

Simulation of Realistic Fur Texture Based on Piece Element Synthesis

by

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Abstract

Realistic fur texture is usually modeled and rendered with three-dimensional software. Due to the complexity and inefficiency of the three-dimensional software, it cannot meet general designers' requirements to represent realistic fur texture quickly. In order to achieve the rapid drawing of real-quality fur, this work proposes an approach to simulating realistic fur texture based on the method of piece elements synthesis. To be specific, we used fur elements to represent fur units, the dislocation and superposition of which formed the fur visual effect. Also, we discuss the validity and feasibility of parameters that can be matched in Adobe Illustrator software. The method applied in this paper can not only meet the timeliness and convenience of designers when representing fur texture, but also visualize fur texture with a high sense of authenticity and diversity.

Introduction

Designers usually need to visualize realistic fur texture when representing product effects in a variety of fields, such as leather product design, fashion design, industrial design, graphic design, etc. Due to the character of fur with huge quantities, tiny radius, complex illumination, and mutual shadow, it is harder and slower to visualize fur texture than other kinds of surface textures.¹⁻⁵ At present, designers use either sketch, bristle brush or map rendering to demonstrate the complexity of fur.^{3,6} However, designers need to pay great attention to drawing the geometric details of fur and to representing the invisible changes of light and shadow among fur, the heavy workload of which cannot meet designers' timeliness requirement when representing fur texture.

There exist several challenging problems for fur simulation in the field of computer graphics, including fur quantities, fur radius, fur density, complex illumination, and mutual shadow.^{2,4,6,14} To represent realistic fur texture, researchers have done plenty of research on fur modeling and fur rendering.

Csuril represented each fur as a polygon patch in 1979.⁷ After that, several researchers were trying to adopt different kinds of geometric elements to represent fur, such as pyramid,⁸ curved cylinder,⁹

trigonal prism,¹⁰ etc. However, geometric details of fur were so complicated that it was difficult to achieve high processing speed based on computers, which resulted in dissatisfaction with real-time simulation. In 1997, Gelder proposed a geometric but simplified method to represent each fur as a polyline.¹¹ Although the technique met both real-time and interaction requirements, it lacked not only satisfactory results of fur authenticity but also a shortage of quantities. In addition to adopting geometric models, Reeves first introduced the particle system in 1983, which used simple but tiny particles as basic elements to represent irregular and fuzzy objects. The particle system originated from hydrodynamics, which had significant advantages in simulating fuzzy objects with highly efficient operation.¹² Since the system applied smoothly in real-time fur simulation, it has been widely used in fur modeling for many analysts and researchers. However, due to countless numbers of fur, using the particle system occupied plenty of computer memory.

In order to reduce the unnecessary costs of the particle system, some studies began to consider the use of texture to represent detailed characteristics of fur. In 1989, Perlin proposed that hyper texture could model phenomena intermediates between shape and texture by using space-filling applicative functions to modulate fur density.¹³ He used pseudo-random function to control volume density of fur to generate fluffy objects. In the same year, Kajiya and Kay proposed a method for rendering scenes with sufficient fur detail via an item called Texel.¹⁴ They represented one piece of fur as one Texel inspired by volume densities mixed with anisotropic lighting models to create realistic short-fur effect. However, the adoption of ray tracing rendering resulted in low velocity.

None of the above algorithms were provided with effective real-time performance. In order to achieve rapid simulation of fur, Meyer proposed a method of layered texture slicing based on Kajiya's foundation in 1998, which could represent objects with complicated micro-structures.¹⁵ In terms of the idea of texture slicing, Lengyel proposed a layered shell approach that used multiple layers of texture to represent fur in 2000.¹⁶ To be specific, he cut one piece of fur into several layers of two-dimensional texture and mapped those layers to different layers of mesh surfaces. At first, Lengyel applied particle systems to fur modeling. Then, he sampled fur modeling to generate shell texture. After that, he blended fur mesh surface

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and adopted an illumination model to simulate the lighting effect of fur, a real-time fluffy surface of an object could be generated. To improve the visual quality of the fur near silhouettes, Lengyel introduced lapped textures in 2001.¹ He used conventional two-dimensional texture maps to improve the appearance of silhouette seamlessly. The method produced fur images at an interactive rate for models of moderate complexity. Additionally, it allowed for real-time modification of viewing and lighting conditions, as well as local control of fur color, length, and direction.

Based on the inspiration of Lengyel's shell textures and lapped textures method, this paper proposes a simulation method for generating realistic fur texture based on the idea of piece element synthesis. Even though the concept originated from the research of Lengyel, it is different from Lengyel's method of simulating fur by layered piece fusion in a three-dimensional environment. That is, the authors adopted a method of displacing and superimposing fur elements in a two-dimensional environment to generate real-texture fur effect. The main contributions were as follows. Firstly, the geometric model of fur elements formed the fur visual effect by dislocation and superposition. Secondly, the volume element was converted into piece elements from three-dimensional simulation to two-dimensional simulation, which helped to reduce the number of calculations when processing real-quality fur. Thirdly, the relevant parameters of fur elements were set to control fur simulation results, so as to facilitate designers to simulate diversified fur texture with high efficiency.

As a new fur simulation method, firstly, a formation mechanism of real-quality fur was introduced based on optical principle. Secondly, in terms of the fur texture formation mechanism, the method of the dislocation and superposition of piece elements was applied to control the fur attributes by setting relevant parameters. Finally, the corresponding relationship between the theoretical parameters and the control parameters was analyzed in Adobe Illustrator software, which in turn verified the feasibility of the fur piece element synthesis method.

Method

Formation of the visual texture of fur

Due to the fact that human perception of the texture of an object comes from visual texture and tactile texture,¹⁷ designers obtain the texture information of a perceived object mainly through their vision. Therefore, in order to accurately visualize the realistic texture of an object, it is of great necessity for designers to focus on the characteristics of an object's surface.

In the field of computer graphics, the simulation of an object's texture is one of the primary research contents for the area of photo-realism graphics. Based on Dana's research, an object's material properties and geometric structure determine its visual effect.¹⁸ According to material properties, fur is constructed of fibers, as shown in Figure 1(a),¹⁹ one single piece of fur has multiple-layer structure, which consists of Medulla in the center, Cortex in the

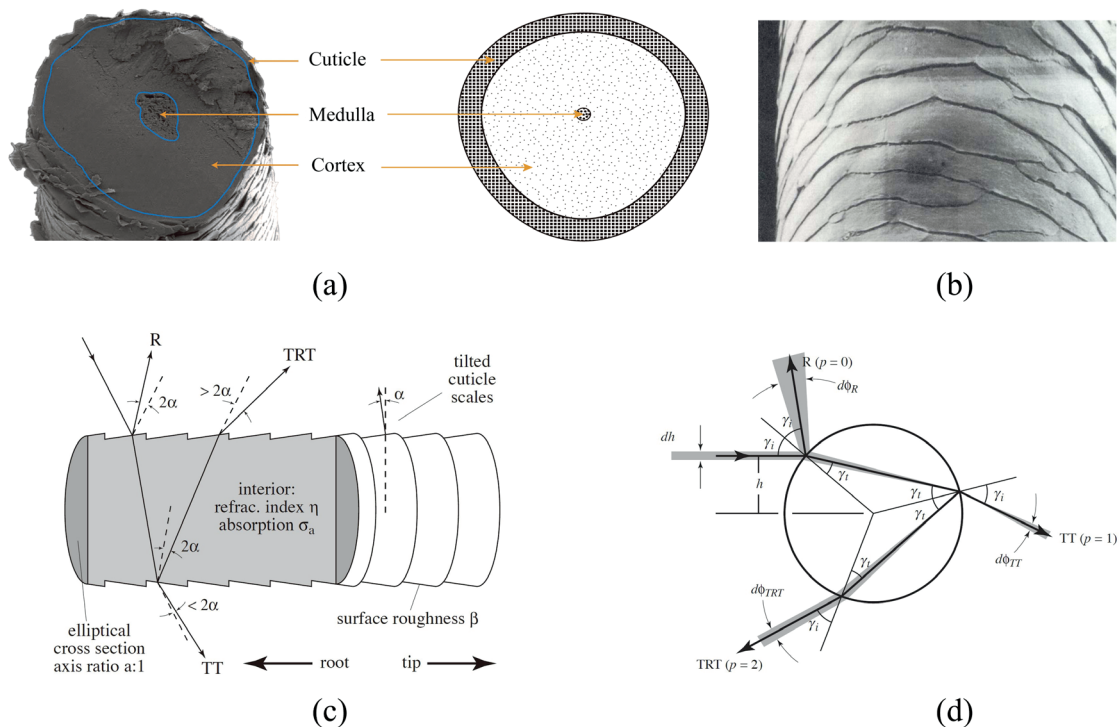


Figure 1. (a) Schematic diagram of a fur fiber cross-section (b) Cuticle scales of fur (c) Schematic diagram of simplified fur fiber model (d) Scattering diagram of light on the circular cross section of fur fiber

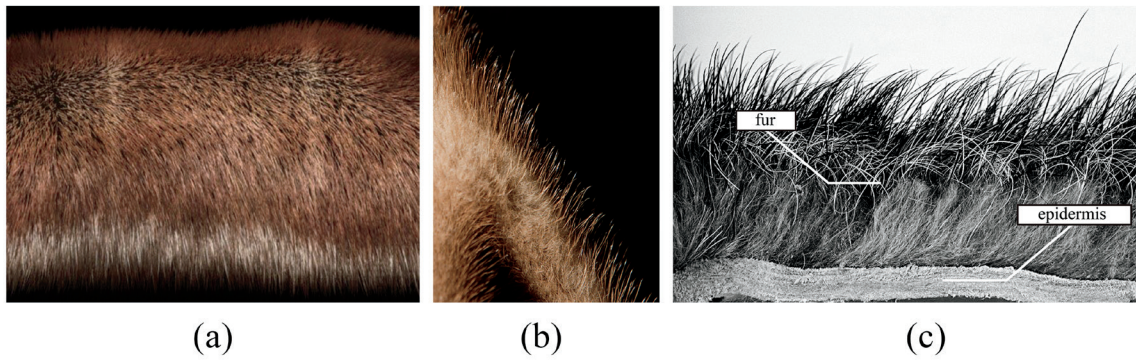


Figure 2. (a) Vertical view of brown mink fur (b) Close-up photos of brown mink fur (c) Cutaway picture of rawhide

interior, and Cuticle in the epidermis. To be specific, after the Cuticle is amplified, as shown in Figure 1(b),¹⁹ pothole micro surfaces are visible, a medium that causes highlights and reflections. In addition, transmission and secondary reflection will occur after the light hits the surface of fur, as shown in Figure 1(c), (d).²⁰ We can generally think of fur as a scattering medium. That is to say, the light emitted vertically from the surface of fur will reach at the bottom part in an exponentially decreasing manner. In light of the vertical view of brown mink fur, as shown in Figure 2(a), (b),²¹ lighter areas tend to be at the tip of fur, while darker areas tend to be at the root of fur. As a result, the characteristics of fur mentioned above together form its texture in human perceptions.

Research hypothesis

According to Jiang’s research on the texture of objects, two factors that could affect the surface of an object are optical characteristics and geometric details.²² In this study, we assume that the fur unit could be formed by plotting piece elements with geometric details and light features, and that fur texture could be formed by displacing and superposing fur unit at a certain distance. As shown in Figure 3, it shows the mechanism of fur vision formation.

Definition of fur texture model

As shown in Figure 2(c), fur and epidermis are the two geometric features of fur texture by analyzing a cutaway picture of rawhide. In order to simulate fur texture, we represented fur and epidermis with geometric figures. In this article, there are three levels defining and analyzing the fur element model.

Low-level parameter—circles and triangles

In terms of the fact that fur and epidermis are the two primary units of fur texture simulation, in order to simplify the method mentioned above, we used circles to represent epidermis and triangles to represent fur. As shown in Figure 4(a), (b), the attributes of circles and triangles include the coordinate of the center point P_i , the radius ξ_i , the coordinate of fur tip vertex T_i^j , and the coordinate of fur root vertex B_i^j and B_i^{j+1} .

Middle-level parameter—fur element

Since real fur grows on epidermis, we need to further define the fur element composed of fur and epidermis, as shown in Figure 4(b). Because the length and thickness of fur as well as the distance between fur pieces exist great differences, the texture of fur presents

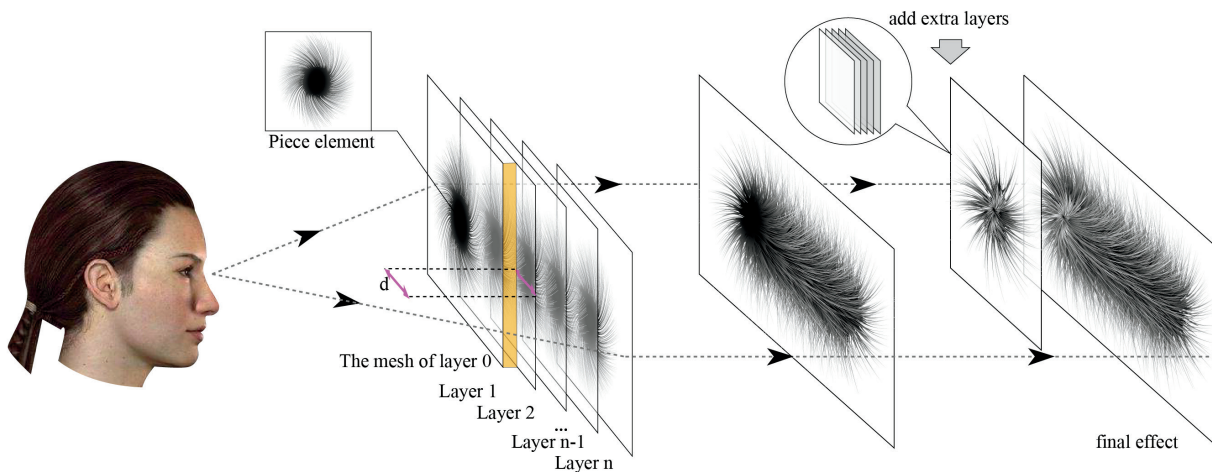


Figure 3. The mechanism of fur vision formation

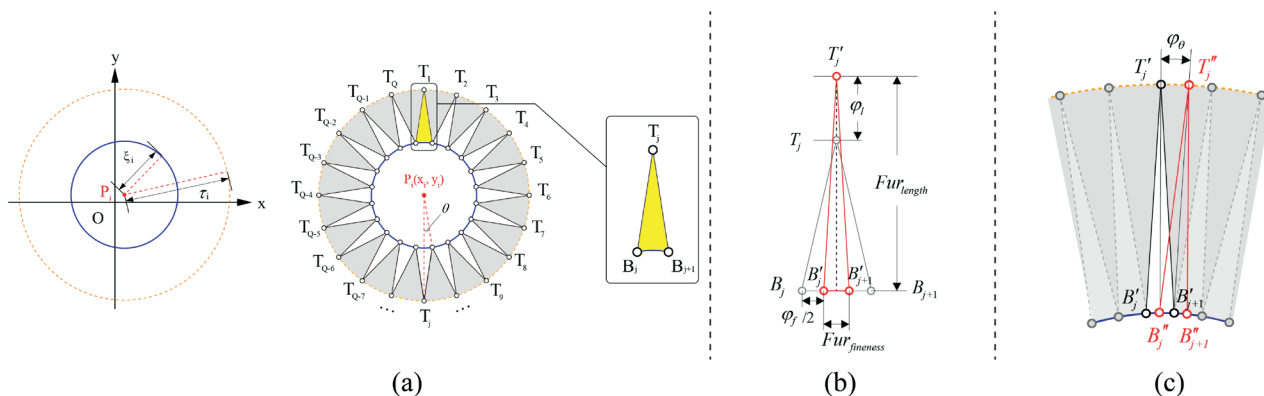


Figure 4. (a) The attributes of the base circle and the outer auxiliary concentric circle of fur piece elements (b) The angular attributes of fur piece elements (c) The principle of random variation of fur difference and related parameter settings

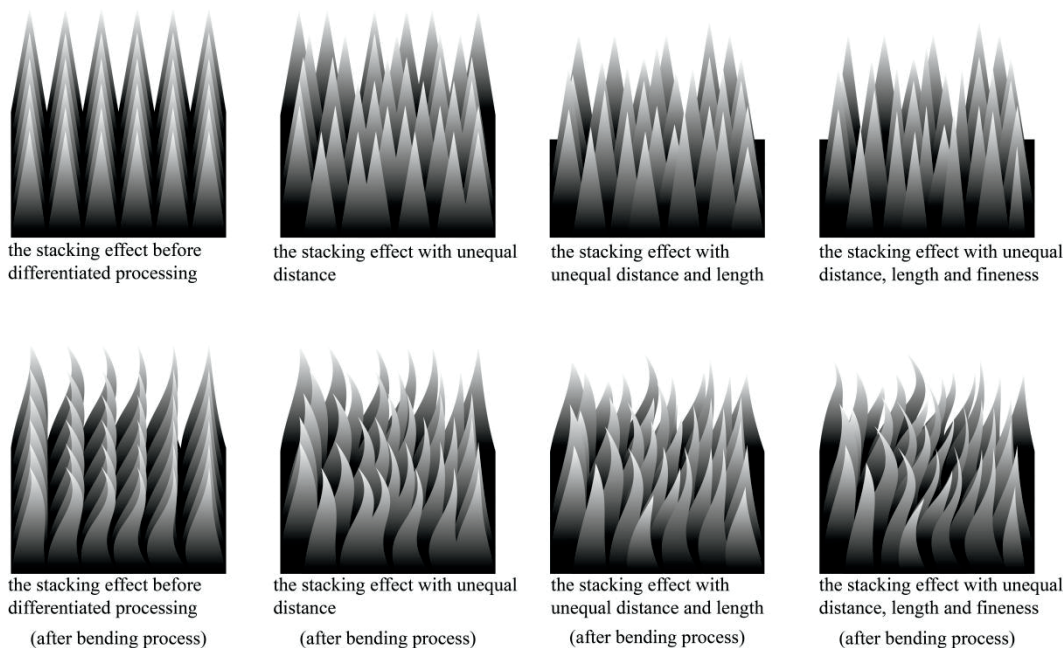


Figure 5. The establishment principle of fur difference

unevenly geometric characteristics. As shown in Figure 5, the effect of fur texture through differentiated processing presents strong irregularities. In this paper, in order to reflect the texture characteristics of real fur, it is necessary to control the difference between fur elements, which is also a significant factor of fur texture simulation. As shown in Figure 4(b), (c), the article defines the degree of difference between each fur piece from random changes in fur length, fur fineness, and fur displacement. The attributes of a composite fur element include the coordinate of the center point of the base circle P_j , the radius of the base circle ξ_i , the radius of the outer auxiliary concentric circle τ_i , the number of vertex of fur piece Num_{vertex} , the length of a single fur piece Fur_{length} , the fineness of a single fur piece $Fur_{fineness}$, the varying quantities of the length of a single fur piece ϕ_j , the varying quantities of the fineness of a single fur piece ϕ_f , the varying quantities of the rotation angle of a single fur piece ϕ_θ .

High-level parameter—fur element arrangement

The key step from the fur piece element to the formation of fur texture is to overlap the piece elements in a dislocation manner, so as to present the visual effect of fur texture. Therefore, based on the definition of one single fur element, it is of great necessity to define multiple high-level parameters to control the stacking arrangement of piece elements. According to the way of dislocation and superposition of fur elements, it can be divided into two categories including off-axis stacking and coaxial stacking, as shown in Figure 6(a). To be specific, off-axis stacking is that the radius of the base circle keeps constant, and the center of the base circle is overlapped by a certain internal distance, leading to an offset stack effect. In contrast, coaxial stacking is that the center of the base circle stays constant, and fur piece elements are gradually scaled and superposed by a certain proportion. As shown in Figure 6(b), each fur piece element has its own parameters, such as the center point

coordinate of the base circle, fur length, fur fineness, and fur density. Also, fur texture model includes several control parameters, such as the superimposition mode of fur piece element, the center point of the base circle, and the spacing among fur elements. At the same time, in order to control the roughness of fur, a control parameter that could randomly adjust the degree of fur difference is added to the fur texture model. In the process of constructing the fur texture model, we need to consider the simulation of optical characteristics of fur, in addition to the representation and definition of geometric details. As shown in Figure 6(c), the light gradually fades from the tip to the root of fur, so that fur has the lowest root brightness and the highest tip brightness. We can simulate the attenuation of light by filling fur piece elements with a black and white gradient from the center of the base circle to the outer auxiliary concentric circle. Due to the difference in the degree of light attenuation on fur under strong and weak light environments, the fur texture displayed quite discrepant. Therefore, it is necessary to set the light attenuation and degree parameters to control the light attenuation. As shown in Figure 6(d), Dg_{max} is the maximum value of the distance between the brightest pixel point and the auxiliary concentric circle edge in the strong light environment, and η is the luminance turning point. While Dl_{max} is the maximum value of the distance between the darkest pixel point and the center of the base circle in the weak light environment, and ϖ is the luminance turning point.

To sum up, key factors in fur texture model include the way of fur element dislocation and superposition, the coordinates of the center point of the base circle-fur element, the distance between fur elements, fur length, fur density, fur fineness, fur difference and light attenuation mitigation.

The texture model of fur is defined as $S = \{\ell, \zeta, \psi\}$, where $\ell = \{\omega, \vartheta, P_0, P_z, \xi, \delta, \rho, \chi, \gamma, \beta_+, \beta_-\}$, is the high-level set, the arrangement of fur piece elements which can be set through user interaction. To be specific, the parameter ω is the stacking mode of fur piece elements, which controls the dislocation and superposition mode of fur. The parameter ϑ is the distance between fur piece elements. When fur piece elements are offset stacking, it represents the distance between the center point of the base circle and the center point of the adjacent one. When piece elements are coaxial stacking, it means the length difference between the radius of the base circle of the adjoining sheet elements and the radius of the outer auxiliary circle. The parameter P_0 is the coordinate of the center point of the initial base circle. The parameter P_z is the coordinate of the center point of the initial base circle. The parameter ξ is the radius of the base circle, which controls the initial radius length of the base circle. The parameter δ is the length of fur, which controls the initial length of fur piece elements. The parameter ρ is the density of fur, which controls the initial density of fur. The parameter χ is the relative fineness of fur, a ratio type value, which controls the fineness of fur. The parameter γ is the fur diversity, which controls the random change on fur elements. The greater the difference is, the larger the roughness of fur texture is. The parameter β_+ is the attenuation mitigation of light on fur in the strong light environment, the larger β_+ is, the smaller Dg_{max} is, and the greater gradient transition is. The parameter β_- is the attenuation intensity of light on fur in the low light environment, the larger β_- is, the smaller Dl_{max} is, and the greater gradient intensity is.

$\zeta = \{FPE_i : i = 1, \dots, Z\}$ is the set of middle-level parameters, known as the set of fur piece elements (FPE). The parameter set

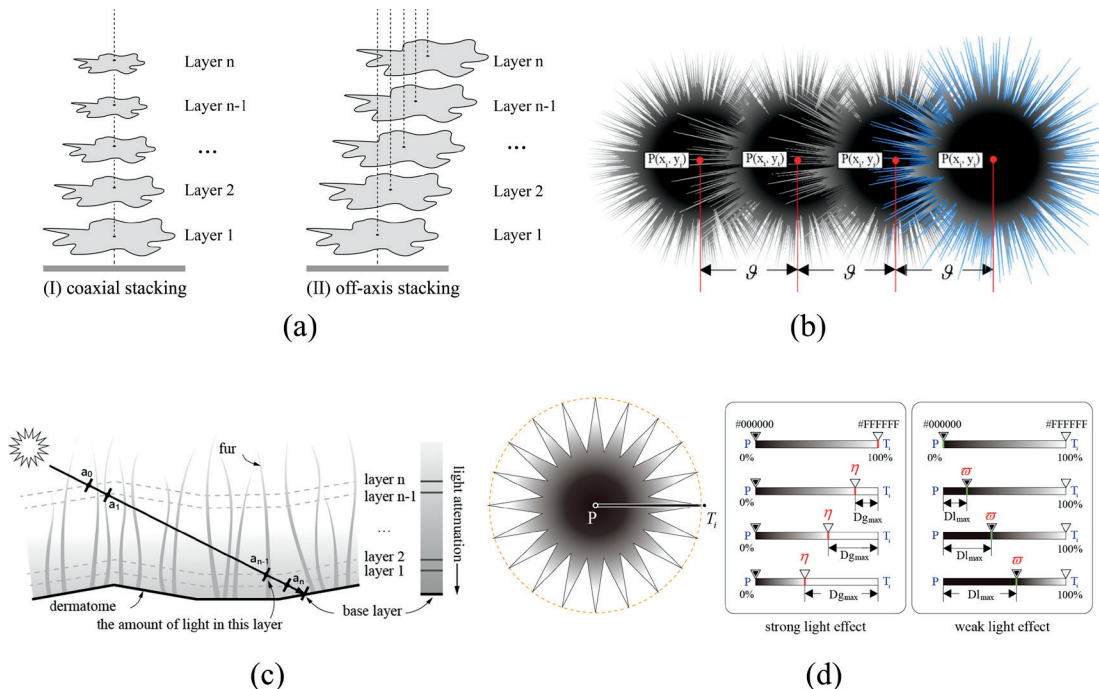


Figure 6. (a) Dislocation and superposition mode of fur piece elements (b) Schematic diagram of off-axis stacking spacing among piece elements (c) Schematic diagram of light attenuation from fur tip to fur root (d) Parameter declaration of light attenuation mitigation

for each fur piece element is $FPE_i = \{P_i, \xi_i, \tau_i, Num_{vertex}, Fur_{length}, Fur_{fineness}, \varphi_l, \varphi_f, \varphi_\theta\}$. To be specific, the parameter P_i is the coordinate of the center point of the base circle. The parameter ξ_i is the radius of the base circle, determined by the three parameters, including the superposition mode parameter ω , the spacing parameter ϑ , and the radius parameter ξ of the base circle in ℓ . The parameter τ_i is the outer auxiliary concentric circle radius, determined by the four parameters, including the radius parameter ξ , the superposition mode parameter ω in ℓ , the spacing parameter ϑ , and the fur length parameter δ . The parameter Num_{vertex} is the number of vertex on the fur piece element, which is determined by ξ_i and ρ . φ_l is the random variable of fur length. φ_f is the random variable of relative fineness of fur. φ_θ is the random variable of fur rotation angle. The parameters φ_l, φ_f , and φ_θ are strongly related to the parameter γ in ℓ , that is, the larger the γ is, the broader the range of changes is in φ_l, φ_f , and φ_θ . Fur_{length} is the length parameter on single fur piece element, which is determined by the fur length parameter δ in ℓ and φ_l in ζ . $Fur_{fineness}$ is the fineness parameter of single fur piece element, determined by the density parameter ρ in ℓ , the relative fineness parameter χ , and the parameter φ_f in ζ .

$\psi = \{Cir_i, Tri_i^j: i=1 \dots, Z; j=1, \dots, Q\}$ is the set of low-level parameters, known as the set of base circle and isosceles triangle. To be specific, each fur piece element contains numbers of fur triangles. The parameter set of each triangle is represented as $Tri_{ij} = \{Tri_{ij}, B_{ij}, B_{ij+1}\}$. T_{ij} is the vertex coordinate of the triangle. B_{ij} and B_{ij+1} are the coordinates of two vertexes at the base of the triangle. The parameter set of each circle is represented as $Cir_i = \{P_i, \xi_i\}$. P_i is the coordinate of the center point of the base circle, ξ_i is the radius of the base circle.

Construction of fur texture model

When building the fur texture model, high-level parameters are sequentially calculated to generate middle-level and low-level parameters. As shown in Figure 6, the main steps of the process are as follows.

Step 1, Set the superposition mode of fur piece elements as the parameter ω , the mathematical definition of ω is shown in Equation (1).

$$\omega = \begin{cases} 0 & \text{coaxial superposition} \\ 1 & \text{off-axis superposition} \end{cases} \quad (1)$$

When the stacking mode is set to coaxial superposition, $\omega = 0$. When the stacking mode is set to off-axis superposition, $\omega = 1$. Taking the off-axis stacking as an example, we first set the coordinate $P_0(x_0, y_0)$ of the center point of the base circle and the coordinate $P_z(x_z, y_z)$ of the center point of the end base circle. Then, we set the spacing parameter ϑ , which could help calculate the coordinate $P_i(x_i, y_i)$ of the base circle's center point of all superimposed piece elements. The calculation formula is shown in Equation (2).

$$\begin{cases} x_i = x_0 + i \cdot \min(\kappa, \vartheta) \cdot \cos\{\arctan[(y_z - y_0)/(x_z - x_0)]\} \\ y_i = y_0 + i \cdot \min(\kappa, \vartheta) \cdot \sin\{\arctan[(y_z - y_0)/(x_z - x_0)]\} \end{cases} \quad (2)$$

In Equation (2), κ is the restriction factor. In order to prevent the spacing parameter ϑ from being overvalued, we set κ value at 0.05. i represents the number of fur piece elements, $i = 1 \dots, Z$.

Step 2, set base circle radius ξ and fur length δ . Calculate outer auxiliary concentric circle radius τ_i . We assume that the radius of the base circle of all piece elements are equal, then the formula for the radius τ_i of the outer auxiliary concentric circle is as follows:

$$\tau_i = \xi_i + \delta \quad (3)$$

Centered on P_i , draw the base circle and outer auxiliary concentric circle with radius ξ_i and τ_i . Then, set fur density ρ and fur fineness χ to calculate the number of vertex of triangles on each piece element. After that, set the number of vertex of triangles as Num_{vertex} , the calculation formula is as follows:

$$Num_{vertex} = \text{int}(2\pi \cdot \xi_i \cdot \rho) = Q \quad (4)$$

In Equation (4), Q is represented as the number of vertex of triangles on one single piece element. Based on the number of vertex of triangles, we could calculate the central angle 2θ through the arc $B_{i,j}$ $B_{i,j+1}$, as shown in Equation (5):

$$2\theta = 2\pi / Num_{vertex} \quad (5)$$

Also, we could calculate the width of fur root Λ , the calculation formula of which is shown in Equation (6):

$$\Lambda = 2 \cdot \xi_i \cdot \sin\theta \cdot \chi \quad (6)$$

In Equation (6), the fineness of fur χ is the ratio parameter, and its value range is 20% to 100%.

Step 3, set the vertex coordinate T_i^1 of the triangle right above the center point of the base circle. We could calculate the vertex T_{ij} of the triangle according to the center point coordinate P_i of the base circle, the radius ξ_i of the base circle, the radius τ_i of the outer auxiliary concentric circle, and the center angle θ of the base circle, as shown in Equation (7):

$$\begin{cases} x_{i,j}^r = x_i + \tau_i \cdot \cos(\pi/2 - 2j\theta + 2\theta) \\ y_{i,j}^r = y_i + \tau_i \cdot \sin(\pi/2 - 2j\theta + 2\theta) \end{cases} \quad (7)$$

In Equation (7), i stands for the number of fur piece elements ($i = 1, \dots, Z$), whereas j represents the number of triangles on each piece element ($j = 1, \dots, Q$). We could calculate the vertex coordinate B_{ij} of the triangle base, as shown in Equation (8):

$$\begin{cases} x_{i,j}^B = x_i + \xi_i \cdot \cos [\pi/2 - (2j-3)\theta - (1-\chi)\theta/2] \\ y_{i,j}^B = y_i + \xi_i \cdot \sin [\pi/2 - (2j-3)\theta - (1-\chi)\theta/2] \end{cases} \quad (8)$$

As shown in Equation (9), we could also calculate the vertex coordinate $B_{i,j+1}$ of the triangle base.

$$\begin{cases} x_{i,j}^B = x_i + \xi_i \cdot \cos [\pi/2 - (2j-1)\theta + (1-\chi)\theta/2] \\ y_{i,j}^B = y_i + \xi_i \cdot \sin [\pi/2 - (2j-1)\theta + (1-\chi)\theta/2] \end{cases} \quad (9)$$

In Equation (9), when $j=Q$, $B_{i,j+1} = B_{i,1}$.

Step 4, set fur difference parameter γ . Based on γ , we could calculate the random variance of fur length, fur fineness, and the rotation angle of each single fur piece element. What is more, we could represent the random variance of one single piece as $SFL_{i,j}$, where i stands for the number of fur piece elements and j stands for fur quantities. As a result, the random variance matrix φ_l of fur length could be expressed as:

$$\varphi_l = \begin{bmatrix} SFL_{1,1} & \cdots & SFL_{1,j} & \cdots & SFL_{1,Q} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ SFL_{i,1} & \cdots & SFL_{i,j} & \cdots & SFL_{i,Q} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ SFL_{z,1} & \cdots & SFL_{z,j} & \cdots & SFL_{z,Q} \end{bmatrix} \quad (10)$$

In Equation (10), $SFL_{i,j} = \text{random}(-\lambda \cdot \delta/2, \lambda \cdot \delta)$, which could take any value between $-\lambda \cdot \delta/2$ and $\lambda \cdot \delta$. The value range of the variance parameter γ is $[0, 100\%]$, the greater the value of γ , the larger the change of $SFL_{i,j}$.

If we represent the fineness random variance of one single fur piece as $SFF_{i,j}$, the fineness random variation matrix φ_f in the fur texture model could be expressed as:

$$\varphi_f = \begin{bmatrix} SFF_{1,1} & \cdots & SFF_{1,j} & \cdots & SFF_{1,Q} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ SFF_{i,1} & \cdots & SFF_{i,j} & \cdots & SFF_{i,Q} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ SFF_{z,1} & \cdots & SFF_{z,j} & \cdots & SFF_{z,Q} \end{bmatrix} \quad (11)$$

In Equation (11), $SFF_{i,j} = \text{random}[-\lambda \cdot \chi, \lambda \cdot (100\% - \chi)]$, which could take any value between $-\lambda \cdot \chi$ and $\lambda \cdot (1 - \chi)$, the larger the value of γ , the greater the variation range of $SFF_{i,j}$.

If we represent the random variable of one single fur piece rotation angle as $SFA_{i,j}$, the fur rotation angle variable matrix φ_θ could be expressed as:

$$\varphi_\theta = \begin{bmatrix} SFA_{1,1} & \cdots & SFA_{1,j} & \cdots & SFA_{1,Q} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ SFA_{i,1} & \cdots & SFA_{i,j} & \cdots & SFA_{i,Q} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ SFA_{z,1} & \cdots & SFA_{z,j} & \cdots & SFA_{z,Q} \end{bmatrix} \quad (12)$$

In Equation (12), $SFA_{i,j} = \text{random}[-\lambda \cdot 2\theta, \lambda \cdot 2\theta]$ which could take any value between $-\lambda \cdot 2\theta$ and $\lambda \cdot 2\theta$. When $SFA_{i,j}$ is negative, the triangle on fur piece elements rotates counterclockwise on the base circle.

When $SFA_{i,j}$ is positive, the triangle on fur piece elements rotates clockwise on the base circle. The greater the value of γ , the larger the variation range of $SFA_{i,j}$.

Combined with $SFL_{i,j}$ and $SFA_{i,j}$, we could calculate the vertex coordinate $T_{i,j}'$ of the triangle after random variation of the fur rotation angle.

$$\begin{cases} x_{i,j}^{T'} = x_i + (\tau_i + SFL_{i,j}) \cdot \cos [\pi/2 - (2j\theta - 2\theta + SFA_{i,j})] \\ y_{i,j}^{T'} = y_i + (\tau_i + SFL_{i,j}) \cdot \sin [\pi/2 - (2j\theta - 2\theta + SFA_{i,j})] \end{cases} \quad (13)$$

Combined with $SFF_{i,j}$ and $SFA_{i,j}$, we could also calculate the vertex coordinates $B_{i,j}'$ and $B_{i,j+1}'$ of the triangle after random variation of the fur relative fineness, as shown in Equation (14) and Equation (15).

$$\begin{cases} x_{i,j}^{B'} = x_i + \xi_i \cdot \cos \{ \pi/2 - (2j-3)\theta - [1 - (\chi + SFF_{i,j})] \theta/2 + SFA_{i,j} \} \\ y_{i,j}^{B'} = y_i + \xi_i \cdot \sin \{ \pi/2 - (2j-3)\theta - [1 - (\chi + SFF_{i,j})] \theta/2 + SFA_{i,j} \} \end{cases} \quad (14)$$

$$\begin{cases} x_{i,j+1}^{B'} = x_i + \xi_i \cdot \cos \{ \pi/2 - (2j-1)\theta + [1 - (\chi + SFF_{i,j})] \theta/2 + SFA_{i,j} \} \\ y_{i,j+1}^{B'} = y_i + \xi_i \cdot \sin \{ \pi/2 - (2j-1)\theta + [1 - (\chi + SFF_{i,j})] \theta/2 + SFA_{i,j} \} \end{cases} \quad (15)$$

Step 5, perform Boolean Operation to obtain the regional elements of star-shaped fur. We fill the gradient from black (at the center of the base circle P_i) to white (at the vertex of triangles $T_{i,j}$), in order to simulate the light decay from the tip to the root of fur. In the strong light environment, we could calculate Dg_{\max} according to the parameter β_+ of light attenuation mitigation, as shown in Equation (16):

$$Dg_{\max} = \tau_i \cdot (1 - \beta_+) \quad (16)$$

Therefore, we could calculate the exact location of the brightness turning point η .

In the weak light environment, we could calculate Dg_{\max} based on the parameter β_- of light attenuation mitigation, as shown in Equation (17):

$$Dl_{\max} = \tau_i \cdot (1 - \beta_-) \quad (17)$$

Therefore, we could figure out the turning point ϖ of brightness.

Based on the five steps mentioned above, we could get the realistic fur texture with dislocated and superposed piece elements. As a result, we could achieve several parametric controls such as superposition mode of fur piece elements, the distance between piece elements, fur length, fur density, fur fineness, fur differences, and fur illumination effect. As shown in Equation (18), the fur element matrix F comprises the radius ξ of base circle, fur length δ , fur density ρ , fur fineness χ , and fur difference γ . Figure 7 shows the shape of fur piece elements under different F effects.

$$F = [\xi \delta \rho \chi \gamma]^T \quad (18)$$

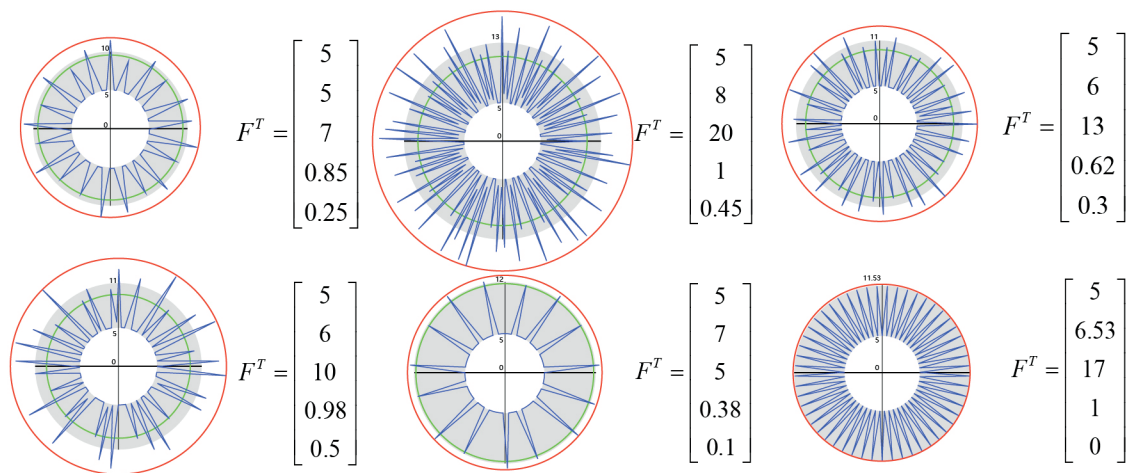


Figure 7. The influence of parameter setting on the geometric contour of fur piece elements

Although parameter values can be set arbitrarily within the value range, different effects on fur simulation result from various kinds of parameter settings such as fur density, fur length, fur fineness, fur diversity, light attenuation, and fur piece element distance. In order to ensure the authenticity of fur, we should take the appropriate value range of each parameter into consideration. Combined with theoretical analysis and experiment research, we could check the value range of each parameter in Table I.

Application

In order to make it easier for designers to adopt the method mentioned above to build a fur texture model, we have explored the possibility of constructing an elemental model of fur piece under the environment of Adobe Illustrator (AI), a general vector drawing software for designers. In AI software, operators could draw various

vector graphics by setting the parameters in each dialog boxes. After several experiments, we found out related tools in AI to build the elemental model of fur piece, including star tool, pucker and bloat effect, roughness effect, blend tool, and gradient tool. To be specific, the control parameters in star tool include inner radius of star, outer radius of star, and star angle points. The control parameters in pucker and bloat effect include pucker value and bloat value. The control parameters in roughness effect include roughen size and roughen detail. The control parameter in blend tool includes blend spacing. The control parameters in gradient tool include gradient type and gradient slider location. By analyzing the influence of the parameter changes on the tools mentioned above and comparing the control parameters in the fur texture model, we have established the corresponding relationship between the fur texture model and the parameters in Adobe Illustrator software in Table II.

Table I
Value range of each parameter in fur texture model

Parameter	Value range	Type
Fur density ρ	[10, 30]	Integer
Fur length δ	[ξ , 4 ξ]	Floating-point
Fur fineness χ	[0.5, 1]	Percentage
Fur diversity γ	[0.2, 0.8]	Percentage
Strong light attenuation mitigation β_+	[0.8, 1]	Percentage
Weak light attenuation mitigation β_-	[0.4, 1]	Percentage
Fur piece element distance ϑ	0.5	Floating-point

Table II
Correspondence between fur texture model and parameters in Adobe Illustrator

Parameters of fur texture model	Related tool in AI	Parameters in AI
The radius of the base circle ξ	Star tool	Inner radius of star
Fur length δ	Star tool	Inner radius of star-Inner radius of star
Fur density ρ	Star tool	Star angle points / 2π Inner radius of star
Fur fineness χ	Pucker & bloat effect	Shrinkage value
Fur diversity γ	Roughen effect	Roughen size
The distance of piece element ϑ	Blend tool	Blend spacing
Strong light attenuation mitigation β_+	Gradient tool	High-brightness gradient slide location
Weak light attenuation mitigation β_-	Gradient tool	Low-brightness gradient slide location

The construction process of fur texture model in AI is shown in Figure 8. First, select a star tool. Set star inner radius, star outer radius and star angle points according to the length and density of fur. Second, use a direct selection tool to round the star's sharp corners, select Pucker & bloat effect, and set shrinkage value of the rounded star according to the relative fineness of fur. Third, select roughness effect. Set roughen size value according to fur variance requirements. Fourth, select gradient tool, set radial gradient from the center of the piece elements to the outermost corner, and set the position of the bright and dark slider according to the light attenuation mitigation. Fifth, replicate and translate fur piece elements. Select blend tool and blend the two piece elements according to the distance of piece elements. Based on the five steps mentioned above, we could produce fur texture realistically based on the dislocated and superposed piece elements.

Results and Discussion

The influence of parameter changes on simulation effect

Based on the method, we could find in Table I that a quite good simulation effect is obtained as long as the distance between piece

elements is 0.5mm. Therefore, we should set mixed distance at 0.5mm and star outer radius length at 15mm in AI. The following results show the influence of changes in star angle points, star outer radius length, roughness size, shrinkage value, and slider position on the final simulation effect of fur.

The influence of the star angle points changes on the effect of fur simulation

Set the roughness to 70, the star outer radius length to 30mm, the shrinkage value to -32%, the mixed distance to 0.5mm, the low-brightness slider displacement to 22%, and the high-brightness slider displacement to 100%. By changing the star corner points (SCP), as shown in Figure 9, the picture shows the fur simulation effect at 10, 15, 20, and 25. By contrast, we can find that the more the number of corners, the greater the fur density.

The influence of the star outer radius length change on the fur simulation effect

Set the star angle point to 15, the roughness to 70, the shrinkage value to -32%, the mixed distance to 0.5mm, the low-brightness slider displacement to 22%, and the high-brightness slider displacement to 100%. By changing the star outer radius length (SORL), as shown in

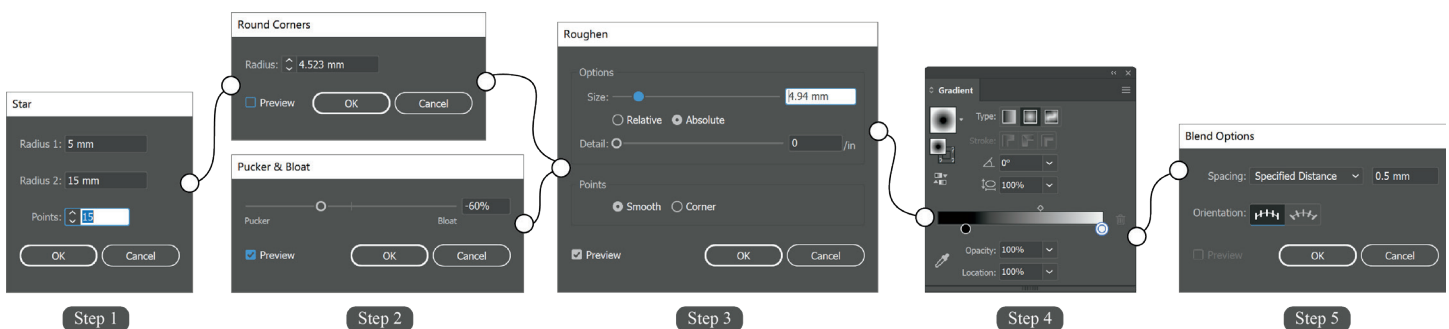


Figure 8. The construction process of realistic fur texture in Adobe Illustrator

Figure 10, the picture shows the fur simulation effect with the outer radius at 35, 40, 45, and 50. By contrast, we can find that the larger the star outer radius length, the greater the fur length.

The influence of the roughness size change on the effect of fur simulation

Set the star angle point to 15, the star outer radius length to 30mm, the shrinkage value to -32%, the mixed distance to 0.5mm, the low-brightness slider displacement to 22%, the high-brightness slider displacement to 100%. By changing the roughness size (RS), as shown in Figure 11, the picture shows the fur simulation effect with the roughness at 20%, 40%, 60%, and 80%. By contrast, we

can find that the greater the roughness, the larger the degree of fur differentiation.

The influence of shrinkage value on the effect of fur simulation

Set the star angle point to 15, the star outer radius length to 30mm, the roughness to 70%, the mixed distance to 0.5mm, the low-brightness slider displacement to 22%, and the high-brightness slider displacement to 100%. By changing the shrinkage value (SV), as shown in Figure 12, the picture shows the fur simulation effect with the shrinkage value at -30%, -40%, -50%, and -60%. By contrast, we can find that with the increase of the absolute value of contraction, the density of fur remains the same, and the relative fineness of fur becomes smaller.

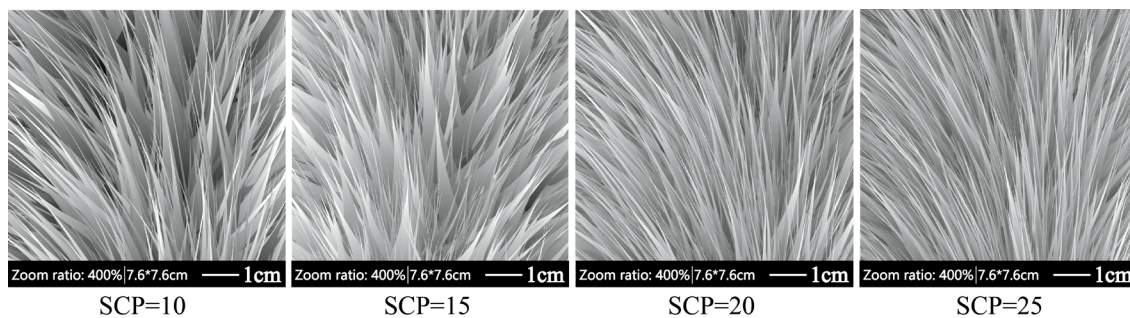


Figure 9. The simulation effect on different star angle points

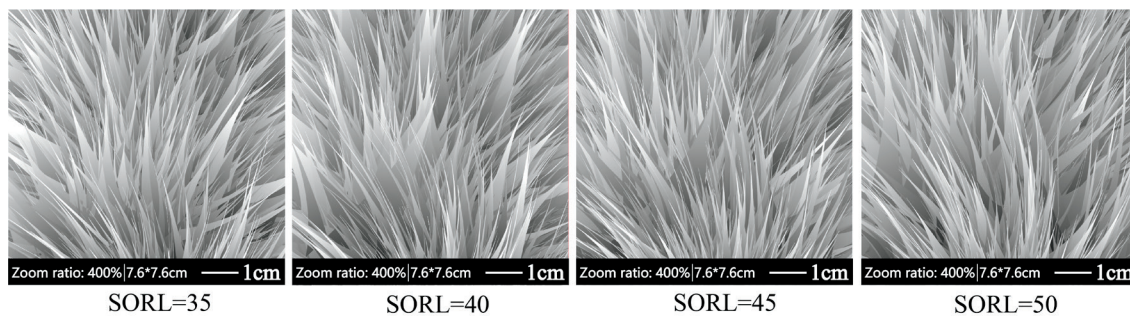


Figure 10. The simulation effect on different star outer radius length

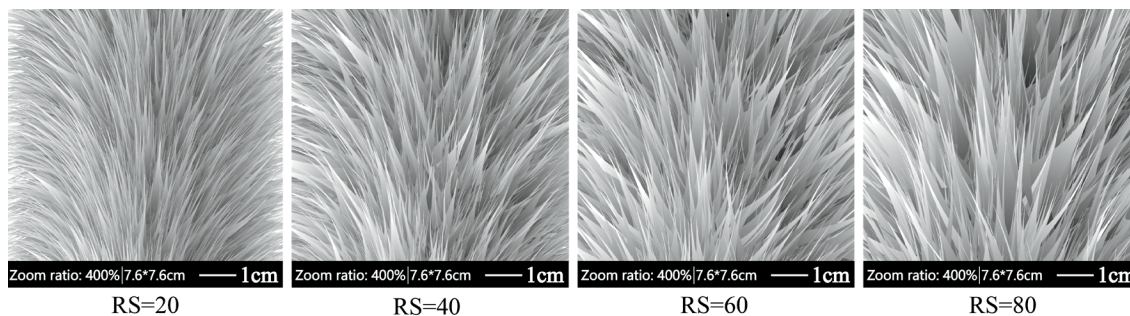


Figure 11. The simulation effect on the change of roughness size

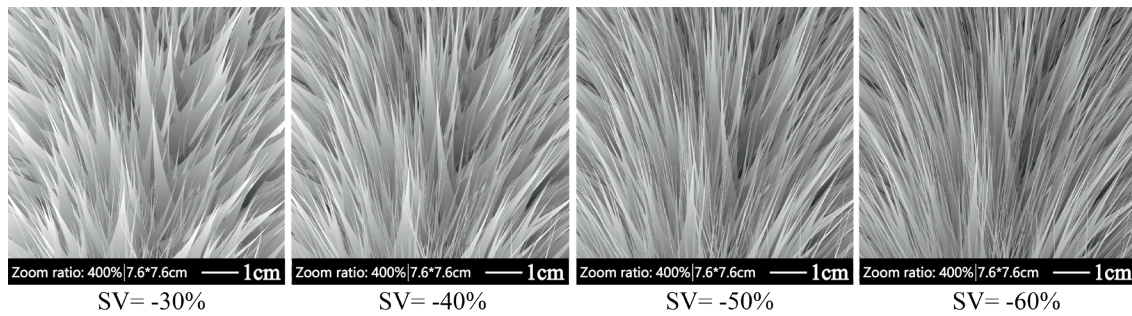


Figure 12. The simulation effect on the change of shrinkage value

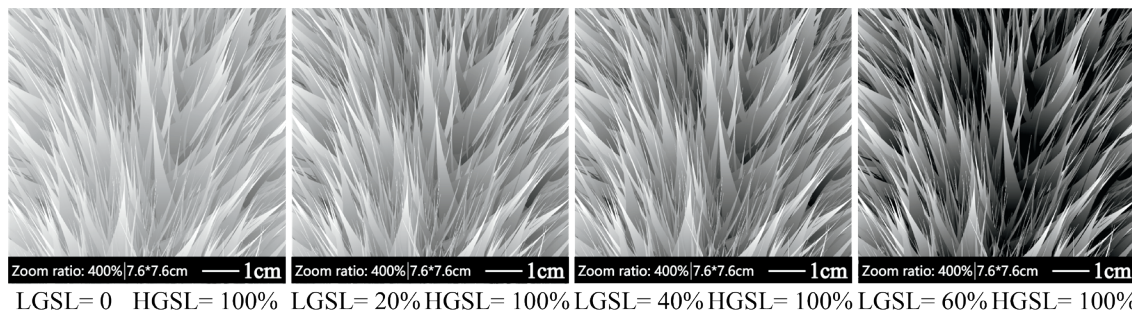


Figure 13. The simulation effect on the location change of the low-brightness gradient slider

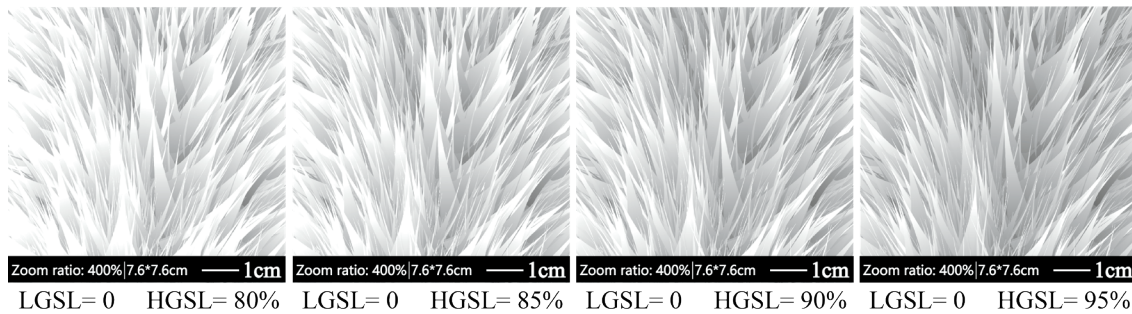


Figure 14. The simulation effect on the location change of the high-brightness gradient slider

The influence of the gradient slider location change on the effect of fur simulation

Set the star angle point to 15, the star outer radius length to 30mm, the roughness to 70%, the mixed distance to 0.5mm, the shrinkage value to -30%, the mode of dislocation and superposition to off-axis dislocation and superposition. By changing the low-brightness gradient slider location (LGSL), as shown in Figure 13, the picture shows the fur simulation effect with the slider location at 0%, 20%, 40%, and 60%. By contrast, we can find that with the increase of the displacement distance of the low-brightness slider, the fur is getting darker. However, by changing the high-brightness gradient slider location (HGSL), as shown in Figure 14, the picture shows the fur simulation effect with the slider location at 80%, 85%, 90%, and 95%.

By contrast, we can find that with the increase of the displacement distance of the high-brightness slider, the fur is getting brighter as a whole.

Discussion

The effectiveness of the method

Known from above, we could adjust the parameters to produce fur texture with different length, density, roughness, and illumination performance, representing a perfect sense of reality. As shown in Figure 15, we use basic geometric primitives to form vector fur, so as to possess all the advantages of vector graphics, such as scalability, compatibility, variability, etc.²³

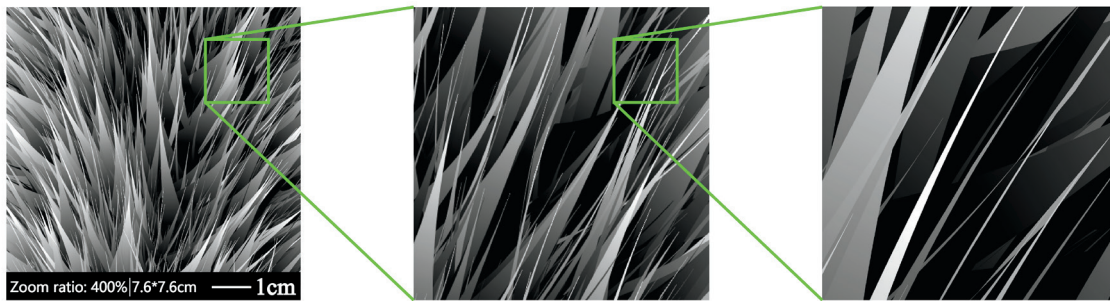


Figure 15. The simulation effect on the magnification factor

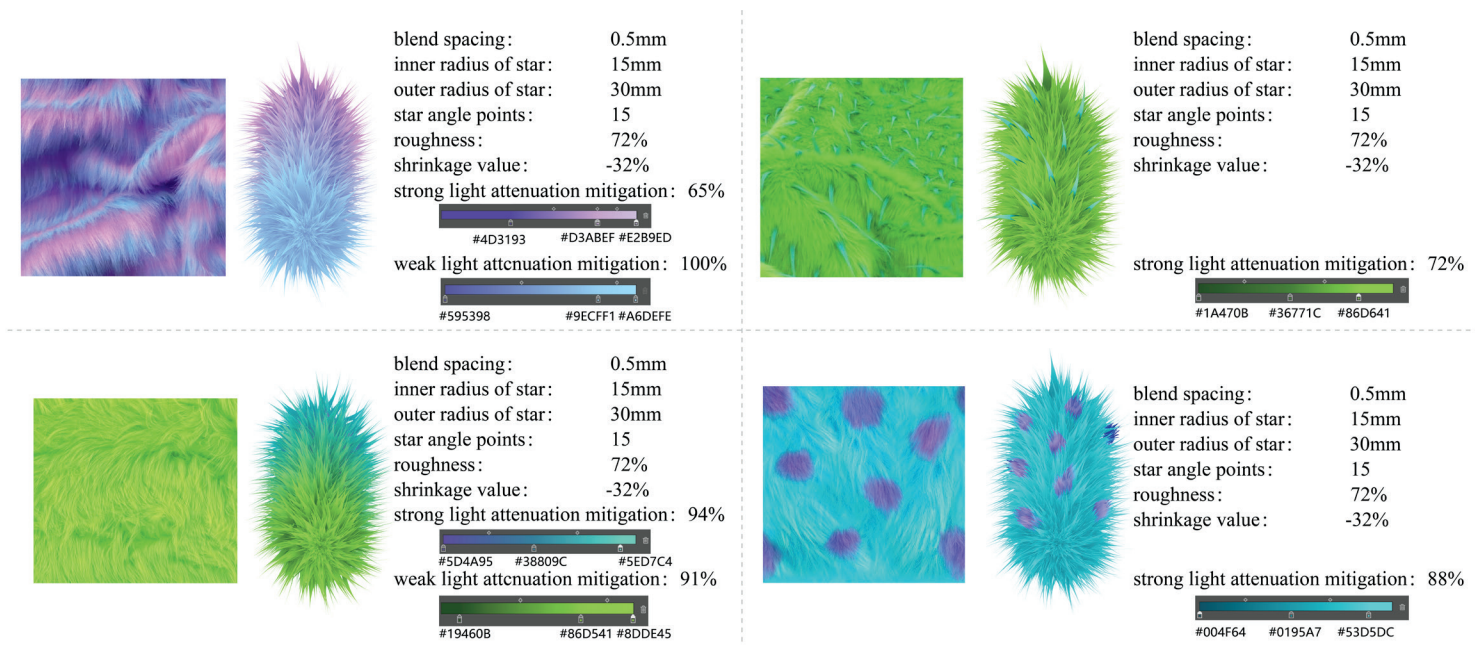


Figure 16. The simulation effects of different kinds of fur

The adaptability of the method

Figure 16 shows the simulation effect of different kinds of fur. Based on the core method of fur piece elements synthesis, we could edit the color of piece elements to generate fur effects with different styles.

Comparisons among the methods

At present, designers use hand-painting, bristle brush, or map rendering to represent fur texture. In this paper, we compared the method of piece-element synthesis with the methods of the bristle brush and the map rendering in many respects, including the advanced principle of fur texture generation, the convenience of user operation, the speed of generation, the rate of modification,

the professional requirements of users, and the restrictions on production styles. We can find from Table III that the bristle brush not only lags in the principle of fur texture formation, but also possesses long drawing time and low speed of modification. Even though the map rendering is more advanced in fur texture generation, it is time-consuming and not convenient for designers to operate the software, which seriously impacts efficiency. By comparing the two methods mentioned above, the piece-element synthesis method could not only quickly generate realistic fur texture, but also achieve real-time modification effect by using parameters to control the attributes of fur texture. What is more, designers could promptly visualize fur texture of different kinds of products by implementing the piece-element synthesis method in Adobe Illustrator.

Table III

The comparisons among different fur simulation methods

Category	Bristle brush	Map rendering	Piece-element synthesis
Principles	Updated	Advanced	Advanced
Convenience for operation	Yes	No	Yes
Generation rate	Low	Low	High
Modification rate	Low	Low	High
Requirements for users	Low	High	Low
Restrictions on styles	Low	High	Low

Conclusion

In this paper, a simulation method for realistic fur texture based on the synthesis of piece elements is proposed. In order to better control the geometric details and optical properties of fur model, we defined a parametric model according to dislocation and superposition of fur piece elements. To be specific, we expressed each fur piece element in circle and triangles. Then, we set the fur difference parameter by controlling the hybrid texture of fur and employed the light attenuation mitigation parameter by simulating the light attenuation. Given the two steps mentioned above, we could obtain the realistic fur texture by dislocating and superposing fur piece elements. At the same time, we have verified the feasibility of the matched parameters in Adobe Illustrator software, which could not only simulate the realistic fur texture, but also generate diversified fur in different types. Due to the uncomplicated illumination calculation and low requirements on computer hardware configuration, designers could quickly generate multiple kinds of fur texture by controlling the relevant parameters.

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