



Inventory Management and Green Marketing Strategies in the Presence of Preservation Technology for Deterioration, under Trade Credit

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KEYWORDS

EOQ model;
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ABSTRACT:

Introduction: This paper presents an economic order quantity framework that combines inventory management, green marketing strategies. The influence of preservation technology on both deterioration and trade credit in the framework of Economic Order Quantity (EOQ) perspective has been examined. As businesses strive for sustainability and environmental responsibility, the intersection of inventory optimization and green marketing becomes increasingly critical. We develop a comprehensive model that accounts for the trade-offs between traditional inventory management practices and sustainable, green marketing initiatives. Our model incorporates the effects of product deterioration, the utilization of preservation technology, and the influence of trade credit on inventory decisions. Through analytical and numerical analyses, we evaluate the proposed model's performance in optimizing inventory decisions while promoting green marketing practices. The findings provide valuable insights for businesses seeking to enhance their environmental performance while maintaining efficient inventory management practices.

Objectives: The objective is to strike a balance between maximizing costs, ensuring product quality, and aligning with environmentally conscious marketing strategies.

Methods: The influence of preservation technology on both deterioration and trade credit in the framework of Economic Order Quantity (EOQ) perspective has been examined. As businesses strive for sustainability and environmental responsibility, the intersection of inventory optimization and green marketing becomes increasingly critical. . Through analytical and numerical analyses, we evaluate the proposed model's performance in optimizing inventory decisions while promoting green marketing practices.

Results: Firstly, concerning the product price, there exists a clear positive correlation between the price and green level, indicating that higher prices lead to an increase in the green level. Additionally, this positive correlation extends to total profit, with higher prices consistently resulting in greater overall profitability. The observation of total profit across price increments underscores the strategy of increasing prices within the observed range, as it is associated with a steady and positive growth in total profit. Examining the parameter Zeta, its impact on both the green level and total profit is less pronounced. There is only a slight variation in the green level as Zeta changes, and the total profit does not exhibit a consistent trend, showing small variations without a clear pattern.

Conclusions: In conclusion, the interplay of pricing, purchasing costs, and demand rates plays a crucial role in determining both the green level and total profit. Understanding these relationships is essential for optimizing the system for maximum profitability and green level.



1. Introduction

Inventory management is a critical aspect of supply chain operations, and the Economic Order Quantity (EOQ) model has long been a fundamental tool in optimizing inventory levels. EOQ helps organizations determine the optimal order quantity that minimizes total inventory costs, striking a balance between ordering and holding costs. In recent times, the integration of environmental considerations and sustainable practices has become imperative for businesses. This has led to the incorporation of green marketing strategies in inventory decision-making processes, aligning with the growing global focus on sustainability.

Green marketing is a key aspect of modern business strategies, reflecting a commitment to environmentally friendly practices and responsible resource management. In our paper, we delve into the realm of green marketing strategies within the EOQ framework. Understanding and implementing such strategies are crucial for businesses aiming to reduce their ecological footprint while maintaining operational efficiency. Our research explores the intersection of traditional inventory management practices and environmentally conscious strategies to enhance the overall sustainability of the supply chain.

Deterioration is an inherent challenge in inventory management, particularly in industries dealing with perishable goods or those sensitive to environmental conditions. Preservation technology plays a pivotal role in mitigating deterioration risks and maintaining product quality. In our study, we explore the impact of preservation technology within the EOQ context, recognizing its significance in sustaining product integrity and minimizing losses due to deterioration. This adds a practical and real-world dimension to our analysis, providing insights for industries where product preservation is a critical concern.

Preservation technology refers to the application of various methods and techniques designed to extend the shelf life and maintain the quality of products over an extended period. In the context of inventory management, preservation technology plays a crucial role in mitigating the effects of deterioration, particularly for perishable goods or items sensitive to environmental factors. The primary aim of preservation technology is to reduce waste, enhance product longevity, and minimize the

environmental impact associated with the disposal of spoiled or deteriorated goods.

One key aspect of preservation technology involves the use of advanced storage and packaging methods to create optimal conditions for maintaining product freshness. Cold storage, controlled atmospheres, and modified packaging materials are examples of preservation technologies commonly employed in industries dealing with perishable goods such as food, pharmaceuticals, and certain chemicals. These methods slow down the chemical and biological processes that lead to deterioration, ensuring that products reach consumers in optimal condition.

The application of preservation technology aligns with the broader goal of sustainable business practices. By reducing the rate of deterioration, companies can lower the amount of unsellable or discarded products, ultimately minimizing waste and its associated environmental footprint. This approach is particularly significant in industries where spoilage is a prevalent concern, as it contributes to resource efficiency and supports the principles of a circular economy by maximizing the use of resources throughout the supply chain.

Furthermore, the reduction of deterioration through preservation technology not only benefits the environment but also enhances economic efficiency. Businesses can optimize their inventory levels, minimize losses, and improve overall supply chain performance. This, in turn, supports the long-term viability and profitability of the company. Overall, the incorporation of preservation technology in inventory management is a strategic and environmentally conscious approach that underscores the importance of balancing economic efficiency with ecological responsibility.

Trade credit, a common financial tool in business transactions, plays a vital role in influencing inventory decisions. The terms and conditions of trade credit agreements can significantly impact the overall cost structure and financial health of a business. Our research incorporates the dimension of trade credit within the EOQ model, emphasizing the interconnectedness of financial decisions and inventory management. By considering trade credit in the EOQ perspective, we aim to provide a holistic understanding of the factors influencing businesses' financial and operational strategies.



Against this backdrop, our study seeks to answer the following research questions:

- How can inventory policies be optimized when product deterioration, preservation investment, and trade credit are jointly considered?
- What role does green marketing play in influencing inventory management decisions in such a framework?
- How do preservation efforts and trade credit terms impact the balance between cost minimization, product quality, and environmental performance?

By framing the study around these questions, we aim to provide a comprehensive model that not only extends the EOQ theory but also delivers actionable insights for businesses seeking to align profitability, sustainability, and customer satisfaction.

Motivation

The motivation of this paper lies in bridging the gap between traditional inventory models and the growing need for sustainable, market-oriented, and financially viable practices. Green marketing influences inventory decisions because environmentally conscious consumers demand lower waste, better product quality, and sustainable sourcing. To support these expectations, preservation technology is essential for reducing deterioration, minimizing waste, and ensuring products remain marketable. At the same time, trade credit directly affects purchasing and holding decisions, linking financial flexibility with operational efficiency.

By integrating these dimensions into the EOQ framework, our study highlights how inventory management decisions are not only cost-driven but also shaped by consumer perception, environmental responsibility, and financing mechanisms. This combined perspective provides both theoretical insights and practical relevance, especially for industries like food, pharmaceuticals, and consumer goods where deterioration, sustainability, and credit terms play a central role.

2. LITERATURE REVIEW

2.1 Integrating Green Marketing and Sustainable Supply Chain Management. The intersection of green marketing and sustainable supply chain management has garnered increasing attention in recent years, reflecting a paradigm shift towards environmentally

conscious business practices. This literature review explores key studies that contribute to this evolving field and align with the themes of our paper, which integrates green marketing strategies, preservation technology, and trade credit within an Economic Order Quantity (EOQ) framework.

Liu, Kasturiratne, and Moizer (2012) propose a hub-and-spoke model for the multi-dimensional integration of green marketing and sustainable supply chain management. Their work emphasizes the need for a coordinated approach that considers various dimensions of sustainability. This framework provides a foundational understanding of the complexities involved in aligning green marketing strategies with supply chain operations. Raza and Faisal (2018) delve into joint pricing and greening effort decisions in the context of inventory models. Their study underscores the importance of considering both pricing and environmental considerations concurrently, aligning with the dual focus of our paper. By integrating pricing decisions with greening efforts, businesses can optimize profitability while meeting sustainability goals. Wang et al. (2019) contribute a green growth perspective to inventory control and supply chain management. Their work emphasizes the symbiotic relationship between green practices and business growth. This perspective is particularly relevant to our paper, which aims to balance environmental responsibility with operational efficiency. Becerra, Mula, and Sanchis (2021) offer a comprehensive review of green supply chain quantitative models for sustainable inventory management. Their study provides insights into various quantitative models, offering a valuable reference for researchers exploring the integration of sustainability into inventory optimization. Ruidas, Seikh, and Nayak (2022) present a production inventory model for green products, considering emission reduction technology investment and green subsidies. This study extends our understanding of the complexities involved in managing green inventory, introducing elements of technology investment and government support.

The work of Saxena et al. (2023a; 2023b) explores reverse logistics models with eco-design, sustainable



production models with stochastic machine breakdowns, and inventory optimization in a green environment. These studies contribute diverse perspectives on incorporating sustainability into different facets of supply chain and inventory management. Handa, Singh, and Punetha (2023) address the integration of green considerations in inventory management for deteriorating items. This study offers insights into managing environmentally sensitive items within the context of imperfect production processes. Finally, Akhtar et al. (2023) employ a tournament differential evolution algorithm to optimize a production inventory model with green levels and expiry time-dependent demand. This work showcases innovative optimization techniques for incorporating green elements into inventory decisions.

Collectively, these studies form a rich body of literature that informs our research by providing insights into the challenges and opportunities associated with integrating green marketing, preservation technology, and trade credit within the EOQ framework.

2.2 Reduction of Deterioration with Preservation Technology

The management of deteriorating inventory is a complex challenge that demands innovative solutions. Preservation technology emerges as a pivotal tool to mitigate the impact of deterioration, offering strategies for extending shelf life, reducing waste, and enhancing overall inventory efficiency. The following literature review examines key studies that contribute insights into the reduction of deterioration through the application of preservation technology.

Mishra (2013) introduces a deteriorating inventory model that incorporates preservation technology, salvage value, and shortages. The study addresses the dynamic nature of inventory management, recognizing the potential benefits of preservation technology in minimizing losses due to deterioration and shortages. Yang, Dye, and Ding (2015) focus on the optimal dynamic allocation of trade credit and preservation technology in a deteriorating inventory model. Their work underscores the interconnectedness of financial strategies and

preservation technology, providing a holistic perspective on managing deteriorating inventory efficiently. Mishra et al. (2017) extend the exploration by incorporating price and stock-dependent demand in an inventory model with controllable deterioration rate. The study emphasizes the importance of preservation technology investment in addressing both demand variability and deterioration control. Shen, Shen, and Yang (2019) contribute a production inventory model for deteriorating items with collaborative preservation technology investment under carbon tax considerations. This study addresses the environmental aspect, highlighting the role of preservation technology in aligning inventory management with sustainability goals. Pervin, Roy, and Weber (2020) delve into a deteriorating inventory model considering preservation technology under price- and stock-sensitive demand. Their study contributes insights into optimizing inventory decisions by considering both market dynamics and preservation technology.

Recently, Sepehri, Mishra, and Sarkar (2021) present a sustainable production-inventory model incorporating imperfect quality, preservation technology, and quality improvement investment. The study emphasizes the integration of preservation technology in a broader sustainability context, considering both quality and environmental impact. Priyamvada, Rini, and Jaggi (2022) explore optimal inventory strategies for deteriorating items with price-sensitive investment in preservation technology. The study focuses on the strategic aspect of preservation technology adoption, aligning it with market dynamics and investment considerations.

Collectively, these studies provide a nuanced understanding of the role of preservation technology in reducing deterioration within inventory management. They address various dimensions, including financial considerations, sustainability goals, and market dynamics, contributing to a comprehensive framework for managing deteriorating inventory effectively. The integration of preservation technology emerges as a crucial element in optimizing inventory decisions and aligning them with broader business objectives.



2.3 EOQ Models with Trade Credit

Efficient inventory management is crucial for businesses, and the integration of trade credit in Economic Order Quantity (EOQ) models presents a dynamic approach to optimize ordering decisions. The following literature review examines key studies that contribute insights into EOQ models with trade credit, providing a comprehensive understanding of the various scenarios and considerations involved.

Jaggi et al. (2015) investigate an EOQ model with allowable shortages under trade credit, exploring different scenarios. The study highlights the impact of trade credit on order quantity and shortage allowances, providing a foundational understanding of the interplay between trade credit terms and inventory decisions. Cárdenas-Barrón et al. (2020) extend the exploration by introducing a comprehensive EOQ model with nonlinear stock-dependent holding cost, nonlinear stock-dependent demand, and trade credit. The study considers the complexities introduced by nonlinearities, broadening the applicability of the EOQ model in realistic inventory management scenarios. Mahata and Mahata (2020) delve into a two-echelon trade credit model with default risk in an EOQ setting for deteriorating items under dynamic demand. The study introduces a risk perspective, recognizing the importance of default risk considerations in trade credit arrangements for managing deteriorating inventory. Handa et al. (2020) contribute a trade credit policy in an EOQ model with stock-sensitive demand and shortages for deteriorating items. This study emphasizes the strategic role of trade credit in addressing stock-sensitive demand and managing shortages, offering practical insights for businesses facing dynamic demand environments. Molamohamadi and Mirzazadeh (2021) explore ordering policies of a deteriorating item in an EOQ model under upstream partial order-quantity-dependent trade credit and downstream full trade credit. The study introduces a nuanced approach considering both upstream and downstream trade credit dynamics, enhancing the model's applicability in supply chain scenarios.

Jayaswal et al. (2022) present a learning EOQ model with a trade credit financing policy for imperfect

quality items under a cloudy fuzzy environment. The study introduces a fuzzy logic-based approach, recognizing uncertainties in quality considerations, and aligns trade credit with the evolving field of cloud-based technologies. Yadav et al. (2023) extend the research by proposing a multi-item EOQ model for deteriorating items with multivariate dependent demand, variable holding cost, and trade credit. The study addresses the complexities arising from managing multiple items with dependencies, providing insights into the simultaneous optimization of inventory decisions. Katariya and Shukla (2023) introduce an EOQ model for deteriorating products with green technology investment and trade credit financing. The study considers the integration of green technology and trade credit in managing deteriorating inventory, aligning with the increasing focus on sustainability in business practices. Jayashri and Umamaheswari (2023) contribute an EOQ inventory model for perishable items with price-dependent demand and preservation technique under upstream and downstream trade credit. The study considers the perishable nature of items, incorporating trade credit in both upstream and downstream scenarios, and introduces the preservation technique to address perishability.

3. Collectively, these studies offer a comprehensive overview of the diverse applications and considerations involved in EOQ models with trade credit. From addressing deteriorating items to incorporating nonlinearities, risk, and sustainability considerations, the literature provides a robust foundation for understanding and optimizing inventory decisions in dynamic business environments.
4. Recent extensions of EOQ inventory models increasingly embed elements such as deterioration control, preservation technology, and trade credit. For example, Kumar, Agrawal & Yadav (2024) developed an EOQ model that incorporates controllable, non-instantaneous deterioration, waste management, carbon-sensitive demand, and trade credit—underscoring how sustainability and financial flexibility can jointly drive inventory efficiency. In another significant contribution, Tripathi & Kaur (2024) focused on price- and time-



sensitive deteriorating demand under trade credit, delivering optimal cycle time and order quantity outcomes within a cost-minimization EOQ framework. Additionally, Yadav et al. (2025) introduced a retail inventory model encompassing preservation technology, green marketing investment, and promotional efforts, advancing the discussion on aligning marketing strategies with inventory preservation. Another 2025 study created an EOQ model for deteriorating items with price- and trade credit-dependent demand, preservation investment, and inflation considerations—explicitly aimed at maximizing profit while mitigating deterioration.

5.

While these studies make important strides, most explore only two or three dimensions—such as trade credit with deterioration or preservation investment with green marketing. In contrast, our paper integrates four critical dimensions—deterioration, preservation technology, trade credit, and green marketing strategies—within a single EOQ framework. This holistic integration not only advances theoretical modeling but also better mirrors the multidimensional challenges modern, sustainability-conscious supply chains face. Our model sheds light on how these factors interact to influence cost minimization, product quality, and environmental performance in real-world settings.

1. Assumptions and Notations

3.1 Assumptions

1. The inventory model is developed for the Economic Order Quantity.
2. In this scenario, the supplier purchases green products from the manufacturer and pays the cost of items depending on the degree of the product's greenness.
3. Green marketing is employed, and the demand rate is assumed to be dependent on the green level.
4. The supplier is provided with a permissible delay in payment by the manufacturer, allowing the supplier to make the payment after the credit period. Additionally, the supplier earns interest on revenue during this period. However, after the specified period, the supplier is obligated to pay interest on the cost of the items.
5. The model is developed for deteriorating items.

6. Preservation technology is utilized to reduce deterioration, thereby also reducing the environmental burden.

3.2 Notation

A	Ordering cost
ξ	Preservation technology cost
C_1	Constant purchasing cost
C_2	Variable purchasing cost
p	Price
g	Green degree
α	Constant demand rate
β	Variable demand rate
h	Holding cost
I_e	Interest earned
I_p	Interest paid
M	Delay in period
ψ_1	Profit function for case 1
ψ_2	Profit function for case 2
$Q(t)$	Inventory level at time t
θ	Deterioration rate
$m(\zeta)$	Reduction in deterioration
y	Cycle length
q	Order quantity

2. MATHEMATICAL MODELLING

This study focuses on investigating an Economic Production Quantity (EPQ) model for multiple items. The key assumptions include the perishable nature of the items and a finite replenishment rate. To minimize the overall cost of the system, a mathematical model has been developed to determine the optimal value of a decision variable.

Here we study a single-item, continuous production–inventory cycle of infinite horizon (we analyze one representative cycle of length T). The objective is to determine cycle timings and investment levels that



minimize the long-run average relevant cost (or equivalently maximize profit) while accounting for time-dependent demand, product deterioration, preservation technology investment, carbon emission cost, partial backlogging of shortages, and the financial effect of trade credit.

The differential equation governing the EOQ model is given in equation (1)

$$Q'(t) + (\theta - m(\zeta))Q(t) = -D, \quad Q(y) = 0 \tag{1}$$

Here (t) denote the on-hand inventory at time t . When there is no production under consideration, the inventory changes only because of customer demand and deterioration. Here the natural deterioration rate is θ and preservation investment ζ reduces that rate by an amount (ζ) (with $0 < (\zeta) < \theta$), then the effective deterioration rate is $\theta - m(\zeta)$.

The solution of the above equation is given as (2)

$$Q(t) = -\frac{De^{m(\zeta)t-t\theta}(-e^{t(-m(\zeta)+\theta)}+e^{y(-m(\zeta)+\theta)})}{m(\zeta)-\theta} \tag{2}$$

After simplifying one can get

$$Q(t) = -\frac{D(-1+e^{(t-y)(m(\zeta)-\theta)})}{m(\zeta)-\theta} \tag{3}$$

By using the equation (3), we can get the Ordering quantity

$$q = Q(0) = -\frac{D(-1+e^{y(-m(\zeta)+\theta)})}{m(\zeta)-\theta} \tag{4}$$

The cost components of the supplier can be calculated as follows

Ordering cost

$$OC = A$$

Cost of Preservation Technology

$$PTC = \zeta$$

Purchasing cost

The supplier needs to purchase green item whose cost can be calculated as follows

$$IC = q(C_1 + C_2g) = \left(-\frac{D(-1 + e^{y(-m(\zeta)+\theta)})}{m(\zeta) - \theta}\right)(C_1 + C_2g)$$

Holding cost

The supplier spends in energy; warehouse etc. to store the items whose cost can be termed as the storage or holding cost. The holding cost of the supplier can be calculated as follows

$$HC = h \int_0^y \left(-\frac{D(-1 + e^{(t-y)(m(\zeta)-\theta)})}{m(\zeta) - \theta}\right) dt = h \frac{D(-1 + e^{y(-m(\zeta)+\theta)} + m(\zeta)y - y\theta)}{(m(\zeta) - \theta)^2}$$

Selling Price

$$SP = pDy$$

Permissible delay

As we have already discussed that the supplier is provided with the delay in payment hence there are two cases arises

Case 1: when the delay period is greater that the cycle length ($y < M$)

In this case supplier doesn't need to pay the interest and he earn interest on its revenue, which can be calculate as

Interest earned

$$IE_1 = pI_e \left(y^2 \frac{D}{2} + yD(M - y)\right)$$

Case 2: When the delay period is less than the cycle length ($y > M$)

In this case supplier need to pay the interest on the products purchased and also earn interest on the revenue

Interest paid

$$IP_2 = (C_1 + C_2g)I_p \int_M^y (D(y - t)) dt = (C_1 + C_2g)I_p \frac{D(y - M)^2}{2}$$

Interest earned



$$IE_2 = pI_e \left(M^2 \frac{D}{2} \right)$$

3. Solution procedure

Here the system is developed for the green product with green marketing hence the demand rate is taken as $D = (\alpha + \beta g)$ and the reduction in the deterioration due to the preservation is taken as $m(\zeta) = \theta(1 - \text{Exp}[-a\zeta])$

By putting the values of the D and $m(\zeta)$ one can rewrite the profit function for case one and case 2 as follows

Average profit for case 1

$$\psi_1 = \frac{1}{y} \left((p_1 + p_2 g) D y + (p_1 + p_2 g) I_e \left(y^2 \frac{D}{2} + y D (M - y) \right) - \left(A + \zeta + \left(-\frac{D(-1+e^{y(-m(\xi)+\theta)})}{m(\xi)-\theta} \right) (C_1 + C_2 g) + h \frac{D(-1+e^{y(-m(\xi)+\theta)+m(\xi)y-y\theta})}{(m(\xi)-\theta)^2} \right) \right) \quad (5)$$

$$\psi_1 = \frac{1}{y} \left(-A - \zeta - \frac{h(\alpha+g\beta)(-1+e^{y(\theta-(1-e^{-a\zeta})\theta)}-y\theta+(1-e^{-a\zeta})y\theta)}{(-\theta+(1-e^{-a\zeta})\theta)^2} + \frac{(-1+e^{y(\theta-(1-e^{-a\zeta})\theta)}) (\alpha+g\beta)(C_1+gC_2)}{-\theta+(1-e^{-a\zeta})\theta} + y(\alpha+g\beta)(p_1+gp_2) + \left((M-y)y(\alpha+g\beta) + \frac{1}{2}y^2(\alpha+g\beta) \right) i_e(p_1+gp_2) \right) \quad (6)$$

$$\frac{d\psi_1}{dy} = -\frac{1}{y} \left(\frac{e^{a\zeta}h(\alpha+g\beta)(-\theta+e^{-a\zeta}y\theta)}{\theta^2} + e^{-a\zeta}y\theta(\alpha+g\beta)(C_1+gC_2) - (\alpha+g\beta)(p_1+gp_2) - \frac{1}{2}(2M-y)(\alpha+g\beta)i_e(p_1+gp_2) + \frac{1}{2}y(\alpha+g\beta)i_e(p_1+gp_2) \right) + \frac{1}{y^2} \left(A + \zeta + \frac{e^{a\zeta}h(\alpha+g\beta)(-e^{a\zeta}+e^{-a\zeta}y\theta-y\theta)}{\theta^2} + \frac{e^{a\zeta}(-1+e^{-a\zeta}y\theta)(\alpha+g\beta)(C_1+gC_2)}{\theta} - y(\alpha+g\beta)(p_1+gp_2) - \right.$$

$$\left. \frac{1}{2}(2M-y)y(\alpha+g\beta)i_e(p_1+gp_2) \right) \quad (7)$$

$$\frac{d\psi_1}{dg} = -\frac{1}{y} \left(\frac{e^{a\zeta}h\beta(-e^{a\zeta}+e^{-a\zeta}y\theta-y\theta)}{\theta^2} + \frac{e^{a\zeta}(-1+e^{-a\zeta}y\theta)(\alpha+g\beta)C_2}{\theta} + \frac{e^{a\zeta}(-1+e^{-a\zeta}y\theta)\beta(C_1+gC_2)}{\theta} - y(\alpha+g\beta)p_2 - \frac{1}{2}(2M-y)y(\alpha+g\beta)i_e p_2 - y\beta(p_1+gp_2) - \frac{1}{2}(2M-y)y\beta i_e(p_1+gp_2) \right) \quad (8)$$

Now putting $\frac{d\psi_1}{dg} = 0$ one can get

$$g = \frac{1}{2\beta\theta(2e^{a\zeta}(-1+e^{-a\zeta}y\theta)C_2+y\theta(-2+(-2M+y)i_e)p_2)} \left(2e^{2a\zeta}h\beta - 2e^{2a\zeta+e^{-a\zeta}y\theta}h\beta + 2e^{a\zeta}hy\beta\theta - 2e^{a\zeta}(-1+e^{-a\zeta}y\theta)\beta\theta C_1 - 2e^{a\zeta}(-1+e^{-a\zeta}y\theta)\alpha\theta C_2 + 2y\beta\theta^2 p_1 + 2My\beta\theta^2 i_e p_1 - y^2\beta\theta^2 i_e p_1 + 2y\alpha\theta^2 p_2 + 2My\alpha\theta^2 i_e p_2 - y^2\alpha\theta^2 i_e p_2 \right) \quad (9)$$

$$\frac{d^2\psi_1}{dg^2} = -\frac{2e^{a\zeta}(-1+e^{-a\zeta}y\theta)\beta C_2 - 2y\beta p_2 - (2M-y)y\beta i_e p_2}{y} \quad (10)$$

Theorem: the profit function will be concave if

$$-\frac{2e^{a\zeta}(-1+e^{-a\zeta}y\theta)\beta C_2 - 2y\beta p_2 - (2M-y)y\beta i_e p_2}{y} < 0$$

Average profit for case 2

$$\psi_2 = \frac{1}{y} \left((p_1 + p_2 g) D y + (p_1 + p_2 g) I_e \left(M^2 \frac{D}{2} \right) - \left(A + \zeta + \left(-\frac{D(-1+e^{y(-m(\xi)+\theta)})}{m(\xi)-\theta} \right) (C_1 + C_2 g) + h \frac{D(-1+e^{y(-m(\xi)+\theta)+m(\xi)y-y\theta})}{(m(\xi)-\theta)^2} + (C_1 + C_2 g) I_p \frac{D(y-M)^2}{2} \right) \right) \quad (11)$$



$$\psi_2 = \frac{1}{y} \left(-A - \zeta - \frac{h(\alpha + g\beta) \left(-1 + e^{y(\theta - (1 - e^{-\alpha\zeta})\theta)} - y\theta + (1 - e^{-\alpha\zeta})y\theta \right)}{(-\theta + (1 - e^{-\alpha\zeta})\theta)^2} + \frac{(-1 + e^{y(\theta - (1 - e^{-\alpha\zeta})\theta)}) (\alpha + g\beta) (C_1 + gC_2)}{-\theta + (1 - e^{-\alpha\zeta})\theta} - \frac{1}{2} (-M + y)^2 (\alpha + g\beta) i_p (C_1 + gC_2) + y(\alpha + g\beta) (p_1 + gp_2) + \frac{1}{2} M^2 (\alpha + g\beta) i_e (p_1 + gp_2) \right) \quad (12)$$

$$\frac{d\psi_2}{dy} = -\frac{1}{y} \left(\frac{e^{\alpha\zeta} h(\alpha + g\beta) (-\theta + e^{-\alpha\zeta} y\theta)}{\theta^2} + e^{-\alpha\zeta} y\theta (\alpha + g\beta) (C_1 + gC_2) - (M - y) (\alpha + g\beta) i_p (C_1 + gC_2) - (\alpha + g\beta) (p_1 + gp_2) \right) + \frac{1}{y^2} \left(A + \zeta + \frac{e^{\alpha\zeta} h(\alpha + g\beta) (-e^{\alpha\zeta} + e^{\alpha\zeta} + e^{-\alpha\zeta} y\theta - y\theta)}{\theta^2} + \frac{e^{\alpha\zeta} (-1 + e^{-\alpha\zeta} y\theta) (\alpha + g\beta) (C_1 + gC_2)}{\theta} + \frac{1}{2} (M - y)^2 (\alpha + g\beta) i_p (C_1 + gC_2) - y(\alpha + g\beta) (p_1 + gp_2) - \frac{1}{2} M^2 (\alpha + g\beta) i_e (p_1 + gp_2) \right) \quad (13)$$

$$\frac{d\psi_2}{dg} = -\frac{1}{y} \left(\frac{e^{\alpha\zeta} h\beta (-e^{\alpha\zeta} + e^{\alpha\zeta} + e^{-\alpha\zeta} y\theta - y\theta)}{\theta^2} + \frac{e^{\alpha\zeta} (-1 + e^{-\alpha\zeta} y\theta) (\alpha + g\beta) C_2}{\theta} + \frac{1}{2} (M - y)^2 (\alpha + g\beta) i_p C_2 + \frac{e^{\alpha\zeta} (-1 + e^{-\alpha\zeta} y\theta) \beta (C_1 + gC_2)}{\theta} + \frac{1}{2} (M - y)^2 \beta i_p (C_1 + gC_2) - y(\alpha + g\beta) p_2 - \frac{1}{2} M^2 (\alpha + g\beta) i_e p_2 - y\beta (p_1 + gp_2) - \frac{1}{2} M^2 \beta i_e (p_1 + gp_2) \right) \quad (14)$$

$$g = \left(\frac{1}{\left(2\beta\theta \left((2e^{\alpha\zeta} (-1 + e^{-\alpha\zeta} y\theta) + (M - y)^2 \theta i_p) C_2 - \theta (2y + M^2 i_e) p_2 \right) \right)} \right) \left(2e^{2\alpha\zeta} h\beta - 2e^{2\alpha\zeta + e^{-\alpha\zeta} y\theta} h\beta + 2e^{\alpha\zeta} h y \beta \theta - \beta \theta \left(2e^{\alpha\zeta} (-1 + e^{-\alpha\zeta} y\theta) + (M - y)^2 \theta i_p \right) C_1 - \alpha \theta \left(2e^{\alpha\zeta} (-1 + e^{-\alpha\zeta} y\theta) + (M - y)^2 \theta i_p \right) C_2 + 2y\beta\theta^2 p_1 + M^2 \beta \theta^2 i_e p_1 + 2y\alpha\theta^2 p_2 + M^2 \alpha \theta^2 i_e p_2 \right) \quad (15)$$

$$\frac{d^2\psi_1}{dg^2} = -\frac{2e^{\alpha\zeta} (-1 + e^{-\alpha\zeta} y\theta) \beta C_2 + (M - y)^2 \beta i_p C_2 - 2y\beta p_2 - M^2 \beta i_e p_2}{y} \quad (16)$$

4. Numerical Analysis

Example: The parametric values of the parameters are taken as follows

$$A = 1000, \zeta = 100, h = 0.5, C_1 = 8, C_2 = 0.5, p_1 = 21, p_2 = 0, y = 2.5, I_e = 0.5, I_p = 0.8, \theta = 0.1, M = 3, \alpha = 500, \beta = 10, a = 0.1$$

By putting the above parametric values in our solution procedure, the optimum results can be obtained as $\psi_1 = 15100.28$ and $g = 5.749$

Example 2: In this example profit for second case has been discussed and where all the parameters are same except the permissible period M . The parametric value of the delay period is taken as 2 for this example but after the careful analysis it is noted that if we increase the delay period the system goes into loss hence it is necessary to increase the selling

price simultaneously if the delay period decrease. Therefore for this example we have taken the selling price $p_1 = 28$ instead of 21.

By using these parametric values the optimal profit and optimal green level for the second cases resulted as $\psi_2 = 14774.5$ and $g = 4.0912$

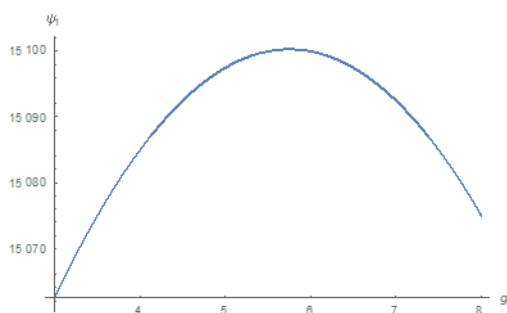


Figure 1: The convexity of the profit function of case 1

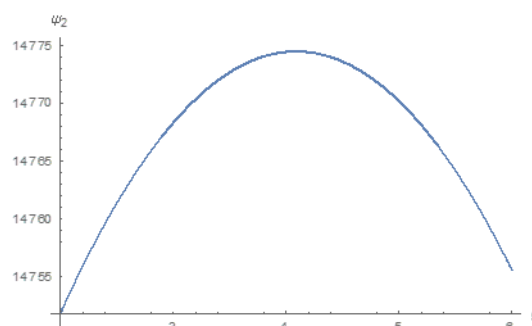


Figure 2: The convexity of profit function of case 2

Sensitivity analysis

Sensitivity of the system with respect to the different parameters is examined in table 1 for case 1 and in table 2 for case 2

Table 1: sensitivity analysis for case 1

Parameter	Parametric values	Green level	Total profit
p_1	18	0.124814	12122.6
	20	3.87479	14072.5
	22	7.62477	16163.2
	24	11.3748	18394.4
	26	15.1247	20766.3
	28	18.8747	23278.8
ζ	60	5.73787	15114.4
	80	5.74836	15108.0
	100	5.74978	15100.3
	120	5.75048	15092.1
	140	5.71875	15095.6
C_1	5	8.74978	16817.8
	6	7.74978	16235.3
	7	6.74978	15662.8
	8	5.74978	15100.3
	9	4.74978	14547.8



	10	3.74978	14005.3
C_2	0.1	128.749	31511.4
	0.2	51.8745	20316.9
	0.3	26.2496	17002.1
	0.4	13.4372	15657.2
	0.5	5.74978	15100.3
	0.6	0.624818	14937.3
α	100	25.7498	5950.27
	200	20.7498	7862.77
	300	15.7498	10025.3
	400	10.7498	12437.8
	500	5.74978	15100.3
	600	0.749782	18012.8
β	9	2.97201	14974.7
	10	5.74978	15100.3
	11	8.02251	15289.
	12	9.91645	15525.
	13	11.519	15797.5
	14	12.8926	16098.5
	15	14.0831	16422.5

Table 2: sensitivity analysis for case 2

Parameter	Parametric values	Green level	Total profit
p_1	25	0.0527067	12587.5
	27	2.745	14026.7
	29	5.43729	15541.2
	31	8.12959	17131.1
	33	10.8219	18796.5
	35	13.5142	20537.2
ζ	60	4.08017	14788.8
	80	4.08983	14782.3
	100	4.09115	14774.5
	120	4.09182	14766.3



	140	4.06025	14769.5
C_1	5	7.09115	16509
	6	6.09115	15920.4
	7	5.09115	15342.3
	8	4.09115	14774.5
	9	3.09115	14217.2
	10	2.09115	13670.2
C_2	0.1	120.456	29777.5
	0.2	47.7279	19425.6
	0.3	23.4852	16408.3
	0.4	11.3639	15224.7
	0.5	4.09115	14774.5
	0.6	-0.757378	14691.1
α	100	24.0911	5603.51
	200	19.0911	7506.26
	300	14.0911	9669.01
	400	9.09115	12091.8
	500	4.09115	14774.5
	600	-0.908854	17717.3
β	9	1.31337	14695.6
	10	4.09115	14774.5
	11	6.36387	14919.1
	12	8.25781	15113
	13	9.86038	15344.7
	14	11.234	15606.2
	15	12.4245	15891.6

Observations :

From above tables following assumptions are obtained.

- There is a positive correlation between the price of the product and the green level. As the price increases, the green level also increases, indicating a direct relationship.
- There is an apparent positive correlation between the price of the product and total profit. Higher prices result in higher total profits, indicating that the pricing strategy is contributing positively to overall profitability.
- The total profit steadily increases with each increment in price. This suggests that, within the observed range,



increasing the price is associated with a consistent growth in total profit.

- There is a slight variation in the green level as Zeta changes. The green level shows a relatively stable trend within the observed range of Zeta values (60 to 140). Overall, there is no clear and consistent trend indicating a strong relationship between Zeta and the green level.
- The total profit does not exhibit a consistent trend with changes in Zeta. The variations in total profit are relatively small, and there is no clear pattern of increase or decrease with Zeta values.
- There is an inverse relationship between the constant purchasing cost per item and the green level. As the constant purchasing cost per item decreases, the green level increases. This suggests that a lower constant purchasing cost is associated with a higher green level.
- There is an inverse relationship between the constant purchasing cost per item and total profit. As the constant purchasing cost per item decreases, the total profit increases. Lower constant purchasing costs contribute to higher total profits.
- There is a clear inverse relationship between the green level dependent purchasing cost per item and the green level. As the green level dependent purchasing cost per item decreases, the green level increases. This indicates that a lower green level dependent purchasing cost is associated with a higher green level.
- There is a complex relationship between the green level dependent purchasing cost per item and total profit. Initially, as the green level dependent purchasing cost per item decreases, total profit increases. However, after a certain point (around 0.5), further decreases in the green level dependent purchasing cost per item do not result in a significant increase in total profit.
- There is an inverse relationship between the constant demand rate and the green level. As the constant demand rate decreases, the green level increases. This suggests that a lower constant demand rate is associated with a higher green level.
- There is an inverse relationship between the constant demand rate and total profit. As the constant demand rate decreases, the total profit increases. Lower constant demand rates contribute to higher total profits.

- There is a positive correlation between the variable demand rate and the green level. As the variable demand rate increases, the green level also increases. This suggests that a higher variable demand rate is associated with a higher green level.

- There is a mixed relationship between the variable demand rate and total profit. For lower variable demand rates (9, 10, 11), the total profit remains relatively stable. Beyond a certain point (12, 13, 14, 15), further increases in the variable demand rate lead to an increase in total profit.

CONCLUSION

The analysis of various parameters reveals insightful patterns and relationships that significantly impact both the green level and total profit in the observed system. Firstly, concerning the product price, there exists a clear positive correlation between the price and green level, indicating that higher prices lead to an increase in the green level. Additionally, this positive correlation extends to total profit, with higher prices consistently resulting in greater overall profitability. The observation of total profit across price increments underscores the strategy of increasing prices within the observed range, as it is associated with a steady and positive growth in total profit.

Examining the parameter Zeta, its impact on both the green level and total profit is less pronounced. There is only a slight variation in the green level as Zeta changes, and the total profit does not exhibit a consistent trend, showing small variations without a clear pattern. This suggests that Zeta may not be a decisive factor in determining the green level or total profit within the observed range.

Moving on to the purchasing cost parameters, an inverse relationship is observed between both constant purchasing cost per item and green level, as well as constant purchasing cost per item and total profit. Lower constant purchasing costs are associated with higher green levels and higher total profits. Similarly, the green level dependent purchasing cost per item exhibits an inverse relationship with the green level, but the relationship with total profit is more complex. Initial decreases in the cost lead to total profit increases, but after a certain point, further reductions do not significantly impact total profit.



Concerning the demand rate parameters, a lower constant demand rate is associated with higher green levels and higher total profits, indicating an inverse relationship. However, the variable demand rate shows a positive correlation with the green level, and its relationship with total profit is more nuanced. For lower variable demand rates, total profit remains relatively stable, while increasing demand rates beyond a certain point lead to an increase in total profit.

In conclusion, the interplay of pricing, purchasing costs, and demand rates plays a crucial role in determining both the green level and total profit. Understanding these relationships is essential for optimizing the system for maximum profitability and green level.

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