nor $\mathcal{A}$ is introduced into the dialogue yet. Then, the *enthymemic theory* is depicted in the upper-right side of the figure. This theory is created after the message $\langle \mathsf{ag}, \mathcal{E} \rangle$ is introduced into the dialogue. Finally, in the lower-right side of the figure, the *complete theory* is depicted, which is created from the dialogue where the original argument $\mathcal{A}$ is exchanged instead.

Notation 6.1. Given a theory $\mathbb{T}_{\mathbb{X}}$ and a framework $\mathbb{F}_{\mathbb{X}}$, where $\mathbb{X} \in \{\mathbb{B}, \mathbb{E}, \mathbb{C}\}$, we denote:

- $\mathcal{K}(\mathbb{X}) = \mathcal{K}_n(\mathbb{X}) \cup \mathcal{K}_p(\mathbb{X})$ as the set of axiomatic and ordinary premises in $\mathbb{T}_{\mathbb{X}}$.
- $R(\mathbb{X}) = R_d(\mathbb{X}) \cup R_s(\mathbb{X})$ as the set of defeasible and strict rules in $\mathbb{T}_{\mathbb{X}}$.

[2]Despite the similarity in nomenclature, the *complete theory* does not have any similarity with the *complete semantics* defined in [8]

- We denote the set of arguments of $\mathbb{F}_\mathbb{X}$ as $A(\mathbb{X})$ and the set of defeats as $D(\mathbb{X})$

Example 6.1. Recall dialogue $\mathbb{D}_{\mathsf{A}}^{\oplus\updownarrow} = [m_1, m_2]$ from Example 4.3 and the valid message $m_a = \langle \mathsf{prop}, \mathcal{E} \rangle$, where $\mathcal{E} = \langle \neg a, \{\neg a \leftarrow b, c\}, \{b\} \rangle$ comes from $\mathcal{A}_1$ depicted in Example 4.5. We denote the *base, enthymemic and complete theories* by considering the following dialogues and their corresponding argumentation theory:

- The *base theory* $\mathbb{T}_\mathbb{B} = \mathbb{AT}_{\mathbb{D}_{\mathsf{A}}^{\oplus\updownarrow}}$.
- The *enthymemic theory* $\mathbb{T}_\mathbb{E} = \mathbb{AT}_{(\mathbb{D}_{\mathsf{A}}^{\oplus\updownarrow})'}$, where $(\mathbb{D}_{\mathsf{A}}^{\oplus\updownarrow})' = [m_1, m_2, m_a]$
- The *complete theory* $\mathbb{T}_\mathbb{C} = \mathbb{AT}_{(\mathbb{D}_{\mathsf{A}}^{\oplus\updownarrow})''}$ where $(\mathbb{D}_{\mathsf{A}}^{\oplus\updownarrow})'' = [m_1, m_2, m'_a]$, such that $m'_a = \langle \mathsf{prop}, \mathcal{A}_1 \rangle$, where $\mathcal{A}_1 = \langle \neg a, \{\neg a \leftarrow b, c,\ c \Leftarrow b, d\}, \{b, \mathbf{d}\} \rangle$

$$\mathbb{T}_\mathbb{B} = \left\{ \begin{array}{lll} x \Leftarrow y, b, & y, & b, \\ \neg y \leftarrow z, a, & \underline{z}, & \underline{a}, \\ \underline{\neg x \Leftarrow \neg y} & & \end{array} \right\} \quad \mathbb{T}_\mathbb{E} = \left\{ \begin{array}{lll} x \Leftarrow y, b, & y, & b, \\ \neg y \leftarrow z, a, & \underline{z}, & \underline{a}, \\ \underline{\neg x \Leftarrow \neg y,} & \underline{c}, & \\ \neg a \leftarrow b, c & & \end{array} \right\} \quad \mathbb{T}_\mathbb{C} = \left\{ \begin{array}{lll} x \Leftarrow y, b, & y, & b, \\ \neg y \leftarrow z, a, & \underline{z}, & \underline{a}, \\ \underline{\neg x \Leftarrow \neg y,} & \mathbf{d}, & \\ \neg a \leftarrow b, c, \quad c \Leftarrow b, d & & \end{array} \right\}$$

Furthermore, Figure 5 depicts the *base, enthymemic and complete frameworks* $\mathbb{F}_\mathbb{B}$, $\mathbb{F}_\mathbb{E}$ and $\mathbb{F}_\mathbb{C}$ respectively following the layout from Figure 4. As the reader may observe, every piece of information in $\mathbb{T}_\mathbb{B}$ is part of $\mathbb{T}_\mathbb{E}$ and $\mathbb{T}_\mathbb{C}$. As such, every argument in $\mathbb{F}_\mathbb{B}$ is also present in $\mathbb{F}_\mathbb{E}$ and $\mathbb{F}_\mathbb{C}$. Then, Figure 5 only describes the structure of the newly created arguments in $\mathbb{F}_\mathbb{E}$ and $\mathbb{F}_\mathbb{C}$.

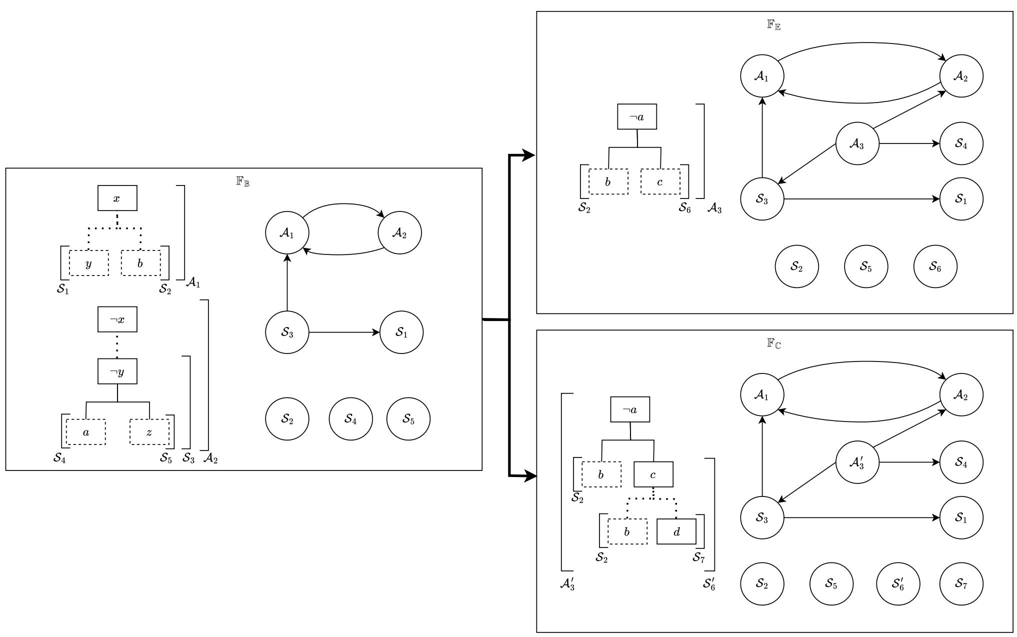

Fig. 5. Arguments obtainable from $\mathbb{T}_\mathbb{B}$,$\mathbb{T}_\mathbb{E}$ and $\mathbb{T}_\mathbb{C}$ as well as their respective argumentation framework.

The example above allows us to illustrate some interesting properties regarding the differences and similarities between the *enthymemic and complete frameworks*. For instance, even though some arguments in $\mathbb{F}_\mathbb{C}$ are not part of $\mathbb{F}_{\mathbb{E}}$ and vice versa, both frameworks share a set of arguments which correspond to the ones in $\mathbb{F}_{\mathbb{B}}$. This property is formalized by the following remark.

Remark 3. If $\mathcal{A} \in A(\mathbb{B})$, then $\mathcal{A} \in A(\mathbb{C}) \cap A(\mathbb{E})$.

In addition, note that the set of knowledge that differs between $\mathbb{T}_{\mathbb{C}}$ and $\mathbb{T}_{\mathbb{E}}$ corresponds exactly to the information present in $\mathcal{A}$ but not in $\mathcal{E}$. In contrast, the knowledge that is part of $\mathbb{T}_{\mathbb{E}}$ but not $\mathbb{T}_{\mathbb{C}}$ corresponds to the information created by $\oplus$.

Remark 4. Given a rule or premise z, if $z \in \mathsf{knowledge}(\mathbb{T}_{\mathbb{E}}) \setminus \mathsf{knowledge}(\mathbb{T}_{\mathbb{C}})$ then $z \in \oplus\mathcal{E} \setminus \mathcal{E}$ and vice-versa. Otherwise, if $z \in \mathsf{knowledge}(\mathbb{T}_{\mathbb{C}}) \setminus \mathsf{knowledge}(\mathbb{T}_{\mathbb{E}})$ then $z \in \mathcal{A} \setminus \mathcal{E}$ and vice-versa.

As we mentioned above, some arguments in $\mathbb{F}_{\mathbb{C}}$ are not part of $\mathbb{F}_{\mathbb{E}}$. Note however that in Example 6.1, some of them have an enthymemic counterpart in $\mathbb{F}_{\mathbb{E}}$, such as $\mathcal{A}_3'$ in $\mathbb{F}_{\mathbb{C}}$ and $\mathcal{A}_3$ in $\mathbb{F}_{\mathbb{E}}$. Even though $\mathcal{A}_3'$ is not part of $A(\mathbb{E})$, $\mathcal{A}_3$ shares its conclusion and some of its contents. In this work, we denote that the latter one is the *proxy argument* of $\mathcal{A}_3'$. Intuitively, given an argument $\mathcal{A}$ in $\mathbb{F}_{\mathbb{C}}$, its *proxy* is the argument in $\mathbb{F}_{\mathbb{E}}$ which shares the most information with $\mathcal{A}$, as well as its conclusion.

Definition 12 (Proxy Argument)**.** Let $\mathcal{A} \in A(\mathbb{C}) \setminus A(\mathbb{E})$, $\mathcal{B} \in A(\mathbb{E})$ and let $\mathcal{E}' = \oplus\mathcal{E} \setminus \mathcal{E}$, we denote that $\mathcal{B}$ is the *proxy argument* of $\mathcal{A}$ iff $\mathcal{B}$ is the maximal argument such that $\mathsf{conc}(\mathcal{B}) = \mathsf{conc}(\mathcal{A})$ and $\mathsf{knowledge}(\mathcal{B}) \setminus \mathcal{E}' \subseteq \mathsf{knowledge}(\mathcal{A})$.

Example 6.2. Recall Figure 5 from Example 6.1. Arguments $\mathcal{A}_3$ and $\mathcal{S}_6$ are the proxy arguments of $\mathcal{A}_3'$ and $\mathcal{S}_6'$ respectively. Note however that some arguments in $\mathbb{F}_{\mathbb{C}}$ do not have a proxy in $\mathbb{F}_{\mathbb{E}}$, such as $\mathcal{S}_7$.

As the example above shows, some arguments in the complete framework do not have their proxy in the enthymemic one. This situation varies depending on the equipped operator (for instance, if the simple operator $\oplus_s$ were to be used in Example 6.1, most of the arguments in $\mathbb{F}_{\mathbb{C}}$ would not have a proxy in $\mathbb{F}_{\mathbb{E}}$). As such, we will now show that if the equipped operator is *connective* and *conclusive*, then every argument in $\mathbb{F}_{\mathbb{C}}$ (except $\mathcal{A}$'s sub-arguments) is either part of $\mathbb{F}_{\mathbb{E}}$, or has a proxy argument in $\mathbb{F}_{\mathbb{E}}$.

Proposition 4. Let $\mathbb{D}_{\mathsf{A}}^{\oplus}$ be a dialogue where every message is *valid*, $\oplus$ is *strongly argumentative* and let $m = \langle \mathsf{ag}, \mathcal{M} \rangle$ be a *valid* message such that $\mathcal{M}$ comes from argument $\mathcal{A}$. Then, for every argument $\mathcal{X} \in (A(\mathbb{C}) \setminus \mathsf{sub}(\mathcal{A})) \cup \{\mathcal{A}\}$, either $\mathcal{X} \in A(\mathbb{E})$ or there exists an argument $\mathcal{X}' \in A(\mathbb{E})$ such that $\mathcal{X}'$ is the proxy argument of $\mathcal{X}$.

Proof. See Appendix A. □

As the reader may have noticed, $\mathcal{A}$ is included in the set of arguments that has a proxy argument in $\mathbb{F}_{\mathbb{E}}$. In the proof of the proposition above, we have shown that the proxy argument of $\mathcal{A}$ corresponds to $\oplus\mathcal{M}$. Said result will be useful for later properties and results, as such, it is formalized in the following remark.

Remark 5. Let $\mathbb{D}_{\mathsf{A}}^{\oplus}$ a dialogue where $\oplus$ is *strongly argumentative*, and $m = \langle \mathsf{ag}, \mathcal{M} \rangle$ be a *valid message* such that $\mathcal{M}$ comes from argument $\mathcal{A}$. Then, $\oplus\mathcal{M} \in A(\mathbb{E})$ is the proxy argument of $\mathcal{A} \in A(\mathbb{C})$.

Following this proposition, we introduce the concept of *Proxy Set*, which will be used in later properties and results. Intuitively, given a set of arguments S in $\mathbb{F}_{\mathbb{C}}$, its proxy set S' in $\mathbb{F}_{\mathbb{E}}$ contains the arguments that are part of S and $\mathbb{F}_{\mathbb{E}}$, and for each of the remaining arguments in S, there exists a proxy argument in S'.

Definition 13 (Proxy Set)**.** Given a set $S \subseteq A(\mathbb{C})$, then $S' = (S \cap A(\mathbb{E})) \cup P$ is the Proxy Set for S where $(\forall \mathcal{X} \in S \setminus A(\mathbb{E}))$ there exists an argument $\mathcal{X}' \in P$ such that, $\mathcal{X}'$ is the proxy argument for $\mathcal{X}$.

Example 6.3. Recall frameworks $\mathbb{F}_{\mathbb{C}}$ and $\mathbb{F}_{\mathbb{E}}$ from Example 6.1. Given the set of arguments $\{\mathcal{A}_2, \mathcal{S}_1, \mathcal{A}_3'\} \subset A(\mathbb{C})$, its proxy set in $\mathbb{F}_{\mathbb{E}}$ is $\{\mathcal{A}_2, \mathcal{S}_1, \mathcal{A}_3\}$.

As we have shown in Proposition 4, and more specifically in Remark 5, $\oplus\mathcal{M}$ is the proxy argument of $\mathcal{A}$. As such, if $\mathcal{A}$ is part of some set S in $\mathbb{F}_{\mathbb{C}}$, then $\oplus\mathcal{E}$ will be part of the proxy set S' of S.

Remark 6. Let $S \subseteq A(\mathbb{C})$ such that $\mathcal{A} \in S$. If $\oplus$ is *strongly argumentative*, then $\oplus\mathcal{M} \in S'$ where $S' \subseteq A(\mathbb{E})$ is the proxy set of S.

We are now ready to explore which properties from the *complete framework* are preserved by the *enthymemic* one. As the reader may recall from sub-section 2.2, the defeat relation is defined by considering if an argument $\mathcal{A}$ attacks $\mathcal{B}$ and if $\mathcal{B} \not\succ \mathcal{A}$ holds, where $\preceq$ is a *preference ordering* over arguments. Given that our aim is not to delve in how said ordering is defined, we will assume from now on that the proxy arguments in $\mathbb{F}_{\mathbb{E}}$ preserves the strength of their complete counterpart in $\mathbb{F}_{\mathbb{C}}$. Then, in the aforementioned sub-section, the Dung Semantics [8] are described. More specifically, a set S is denoted as conflict-free if it does not contain any pair of arguments which defeat each other. We will now show that if $\mathcal{A}$ is part of a conflict-free set S, $\oplus$ is *strongly argumentative* and m is a valid message, then S's proxy set is also conflict-free.

Theorem 2. Let $\mathbb{D}^{\oplus}_{\mathsf{A}}$ a dialogue where $\oplus$ is *strongly argumentative*, and $m = \langle \mathsf{ag}, \mathcal{M} \rangle$ be a *valid message* such that $\mathcal{M}$ comes from argument $\mathcal{A}$. Let $S \subseteq A(\mathbb{C})$ be a conflict-free set such that $\mathcal{A} \in S$, then the proxy set S' of S is also conflict-free.

Proof. See Appendix A. □

Before we delve into *acceptability* and *admissibility*, we need to introduce the concept of non-weakened messages. Given an argument $\mathcal{A}$ and a *valid message* $m = \langle \mathsf{ag}, \mathcal{M} \rangle$ such that $\mathcal{M}$ is based of $\mathcal{A}$, we denote that m is a non-weakened message if it contains all strict knowledge that is in conflict with other arguments in $\mathbb{F}_{\mathbb{C}}$.

Definition 14 (Non-weakened message)**.** Let $\mathbb{D}^{\oplus}_{\mathsf{A}}$ be a dialogue and let $m = \langle \mathsf{ag}, \mathcal{M} \rangle$ be a *valid* message such that $\mathcal{M}$ comes from $\mathcal{A}$. Then, m is a *non-weakened* message if $\forall \mathcal{B} \in \mathsf{sub}(\mathcal{A})$ such that $(\mathcal{B}, C) \in D(\mathbb{C})$ one of the following complies:

- $(\mathsf{topRule}(\mathcal{B}) = x \leftarrow Y) \rightarrow (x \leftarrow Y \in \mathsf{strRules}(\mathcal{M}))$, or
- $\mathsf{conc}(\mathcal{B}) \in \mathsf{axPremises}(\mathcal{A}) \rightarrow \mathsf{conc}(\mathcal{B}) \in \mathsf{axPremises}(\mathcal{M})$

Recall that an argument $\mathcal{X}$ is acceptable with respect to a set S if for every defeater of $\mathcal{X}$, there exists an argument in S which defeats it. We will show that under certain conditions, the enthymemic framework preserves said property. More specifically, we show that if $\mathcal{A}$ is acceptable w.r.t a set S in the complete framework, then $\oplus\mathcal{M}$ will be acceptable w.r.t the proxy set of S. In order for this to hold, the following conditions must be complied: First, $\oplus$ must be must be *strongly argumentative*. Next, m must be a non-weakened message. Furthermore, every sub-argument of $\mathcal{A}$ which defends it must be part of $\oplus\mathcal{M}$. Finally, $\oplus\mathcal{M}$'s contents cannot be shared with other arguments which attack $\oplus\mathcal{M}$.

Theorem 3. Let $m = \langle \mathsf{ag}, \mathcal{M} \rangle$ be a *non-weakened* message such that $\mathcal{M}$ comes from argument $\mathcal{A}$, let $\mathbb{D}^{\oplus}_{\mathsf{A}}$ be a dialogue where $\oplus$ is *strongly argumentative*. If $\mathcal{A}$ is acceptable w.r.t S in $\mathbb{F}_{\mathbb{C}}$, $S \cap \mathsf{sub}(\mathcal{A}) \subseteq \mathsf{sub}(\oplus\mathcal{M})$ and $\nexists X \in \mathsf{sub}(\oplus\mathcal{M}) \cap \mathsf{sub}(\mathcal{B})$ such that $(\mathcal{B}, \oplus\mathcal{M}) \in D(\mathbb{E})$. Then, $\oplus\mathcal{M}$ is acceptable w.r.t. S', where S' is the proxy set of S.

Proof. See Appendix A. □

Finally, the last semantic property preserved by the enthymemic framework is *admissibility*. Recall that a set S is considered *admissible* if it is conflict-free and each argument in S is defended by said set. We will show that under certain conditions, if $\mathcal{A}$ is part of an admissible set S in the complete framework, then $\oplus\mathcal{M}$ is part of the admissible set S', which is the proxy set of S. In order for this property to hold, the following must comply: First, $\oplus$ must be *strongly argumentative*. Then, m must be a non-weakened message. Next, ll of sub-arguments of $\mathcal{A}$

are part of S, must be part of $\oplus\mathcal{M}$. Lastly, $\oplus\mathcal{M}$ must not provide knowledge to an argument which attacks an argument in S'.

Theorem 4. Let $m = \langle \mathsf{ag}, \mathcal{M} \rangle$ be a *non-weakened* message such that $\mathcal{M}$ comes from argument $\mathcal{A}$, let $\mathbb{D}^{\oplus}_{\mathsf{A}}$ be a dialogue where $\oplus$ is *strongly argumentative*. Given an admissible set $S \subseteq A(\mathbb{C})$ such that $\mathcal{A} \in S$, then the proxy set S' of S is an admissible set in $\mathbb{F}_{\mathbb{E}}$ if $\mathsf{sub}(\mathcal{A}) \cap S \subseteq \mathsf{sub}(\oplus\mathcal{M})$ and $(\forall \mathcal{Y} \in A(\mathbb{C}))$ if $(\mathcal{Y}, \mathcal{X}) \in D(\mathbb{E})$ then $\mathsf{sub}(\oplus\mathcal{M}) \cap \mathsf{sub}(\mathcal{Y}) = \emptyset$.

Proof. See Appendix A. □

As stated above, some conditions have to be met in order for the semantic properties to hold. We believe that said conditions could aid an agent when defining strategies for enthymemes' exchange. For instance, if an agent detects that by exchanging $\mathcal{A}$ said argument is part of an admissible set S in $\mathbb{F}_{\mathbb{C}}$, and if said agent wants to preserve admissibility (*i.e.,* that $\oplus\mathcal{M}$ is also part of an admissible set), then it should consider the conditions above when selecting which premises to omit in $\mathcal{M}$ (*e.g.,* it should preserve every premise that is part of $\mathsf{sub}(\mathcal{A}) \cap S$).

To sum up, in this section we defined the concept of *base, complete, enthymemic theory and framework*. We have also shown that under some restrictions, most arguments from the *complete framework* are either part of the *enthymemic framework*, or have an enthymemic counterpart in the *enthymemic framework*. Finally, we have shown that under some conditions, the enthymemic preserves some semantic properties of the complete one: (i) if $\mathcal{A}$ is part of a conflict-free set S, then the proxy set of S is also conflict-free, (ii) if $\mathcal{A}$ is acceptable w.r.t a set S, then $\oplus\mathcal{M}$ is acceptable w.r.t S' (where S' is the proxy set of S), and (iii) Given a admissible set S such that $\mathcal{A}$ is part of S, then its proxy set S' is also admissible.

7 Related Work

In this section, we will discuss some related work in the literature and contrast it with our proposal. As far as we know, most approaches in the area either (i) provide the means to explicitly complete enthymemes [28, 27, 30, 22, 16]; (ii) use common knowledge (which is known beforehand by the participants of the dialogue) as the means to fill the gaps of missing information in said enthymemes [20]; or (iii) use the mental model of the participants involved to encode and decode the uttered enthymemes [6]. In either case, the mechanism used in those works is a fixed one. In contrast, our approach defines an abstract mechanism for assumptions generation, and we propose a set of principles which represents some rationality criteria based in the Grice's Maxims and ARG Conditions. As far as we know, no other work in the area has explored this kind of approach when dealing with enthymemes. Furthermore, we formally proved that at least, one concrete operator (*i.e.,* the needed assumptions operator $\oplus_{\updownarrow}$) complies with the proposed principles. Also, we have shown that if a concrete operator complies with these principles, the resulting dialogue preserves some semantic properties of the one in which complete arguments are sent.

As mentioned in Section 3, some works in the literature have explored enthymemic dialogues and their representation. First, works such as [7, 6, 14, 22] use deductive and logic-based argumentation as the underlying enthymemic representation. In contrast, [16] uses the DeLP structured argumentation formalism to represent both the involved agents, as well as the exchanged enthymemes. Finally, other works like [13, 28, 29, 27, 30] adapt the ASPIC+ argumentation formalism to accommodate enthymemic arguments. As the reader may recall from Section 1, in [24], enthymemes are described by the following definition: "An argument is said to be an enthymeme if there are premises needed to make the argument valid that are only tacitly, but not explicitly stated or advanced as part of the argument.". This definition only refers to *backward-extendable* enthymemes (*i.e.,* arguments with unstated premises). Note however that there are different definitions for enthymemes in the literature, such as the one in [26]: "An enthymeme, in current usage, is an argument that has one or more premises, or possibly a conclusion, not explicitly stated in the text, but that needs to have these propositions explicitly stated to extract the complete argument from the text.”. This definition allow for more kinds of enthymemes to be depicted, such as *forward-extendable* enthymemes (*i.e.,* enthymemes in which the claim is unstated). From the aforementioned works, some accommodate only *backward-extendable* enthymemes [16, 7, 14, 13]. Others like [29, 30] only accommodate *forward-extendable* enthymemes. Finally, works like [28, 27, 6, 22] provide a representation that allows for both *backward-extendable* and *forward-extendable* enthymemes. In this work, we provided a representation of arguments and enthymemes by encapsulating the original ASPIC+ arguments’ contents into tuples similarly to [16]. Our motivation of using this representation is twofold: First, given that we are interested in how assumptions are created, we decided that said assumptions should be represented as pieces of knowledge. As such, we thought that encapsulating the ASPIC+ arguments’ contents into tuples was a sensible approach to analyze how the assumptions could lead to the creation of new arguments. On the other hand, given that this formalization is similar to [16], our aim in future work is to provide a framework for enthymeme’s representation that would allow for agents to exchange enthymemes independently of how their knowledge is represented (*e.g.,* ASPIC+, DeLP or ABA). Note however that this formalization allows only for *backward-extendable* enthymemes. We recognize that this is a limitation of our work and we desire to explore how to accommodate more kinds of enthymemes in future work.

In [20], they introduce an argumentation framework for multi-agent systems, in which agents can exchange enthymemic messages by omitting information that is considered part of a *common knowledge set*. Said knowledge, which is assumed to exist beforehand as part of organizational knowledge, is used in conjunction with Argumentation Schemes (*i.e.,* common reasoning patterns from which arguments can be instantiated) to guide argument reconstruction. As such, an agent can exchange an enthymemic message by first creating a complete argument, and then removing both the organizational knowledge as well as the rules involved in the Argumentation Scheme used. The recipient of said message can then reconstruct the original argument by using information from the *common knowledge set* (which is the union of the organizational knowledge set and the argumentation schemes set).

Our proposal differs with theirs in several ways: Firstly, our definition of assumptions operator is abstract enough to encapsulate their mechanism for argument reconstruction. As mentioned above, their proposal guarantees that the recipient of an enthymeme can decode the exact argument from which it was created. Even though we cannot guarantee the same results, as shown in Section 6, our proposal guarantees that under certain conditions, most arguments in the complete dialogue have an enthymemic counterpart in the enthymemic dialogue. Another point of comparison is the fact that both their and our work use the Grice’s Maxims as a point of inspiration. In their case, they show that the proposed framework complies with some of them. In contrast, we designed the principles presented in Section 5 based on said maxims, as well as the ARG conditions of cogent arguments. Furthermore, we show that an operator that complies with the aforementioned principles will yield a cogent argument under our interpretation of the ARG conditions.

In [6], the authors define a framework for handling enthymemes inspired by some concept of *Relevance Theory*. In their work, they provide a mechanism for encoding enthymemes from an intended argument by omitting information from the *cobase* (*i.e.,* information that the sender believes the recipient knows), and a mechanism for decoding enthymemes into arguments using the same logic (*i.e.,* using information from its *cobase* to create the newly formed argument).

When comparing their proposal with our Needed Assumptions Operator, some interesting differences arise: In some optimal scenarios their proposal behaves perfectly. For instance, if both agents share the same *cobase*, then the deconstructed argument is the same as the one intended by the sender of the enthymeme. Even though this is a desirable result, when the agents’ *cobase* differ, then there are no guarantees that the argument created is similar to the one intended by the sender. Furthermore, if the recipient’s *cobase* is not adequate, there are no guarantees that the recipient can decode (and thus, deconstruct) an argument at all. In contrast, the needed assumptions operator always guarantees that an argument can be created from any *valid enthymemic message* (Theorems 1, 3), although it may not be the same as the original argument. As mentioned above, their approach allows the sender to construct enthymemes with $\top$ as its conclusion, which means that they allow for enthymemes with unstated claims to be sent and subsequently deconstructed into arguments. Even though we have not delved in how to deal with enthymemes with unstated claims, we are very interested in said subject and aim to explore it in future work.

In [17], the author provides a framework in which common knowledge is used to decode arguments from enthymemes (similarly to [20]), but also provides the means to deal with mistaken attacks which could occur if an agent decodes "incorrectly" an argument. In his work, each agent has an *enthymeme-based abstract framework* which contain *certain* attacks (between complete arguments) and *questionable* attacks (attacks that involve at least one enthymeme), and can receive additional information regarding attacks or acceptance of arguments in said framework (*e.g., argument X should attack Y*). The newly added information then modifies the agent's AF, which leads to modifications in the internal structure of the enthymemes involved (in order to comply with the newly received restriction). Even though we consider this work to be an interesting extension to [20], by using common knowledge as the means to complete enthymemes (both initially and after modifications of the AF), it has the same drawbacks as [20]: It assumes that the common knowledge set exists and is adequate, which makes it a less general solution. Furthermore, their work does not address how an enthymeme that cannot be decoded into an argument should be dealt with.

In [25], Walton defines a theory for enthymemes and their use in dialogues. In his work, the dialogue system CBVK is defined, which deals with enthymeme by not only considering a common knowledge set and the set of argumentation schemes (as seen in [20]), but also using the commitments of the participants involved. The commitments store of an agent represents the knowledge that said agent uttered (and thus, committed to) during the dialogue. Even though Walton's proposal is an informal one (that is, their work does not formally define the mechanism which encodes an argument from an enthymeme), we consider it to be a step forward in the area. Note however that its usage should also contemplate the cases in which the common knowledge set, the schemes and the commitment store are not sufficient to decode an enthymeme, by incorporating a mechanism that generates assumptions based solely on the information of the enthymeme exchanged, such as the one presented in this work.

As briefly mentioned above, Xydis's works have explored enthymemic dialogues and their representation based on the ASPIC+ formalism [28, 29, 27, 30]. In [28], a dialogue system for enthymemic dialogues is defined, in which agents exchange enthymemes and have the means to provide both backward and forward extensions of said enthymemes, as well as provide the means to handle misunderstandings during said dialogues. [29] only provides the means to handle forward-extendable enthymemes, but they show that under some restrictions, there is a correspondence (**SC** correspondence) between the status of the moves made during a dialogue and the status of the arguments instantiated by the contents of said moves. This work is then extended in [30] to provide a more flexible locution for forward extensions of enthymemes, in which agents can reveal missing parts of the enthymemes in parts rather than all at once. Furthermore, said work defines a rational strategy to generate dialogues in which the aforementioned **SC** correspondence is preserved. Finally, in [27], the **SC** correspondence is proven for a dialogue system in which agents can exchange enthymemes and have the means to address possible misinterpretations that arise from the use of said enthymemes. Our work differs to theirs in several aspects: First, although we define a dialogue system for handling enthymemes, ours is limited in terms of the amount of locutions defined (For instance, we do not define locutions for handling misunderstandings nor to provide forward extensions). Secondly, our representation of enthymemes differs from theirs. In our work, enthymemes and arguments are built based on the contents of ASPIC+ arguments. In contrast, their work defined enthymemes as a forest of trees, which is the result of pruning some of the ASPIC+ arguments' nodes and edges. Finally, our work does not aim to prove the **SC** correspondence, rather to define the concept of assumptions operators, and provide guidelines to create good operators.

Finally, [16] introduces a dialogue system for persuasion dialogues in which agents exchange enthymemes in order to defend their viewpoint. Since their focus was on the definition of a dialogue system for persuasion dialogues, their proposal provides the means to request and provide completions of the exchanged enthymemes. Furthermore, their work shows that under certain conditions, the participants involved can acquire the necessary information to align their mental model to the outcome of the dialogue. In contrast, our aim was to define principles that the assumptions operators should comply with. As such, the dialogue system presented in this work has fewer types of moves. Furthermore, rather than delving into how agents behave during persuasion dialogues, our work aims to show that an operator that complies with the aforementioned principles allows for dialogues in which the semantic outcome is similar to the one in which complete arguments are exchanged.

To sum up, as far as we know, most works deal with enthymemic messages by providing a fixed mechanism which completes them via common knowledge, commitments made during the dialogue, mental model of the participants involved, or a combination of them. In contrast, we aim to define some principles which every desirable mechanism should comply. We then present a concrete operator (*i.e.,* the needed assumptions operator) which complies with those principles. It is important to remark that every approach has its advantages and drawbacks, which depend on the context in which it is used. For instance, if the dialogue involve two agents that have not interacted before, it is not sensible to use a mechanism which involves common knowledge (in particular if those agents differ in how that knowledge is represented). When dealing with enthymemes in dialogues, there should exist a mechanism which uses both common knowledge, and also generates assumptions when the said knowledge is not sufficient or adequate. In future work, we aim to explore how to combine both approaches.

8 Conclusions

In this paper we proposed the concept of *assumptions operator* which formalizes a mechanism for generating assumptions from enthymemes, in order to decode arguments in enthymemic-based dialogues. Also, we proposed two group of principles, *basic* and *argumentative*, that serve as guidelines to the construction of such kind of operators. Those principles were inspired by Grice's *Maxims of conversation* [12], and also by Govier's *ARG conditions of cogent arguments* [11]. We showed that if a concrete operator complies with a subset of these principles, the result of its application will always yield an argument. To introduce the mentioned set of principles, and explore how operators adhere to them, we have introduced a formalization of enthymemes in ASPIC+ alongside a simple enthymeme-based dialogue system equipped with an assumptions operator.

In Section 6, we defined a framework to contrast the dialogue in which enthymemes are exchanged and the one in which complete arguments are sent instead. Under this framework, we have shown that if the operator adheres to the aforementioned principles, the former dialogue preserves some semantic properties (*conflict-freeness, acceptability, admissibility*) from the latter one. This is of utmost importance, as it guarantees that if an agent exchanges an enthymeme, the dialogue's outcome will be similar to that of exchanging complete arguments.

Future work has multiple directions. Firstly, we defined a set of principles for general argumentative dialogues. Our intention is to expand those principles for more specific types of dialogues, such as *negotiation, inquiry or information-seeking* dialogues. Secondly, as we mentioned in Section 7, we are interested in defining operators that utilize external sources of information, such as commitment stores, common knowledge sets and so on. Finally, as stated in Section 6, the conditions of Theorems 2,3, and 4 can be employed to define strategies that agents may use during argumentative dialogues. We are highly interested in defining said strategies for collaborative-competitive dialogues, such as *persuasion dialogues.*

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A Proofs

This appendix contains all proofs for every proposition and theorem shown during this paper.

Proposition 1. The needed assumptions operator $\oplus_{\updownarrow}$ is a *basic operator.*

Proof. First of all, by $\oplus_{\updownarrow}$ definition, we know that $\text{conc}(\oplus_{\updownarrow}\mathcal{M}) = \text{conc}(\mathcal{M}) = h$, as such, we can trivially prove that $\oplus_{\updownarrow}$ is *coherent.* Furthermore, we know that $\oplus_{\updownarrow}\mathcal{M} = \langle h, R \cup R', F \cup F' \rangle$, which means that $\text{rules}(\mathcal{M}) \subseteq \text{rules}(\oplus_{\updownarrow}\mathcal{M})$, and $\text{premises}(\mathcal{M}) \subseteq \text{premises}(\oplus_{\updownarrow}\mathcal{M})$, which means that $\mathcal{M} \subseteq \oplus_{\updownarrow}\mathcal{M}$, which lead us to conclude that $\oplus_{\updownarrow}$ is also *lossless.*

Finally, given that $\mathcal{M} \in \mathbb{E}$, there exists at least one literal $l \in \text{baseless}(\mathcal{M})$ (Remark 2). As such, by $\oplus_{\updownarrow}$'s definition, $\oplus\mathcal{M} \setminus \mathcal{M}$ is a non-empty set (there must exist at least one rule or premise that addresses l), where each rule is defeasible and each premise is ordinary. As such, we can conclude that $\oplus_{\updownarrow}$ is *additive* too.

□

Theorem 1. Let $\oplus$ be a *connective, conclusive* and *concise* operator, $m = \langle \mathsf{ag}, \mathcal{E} \rangle$ a *valid message* such that $\mathbb{D}^{\oplus}_{\mathsf{A}} = [m_1, \ldots, m]$, $\mathcal{E} \in \mathbb{E}$. Then, given $\mathbb{AF}_{\mathbb{D}} = \langle A, D \rangle$, there is one and only one ASPIC+ argument $\mathcal{X} \in A$ such that $\mathsf{conc}(\mathcal{X}) = \mathsf{conc}(\oplus\mathcal{E})$, $\mathsf{knowledge}(\mathcal{X}) = \oplus\mathcal{E}$.

Proof. Let us consider a *valid message* $m = \langle \mathsf{ag}, \mathcal{E} \rangle$ such that $\mathbb{D}^{\oplus}_{\mathsf{A}} = [m_1, \ldots, m]$, $\mathcal{E} \in \mathbb{E}$, and an operator $\oplus$ that is *connective, concise* and *conclusive.* Then, given the argumentation theory $\mathbb{AT}_{\mathbb{D}}$ and the argumentation framework $\mathbb{AF}_{\mathbb{D}} = \langle A, D \rangle$ from $\mathbb{D}^{\oplus}_{\mathsf{A}}$, we aim to prove that there is one and only one ASPIC+ argument $\mathcal{X} \in A$ such that (i) $\mathsf{conc}(\mathcal{X}) = \mathsf{conc}(\oplus\mathcal{E})$, and (ii) $\mathsf{knowledge}(\mathcal{X}) = \oplus\mathcal{E}$.

First of all, by Definition 7, we know that $\oplus\mathcal{E} \subseteq \mathsf{knowledge}(\mathbb{D}^{\oplus}_{\mathsf{A}})$, furthermore, given that $\oplus$ is conclusive, we know that there exists an ASPIC+ argument $\mathcal{X} \in A$ such that $\mathsf{conc}(\mathcal{X}) = \mathsf{conc}(\oplus\mathcal{E})$, thus we can conclude that (i) holds. Furthermore, given that $\oplus$ is *conclusive*, we know that $\mathcal{X}$ is an argument such that its knowledge is a subset of the knowledge in AT, being AT an argumentation theory created from $\oplus\mathcal{E}$'s contents. As such, we can deduce that $\mathcal{X}$ is solely formed by $\oplus\mathcal{E}$'s contents, that is: $\mathsf{knowledge}(\mathcal{X}) \subseteq \oplus\mathcal{E}$. Then, we assume for contradiction sake that $\mathsf{knowledge}(\mathcal{X}) \subset \oplus\mathcal{E}$, that is, there is at least one premise or rule $x \in \oplus\mathcal{E} \setminus \mathsf{knowledge}(\mathcal{X})$. Then, given the fact that $\mathcal{E}$ is part of a valid message (which means that $\mathcal{E}$ comes from an argument with no irrelevant rules or premises), and the fact that $\oplus$ is *concise*, we can conclude that every premise in $\mathsf{knowledge}(\mathcal{X})$ is connected to its claim. As such, if we were to add x to $\mathsf{knowledge}(\mathcal{X})$, x would be irrelevant to $\mathscr{A}$'s claim and thus, $\mathsf{disconnected}(\oplus\mathcal{E}) \neq \emptyset$, which contradicts the fact that $\oplus$ is *connective.* This contradiction arises from assuming that $\mathsf{knowledge}(\mathcal{X}) \subset \oplus\mathcal{E}$, and then we can conclude that $\mathsf{knowledge}(\mathcal{X}) = \oplus\mathcal{E}$ and by (i), that $\mathsf{conc}(\mathcal{X}) = \mathsf{conc}(\oplus\mathcal{E})$.

As the reader may have notice, we have proven the *existence* of an ASPIC+ argument $\mathcal{X}$, but not its uniqueness. Let us assume for contradiction sake that there exists two ASPIC+ arguments $\mathcal{X}, \mathcal{Y} \in A$ such that $\mathsf{knowledge}(\mathcal{X}) = \oplus\mathcal{E}$, $\mathsf{knowledge}(\mathcal{Y}) = \oplus\mathcal{E}$ and $\mathcal{X} \neq \mathcal{Y}$. In order for $\mathcal{X} \neq \mathcal{Y}$ to hold, either $\mathsf{knowledge}(\mathcal{X}) \neq \mathsf{knowledge}(\mathcal{Y})$, which is not possible given that $\mathsf{knowledge}(\mathcal{X}) = \mathsf{knowledge}(\mathcal{Y}) = \oplus\mathcal{E}$, or there exists two sub-arguments $\mathcal{S}_1, \mathcal{S}_2 \in \mathsf{sub}(\mathcal{X}) \cap \mathsf{sub}(\mathcal{Y})$ with the same conclusion such that they are "swapped" in terms of $\mathcal{X}$ and $\mathcal{Y}$ structure, as shown in figure 6.

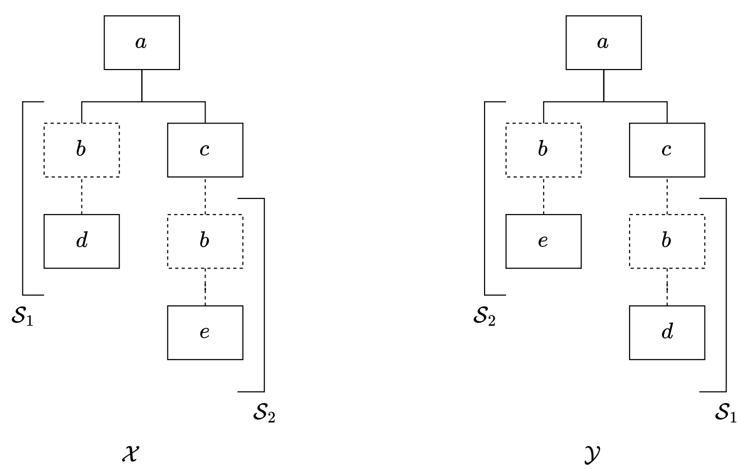

Fig. 6. Two different ASPIC+ arguments $\mathcal{X}, \mathcal{Y}$ such that $\mathsf{knowledge}(\mathcal{X}) = \mathsf{knowledge}(\mathcal{Y}) = \oplus\mathcal{E}$.

Observe that, for this to hold, there must exist two pieces of knowledge $k_1, k_2 \in \oplus\mathcal{E}$ that entail the same literal (*e.g.,* rules $b \Leftarrow d$ and $b \Leftarrow e$ shown in the figure). This could arise in two scenarios: Either $\mathcal{E}$ is based on ASPIC+ arguments holding that, or it is generated by the operator. In the former case, we know that m is *valid*, and as such, $\mathcal{E}$ must not come from an ASPIC+ argument where this situation arises. In the latter, given that $\oplus$ is *concise*, we can guarantee that $\oplus\mathcal{E}$ cannot contain two pieces of knowledge that entail the same conclusion. As such, is not possible that arguments $\mathcal{X}, \mathcal{Y}$ exist simultaneously. In conclusion, not only the ASPIC+ argument $\mathcal{X}$ does exist in the basis of $\mathbb{AT}_{\mathbb{D}}$, but also it is the only one such that $\mathsf{conc}(\mathcal{X}) = \mathsf{conc}(\oplus\mathcal{E})$ and $\mathsf{knowledge}(\mathcal{X}) = \oplus\mathcal{E}$. □

Proposition 2. Let $\mathbb{D}^{\oplus}_{\mathsf{A}} = [m_1, m_2, \ldots, m_n]$ be a dialogue where every message m_i $(i \in [1, n])$ is *valid*, and $\oplus$ is an *argumentative operator*. Then, given $\mathbb{AF}_{\mathbb{D}} = \langle A, D\rangle$ it holds that $(\forall x \in \mathsf{knowledge}(\mathbb{D}^{\oplus}_{\mathsf{A}}))(\exists \mathcal{X} \in A)\ x \in \mathsf{knowledge}(\mathcal{X})$.

Proof. Consider a dialogue $\mathbb{D}^{\oplus}_{\mathsf{A}} = [m_1, \ldots, m_n]$ where each message m_i is valid, and the argumentation framework and $\mathbb{AF}_{\mathbb{D}} = \langle A, D\rangle$ from $\mathbb{AT}_{\mathbb{D}}$. By Definition 7, we know that:

$$\mathsf{knowledge}(\mathbb{D}^{\oplus}_{\mathsf{A}}) = \left\{\oplus\mathcal{E} \mid \mathcal{E} \in \mathbb{E}, \langle \mathsf{ag}, \mathcal{E}\rangle \in \mathbb{D}^{\oplus}_{\mathsf{A}}\right\} \cup \left\{\mathcal{A} \mid \mathcal{A} \in \mathbb{A}, \langle \mathsf{ag}, \mathcal{E}\rangle \in \mathbb{D}^{\oplus}_{\mathsf{A}}\right\}$$

Then, given a premise or rule $x \in \mathsf{knowledge}(\mathbb{D}^{\oplus}_{\mathsf{A}})$ we can distinguish two scenarios. In the first one, $x \in \mathcal{A}$, $\mathcal{A} \in \mathbb{A}$ and $\langle \mathsf{ag}, \mathcal{A}\rangle \in \mathbb{D}^{\oplus}_{\mathsf{A}}$. As described above, $\mathcal{A} \subseteq \mathsf{knowledge}(\mathbb{D}^{\oplus}_{\mathsf{A}})$ and by Remark 1, there exists an ASPIC+ argument $\mathcal{X} \in A$ such that $\mathsf{knowledge}(\mathcal{X}) = \mathcal{A}$, and in conclusion, $x \in \mathsf{knowledge}(\mathcal{X})$. On the other hand, consider $x \in \oplus\mathcal{E}, \mathcal{E} \in \mathbb{E}$, and $\langle \mathsf{ag}, \mathcal{E}\rangle \in \mathbb{D}^{\oplus}_{\mathsf{A}}$. Recall that $\oplus$ is *argumentative*, and thus by Theorem 1, we know that there exists an ASPIC+ argument $\mathcal{X} \in A$ such that $\mathsf{knowledge}(\mathcal{X}) = \oplus\mathcal{E}$, then, we can conclude that $x \in \mathsf{knowledge}(\mathcal{X})$. In both scenarios, we have shown that for every premise or rule $x \in \mathsf{knowledge}(\mathbb{D}^{\oplus}_{\mathsf{A}})$, exists an ASPIC+ argument $\mathcal{X} \in A$ such that $x \in \mathsf{knowledge}(\mathcal{X})$. □

Proposition 3. The *needed assumptions operator* $\oplus_{\updownarrow}$ is a *strongly argumentative operator*.

Proof. To begin this proof, let us consider $\mathcal{E} \in \mathbb{E}$, $\mathcal{E} = \langle h, R, F\rangle$ such that $m = \langle \mathsf{ag}, \mathcal{E}\rangle$ is a valid message. In addition, recall Definition 5 where the *needed assumptions operator* is defined: Given $\mathcal{E}$ and $\oplus_{\updownarrow}$, its application yields $\oplus_{\updownarrow}\mathcal{E} = \langle h, R \cup R', F \cup F'\rangle$ where:

- If $\mathsf{disconnected}(\mathcal{E}) = \emptyset$ then $R' = \emptyset, F' = \{x \mid x \in \mathsf{baseless}(\mathcal{E}) \cap \mathsf{relevant}(\mathcal{E})\}$
- Otherwise, $R' = \{x \Leftarrow Y \mid x \in \mathsf{baseless}(\mathcal{E}) \cap \mathsf{relevant}(\mathcal{E}),\ Y = \mathsf{disconnected}(\mathcal{E})\}$, $F' = \{x \mid x \in \mathsf{baseless}(\mathcal{E}) \setminus \mathsf{relevant}(\mathcal{E})\}$

We have already shown in Proposition 1 that $\oplus_{\updownarrow}$ is a *basic operator*. We will now show that $\oplus_{\updownarrow}$ is *consistent, strongly connective*, *concise* and *conclusive*: First, given that m is a valid message, we know that $\mathsf{literals}(\mathcal{E})$ is consistent. Note also that $\mathsf{baseless}(\mathcal{E}) \cup \mathsf{disconnected}(\mathcal{E}) \subseteq \mathsf{literals}(\mathcal{E})$. By $\oplus_{\updownarrow}$ definition, we know that $\mathsf{literals}(\oplus_{\updownarrow}\mathcal{E} \setminus \mathcal{E}) = \mathsf{baseless}(\mathcal{E}) \cup \mathsf{disconnected}(\mathcal{E})$ (i), and thus $\oplus_{\updownarrow}$ does not add any new literal to $\oplus_{\updownarrow}\mathcal{E}$. As such, $\mathsf{literals}(\oplus_{\updownarrow}\mathcal{E})$ cannot be inconsistent, and then $\oplus_{\updownarrow}$ is a *consistent* operator.

Secondly, to show that $\oplus_{\updownarrow}$ is *strongly connective*, we have to consider two cases: First, if we assume that $\mathsf{disconnected}(\mathcal{E}) = \emptyset$, by $\oplus_{\updownarrow}$'s definition, an ordinary premise f will be added for each literal f that is both baseless and relevant, that is, no disconnected literals are added. Then, we can conclude that $\mathsf{disconnected}(\oplus_{\updownarrow}\mathcal{E}) = \emptyset$. On the other hand, if we assume that $\mathsf{disconnected}(\mathcal{E}) \neq \emptyset$, then by $\oplus_{\updownarrow}$'s definition, $\oplus_{\updownarrow}\mathcal{E}$ contains at least one rule $x \Leftarrow \mathsf{disconnected}(\mathcal{E})$ (note that $\exists x \in \mathsf{baseless}(\mathcal{E}) \cap \mathsf{relevant}(\mathcal{E})$, given that every $\mathcal{E} \in \mathbb{E}$ contain at least one baseless literal by Remark 2). As such, every literal $l \in \mathsf{disconnected}(\mathcal{E})$ is now part of the body of a rule in $\oplus_{\updownarrow}\mathcal{E}$. Furthermore, as we have shown before, no new literals are added to $\oplus_{\updownarrow}\mathcal{E}$ (recall (i) from the above paragraph), and no rule is removed from $\mathcal{E}$ (given that $\oplus_{\updownarrow}$ is *lossless*, as shown above). Then, we can conclude that $\text{disconnected}(\oplus_{\updownarrow}\mathcal{E}) = \emptyset$, and in conclusion, $\oplus_{\updownarrow}$ is *strongly connective*.

Next, we show that $\oplus_{\updownarrow}$ is a *concise* operator. We assume for contradiction sake that this is not the case, that is: We assume that there are two pieces of information in $\oplus\mathcal{E}$ that entail the same literal. We assume without loss of generality that both are defeasible rules, that is: $\{x \Leftarrow Y, x \Leftarrow Z\} \subseteq \oplus_{\updownarrow}\mathcal{E},\ Y \neq Z$. Given that m is a valid message, we know that $\{x \Leftarrow Y, x \Leftarrow Z\} \nsubseteq \mathcal{E}$. Then, in order for the above to hold, we distinguish two scenarios: In the former one, $x \Leftarrow Y \in \mathcal{E},\ x \Leftarrow Z \in \oplus_{\updownarrow}\mathcal{E} \setminus \mathcal{E}$ (or vice-versa). Observe that, if $x \Leftarrow Y \in \mathcal{E}$, then $x \notin \text{baseless}(\mathcal{E})$, and in turn, it is not possible (by $\oplus_{\updownarrow}$ definition) that $x \Leftarrow Z \in \oplus_{\updownarrow}\mathcal{E}$. In the latter scenario, $\{x \Leftarrow Y, x \Leftarrow Z\} \subseteq \oplus_{\updownarrow}\mathcal{E} \setminus \mathcal{E}$, that is, both rules are created by $\oplus_{\updownarrow}$. Observe that, by $\oplus_{\updownarrow}$ definition, if both rules were created, then $Y = Z = \text{disconnected}(\mathcal{E})$, which contradicts the fact that $Y \neq Z$. In both scenarios we have shown that a contradiction arises from assuming that $\oplus_{\updownarrow}$ is not a *concise* operator, as such, we can conclude that $\oplus_{\updownarrow}$ complies to said principle.

Finally, we show that $\oplus_{\updownarrow}$ is *conclusive*. By $\oplus_{\updownarrow}$'s definition we know that for every baseless literal $l \in \text{baseless}(\mathcal{E})$, there will exist a rule $l \Leftarrow Y \in \oplus_{\updownarrow}\mathcal{E}$ or an ordinary premise $l \in \oplus_{\updownarrow}\mathcal{E}$. Given that no new literals are added to $\oplus_{\updownarrow}\mathcal{E}$, we can conclude that $\text{baseless}(\oplus_{\updownarrow}\mathcal{E}) = \emptyset$. Then, if we were to create an argumentation theory $\mathbb{AT}$ with $\oplus_{\updownarrow}\mathcal{E}$'s contents, then an ASPIC+ argument $\mathcal{A}$ can be created on the basis of $\mathbb{AT}$ such that $\text{conc}(\mathcal{A}) = \text{conc}(\oplus\mathcal{E})$, and then we conclude that $\oplus_{\updownarrow}$ is *conclusive*.

To sum up, we have shown that $\oplus_{\updownarrow}$ is *consistent*, *strongly connective*, *concise* and *conclusive*, and thus, $\oplus_{\updownarrow}$ is a *strongly argumentative operator*. □

Proposition 4. Let $\mathbb{D}^{\oplus}_{\mathsf{A}}$ be a dialogue where every message is *valid*, $\oplus$ is *strongly argumentative* and let $m = \langle \mathsf{ag}, \mathcal{M} \rangle$ be a *valid* message such that $\mathcal{M}$ comes from argument $\mathcal{A}$. Then, for every argument $\mathcal{X} \in (A(\mathbb{C}) \setminus \text{sub}(\mathcal{A})) \cup \{\mathcal{A}\}$, either $\mathcal{X} \in A(\mathbb{E})$ or there exists an argument $\mathcal{X}' \in A(\mathbb{E})$ such that $\mathcal{X}'$ is the proxy argument of $\mathcal{X}$.

Proof. Consider an arbitrary argument $\mathcal{X} \in A(\mathbb{C})$, then, either one of the following must comply:

(1) $\mathcal{X} = \mathcal{A}$.
(2) $\mathcal{X}$ is an argument which was present before move m, that is: $\mathcal{X} \in A(\mathbb{B})$.
(3) $\mathcal{X}$ is neither $\mathcal{A}$ nor an argument present before move m, that is: $\mathcal{X} \in A(\mathbb{C}) \setminus A(\mathbb{E}),\ \mathcal{X} \neq \mathcal{A}$.

We will show that for every scenario, either $\mathcal{X} \in A(\mathbb{E})$ or there exists $\mathcal{X}' \in A(\mathbb{E})$ such that $\mathcal{X}'$ is the proxy argument of $\mathcal{X}$.

Case 1: $\mathcal{X} = \mathcal{A}$.

In this scenario, $\mathcal{X} = \mathcal{A}$ is the original argument from which $\mathcal{M}$ is based on. Observe that $\oplus$ is *conclusive, concise* and *connective*, and that m is a *valid* message. Then, following Theorem 1, we know that there exists an argument $\mathcal{Y} \in A(\mathbb{E}),\ \text{knowledge}(\mathcal{Y}) = \oplus\mathcal{M}$ and $\text{conc}(\mathcal{Y}) = \text{conc}(\mathcal{A})$.

Then, we assume for contradiction sake that $\oplus\mathcal{M}$ is not the proxy argument of $\mathcal{A}$. That is, there exists $\mathcal{X}' \in A(\mathbb{E})$ such that (i) $\text{knowledge}(\mathcal{X}') \setminus (\oplus\mathcal{M} \setminus \mathcal{M}) \subseteq \mathcal{A}$ and (ii) $\oplus\mathcal{M} \subset \text{knowledge}(\mathcal{X}')$. Then, we can distinguish two scenarios. In the first one $\text{knowledge}(\mathcal{X}') \cap (\oplus\mathcal{M} \setminus \mathcal{M}) = \emptyset$. Note that, in order for $\text{knowledge}(\mathcal{X}') \cap (\oplus\mathcal{M} \setminus \mathcal{M}) = \emptyset$ and $\oplus\mathcal{M} \subset \text{knowledge}(\mathcal{X}')$ to hold simultaneously, either $\oplus\mathcal{M} \subset \mathcal{M}$ (which is not possible given that $\oplus$ is *additive* and *lossless*), or $\oplus\mathcal{M} = \mathcal{M}$, which would mean that $\mathcal{M} \in \mathbb{A}$, which contradicts the premise of this proposition (that $\mathcal{M}$ is an enthymeme from $\mathcal{A}$). As such, this scenario is not possible.

Then, in the other scenario, $\text{knowledge}(\mathcal{X}') \cap (\oplus\mathcal{M} \setminus \mathcal{M}) \neq \emptyset$. By (ii), we know that $\oplus\mathcal{M} \subset \text{knowledge}(\mathcal{X}')$ and then, $\mathcal{M} \cup (\oplus\mathcal{M} \setminus \mathcal{M}) \subset \text{knowledge}(\mathcal{X}')$. Consider an arbitrary rule or premise $z \in \text{knowledge}(\mathcal{X}') \setminus \oplus\mathcal{M}$. Given that $z \notin \oplus\mathcal{M}$, then $z \notin \mathcal{M}$ and $z \notin (\oplus\mathcal{M} \setminus \mathcal{M})$. Note however that, by (i) $\text{knowledge}(\mathcal{X}') \setminus (\oplus\mathcal{M} \setminus \mathcal{M}) \subseteq \mathcal{A}$ and then $z \in \mathcal{A}$. Note that, by Remark 4, given that $z \in \mathcal{A}, z \notin \mathcal{M}$, then $z \notin \text{knowledge}(\mathbb{T}_{\mathbb{E}})$ which then contradicts the fact that $z \in \mathsf{knowledge}(\mathcal{X}')$ (recall that $\mathcal{X}' \in A(\mathbb{E})$). In both scenarios, a contradiction arises by the fact that we assume that $\oplus\mathcal{M}$ is not the proxy argument of $\mathcal{A}$, as such, the opposite must hold.

Case 2: $\mathcal{X} \in A(\mathbb{B})$.

In this scenario, by Remark 3, we can conclude that $\mathcal{X} \in A(\mathbb{E})$ and thus the property holds.

Case 3: $\mathcal{X} \neq \mathcal{A}, \mathcal{X} \in A(\mathbb{C}) \setminus A(\mathbb{E})$.

In this scenario, we will show that a proxy argument of $\mathcal{X}$ exists in $A(\mathbb{E})$. That is, we show that an argument $\mathcal{X}'$ is the proxy of $\mathcal{X}$, as such, complies with (i) $\mathsf{conc}(\mathcal{X}) = \mathsf{conc}(\mathcal{X}')$ and (ii) $\mathcal{X}'$ is the maximal argument that complies with $\mathsf{knowledge}(\mathcal{X}') \setminus (\oplus\mathcal{M} \setminus \mathcal{M}) \subseteq \mathsf{knowledge}(\mathcal{X})$.

First, if $\mathcal{X} \in A(\mathbb{C}) \setminus A(\mathbb{E})$, then there exists an argument $\mathcal{S}$ such that $\mathcal{S} \in (\mathsf{sub}(\mathcal{A}) \cap \mathsf{sub}(\mathcal{X})) \setminus \mathsf{sub}(\oplus\mathcal{M})$. Then, there exists a rule $r \in \mathcal{X}$ such that $\mathsf{conc}(\mathcal{S})$ is part of the body of r and r is part of $\mathbb{T}_{\mathbb{E}}$. Then, by Proposition 2, we know that there exists an argument $\mathcal{X}' \in A(\mathbb{E})$ such that $r \in \mathcal{X}'$, and another argument $\mathcal{S}' \in A(\mathbb{E})$ with conclusion $\mathsf{conc}(\mathcal{S})$. As such, we know that $\mathcal{X}' = (\mathcal{X} \setminus \mathcal{S}) \cup \mathcal{S}'$ and then, $\mathsf{conc}(\mathcal{X}') = \mathsf{conc}(\mathcal{X})$, thus $\mathcal{X}'$ complies with condition (i) stated above.

On the other hand, we will assume for contradiction sake that $\mathsf{knowledge}(\mathcal{X}') \setminus (\oplus\mathcal{M} \setminus \mathcal{M}) \nsubseteq \mathcal{X}$. This means that there exists a rule or premise z such that $z \in \mathsf{knowledge}(\mathcal{X}'), z \notin (\oplus\mathcal{M} \setminus \mathcal{M}), z \notin \mathcal{X}$. If $z \in \mathsf{knowledge}(\mathcal{X}')$ then $z \in \mathsf{knowledge}(\mathbb{T}_{\mathbb{E}})$, and if $z \notin \mathsf{knowledge}(\mathcal{X})$ then $z \notin \mathsf{knowledge}(\mathbb{T}_{\mathbb{C}})$. As such, $z \in \mathsf{knowledge}(\mathbb{T}_{\mathbb{E}}) \setminus \mathsf{knowledge}(\mathbb{T}_{\mathbb{C}})$ and by Remark 4, $z \in (\oplus\mathcal{M} \setminus \mathcal{M})$ which contradicts what we stated above. This contradiction arises from assuming that $\mathsf{knowledge}(\mathcal{X}') \setminus (\oplus\mathcal{M} \setminus \mathcal{M}) \nsubseteq \mathcal{X}$, as such, we can conclude that $\mathsf{knowledge}(\mathcal{X}') \setminus (\oplus\mathcal{M} \setminus \mathcal{M}) \subseteq \mathcal{X}$. Then, we have proven that conditions (i) and (ii) holds, and then, that argument $\mathcal{X}'$ is the proxy argument of $\mathcal{X}$ under this scenario.

In conclusion, we have shown that under every possible scenario (cases 1-3), either $\mathcal{X}$ is part of $A(\mathbb{E})$ or there exists an argument $\mathcal{X}'$ which is the proxy argument of $\mathcal{X}$.

□

Theorem 2. Let $\mathbb{D}_{\mathsf{A}}^{\oplus}$ a dialogue where $\oplus$ is *strongly argumentative*, and $m = \langle \mathsf{ag}, \mathcal{M} \rangle$ be a *valid message* such that $\mathcal{M}$ comes from argument $\mathcal{A}$. Let $S \subseteq A(\mathbb{C})$ be a conflict-free set such that $\mathcal{A} \in S$, then the proxy set S' of S is also conflict-free.

Proof. Consider the proxy set S' of S, which is defined as $S' = (S \cap A(\mathbb{E})) \cup P$, where P contains one proxy argument for each one in $S \setminus A(\mathbb{E})$. To show that the property holds, we must show that:

(1) $\oplus\mathcal{M} \in S'$.
(2) $S' \setminus \{\oplus\mathcal{M}\}$ is conflict-free.
(3) No argument $\mathcal{X} \in S'$ attacks $\oplus\mathcal{M}$.
(4) $\oplus\mathcal{M}$ does not attack any argument in S'

First, (1) is trivially addressed by Remark 6.

Then, we will show that $S' \setminus \{\oplus\mathcal{M}\}$ is conflict-free. We assume for contradiction sake that this does not hold, which means that $\exists \mathcal{X}, \mathcal{Y} \in S',\ (\mathcal{X}, \mathcal{Y}) \in D(\mathbb{E})$. In order for $\mathcal{X}$ to defeat $\mathcal{Y}$, $\mathcal{X}$ must successfully attack $\mathcal{Y}$, that is: either (a) $\mathsf{conc}(\mathcal{X}) = -\mathsf{conc}(\mathcal{Y})$, or (b) $\mathsf{conc}(\mathcal{X}) = -\mathsf{conc}(\mathcal{S}),\ \mathcal{S} \in \mathsf{sub}(\mathcal{Y})$.

If (a) holds, given that every argument in S' is either part of S or a proxy argument for one in S (and as such, it preserves its conclusion), there would exist arguments $\mathcal{X}', \mathcal{Y}' \in S$ such that $(\mathcal{X}', \mathcal{Y}') \in D(\mathbb{C})$, and then, S would not be conflict-free, which contradicts the premise of this theorem. Scenario (b) follows the same reasoning: First of all, it cannot be that $\mathcal{X}, \mathcal{Y} \in (S \cap A(\mathbb{E}))$ because S would not be conflict-free neither. Then, the only possibility is that $\mathcal{Y}$ is formed by some sub-argument of $\oplus\mathcal{M}$ and $\mathcal{X}$ attacks said sub-argument. In more formal terms: $\mathsf{conc}(\mathcal{X}) = -\mathsf{conc}(\mathcal{S}),\ \mathcal{S} \in \mathsf{sub}(\oplus\mathcal{M}) \cap \mathsf{sub}(\mathcal{Y})$. Note however that this is not possible either: Recall that $\oplus$ is *strongly-connective* and then $\mathsf{literals}(\oplus\mathcal{M}) \subseteq \mathsf{literals}(\mathcal{A})$. This means that if $\mathcal{X}$ were to attack, and in turn, defeat $\mathcal{S}$, then said defeat should also appear in $D(\mathbb{C})$, which would contradict that S is conflict-free. In either case, both scenarios show that is not possible that $\exists \mathcal{X}, \mathcal{Y} \in S',\ (X, Y) \in D(\mathbb{E})$, which let us conclude that (2) $S' \setminus \{\oplus\mathcal{M}\}$ is a conflict-free set.

Then, we address (3) and (4). Recall that S is conflict free, and as such, the set $C = \{\mathsf{conc}(\mathcal{X}) \mid \mathcal{X} \in (S \cup \mathsf{sub}(\mathcal{A}))\}$ is consistent. Then, consider that $\oplus$ is *strongly-connective*, as such $\mathsf{literals}(\oplus\mathcal{M}) \subseteq \mathsf{literals}(\mathcal{A})$. Furthermore, by Definition 13, we know that every argument in S' is either part of S or a proxy argument of one in S. As such, the set $C' = \{\mathsf{conc}(\mathcal{X}) \mid \mathcal{X} \in S' \cup \mathsf{sub}(\oplus\mathcal{M})\}$ is a subset of C and therefore C' is also consistent. In conclusion, neither $\oplus\mathcal{M}$ attacks an argument in S' nor vice versa (3,4).

By addressing (1-4), we have shown that if S is conflict-free and $\mathcal{A} \in S$, then the proxy set S' of S is also conflict-free.

□

Theorem 3. Let $m = \langle \mathsf{ag}, \mathcal{M} \rangle$ be a *non-weakened* message such that $\mathcal{M}$ comes from argument $\mathcal{A}$, let $\mathbb{D}^{\oplus}_{\mathsf{A}}$ be a dialogue where $\oplus$ is *strongly argumentative*. If $\mathcal{A}$ is acceptable w.r.t S in $\mathbb{F}_{\mathbb{C}}$, $S \cap \mathsf{sub}(\mathcal{A}) \subseteq \mathsf{sub}(\oplus\mathcal{M})$ and $\nexists X \in \mathsf{sub}(\oplus\mathcal{M}) \cap \mathsf{sub}(\mathcal{B})$ such that $(\mathcal{B}, \oplus\mathcal{M}) \in D(\mathbb{E})$. Then, $\oplus\mathcal{M}$ is acceptable w.r.t. S', where S' is the proxy set of S.

Proof. Let us begin this proof by stating some premises:

(1) $m = \langle \mathsf{ag}, \mathcal{M} \rangle$ is a *non-weakened* message (Def. 14).
(2) $\oplus$ is a *strongly argumentative* operator.
(3) $\mathcal{A}$ is acceptable w.r.t. S in $\mathbb{F}_{\mathbb{C}}$.
(4) $S \cap \mathsf{sub}(\mathcal{A}) \subseteq \mathsf{sub}(\oplus\mathcal{M})$.
(5) $\nexists \mathcal{X} \in (\mathsf{sub}(\oplus\mathcal{M}) \cap \mathsf{sub}(\mathcal{B}))$ such that $(\mathcal{B}, \oplus\mathcal{M}) \in D(\mathbb{E})$.
(6) S' is the proxy set of S.

Then, we assume for contradiction's sake that $\oplus\mathcal{M}$ is not acceptable w.r.t. any set S' in $\mathbb{F}_{\mathbb{E}}$. This means that $\exists \mathcal{Y} \in A(\mathbb{E})\ (\mathcal{Y}, \oplus\mathcal{M}) \in D(\mathbb{E})$, and $\nexists \mathcal{Z} \in A(\mathbb{E})\ (\mathcal{Z}, \mathcal{Y}) \in D(\mathbb{E})$. In order for this to occur, one of the following should comply: Either (1) $\mathcal{Y} \in A(\mathbb{E}) \setminus A(\mathbb{C})$ (that is, $\mathcal{Y}$ is a newly created argument in $\mathbb{F}_{\mathbb{E}}$), (2) $\nexists \mathcal{Z} \in A(\mathbb{E})\ (\mathcal{Z}, \mathcal{Y}) \in D(\mathbb{E})$ ($\oplus\mathcal{M}$ has no defenders in $A(\mathbb{E})$), or (3) $(\mathcal{Y}, \mathcal{A}) \notin D(\mathbb{C})$ and $(\mathcal{Y}, \oplus\mathcal{M}) \in D(\mathbb{E})$ (that is, $\mathcal{Y}$ defeats $\oplus\mathcal{M}$ but not $\mathcal{A}$).

Case 1: $\mathcal{Y} \in A(\mathbb{E}) \setminus A(\mathbb{C})$

In this case, $\mathcal{Y}$ is part of the *enthymemic framework* $\mathbb{F}_{\mathbb{E}}$ but not part of the *complete* one $\mathbb{F}_{\mathbb{C}}$, which means that part of $\mathcal{Y}$ is made from $\oplus\mathcal{M}$'s contents, which in turn means that $\mathsf{sub}(\mathcal{Y}) \cap \mathsf{sub}(\oplus\mathcal{M}) \neq \emptyset$. Observe that this is not possible, given that it would contradict Premise 5.

Case 2: $\nexists \mathcal{Z} \in S'\ (\mathcal{Z}, \mathcal{Y}) \in D(\mathbb{E})$

In this scenario, no argument in S' defends $\oplus\mathcal{M}$. $\mathcal{A}$ was acceptable w.r.t. S. As such, there exists an argument $\mathcal{Z} \in S,\ (\mathcal{Z}, \mathcal{Y}) \in D(\mathbb{C})$. Now, if $\mathcal{Z} \notin A(\mathbb{E})$, it would mean that either $\mathcal{Z} \in \mathsf{sub}(\mathcal{A}) \setminus \mathsf{sub}(\oplus\mathcal{M})$ (which is not possible given that it contradicts premise 4), or either $\mathcal{Z} \cap \mathsf{sub}(\mathcal{A}) \neq \emptyset$. In this latter case, by Theorem 4 (and the fact that $\mathcal{Z} \notin \mathsf{sub}(\mathcal{A})$) there exists an argument $\mathcal{Z}' \in A(\mathbb{E})$ which is the proxy argument of $\mathcal{Z}$. Furthermore, given that $\mathcal{Z}'$ is the proxy argument of $\mathcal{Z}$ and given that $\mathcal{Z} \in S$, then $\mathcal{Z}' \in S'$ (Definition 13). In conclusion, $(\mathcal{Z}', \mathcal{Y}) \in D(\mathbb{E})$ and thus, it will defend $\oplus\mathcal{M}$.

Case 3: $(\mathcal{Y}, \mathcal{A}) \notin D(\mathbb{A})$ and $(\mathcal{Y}, \oplus\mathcal{M}) \in D(\mathbb{E})$

This last scenario could occur for a few reasons: First, $\mathcal{Y}$ could attack a literal which is part of $\oplus\mathcal{M}$ but not part of $\mathcal{A}$, that is: $\overline{\mathsf{conc}(\mathcal{Y})} \in \mathsf{literals}(\oplus\mathcal{M}) \setminus \mathsf{literals}(\mathcal{A})$. As the reader may recall $\mathsf{literals}(\mathcal{M}) \subseteq \mathsf{literals}(\mathcal{A})$, and then given that $\oplus$ is *strongly-connective*, then $\mathsf{literals}(\oplus\mathcal{M}) \subseteq \mathsf{literals}(\mathcal{A})$. As such $\mathsf{literals}(\oplus\mathcal{M}) \setminus \mathsf{literals}(\mathcal{A}) = \emptyset$ and thus, this scenario is not possible. On the other hand, it could happen that there exists an strict rule $\overline{\mathsf{conc}(\mathcal{Y})} \leftarrow B \in \mathcal{A} \setminus \oplus\mathcal{M}$ or an axiomatic premise $\overline{\mathsf{conc}(\mathcal{Y})} \in \mathsf{axPremises}(\mathcal{A}) \setminus \mathsf{axPremises}(\oplus\mathcal{M})$ such that, the attack arises due to the fact that the strict rule became a defeasible one, or the axiomatic premise became an ordinary one. This is not possible neither, given that it would contradict Premise 1.

In every scenario, we have proven that is not possible that $\exists \mathcal{Y} \in A(\mathbb{E})$ $(\mathcal{Y}, \oplus\mathcal{M}) \in D(\mathbb{E})$, and $\nexists \mathcal{Z} \in A(\mathbb{E})$ $(\mathcal{Z}, \mathcal{Y}) \in D(\mathbb{E})$ hold simultaneously. As such, we have proven that if premises 1-6 holds, then $\oplus\mathcal{M}$ is acceptable w.r.t S' in $\mathbb{F}_{\mathbb{C}}$, where S' is the proxy set of S.

□

Theorem 4. Let $m = \langle \mathsf{ag}, \mathcal{M} \rangle$ be a *non-weakened* message such that $\mathcal{M}$ comes from argument $\mathcal{A}$, let $\mathbb{D}^{\oplus}_{\mathsf{A}}$ be a dialogue where $\oplus$ is *strongly argumentative*. Given an admissible set $S \subseteq A(\mathbb{C})$ such that $\mathcal{A} \in S$, then the proxy set S' of S is an admissible set in $\mathbb{F}_{\mathbb{E}}$ if $\mathsf{sub}(\mathcal{A}) \cap S \subseteq \mathsf{sub}(\oplus\mathcal{M})$ and $(\forall \mathcal{Y} \in A(\mathbb{C}))$ if $(\mathcal{Y}, \mathcal{X}) \in D(\mathbb{E})$ then $\mathsf{sub}(\oplus\mathcal{M}) \cap \mathsf{sub}(\mathcal{Y}) = \emptyset$.

Proof. We will show that given an admissible set $S \subseteq A(\mathbb{C})$ such that $\mathcal{A} \in S$, then its proxy set S' (which contains $\oplus\mathcal{M}$ by Remark 6) is also admissible if the following conditions are met:

(1) $\mathsf{sub}(\mathcal{A}) \cap S \subseteq \mathsf{sub}(\oplus\mathcal{M})$.
(2) $(\forall \mathcal{Y} \in A(\mathbb{C}))$ if $(\mathcal{Y}, \mathcal{X}) \in D(\mathbb{E})$ then $\mathsf{sub}(\oplus\mathcal{M}) \cap \mathsf{sub}(\mathcal{Y}) = \emptyset$.

As the reader may recall from Sub-section 2.2, X is an admissible set iff it is conflict-free and every argument in it is acceptable w.r.t X. Then, given that S is admissible, it is also conflict-free. As the reader may recall, Theorem 2 dictates that if S is conflict-free and $\mathcal{A} \in S$, then its proxy set S' is also conflict-free in $\mathbb{F}_{\mathbb{E}}$.

Then, we need to show that every argument in S' is acceptable w.r.t itself. From Definition 13, we know that:

$$S' = (S \cap A(\mathbb{E})) \cup P \text{ where } \forall \mathcal{X} \in (S \setminus A(\mathbb{E})) \text{ the proxy argument of } \mathcal{X} \text{ is in } P, \text{ and } \oplus\mathcal{M} \in S'.$$

First, we show that $\oplus\mathcal{M}$ is acceptable w.r.t S'. As the reader may recall, $\mathcal{A}$ is acceptable w.r.t S and then, by Theorem 3, we can then conclude that $\oplus\mathcal{M}$ is acceptable w.r.t S' (observe that condition 1 of this theorem is the same as condition 4 of Theorem 3, and that condition 2 of this theorem is a strengthened version of condition 5 in Theorem 3).

Next, consider an arbitrary argument $\mathcal{X} \in S' \setminus \{\oplus\mathcal{M}\}$. We will assume for contradiction sake that it is not acceptable w.r.t. S'. This would mean that there exists an argument $\mathcal{Y} \in A(\mathbb{E})$ such that $(\mathcal{Y}, \mathcal{X}) \in D(\mathbb{E})$ and $(\nexists \mathcal{Z} \in S')(\mathcal{Z}, \mathcal{Y}) \in D(\mathbb{E})$. If a delve a bit deeper, we can determine four different scenarios depending on whether $\mathcal{X}$ was part of S or not (that is, $\mathcal{X} \in S \cap A(\mathbb{E})$), and whether $\mathcal{Y}$ is a newly created argument in $\mathbb{F}_{\mathbb{E}}$. Then, we will analyze each of those scenarios:

Case 1: $\mathcal{X} \in (S \cap A(\mathbb{E}))$, $\mathcal{Y} \in (A(\mathbb{C}) \cap A(\mathbb{E}))$

In this scenario $(\mathcal{Y}, \mathcal{X}) \in D(\mathbb{E})$ and both arguments are part of $\mathbb{F}_{\mathbb{C}}$ and $\mathbb{F}_{\mathbb{E}}$, as such $(\mathcal{Y}, \mathcal{X}) \in D(\mathbb{C}) \cap D(\mathbb{E})$. As the reader may recall, $\mathcal{X}$ is acceptable w.r.t. S in $\mathbb{F}_{\mathbb{C}}$, as such, there exists an argument $\mathcal{Z} \in S$ which defends $\mathcal{X}$. In addition, recall that S' is a proxy set of S, which means that for every argument in S, there is a proxy argument in S' (In this case, we do not care if it is the same argument or just a proxy one). That is, $(\exists \mathcal{Z}' \in S')$ $\mathsf{conc}(\mathcal{Z}') = \mathsf{conc}(\mathcal{Z})$ and in conclusion $(\mathcal{Z}', \mathcal{Y}) \in D(\mathbb{E})$, which contradicts the fact that $\mathcal{X}$ is not acceptable w.r.t S'.

Case 2: $\mathcal{X} \in (S \cap A(\mathbb{E}))$, $\mathcal{Y} \notin (A(\mathbb{C}) \cap A(\mathbb{E}))$

In this scenario, $\mathcal{Y}$ is not part of $\mathbb{F}_{\mathbb{C}}$, which means that some of it is created by $\oplus\mathcal{M}$'s contents. Observe that this cannot happen due to restriction 2 of this theorem.

Case 3: $\mathcal{X} \notin (S \cap A(\mathbb{E}))$, $\mathcal{Y} \in (A(\mathbb{C}) \cap A(\mathbb{E}))$

In this scenario, an existing argument $\mathcal{Y}$ defeats a newly created argument $\mathcal{X} \in S'$. First of all, observe that if $\mathcal{X}$ is a new argument, then $\mathcal{X} \in P$. As such, by Definition 13, there exists an argument $\mathcal{X}' \in A(\mathbb{C})$ such that $\mathcal{X}$ is a proxy argument for $\mathcal{X}'$. Then, we can distinguish two sub-scenarios by observing whether $\mathcal{Y}$ defeats $\mathcal{X}'$ in $\mathbb{F}_{\mathbb{C}}$.

Let us delve in what would happen if $(\mathcal{Y}, \mathcal{X}') \in D(\mathbb{C})$ holds. Firstly, recall that S is an admissible set, which means that $\mathcal{X}'$ is acceptable w.r.t. S. As such, there exists an argument $\mathcal{Z} \in S$ which defends $\mathcal{X}'$. Then, as we have shown before, there will exist a proxy argument $\mathcal{Z}' \in S$ which defends $\mathcal{X}$ in $\mathbb{F}_{\mathbb{E}}$, which contradicts the fact that $\mathcal{X}$ is not acceptable w.r.t. S'.

On the other hand, if $(\mathcal{Y}, \mathcal{X}') \notin D(\mathbb{C})$, then $\mathcal{Y}$ attack, and thus defeat the "new" part of $\mathcal{X}$, that is: $(\exists \mathcal{C} \in \text{sub}(\mathcal{X}) \cap \text{sub}(\oplus\mathcal{M}))$ such that $(\mathcal{Y}, \mathcal{C}) \in D(\mathbb{E})$. As the reader may recall, $\oplus$ is *strongly-connective*, as such: $\text{conc}(\mathcal{C}) \in \text{literals}(\mathcal{A})$ ($\oplus$ does not add new literals, so $\text{conc}(\mathcal{C})$ should be part of $\mathcal{A}$), which means that there exists an argument $\mathcal{C}' \in \text{sub}(\mathcal{A})$ where $\text{conc}(\mathcal{C}') = \text{conc}(\mathcal{C})$. Then, if $(\mathcal{Y}, \mathcal{C}') \in D(\mathbb{C})$ and $\mathcal{C}' \in \text{sub}(\mathcal{A})$, we can deduce that $(\mathcal{Y}, \mathcal{A}) \in D(\mathbb{C})$. Given that $\mathcal{A}$ is acceptable w.r.t. S, we can follow theorem 3 and conclude that $\oplus\mathcal{M}$ is acceptable w.r.t. S' and then, $\mathcal{C}$ is defended by some $\mathcal{Z} \in S'$ (that is, $(\mathcal{Z}, \mathcal{Y}) \in D(\mathbb{E})$) and then, $\mathcal{X}$ would be defended by $\mathcal{Z} \in S'$, which contradicts the fact that $\mathcal{X}$ is not acceptable w.r.t. S'. Note however, that there is other possible scenario in which $(\mathcal{Y}, \mathcal{C}') \notin D(\mathbb{C})$ but $(\mathcal{Y}, \mathcal{C}) \in D(\mathbb{E})$: If $\text{conc}(\mathcal{Y})$ is in conflict with an strict rule or axiomatic premise in $\mathcal{C}'$, then a defeat will not arise in $\mathbb{F}_{\mathbb{C}}$, but if said rule or premise were to be replaced in $\mathcal{C}$ by a defeasible rule or ordinary premise, then $(\mathcal{Y}, \mathcal{C}) \in D(\mathbb{E})$. Note however that this scenario is not possible neither, given that m is a *non-weakened* message.

Case 4: $\mathcal{X} \notin (S \cap A(\mathbb{E}))$, $\mathcal{Y} \notin (A(\mathbb{C}) \cap A(\mathbb{E}))$

Similarly to *case 2*, in this scenario $\mathcal{Y}$ is created by $\oplus\mathcal{M}$'s contents, which cannot be possible given that it contradicts restriction 2 of this theorem.

Then, we have shown that for every scenario, that every argument $\mathcal{X} \in S'$ is acceptable w.r.t. S'. In addition, we have also proven that S' is a conflict-free set. As such, we can conclude that given an admissible set S and if certain restrictions are met, then S' is also an admissible set. □

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