



## An inventory model with screening of items through machinery system for supply chain under fuzzy logic system

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(Received: 16 July 2025

Revised: 20 August 2025

Accepted: 29 September 2025)

### KEYWORDS

EOQ, machine learning based inspection, imperfect quality items, and carbon emission.

**ABSTRACT:** These days it is very difficult to produce 100% good quality items. Hence, delivered lot may have some defective items in the lot. Thus, inspection of lot becomes necessary. The machinery system reduces labour cost and it is more beneficial in the inventory management. Day by day labour cost increases due to effect of human need increase and retailer faces some economic issues. During manufacturing and production, some produced products are not in good shape teemed as defective items. The buyer inspects the whole received lot from the seller through inspection machine and includes screening cost. The screening cost is the sum of various expenditures of machinery system. In this paper, we deal with an inventory model with pullulated free demand rate under eco-friendly environment. Now days a lot of carbon units emit from the different sources like factories, firms and some other production factories in the form of poisonous gases and these gases effect the neat and clean environment. Due to the carbon emissions carbon dioxide, sulphur dioxide and methane are more responsible for these issues and some cities like Delhi, Noida and Ghaziabad more effected currently on account of such reason. There are more problems create related to the environment like air pollution, water pollution and the quality of the eco-friendly foods deteriorate as soon as.

The Government tries to control such of issues by including of some cost like pollution cost, carbon emission cost ect. Finally, we minimized the inventory cost with respect to the order quantity for the eco-friendly foods and has been shown the numerical example for the justification of the proposed model. The sensitivity observation has been presented for the ordering policies as well as decision makers and future scope of this scenario also presented.

**ntroduction:** In this paper Hill's model (1997) has been considered as a base model and we have added fuel cost and the emission tax to understand the impact of environmental issues on the inventory model. Total cost has been minimized with the help of derivative method and optimal order quantity is obtained by minimizing the total cost. Numerical example is provided with sensitivity analysis to know the robustness of the model.

**Objectives:** Total cost has been minimized with the help of derivative method and optimal order quantity is obtained by minimizing the total cost. Numerical example is provided with sensitivity analysis to know the robustness of the model.

**Methods:** We adopt the property of maxima and minima property for the calculation of integrated fuzzy cost,

if  $\frac{d\tilde{\Psi}(Q)}{dQ} = 0$ , we calculated the value of lot size,  $(Q)$  by using of Mathematica 9.0, and suppose it is

$Q = Q^*$  (consider) and now we calculates  $\frac{d\tilde{\Psi}(Q)}{dQ}$ , and if we get  $\frac{d\tilde{\Psi}(Q)}{dQ} \geq 0$ , then  $Q = Q^*$  is

called optimal value of  $Q$  and denoted by  $Q^*$ . The convexity of joint total fuzzy cost for supply has been



shown in the figure with help of Mathematica 9.0 software.

Results: From Table-3, we can conclude that the percentage of defective items increase then optimal shipment lot size increase while joint total cost decreases. From Table-4, it can easily be analysed that as production rate increase, the order quantity almost constant and joint total cost increases. From Table-5, it can easily be analysed that as the number of truck increases with keeping other model parameter constant then order quantity and Joint total cost increases.

Conclusions The machine learning algorithm is good tools for the reduction of labour cost as well the total cost is decreasing which includes. inspection cost, holding cost, setup cost, item cost, transportation cost, emission tax cost.

## 1. Introduction

Inventory is the complete list of items such as goods in property. Inventory includes the raw material, partially manufactured items, prior to sale and departure from the manufacturing product. Inventory is widely used in public as well as private sector organization. Proposed strategy of Hill (1997) refers to the shipment needs to be delivered to retailer during the production segment of manufacturer cycle. In addition to Hill (1997) transportation cost and emission cost was added by Aljazzar et al. (2017). Where transportation cost & emission through transportation was defined in detail by Gurtu et al.(2015) and manufacturer emission was defined by jaber et al. (2013). Whenever we purchase any item from the market, we must be inspecting the item because of many defects in between them (Jaggi et al., 2011). Also in supply chain model it affects the demand when there are imperfect items available in the lot (Yadav et al. 2018). In this paper we have studied the impact of incorporating the imperfect quality items in the lot to the total cost which includes setup cost, holding cost, transportation cost, production cost, interest earning & emission. If the total cost increases or decreases it directly impact the factors included in total cost. So here we have observed the changes in the total lot size and changes in total cost if there are some imperfect qualities in the lot delivered.

## 2 Notation

The following notations have been given under below with meaning description

$Q$  = Order lot (decision variable) (units)

$D$  = Retailer's demand rate (units/year)

$\tilde{D}$  = Retailer's fuzzy demand rate (units/year)

$P$  = Manufacturer's production rate where ( $D < P$ )

$A_r$  = Ordering cost for reailer (\$/shipment)

$C_r$  = Item unit cost for reailer (\$/unit)

$h_r$  = Holding cost for reailer (\$/unit/year)

$I_S$  = Screening cost of machinery system (\$/unit)

$s_r$  = Storage holding cost for for reailer (\$/unit/year)

$A_m$  = Ordering cost for manufacturer (\$/shipment)

$C_m$  = Item unit cost for manufacturer (in \$)

$h_m$  = Holding cost for manufacturer (\$/unit/year)

$s_m$  = Storage holding cost for for reailer (\$/unit/year)

$\lambda$  = Shipment lot size multiplier in a vendor cycle

$T_m$  = Manufacturer's cycle time =  $\lambda Q/D$  (year)

$\delta$  = Percentage of defective items

$T_e$  = Transport emission tax (in \$)

$T_{ci}$  = Emission tax rate (\$/Ton)

$T_{cap}$  = Truck capacity (in integer)

$\eta$  = Number of trucks per shipment ( $Q/T_{cap}$ )

$E_m$  = Manufacturing emission (g/unit)

$d$  = Distance traveled in one side (Km)

$TAC_R$  = Retailer's total annual cost (in \$)

$TAC_m$  = Manufacturer's total annual cost (in \$)

$\Psi(Q)$  = Integrated total annual cost (in \$) under crisp model

$\tilde{\Psi}(Q)$  = Total integrated fuzzyfied cost (in \$) under fuzzy environment



$\tilde{\Psi}(Q)$  = Total integrated defuzzified cost (in \$) under fuzzy environment

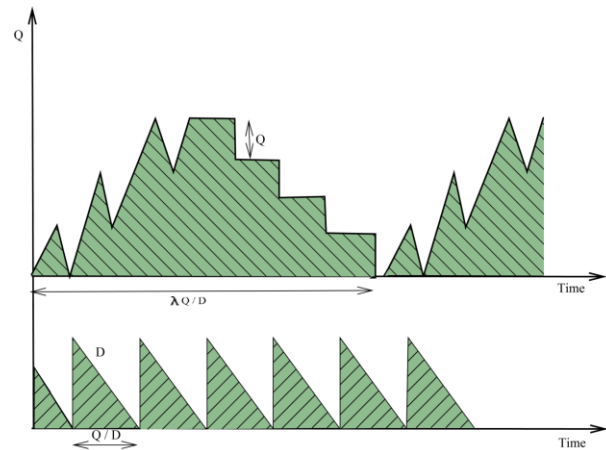
**2. Assumptions**

- The rate of demand obeys the effect of fuzzy environment.
- The demand rate is taken as triangular fuzzy number
- Shortages are not permitted in this model.
- The rate of demand is less than the rate of production.
- The capacity of all trucks is same and uses same path for each truck during supply chain.
- Single type of items is used during this supply chain.
- The cost of holding units is included during non-production phase.
- Delivered lot has some defective items.
- Imperfect quality items are sold at low price in another market.
- Manufacturer emission cost will be fixed during supply chain
- The cost related to holding units is termed as financial cost which will be same during supply chain.
- The financial cost for retailer is greater than the financial cost of manufacturer at the retailer's end.

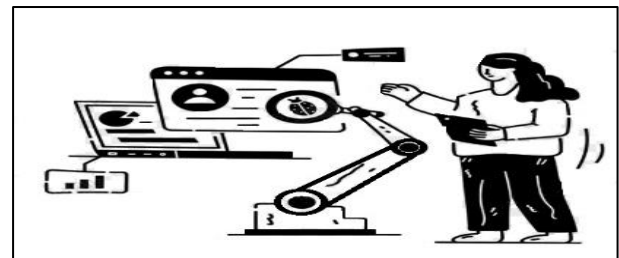
**3. Proposed Model**

Our proposed model follows the model mythology of Hill (1997) and its inventory representation has been depicted in figure 1. Manufacturer prepared  $Q$  units as per demanded retailer. It is suppose that  $D$  is the rate of demand for retailer. In this order, we studied the model of Jaber et. al. (2013) which presented for the manufacturer emissions which are equal to  $E_m = aP^2 - bP + c$ . Gurtu

et al. (2015) explained the carbon emission cost associated with transportation and transport with the help of formula  $(2d eT_{ci})$ .



**Figure 1: Working process of inventory**



**Figure 2: Screening of items through machinery system**

**.3.1 Retailer's total annual cost**

In this section, we calculated the total annual cost for the retailer which is the sum of the Screening cost  $(IC_s = I_s Q)$ , setup cost  $(SC = \frac{(A_r)D}{Q})$ , item cost  $(IC = C_r D)$ , holding cost  $(h_r + S_r) \left[ \frac{(1-\delta)Q}{2} + \frac{\delta Q^2}{17520} \right]$ , and transportation cost  $(TC = \frac{\eta D A_0}{Q} + \frac{\eta A_1 (2\lambda n - \lambda - 1)}{2})$  and emission tax  $(ET = \frac{\eta D T_e}{Q})$ .

The Retailer's annual cost  $(TAC_R)$  can be defined as given below;

$$TAC_R = SC + IC + IHC + TC + ET + IC_s$$

$$TAC_R = I_s Q + \frac{(A_r)D}{Q} + C_r D + (h_r + S_r) \left[ \frac{(1-\delta)Q}{2} + \frac{\delta Q^2}{17520} \right] + \frac{\eta D A_0}{Q} + \frac{\eta A_1 (2\lambda n - \lambda - 1)}{2} + \frac{\eta D T_e}{Q} \quad (1)$$



### 3.2 Manufacturer's total annual cost

In this section, we calculated the total annual cost for the retailer which is the sum of the setup cost ( $SC_m = \frac{(A_m)D}{\lambda Q}$ ), item cost ( $IC_m = C_m D$ ), holding cost ( $IHC_m = (h_m + S_m) \frac{Q}{2} \left[ \frac{2D}{P} + \frac{\lambda(P-D)}{P} - 1 \right]$ ), and transportation cost ( $TC_m = \frac{\eta D A_0}{Q} + \frac{\eta A_1 (2\lambda n - \lambda - 1)}{2}$ ) and emission tax ( $ET_M = E_m D T_{ci}$ ). The manufacturer's annual cost ( $TAC_m$ ) can be defined as given below;

$$TAC_m = SC_m + IC_m + IHC_m + TC_m + ET_m$$

$$TAC_m = \frac{(A_m)D}{\lambda Q} + C_m D + (h_m + S_m) \frac{Q}{2} \left[ \frac{2D}{P} + \frac{\lambda(P-D)}{P} - 1 \right] + \frac{\eta D A_0}{Q} + \frac{\eta A_1 (2\lambda n - \lambda - 1)}{2} + E_m D T_{ci} \quad (2)$$

### 3.3 Integrated total annual cost for supply chain system

In this section, we have calculated total joint cost for supply chain which is the sum of retailer's annual cost and manufacturer's cost. From the equation (1) and (2), we get

$$\Psi(Q) = I_s Q + \frac{(A_m + \lambda A_r)D}{\lambda Q} + D(C_r + C_m) + (h_r + S_r) \left[ \frac{(1-\delta)Q}{2} + \frac{\delta Q^2}{17520} \right] + \frac{(h_m + S_m)Q}{2} \left\{ \frac{2D}{P} + \frac{\lambda(P-D)}{P} - 1 \right\} + \frac{\eta D A_0}{Q} + \frac{\eta A_1 (2\lambda n - \lambda - 1)}{2} + \frac{\eta D T_e}{Q} + E_m D T_{ci} \quad (3)$$

### 3.4 Integrated total annual cost for supply chain system under fuzzy environment

Let us suppose that the demand rate is taken as triangular fuzzy number is  $\tilde{D} = (d_1, d_2, d_3)$  and its  $\alpha$ -cut of  $\tilde{D}$  is represented by  $D(\alpha)$  and the value of  $D(\alpha)$  can be defined as  $D(\alpha) = [D_L(\alpha), D_U(\alpha)]$ , where  $D_L(\alpha) = d_1 + (d_2 - d_1)\alpha$  and  $D_U(\alpha) = d_3 - (d_1 - d_2)\alpha$ .

The equation (3) fuzzified the total integrated cost, we get

$$\tilde{\Psi}(Q) = I_s Q + \frac{(A_m + \lambda A_r)\tilde{D}}{\lambda Q} + \tilde{D}(C_r + C_m) + (h_r + S_r) \left[ \frac{(1-\delta)Q}{2} + \frac{\delta Q^2}{17520} \right] + \frac{(h_m + S_m)Q}{2} \left\{ \frac{2\tilde{D}}{P} + \frac{\lambda(P-\tilde{D})}{P} - 1 \right\} + \frac{\eta \tilde{D} A_0}{Q} + \frac{\eta A_1 (2\lambda n - \lambda - 1)}{2} + \frac{\eta \tilde{D} T_e}{Q} + E_m \tilde{D} T_{ci} \quad (4)$$

Now, defuzzified the equation (4) by using of singed distance method, we get

Integrated total fuzzy cost for the supply chain,

$$\left( \tilde{\Psi}(Q), \tilde{0} \right) = I_s Q + \frac{(A_m + \lambda A_r)(4D + d_1 - d_2)}{4\lambda Q} + \frac{(C_r(4D + d_1 - d_2) + C_m(4D + d_1 - d_2))}{4} + (h_r + S_r) \left[ \frac{(1-\delta)Q}{2} + \frac{\delta Q^2}{17520} \right] + \frac{(h_m + S_m)Q}{2} \left\{ \frac{2(4D + d_1 - d_2)}{4P} + \frac{\lambda(3P - (4D + d_1 - d_2))}{4P} - 1 \right\} + \frac{(4D + d_1 - d_2)\eta A_0}{4Q} + \frac{\eta A_1 (2\lambda n - \lambda - 1)}{2} + \frac{\eta(4D + d_1 - d_2)T_e}{4Q} + \frac{E_m(4D + d_1 - d_2)T_{ci}}{4} \quad (5)$$

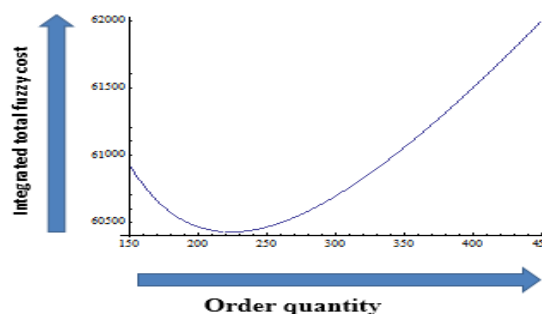


Figure 3: The convexity of integrated total fuzzy cost

### 3.4 Numerical example

The input parameters have been taken from the contribution of Jaber et al. (2013) and Gurtu et al. (2015) and presented in the Table 1

Table 1 Input parameters

Variable	Value	Variable	Value
$(D - d_2, D, D + d_1)$	(995,1000,1020) units per year	$T_{cap}$	80



P	3200	units per year	$\delta$	0.02
$A_m$	\$200	per order	$T_{ci}$	30
$C_m$	\$15	per unit	$T_e$	$(2)(d)(0.00077344)(30)$
$h_m$	\$ 3	per unit per year	a	0.0000003
$s_m$	\$ 9	per unit per year	b	0.0012
$k_m$	0.001		c	1.4
$A_r$	\$ 30	per order	$E_m$	$aP^2-bP+c$
$C_r$	\$ 20	per unit	$\eta$	3
$h_r$	\$ 4	per unit per year	$d_1$	20 units
$s_r$	\$ 12	per unit per year	$d_2$	5 units
$k_r$	0.002		$Q^*$	223 units
T	5		$\tilde{\Psi}(Q)$	60425 (\$)
d	500			
$\lambda$	2			

For the calculation, we used Mathematica software for the optimization of order quantity and total integrated cost for supply chain. The optimal order quantity of this proposed model is  $Q^* = 223$  units and the optimal integrated cost with respect to this optimal order quantity is  $\Psi(Q^*) = 60425$  \$. The present study is also compared with previous study and has been shown in the Table 2.

### 3.5 Sensitivity analysis

**Sensitive analysis is a vital constituent of research which analyzes the decision which is variable with respect to the model parameters which have been given below**

1. From Table-3, we can conclude that the percentage of defective items increase then optimal shipment lot size increase while joint total cost decreases.
2. From Table-4, it can easily be analysed that as production rate increase, the order quantity almost constant and joint total cost increases.
3. From Table-5, it can easily be analysed that as the number of truck increases with keeping other model parameter constant then order quantity and Joint total cost increases.

Table 2: Comparison of base model with present model

Model	Percentage defective item	Order quantity	Integrated total fuzzy cost
Base Paper	0.02	224	60517.



Proposed Model	0.02	223	60425
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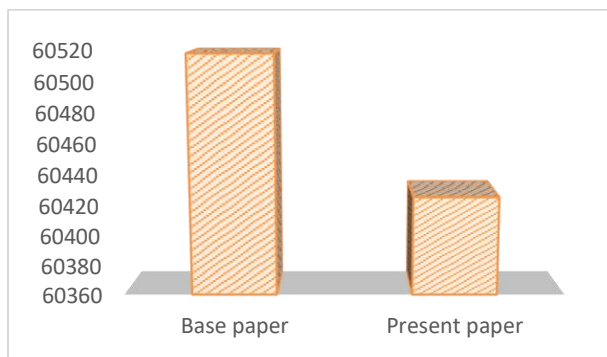


Figure 4: Graphical representation of present model with base model under integrated cost

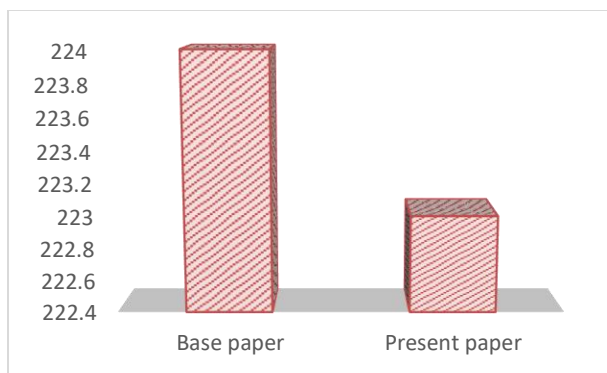


Figure 5: Graphical representation of present model with base model under lot size

#### 4. Conclusion

The purpose of this paper is to analyse the impact of incorporating the machinery system as well as using the machine learning algorithm for the inspection of imperfect quality items in the lot. We observed that the machine learning system reduces the labour cost in the production and manufacturing of products. The machine learning algorithm is good tools for the reduction of labour cost as well the total cost is decreasing which includes inspection cost, holding cost, setup cost, item cost, transportation cost, emission tax cost. Findings together with mathematical analysis clearly suggested that the presence carbon emissions and imperfect items had an affirmative impact on retailer ordering policy. This paper can be improved for more sensible positions such as supply reliant and stochastic demand with partial-trade credit, etc The research emphasizes the importance of integrating sustainability into supply chain operations and offers practical guidance for stakeholders. Future directions for this work include expanding the model to address dynamic factors such as fluctuating carbon taxes, incorporating advanced technologies like block chain for emission traceability, and exploring the social dimensions of sustainability, such as labor conditions and community impacts. These advancements would further strengthen the model, ensuring it remains adaptable and comprehensive in addressing the evolving challenges of sustainable supply chain management.

Table 3 Variable effect of percentage defective items on the lot size and integrated total fuzzy cost

Percentage defective items	Order quantity	Integrated total fuzzy cost
0.01	223	60443
0.02	223	60425
0.03	224	60407



**Table 4: Variable effect of production rate with fixed fuzzy demand on the order quantity and integrated total fuzzy cost**

Fuzzy Demand rate	Production rate	Order quantity	Integrated total fuzzy cost
(995,1000,1020)	3200	223	60443
(995,1000,1020)	4200	223	91023
(995,1000,1020)	7200	223	29111

**Table 5: Variable effect of number of truck on the order quantity and integrated total fuzzy cost**

Number of truck	Order quantity	Integrated total fuzzy cost
2	191	59452
3	227	60407
4	252	61281

**Acknowledgement:** This work was conducted independently and does not represent or relate to the author's (1, 2) position at Amazon Web Services Inc. or any of its affiliates.

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