Journal of Chemical Health Risks

www.jchr.org

JCHR (2025) 15(6), 1963-1974 | ISSN:2251-6727

Beal's Fuzzy Relations and Its Applications in Decision Making Problems

Dr. K. Balasubramaniyan[1] *C.Dilly Rani [2]

[1]Assistant Professor, Department of Mathematics, Annamalai University, Chidambaram.
Deputed to Arignar Anna Government Arts College, Vadachennimalai-636121, Attur, Tamilnadu, India.

[2] Research Scholar, Department of Mathematics, Annamalai University, Chidambaram.
Assistant Professor, J.J.College of Engineering and Technology, Tiruchirappalli-600009, Tamilnadu, India.

(Received: 27 October 2025 Revised: 05 November 2025 Accepted: 04 December 2025)

KEYWORDS

Fuzzy set, Beal's fuzzy relation, semi-ring, hemi-ring, poset, lattice, Boolean algebra, composition, decision making.

ABSTRACT:

Introduction: This study introduces the conception of Beal's fuzzy relation (BFR), which will help to remove certain limitations associated with Pythagorean (PFR), fermatean (FFR) and intuitionistic fuzzy relation (IFR). Some basic operations in this regard are given for Beal's fuzzy relations (BFR). The set of all Beal's fuzzy relations leads to several algebric structures for these operations (semi-ring, and bounded distributive lattice). Additionally, an application of the designed approach is proposed for entropy measure orthopairean TOPSIS algorthim. Moreover, the comparative analysis of the proposed method with some existing ideas is also presented by given several examples to authenticate the feasibility and superiority of the proposed approach.

1. Introduction

The theory of fuzzy sets was first introduced by Zadeh [32] in 1965. Fuzzy sets are the most appropriate theory for dealing with uncertainty. After the introduction of the concept of fuzzy sets by Zadeh [32], several researchers conducted research on the generalizations of the notions of fuzzy sets with huge applications in computer science, artificial intelligence, control engineering, robotics, automata theory, decision theory, finite state machine, graph theory, logic, operations research, and many branches of pure and applied mathematics. For example, Xie et al. applied fuzzy set theory to the switching method [23]. In many decision support systems, modeling uncertainties is crucial to address real-life issues in the present scientific era. Due to the continuous increase in socioeconomic environment complications and insouciant knowledge or incomplete information, it is difficult for decision-makers to make the best judgement. The problem of incomplete information has become a significant issue for the last few years. Many theories have been presented to effectively deal with these problems, such as the probability theory, fuzzy set (FS) theory, intuitionistic fuzzy set (IFS) theory, rough set theory and soft set theory, etc. In 2013, Yager [27-29] introduced the Pythagorean fuzzy set and compared it with the intuitionistic fuzzy set. Al-Shami et al. [2,12] proposed the concept of n^{th} power root FSs and provided some applications.

Binary relations become crucial in both pure and applied mathematics and are often used in real-life decision-making. The notion of fuzzy relation (FR) was introduced by Zadeh [32]. FRs have applications in diverse areas, such as fuzzy modeling [15], fuzzy control [14], uncertain reasoning [18], neural networks [17], pattern recognition [20], artificial intelligence [25], and decision-making [[1],[21]]. FRs and their properties can be found in [[18] [19]]. A detailed overview of FSs and FRs studied by Wang et.al.[22]. The conception of intuitionistic fuzzy relations (IFRs) was projected by Atanassov [3]. Many scholars developed the theory and applications of IFRs. For instance, in [[5], [6], Burillo and Bustince examined certain IFR features using the t-norm and t-conorm. Bustince [16] developed divergent results for constructing IFRs on a set with predetermined

Journal of Chemical Health Risks
www.jchr.org
JCHR (2025) 15(6), 1963-1974 | ISSN:2251-6727

properties that permit us to construct reflexive, symmetric, anti symmetric, perfect anti symmetric, and transitive IFRs from FRs with the similar characteristics. With the composition of IFRs specified in [10], Kumar and Gangwal [16] have established an application of IFR in medical diagnosis. Zhang et al. [31] created the idea of Pythagorean fuzzy relation (PFR) to examine the roughness of a PFS with an application to a decision-making problem in mergers and acquisitions. This study introduces the conception of Beal's fuzzy relation (BFR), which will help to remove certain limitations associated with Pythagorean (PFR), fermatean (FFR) and intuitionistic fuzzy relation (IFR). Some basic operations in this regard are given for Beal's fuzzy relations (BFR). The set of all Beal's fuzzy relations leads to several algebric structures for these operations (semi-ring, and bounded distributive lattice). Additionally, an application of the designed approach is proposed for orthopairean TOPSIS algorthim. Moreover, the comparative analysis of the proposed method with some existing ideas is also presented by given several examples to authenticate the feasibility and superiority of the proposed approach..

2. PRELIMINARIES AND VARIOUS CONCEPTS

2.1 Definition [Fuzzy set]: Let X be a non-empty set. Then an uncertainty on X is defined by $J_A: X \to [0,1]$. J_A is called the membership function. $J_A(X)$ is said to be a membership grade of X in J_A. we also write $J_A = \{(x, J_A(x))/ \, x \in X\}$.

Example: Let $X = \{a, b, c, d\}$ *and* $J_A: X \to [0,1]$ defined by $J_A(a) = 0, J_A(b) = 0.6, J_A(c) = 0.3,$ $J_A(d) = 1.$

2.2 Definition [Intuitionistic fuzzy set]: An intuitionistic fuzzy set A in X is defined as $A = \{\langle x, J_A(x), K_A(x)\rangle \, / \, x \in X\}$, where $J_A: X \to [0,1]$ and $K_A: X \to [0,1]$ are respectively degree of membership and degree of non-membership for every $x \in X$ with $0 \le J_A(x) + K_A(x) \le 1$ and $\prod_A(x) = [1 - (J_A(x) + K_A(x))]$ is the degree of indeterminacy of $x \in X$.

2.3 Definition [Pythagorean fuzzy set]:

A Pythagorean uncertainty set A on a set X is defined by $A = \{(x, J_A(x), K_A(x))/x \in X\}$ where $J_A: X \to [0,1]$ is degree of membership x, respectively which fulfill the condition $0 \le J_A^2(x) + K_A^2(x) \le 1$. The degree of indeterminacy is $\prod_A(x) = \sqrt[4]{1 - (J_A(x)^2) - (K_A(x)^2)}$.

2.4 Definition [(3,2) - fuzzy set]: A (3,2) - fuzzy set on a nonempty set 'X' is expressed as a domain $A = \{(x, J_A(x), K_A(x))/ \, x \in X\}$, where such that $0 \le J_A(x)^3 + K_A(x)^2 \le 1$. The degree of indeterminacy of x to A is $\prod_A(x) = \sqrt[5]{1 - (J_A(x)^3) - (K_A(x)^2)}$.

2.5 Definition [Fermatean fuzzy set]: Let X be a universe discourse A. Fermatean uncertainty set "A" in X is an object having the form $A = \{(x, J_A(x), K_A(x))/ \, x \in X\}$ where $J_A(x): X \to [0,1]$ and $K_A(x): X \to [0,1]$ including the condition $0 \le J_A(x)^3 + K_A(x)^3 \le 1 \; \forall x \in X$.The number $J_A(x)$ satisfies the degree of membership and $K_A(x)$ indicate the non-membership of the element 'x' in the set A. $\prod_A(x) = \sqrt[6]{1 - (J_A(x)^3) - (K_A(x)^3)}$ is to find out as the degree of indeterminacy of 'x' to A.

2.6 Definition [Beal's fuzzy set]: A Beal's fuzzy set (BFS) on a universal set 'X' is a set of 3-tuples of the form $A = \{(x, J_A(x), K_A(x))/ \, x \in X\} \ldots . (1)$ where $J_A(x)$ and $K_A(x)$ represent the membership and non-membership degrees of $x \in X$ and satisfy the condition $0 \le J_A(x)^m + K_A(x)^n \le 1$ for all $x \in X, m, n \ge 4$ and the degree of indeterminancy of x to A is $\prod_A(x) = \sqrt[m+n]{1 - [(J_A(x)^m) - (K_A(x)^n)]}$.

2.7 Definition [Partial order]: A partially ordered set (Poset) is a set in which a binary relation $a \le b$ is defined which satisfies for all a, b, c the following conditions,

P_1: *For all* X, $a \le b$.

P_2: *If* $a \le b$ *and* $b \le a$, *then* $a = b$.

Journal of Chemical Health Risks

www.jchr.org

JCHR (2025) 15(6), 1963-1974 | ISSN:2251-6727

$P_3: If\ a \leq b\ and\ b \leq c, then\ a \leq c.$

2.8 Definition: (i) A mapping $f: P \to Q$ is called order preserving, if $a \leq b$ implies $f(a) \leq f(b)$

(ii) A mapping $g: P \to Q$ is called order reversing (antitone) if and only if $a \leq b$ and $g(b) \leq g(a)$.

2.9 Definition: A lattice is a poset in which $a \wedge b = \inf(a, b)\ and\ a \vee b = \sup(a, b)$ exists for any pair of elements a and b. A sublattice of a lattice 'L' is a subset of X and L such that $a, b \in X$ implies $a \wedge b \in X$ and $a \vee b \in X$. A lattice 'L' is complete is said to be distributive, if

$D_1: a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$

$D_2: a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$, for all $a, b, c \in L$.

2.10 Definition: A Pythagorean fuzzy relation from $u_1\ to\ u_2$ is a fuzzy set of $u_1 \times u_2$, that is,

the following Pythagorean expression; $= \{((u_1, u_2), < J(u_1, u_2), K(u_1, u_2) >): u_1 \in U_1, u_2 \in U_2\}$

where $J, K: U_1 \times U_2 \longrightarrow [0,1]$ are membership degree and non-membership degree constrained

to obey $0 \leq (J(u_1, u_2))^2 + (K(u_1, u_2))^2 \leq 1$ for all $(u_1, u_2) \in U_1 \times U_2$.

2.11 Definition: A non-void set S along with an associative binary operation $*$ defined on S is termed a semi group. It is generally symbolized as $(S,*)$.

2.12 Definition: A semi group $(S,*)$ is said to be;

(i) Monoid, if there exists an object $e \in S$ such that $e * a = a * e = a$ for all $a \in S$.

(ii) Idempotent, if $a * a = a$ for $a \in S$.

2.13 Definition: A non-void set S with two binary operations '+' and '.' is called a semi ring, if;

$(i)(S, +)$ is a semigroup

$(ii)(S,\cdot)$ is a semigroup

(iii) Multiplication in distributive over addition from either sides.

$(iv) a.(b + c) = (a.b) + (a.c)$

$(v)\ (b + c).a = (b.a) + (c.a)$ for all $a, b, c \in S$.

Usually a semi ring is represented by $(S, +, \cdot)$. Further, $(S, +, \cdot)$ is named commutative

semi ring, if $(S,\cdot)$ is a commutative semigroup (i.e) $a.b = b.a$ for all $a, b \in S$.

A semi ring $(S, +, \cdot)$ is said to be an identity element 'e', if for each $a \in S$, we have

$a.e = e.a = a$ for some $e \in S$.

A semi ring $(S, +, \cdot)$ is said to be a zero element 0, if for each $a \in S$, $a + 0 = 0 + a = a$,

$a.0 = 0.a = 0$ for some $0 \in S$.

2.14 Definition: A semi ring $(S, +, \cdot)$ with a zero element '0' and commutative addition is called a hemi ring. Next, we recall our idea of bounded lattice taken from $(S, +, \cdot)$.

2.15 Definition: A relation 'R' on a non-empty set S is called a partial order, if 'R' is

$(i)\ Reflexive: \forall a \in S, (a, a) \in R.$

$(ii) Anti - symmetric: if\ \forall a, b \in S,$

$(a, b) \in R\ and\ (b, a) \in R, then\ a = b.$

$(iii) Transitive: if\ \forall a, b, c \in S, (a, b) \in Rand$

$(b, c) \in R, then\ (a, c) \in R.$

Usually the symbol used for a partial order is $\leq$.

A non-empty set S with a partial order $\leq$ is called a partially ordered set (poset) and it is denoted by $(S, \leq)$.

2.16 Definition: Let $(S, \leq)$ be a POSET and $B \subseteq S$. An object $x \in S$ is called;

(i) a lower bound of B, if $x \leq a, \forall\ a \in B$.

(ii) an upper bound of B, if $a \leq x\ \forall\ a \in B$.

(iii) a greatest lower bound (GLB) of B, if there exist $x' \in S$ such that $x' \leq a\ \forall\ a \in B, then$

$x' \leq x.$

(iv) a least upper bound (LUB) of B, if there exist, $x' \in S$ such that $a \leq x'\ \forall\ a \in B, then$

$x' \leq x.$

Journal of Chemical Health Risks
www.jchr.org
JCHR (2025) 15(6), 1963-1974 | ISSN:2251-6727

2.17 Definition: A poset $(S, \leq)$ is called a lattice, if $a \vee b \in S$ and $a \wedge b \in S$ for all $a, b \in S$,
where $a \vee b$ denotes the supremum(LUB) of a and b, and $a \wedge b$ denotes the infimum(GLB) of a and b.

2.18 Definition: A lattice $(S, \leq)$ is called distributive, if $\forall\ a, b, c \in X$;
$(i) a \vee (\ b \wedge c) = (\ a \vee b) \wedge (a \vee c)$
$(ii) a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$.

2.19 Definition: Let $(S, \leq)$ be a lattice and $0,1 \in S$. Then
(i) 0 is called a bottom element of S, if $x \wedge 0 = 0\ and\ x \vee 0 = x\ \forall\ x \in S$.
(ii)1 is called a top element of S, if $x \wedge 1 = x\ and\ x \vee 1 = 1\ \forall\ x \in X$.
A lattice $(S, \leq)$ is called a bounded lattice, if it has a top element 1 and a bottom element 0.
Also, a lattice $(S, \leq)$ is called bounded lattice if it is bounded and distributive as well.

3 . BEAL'S FUZZY RELATION

It is well know that a binary relation plays a vital role in both pure and applied sciences.
In this section, we establish a novel idea of beal's fuzzy relation. Also, discuss some basic set theoretic operations of beal's fuzzy relations.

3.1 Definition: A Beal's fuzzy relation R from u_1 and u_2 is a mathematical structure having the following form $R = \{((u_1, u_2), < J(u_1, u_2), K(u_1, u_2) >)/u_1 \in U_1, u_2 \in U_2\}$ .... (1)

where the mapping $J, K: u_1 \times u_2 \longrightarrow [0,1]$ are denoting the membership degree and non-membership degree from $U_1\ to\ U_2$, respectively, constrained to obey the following condition:

$$0 \leq (J((u_1, u_2))^m + (K(u_1, u_2))^n \leq 1 \ldots.. \quad (2)$$

for all $(u_1, u_2) \in U_1 \times U_2\ and\ m, n \geq 4$. The fuzzy relation $\Delta = U_1 \times U_2 \longrightarrow [0,1]$ associated with each beal's fuzzy relation, where

$$\Delta(u_1, u_2) = \sqrt[m+n]{1 - [(J((u_1, u_2))^m + (K(u_1, u_2))^n]}$$

represent the hesitation index (HI) for the sake of simplicity, we shall use the following notation for an beal's fuzzy relation from $U_1\ to\ U_2$.

$$\delta = < J(u_1, u_2), K(u_1, u_2) >$$

from now onward, we shall apply the notation BFR($U_1 \times U_2$) to represent the collection of all beal's fuzzy relations from $U_1\ to\ U_2$. From the definition(3.1) it is clear that beal's fuzzy relation Ω is simply a BFR($u_1 \times u_2$).

3.2 Remark: The definition (3.1) of beal's fuzzy relation can be extended to 'n' universes
$U_1 \times U_2 \times U_3 \ldots. \times U_n$ in similar fashion.

3.3 Definition: Let $\delta =< J(u_1, u_2), K(u_1, u_2) >$ be a beal's fuzzy relation from $U_1\ to\ U_2$, where
$U_1 = \{\ s_1, s_2, \ldots., s_p\}\ and\ U_2 = \{t_1, t_2, \ldots. t_q\}$ be finite universe.
Consider$\delta =< J(u_1, u_2), K(u_1, u_2) > =$
$< p_{ij}\ , q_{ij} >_{m \times n}$ satisfying $0 \leq (p_{ij})^m + (q_{ij})^n \leq 1$,
where $m, n \geq 4\ and\ i, j\ where\ 1 < i < p\ and\ 1 < j < q$. Then a beal's fuzzy relation Ω can be represented through a matrix as;

$$S = \begin{pmatrix} < p_{11}, q_{11} > < p_{12}, q_{12} > \cdots < p_{1n}, q_{1n} > \\ < p_{21}, q_{21} > < p_{22}, q_{22} > \cdots < p_{2n}, q_{2n} > \\ . \\ . \\ . \longrightarrow \\ < p_{m1}, q_{m1} > < p_{m2}, q_{m2} > \cdots < p_{mn}, q_{mn} > \end{pmatrix}$$

Following set theoretic operations can be considered for Beal's fuzzy realtions.

3.4 Definiton: Let $\delta = < J_1(u_1, u_2), K_1(u_1, u_2) >$ $and\ \delta_2 =< J_2(u_1, u_2), K_2(u_1, u_2) >$ be two beal's fuzzy relations from $U_1\ to\ U_2$.Then for all $u_1, u_2 \in U_1\ \times\ U_2$, we have
$(i)\ \delta_1 \subseteq \delta_1$ if and only if $J_1(u_1, u_2) \leq J_2(u_1, u_2)$ and $K_1(u_1, u_2) \geq K_2(u_1, u_2)$.
$(ii)\ \delta_1 \cup \delta_2 =< J_1(u_1, u_2) \vee J_2(u_1, u_2) >$;

$< K_1(u_1, u_2) \wedge K_2(u_1, u_2) >$.

$(iii)\ \delta_1 \cap \delta_2 = < J_1(u_1, u_2) \wedge J_2(u_1, u_2) >$;

$< K_1(u_1, u_2) \vee K_2(u_1, u_2) >$

$(iv)\ \delta_1^{-1} = < (J_1)^{-1}(u_2, u_1), (K_1)^{-1}(u_2, u_1) >$ in a beal's fuzzy relation from $U_2\ to\ U_1$, where $(J_1)^{-1}(u_2 u_1) = J_1(u_1, u_2)$ and $(K_1)^{-1}(u_2, u_1) = K_1(u_1, u_2)$.

$(v)\ \delta_1^c = < K_1(u_1, u_2), J_1(u_1, u_2) >$ where C is the complement.

3.5 Proposition: If $\delta_1\ \delta_2 \in BFR(U_1 \times U_2)$, then

$(i)\ \delta_1 \cup \delta_2, \delta_1 \cap \delta_2 \in BFR(U_1 \times U_2)$

$(ii)\ \delta_1^{-1} \in BFR(U_1 \times U_2)$.

The proof is straight forward.

3.6 Proposition: Let $\delta_1\ \delta_2 \in BFR(U_1 \times U_2)$. Then the following axiom are exist.

$(i)\ \delta_1 \subseteq \delta_2$ implies that $\delta_1^{-1} \subseteq \delta_2^{-1}$.

$(ii)(\delta_1 \cup \delta_2)^{-1} = \delta_1^{-1} \cup \delta_2^{-1}$

$(iii)(\delta_1 \cap \delta_2)^{-1} = \delta_1^{-1} \cap \delta_2^{-1}$.

$(iv)\ (\delta_1^{-1})^{-1} = \delta_1$.

3.7 Defintion: In $BFR \in (U_1 \times U_2)$, we rerpresent and define full BFR and null BFR as

$$T_{BFR} = \{(u_1, u_2), < T(u_1, u_2), \perp (u_1, u_2) > / u_1 \in U_1, u_2 \in U_2\}$$

$$\perp_{BFR} = \{(u_1, u_2), <\perp (u_1, u_2), T(u_1, u_2) > / u_1 \in U_1, u_2 \in U_2\}$$

where $T(u_1, u_2) = 1, for\ all\ (u_1, u_2) \in U_1 \times U_2$.

$\perp (u_1, u_2) = 0, for\ all\ (u_1, u_2) \in U_1 \times U_2$.

In view of the definition(3.3) and definition (3.4), we get the following the subsequent result.

3.8 Proposition: Let $\delta_1, \delta_2, \delta_3 \in BFR(U_1 \times U_2)$. Then the following results obtained

(1) $\delta_1 \cup \perp_{BFR} = \delta_1$

(2) $\delta_1 \cap \perp_{BFR} = \perp_{BFR}$

(3) $\delta_1 \cup T_{BFR} = T_{BFR}$

(4) $\delta_1 \cap T_{BFR} = \delta_1$

(5) $\delta_1 \cup \delta_1 = \delta_1$

(6) $\delta_1 \cap \delta_1 = \delta_1$

(7) $\delta_1 \cup \delta_2 = \delta_2 \cup \delta_1$

(8) $\delta_1 \cap \delta_2 = \delta_2 \cap \delta_1$

(9) $(\delta_1 \cup \delta_2) \cup \delta_3 = \delta_1 \cup (\delta_2 \cup \delta_3)$

(10) $(\delta_1 \cap \delta_2) \cap \delta_3 = \delta_1 \cap (\delta_2 \cap \delta_3)$

Proof: The result is obvious.

The above result gives the following algebraic structure.

3.9 Corollary: The pairs (BFR$(u_1 \times u_2)$,∪) and (BFR $(u_1 \times u_2)$,∩) are idempotent, commutative monoids with identity elements $\perp_{BFR}$ and T_{BFR} respectively.

3.10 Proposition: Let $\delta_1, \delta_2, \delta_3 \in BFR(u_1 \times u_2)$. Then the following assertions hold;

$(i)\ \delta_1 \cup (\delta_2 \cap \delta_3) = (\delta_1 \cup \delta_2) \cap \delta_3$

$(ii)\ \delta_1 \cap (\delta_2 \cup \delta_3) = (\delta_1 \cap \delta_2) \cup \delta_3$

$(iii) If\ \delta_1 \subseteq \delta_2\ and\ \delta_3 \subseteq \delta_{1,}\ then\ \delta_2 \cup \delta_3 \subseteq \delta_1$

$(iv) If\ \delta_1 \subseteq \delta_2\ and\ \delta_1 \subseteq \delta_{3,}\ then\ \delta_1 \subseteq \delta_2 \cap \delta_3$.

Proof: Straight forward

From the proposition(3.8), we have the following corollaries.

3.11 Corollary: The following statements are true:

$(i)(BFR(u_1 \times u_2)$,∪,∩) is a commutative semiring with identity T_{BFR} and zero elemnet $\perp_{BFR}$.

$(ii)(BFR(u_1 \times u_2)$,∪,∩) is a commutative semiring with identity element $\perp_{BFR}$ and zero element T_{BFR} the above corollary, gives rise to the following result.

3.12 Corollary: The following statements hold;

$(i)(BFR(u_1 \times u_2)$,∪,∩) is a hemiring with zero elemnet $\perp_{BFR}$.

$(ii)(BFR(u_1 \times u_2)$,∪,∩) is a hemiring with zero elemnet T_{BFR}.

The following lemma is very important which gives rise to a lattice adaption associated with beal's fuzzy relations.

3.13 Lemma: Let $\delta_1, \delta_2, \delta_3 \in BFR(u_1 \times u_2)$.Then

$(i)\ \delta_1 \subseteq \delta_1$.

$(ii) If\ \ \delta_1 \subseteq \delta_2\ and\ \delta_2 \subseteq \delta_1, then\ \delta_1 = \delta_2$.

$(iii) If\ \delta_1 \subseteq \delta_2\ and\ \delta_2 \subseteq \delta_1 \subseteq \delta_3, then\ \delta_1 \subseteq \delta_3.$

Proof: Straight forward .

From proposition (3.5) – (3.10) , and lemma (3.13) we have the following important result.

3.14 Proposition: The following asserations hold;

$(i)(BFR(u_1 \times u_2),\cup,\cap)$ is a POSET

$(ii)(BFR(u_1 \times u_2),\cup,\cap)$ is a bounded distribution lattice with top element T_{BFR} and bottom element $\perp_{BFR}$.

Proof:(i)From the above Lemmma(3.13), it is clear that the order is a partial order on $BFR(u_1 \times u_2)$.

(ii) The partially ordered set $(BFR(u_1 \times u_2),\subset)$ is a lattice because each pair $\delta_1, \delta_2 \in BFR(u_1 \times u_2)$ has a GLB of $\delta_1 \cap \delta_2$ and a LUB of $\delta_1 \cup \delta_2$. Now we define the composition of two BFRS and investigate its relevant properties.

3.15 Definition: Let $\delta_1 =< J_1(u_1,u_2), K_1(u_1,u_2) >$ be BFR from $u_1\ to\ u_2$ and $\delta_2 =< J_2(u_1,u_2), K_2(u_1,u_2) >$ be a BFR from $u_2\ to\ u_3$. we define the composition of δ_1 and δ_2 as follows.

$\delta_1 \circ \delta_2 =< (J_1 \circ J_2(u_1,u_2), K_1 \circ K_2(u_1,u_2)) >$ where

$(J_1 \circ J_2)^m(u_1,u_2)$

$= \max_{u_2 \in U_2} (\min\{J_1^m(u_1,u_2), J_2^m(u_2,u_3)\})$

$(K_1 \circ K_2)^n(u_1,u_2)$

$= \min_{u_2 \in U_2} (\max\{K_1^n(u_1,u_2), K_2^n(u_2,u_3)\})$

for all $(u_1,u_2) \in u_1 \times u_3$.

3.16 Lemma: Let $x,y \in [0,1]$ and $m,n \geq 4$ then $[\vee_{max}(x \wedge y)]^m \leq \vee_{max}(x^m \wedge y^m)$.

Proof: we will show only for $m = 5$. Similar proof can follow for an arbitary value of m.

As $x,y \in [0,1]$. So $x \wedge y \leq x$ and $x \wedge y \leq y$.

Therefore $(x \wedge y)^5 \leq x^5\ and\ (x \wedge y)^5 \leq y^5$.

Thus $(x \wedge y)^5 \leq x^5 \wedge y^5$. It follows that $\vee_{max}(x \wedge y)^5 \leq \vee_{max}(x^5 \wedge y^5)$. This proves that $\vee_{max}(x \wedge y)^5 \leq \vee_{max}(x^5 \wedge y^5)$.

The following result illustrates that the composition of two BFR's is also a BFR's.

3.17 Proposition: If $\delta_1 \in BFR(u_1 \times u_2)$ and $\delta_2 \in BFR(u_2 \times u_3)$, then $\delta_1 \circ \delta_2 \in BFR(u_1 \times u_3)$

Proof: we will show that $0 \leq \left((J_1 \circ J_2)(u_1,u_3)\right)^m + \left((K_1 \circ K_2)(u_1,u_3)\right)^n \leq 1$

for all $(u_1,u_2) \in U_1 \times U_3$. Since $0 \leq \left(J_1(u_1,u_2)\right)^m + \left(K_1(u_1,u_2)\right)^n \leq 1$ for all $(u_1,u_2) \in U_1 \times U_2$ and $0 \leq \left(J_2(u_1,u_2)\right)^m + \left(K_2(u_1,u_3)\right)^n \leq 1$ for all $(u_1,u_3) \in U_1 \times U_3$.

It follows that

$$\left(J_1(u_1,u_2)\right)^m \leq 1 - \left(K_1(u_1,u_2)\right)^n\ and\ \left(J_2(u_1,u_2)\right)^m \leq 1 - \left(K_2(u_1,u_2)\right)^n$$

Let $(u_1,u_3) \in U_1 \times U_3$. Then by using the definition (3.15) $\left((J_1 \circ J_2)(u_1,u_3)\right)^m =$ $\vee_{u_2 \in U_2} \min\{J_1^n(u_1,u_2), J_2(u_2,u_3)\}^m$.

$\leq \vee_{u_2 \in U_2} [\{\min(J_1(u_1,u_2))^m, \left(J_2(u_2,u_3)\right)^m\}$ by lemma(3.16).

$\leq \vee_{u_2 \in U_2} [\{\min\left(J_1(u_1,u_2)\right)^m, \left(J_2(u_2,u_3)\right)^m\}$

$\leq \vee_{u_2 \in U_2} [\min(1 - (K_1(u_1,u_2))^n, 1 - \left(K_2(u_2,u_3)\right)^n\}$

$\leq \vee_{u_2 \in U_2} [\max(1 - (K_1(u_1,u_2))^n, 1 - \left(K_2(u_2,u_3)\right)^n\}$

$= 1 - \wedge_{u_2 \in U_2} \max\{\left(J_1(u_1,u_2)\right)^m, \left(J_2(u_2,u_3)\right)^m\}$

Hence $= 1 - \left((K_1 \circ K_2)(u_1,u_3)\right)^n$.

3.18 Theorem: Suppose $\delta_1 \in BFR(u_1 \times u_2)$ and $\delta_2 \in BFR(u_2 \times u_3)$, then

$(\delta_1 \circ \delta_2)^{-1} = \delta_2^{-1} \circ \delta_1^{-1}$.

Proof: Let $(u_3,u_1) \in U_3 \times U_1$. In the light of the definition 3.4 in (iv) and s definition (3.15)

$$(J_1^m \circ J_2^m)^{-1}(u_3,u_1) = (J_1^m \circ J_2^m)(u_1,u_3)$$

$$= \vee_{u_2 \in U_2} \min\{J_1^{-1^m}(u_2,u_1), J_1^{-1^m}(u_3,u_2)\}$$

Journal of Chemical Health Risks
www.jchr.org
JCHR (2025) 15(6), 1963-1974 | ISSN:2251-6727

$= \vee_{u_2 \in U_2} \min\{J_1^{-1^m}(u_2,u_1), J_1^{-1^m}(u_3,u_2)\}$

$= ((J_2^m)^{-1} \circ (J_1^m)^{-1}(u_3,u_1))$

Similarly, it can be showed that

$(K_1^n \circ K_2^n)^{-1}(u_3,u_1) = \left((K_2^n)\right)^{-1} \circ (K_1^n)^{-1})(u_3,u_1).$

3.19 Theorem: If $\delta_1 = < J_1(u_1,u_2), K_1(u_1,u_2) >$ is a beal's fuzzy relation from U_1 to U_2 and

$\delta_2 = < J_2(u_1,u_2), K_2(u_1,u_3) >$ be two beal's fuzzy relations from

U_2 to U_3 such that $\delta_1 \subseteq \delta_2$. Then

(i) $\delta \circ \delta_1 \subseteq \delta \circ \delta_2$.

(ii) $\delta_1 \circ \delta_1 \subseteq \delta_2 \circ \delta_2$.

Proof: (i) Let $\delta_1 \subseteq \delta_2$. Using definition (3.15) and definition (3.4) , we have $(J \circ J_1)^m(u_1,u_2) = \vee_{u_2 \in U_2} \min\{J^m(u_1,u_2), J_1^m(u_1,u_2)\}$

$\leq \vee_{u_2 \in U_2} \min\{J^m(u_1,u_2), J_2^m(u_2,u_3)\}$

$= (J \circ J_2)^m(u_1,u_2)$

In the similar situation, one can explain that

$(J \circ J_1)^m(u_1,u_3) \geq (J \circ J_2)^m(u_1,u_3)$.

(ii) From part (i), we have

$\delta_1 \circ \delta_1 \subseteq \delta_2 \circ \delta_2 \subseteq \delta_2 \circ \delta_3$.

3.20 Theorem: If δ_1 , $\delta_2 \in BFR(u_1 \times u_2)$ with $\delta_1 \subseteq \delta_2$ and $\delta \in BFR(u_2 \times u_3)$, then

$\delta_1 \circ \delta \subseteq \delta_2 \circ \delta$.

Proof: Analogous to the proof of the theorem(3.19).

3.21 Theorem: Let $\delta_1 \in BFR(u_1 \times u_2)$,

$\delta_2 \in BFR(u_2 \times u_3)$ and $\delta_3 \in BFR(u_3 \times u_1)$. Then

$\delta_1 \circ (\delta_2 \circ \delta_3) = (\delta_1 \circ \delta_2) \circ \delta_3$.

Proof: Let $u_1 \in U_1, u_2 \in U_2$. Then by definition (3.15) , it shows that

$(J_1 \circ J_2) \circ J_3)^m = \vee_{u_2 \in U_2} \min\{(J_1 \circ J_2)^m(u_2,u_3), J_3^m(u_3,u_4)\}$ =

$\vee_{u_3 \in U_3} \vee_{u_2 \in U_2} \min\{J_1^m(u_1,u_2), J_2^m(u_2,u_3), J_3^m(u_3,u_4$

$= \vee_{u_2 \in U_2} \vee_{u_2 \in U_2} \min\{J_1^m(u_1,u_2), J_2^m(u_2,u_3), J_3^m(u_3,u_4$

$\vee_{u_2 \in U_2} \min\{J_1^m(u_1,u_2) (\vee_{u_3 \in U_3} \min\{J_2^m(u_2,u_3), J_3^m(u_3,u_4)\}$

$= \vee_{u_2 \in U_2} \min\{J_1^m(u_1,u_2), (J_2 \circ J_3)^m(u_2,u_4)\}$

$= (J_1^m \circ (J_2 \circ J_3)^m)(u_1,u_4)$

Similarly,

$\left((J_1 \circ J_2) \circ J_3\right)^m(u_1,u_4) = \left(J_1 \circ (J_2 \circ J_3)\right)^m(u_1,u_4)$

In the subsequent result, the distributive laws of composition over union and intersection are given.

3.22 Theorem: Let $\delta \in BFR(u_1 \times u_2)$ and δ_1 , $\delta_2 \in BFR(u_2 \times u_3)$. Then the following statements are hold;

(i) $\delta \circ (\delta_1 \cup \delta_2) = (\delta \circ \delta_1) \cup (\delta \circ \delta_2)$

(ii) $\delta \circ (\delta_1 \cap \delta_2) = (\delta \circ \delta_1) \cap (\delta \circ \delta_2)$

Proof: we shall prove only $(i) and$ (ii) proved the following same lines

(i) Let $(u_1,u_3) \in U_1 \times U_3$. From definition (3.15) and definition (3.4) (i), it follows that

$(J \circ (J_1 \cup J_2)^m(u_1,u_3) =$

$\vee_{u_2 \in U_2} \min\{J^m(u_1,u_2), (J_1 \cup J_2)^m(u_2,u_3)\}$

$= \vee_{u_2 \in U_2} \min\{J^m(u_1,u_2), J_1^m(u_2,u_3) \vee J_2^m(u_3,u_4$

$\vee_{u_2 \in U_2} \max\{\min\{J^m(u_1,u_2), J_1^m(u_1,u_2)\},$

$\min\{J^m(u_1,u_2), J_2^m(u_2,u_3)\}\}$

$=\max(\vee_{u_2 \in U_2} \min\{J^m(u_1,u_2),$

$J_1^m(u_1,u_2)), \vee_{u_2 \in U_2} \min\{J^m(u_1,u_2), J_1^m(u_2,u_3)\}$

$= \max\{(J \circ J_1)^m(u_1,u_3), (J \circ J_2)^m(u_1,u_3)\}$

In the similar way, it can be proved that

$(J \circ (J_1 \cup J_2)^m(u_1,u_3)$

$= \min\{(J \circ J_1), (J \circ J_2)^m)(u_1,u_3)$.

Hence the proof.

3.23 Theorem: Let δ_1 , $\delta_2 \in BFR(u_1 \times u_2)$ and $\delta \in BFR(u_2 \times u_3)$ then the following assertion hold;

(i) $(\delta_1 \cup \delta_2) \circ \delta_3 = (\delta_1 \circ \delta_3) \cup (\delta_2 \circ \delta_3)$

$(ii)(\delta_1 \cap \delta_2) \circ \delta_3 = (\delta_1 \circ \delta_3) \cap (\delta_2 \circ \delta_3)$.

Proof: Analogous to the proof of theorem (3.22).

3.24 Lemma: Let $\delta \in BFR(u_1 \times u_1)$. Then

$(i) T_{BFR} \circ \delta = T_{BFR}$

(ii) $\perp_{BFR} \circ \delta = \perp_{BFR}$.

Proof: From theorem (3.22) Lemma (3.24).

3.25 Proposition: The result is obtained theorem (2.23) Triplet (BFR$(U_1 \times U_2), U, 0)$ is

Journal of Chemical Health Risks
www.jchr.org
JCHR (2025) 15(6), 1963-1974 | ISSN:2251-6727

(i) A semi ring identity element $T_{BFR} \in BFR(u_1 \times u_1)$ and zero element $\perp_{BFR} \in BFR(u_1 \times u_1)$.

(ii) A hemi ring with zero element $\perp_{BFR} \in BFR(u_1 \times u_1)$.

Proof: Straight forward.

4. BEAL'S FUZZY ENTROPY MEASURES

In this section, first we give the conditions of entropy for beal's fuzzy set. It is the generalization of intuitionistic fuzzy sets and Pythagorean fuzzy sets respectively. The set of beal's fuzzy set in X is denoted by BFS(X).

4.1Definition A map $M: BFS(X) \longrightarrow [0,1]$ is called as entropy on X if 'M' satisfies the following conditions:

$(M_1): 0 \leq M(\overline{A}) \leq 1;$

$(M_2): M(\overline{A}) = 0,$ if and only if $\overline{A}$ is a crisp set.

$(M_3): M(\overline{A}) = 1$,if and only if $J_{\overline{A}}^{m}(x_i) = K_{\overline{A}}^{n}(x_i), x_i \in X$

$(M_4): M(\overline{A}) \leq M(\overline{B})$ if and only if $\overline{A} \leq B$;

$(M_5): M(\overline{A}) = M(\widetilde{A^c})$, where $\widetilde{A^c}=\{(x, K_{\overline{A}}^{n}(x), J_{\overline{A}}^{m}(x) / x \in X\}$

This idea to find out the vagueness from a fuzzy set and its negation were introduced by Yager. In ranking alternatives using an TOPSIS algorthim, we use distance measure to obtain the distance between every alternatives to positive ideal solution and negative ideal solution respectively.

Let us consider $X = \{x_1, x_2, \ldots . x_n\}$ be a fixed discrete universe of discourse. Then the distance between two BFSS $\tilde{A}$ and $\tilde{B}$ is defined as

$$d(\overline{A}, \overline{B}) = \left[\frac{1}{2n}\left(\sum_{i=1}^{n}\left|J_{\overline{A}}^{m}(x_i) - J_{\overline{B}}^{m}(x_i)\right|^{m} + \left|K_{\overline{A}}^{n}(x_i) - K_{\overline{B}}^{n}(x_i)\right|^{n} + \left|\Pi_{A_i}^{m}(x_i) - \Pi_{B_i}^{m}(x_i)\right|^{\frac{1}{m+n}}\right)\right]^{\frac{1}{m+n}} \ldots \ldots (1)$$

Usually, in various practical life setting applications and ranking of alternatives weight vector $\overline{w}$ of the number $x \in X$ is considered. Therefore, we assign weights in equation (1) and created weighted distance measure for BFSS.

Assume that the weight of every element $x_i \in X$ is $\overline{w}_i(i = 1,2, \ldots n)$ such that $\sum_{i=1}^{n} w_i = 1,$ where $0 \leq w_i \leq 1.$

If we replace $\overline{B}$ by $\widetilde{A^c}$ in equation (1) reduces to distance between $\tilde{A}$ and its complement $\widetilde{A^c}$ as

$$d(\overline{A}, \overline{B}) = \left[\frac{1}{n}\sum_{i=1}^{n}\left|J_{A_i}^{m}(x_i) - K_{A_i}^{n}(x_i)\right|^{\frac{1}{m+n}}\right]^{\frac{1}{m+n}} \rightarrow (2)$$

Based on equation(2), we will use equation(2) to form a new entropy for Beals fuzzy set

$$l_{d_1}(\overline{A}) = 1 - d(\overline{A}, \overline{A^c}) = 1 - \left[\frac{1}{n}\sum_{i=1}^{n}\left|J_{A_i}^{m}(x_i) - K_{A_i}^{n}(x_i)\right|^{\frac{1}{m+n}}\right]^{\frac{1}{m+n}} \rightarrow (3)$$

4.2 Theorem Let $X = \{x_1, x_2, \ldots . x_n\}$ be a fixed discrete universe of discourse. Then the suggested entropy $l_{d_1}(\tilde{A})$ for BFSS satisfies the conditions $(M_1) - (M_3)$ in definition (4.1).

Now let us consider $X = \{x_1, x_2, \ldots . x_n\}$ be a finite universe of discourse and $\tilde{A}$ be a BFSS on X. Then a new entropy measure of $\tilde{A}$ is defined as

$$l_{d_2}(\overline{A}) = 1 - \frac{2}{n}\sum_{i=1}^{n}\frac{\left|J_{\overline{A}}(x_i) - K_{\overline{A}}(x_i)\right|}{1 + \left|J_{\overline{A}}(x_i) - K_{\overline{A}}(x_i)\right|} - - - -(4).$$

4.3 Theorem Let $X = \{x_1, x_2, \ldots . x_n\}$ is a fixed set, the proposed entropy $l_{d_2}(\overline{A})$ satisfies the condition $(M_1) - (M_5)$ to in definition (4.1). Finally, we propose new and intuitive entropy for BFSS based on the division of min and max operations. Let us consider $X = \{x_1, x_2, \ldots . x_n\}$ be a fixed and $\overline{A}$ be a BFSS on X. Then, a new entropy measure $\overline{A}$ is explained as

$$l_{min/max}(\overline{A}) = \frac{1}{n}\sum_{i=1}^{n}\frac{\min\left(J_{\overline{A}}^{m}(x_i), K_{\overline{A}}^{n}(x_i), \Pi_{A_i}^{m+n}(x_i)\right)}{\max\left(J_{\overline{A}}^{m}(x_i), K_{\overline{A}}^{n}(x_i), \Pi_{A_i}^{m+n}(x_i)\right)} - - - -(5)$$

4.4 Theorem Suppose that $X = \{x_1, x_2, \ldots . x_n\}$ be a fixed set, the proposed entropy $l_{min/max}$ satisfies the conditions $(M_1) - (M_3)$ in definition (4.1).

Journal of Chemical Health Risks
www.jchr.org
JCHR (2025) 15(6), 1963-1974 | ISSN:2251-6727

Example: Consider $\tilde{A} = \overline{A}\{(x, 0.6520, 0.7854, 0.6201)\}$

$\tilde{B} = \overline{B}\{(x, 0.8130, 0.5916, 0.6346\}$

$\widetilde{C} = \overline{C}\{(x, 0.1990, 0.9930, 0.2351)$

are three BFSS in the singleton universe of discourse $X = \{x\}$, then the entropy measure for these BFSS. Using proposed equation (3) and (5) in table in for $m \geq 3 and\ n \geq 3$.

BFSS	l_{d_1}	l_{d_2}	$l_{min/max}$
$\tilde{A}$	0.7927	0.6566	0.4922
$\tilde{B}$	0.6697	0.5034	0.3853
$\tilde{C}$	0.0287	0.0146	0.0080

Table-1 Entropy measure of three BFSS.

5. PROPOSED METHOD-ORTHOPAIREAN TOPSIS ALGORITHM

In this section, we develop multi criteria decision making problems in a beal's fuzzy environment. Hwang and Yoon first introduced the Topsis method to handle the problems related to MCDM. Usually, the available information related to daily life setting involving multicriteria decision making processes are mostly in exact or vagueness. So it is very difficult to come up the intuitively acceptable decision without using excellent method. Therefore we utilize orthopairean TOPSIS method based on proposed entropy measures equation (3) and equation(5) to tackle problems containing complex multi criterian decision making process associated to practical life.

we consider that there are m alternatives and wish to calculate them on n criteria. Assume $A = \{A_1, A_2, A_3 \ldots . A_m\}$ with $i = 1,2, \ldots m$ and assume that the set of criteria for the alternatives be denoted by Q_j with $j = 1,2, \ldots n$. Our objective is to pick the most efficient alternative among the given set of alternatives. The construction steps for the orthopairian TOPSIS based on suggested entropy measure equation (3) and equation (5) are given as:

Step-1: Construction of Beal's Fuzzy Decision Matrix

In this case, the alternatives A_i forced to criteria Q_j is denoted by Beal's fuzzy value membership supports, K_{ij}denotes the non-membership grades and π_{ij} stand for the degree of interdeterminacy against the alternatives A_i to the criteria Q_j with the condition that $0 \leq J_{ij}^m, K_{ij}^n, \pi_{ij}^{m+n} \leq 1$ and $J_{ij}^m + K_{ij}^n + \pi_{ij}^{m+n}$=1.

The Beal's fuzzy decision matrix (BFDM) is represented by,

$$R = A_{ij} = \begin{bmatrix} (J_{11}, K_{11}, \pi_{11}) & (J_{12}, K_{12}, \pi_{12}) & (J_{1n}, K_{1n}, \pi_{1n}) \\ (J_{21}, K_{21}, \pi_{21}) & (J_{22}, K_{22}, \pi_{22}) & ((J_{2n}, K_{2n}, \pi_{2n}) \\ (J_{m1}, K_{m1}, \pi_{m1)} & (J_{m2}, K_{m2}, \pi_{m2}) & (J_{mn}, K_{mn}, \pi_{mn}) \end{bmatrix}$$

Step-2: Determination of Weights Criteria

The weighted criteria is $w_j = \frac{\widetilde{M_J}}{\sum_{j=1}^{n} \widetilde{M_J}}$ where $\widetilde{M_J} = \frac{1}{P} \sum_{j=1}^{n} \Delta_{ij}$ with the condition that $0 \leq \widetilde{w_J} \leq 1$ provided that $\sum_{j=1}^{n} \widetilde{w_J}' s$=1.

Step-3: Utilize the score function to determine the Beal's fuzzy positive ideal solution and Beal's fuzzy negative ideal solution.

Let F_1 and F_2 be the sets of benefit and non-benefit in the criteria Q_j with the principle of TOPSIS. we define beal's fuzzy positive and negative ideal solutions is given by

$$A^+ = \{< Q_j, (J_j^+, K_j^+, \pi_j^+) >\} where\ (J_j^+, K_j^+, \pi_j^+) = (1,0,0), j \in F_1;$$

$$A^- = \{< Q_j, (J_j^-, K_j^-, \pi_j^-) >\} where\ (J_j^-, K_j^-, \pi_j^-) = (0,1,0), j \in F_2.$$

Step-4: Use the distance formula, to calculate the weighted distances between two ideal solutions

$$R^+(A_i) = \left[\frac{1}{2}\sum_{j=1}^{n} w_j\left(\left|1 - J_{ij}^m\right|^m + \left|K_{ij}^n\right|^n + \left|\pi_{ij}^{m+n}\right|\right)\right]^{\frac{1}{m+n}}$$

$$R^-(A_i) = \left[\frac{1}{2}\sum_{i=1}^{n} w_j\left(\left|J_{ij}\right|^m + (\left|1 - k_{ij}^n\right|^n + \left|\pi_{ij}^{m+n}\right|\right)^{m+n}\right]^{\frac{1}{m+n}}$$

where $i = 1,2,3, \ldots m\ and\ j = 1,2,3 \ldots n$.

Journal of Chemical Health Risks
www.jchr.org
JCHR (2025) 15(6), 1963-1974 | ISSN:2251-6727

Step-5: Calculation of Relative Closeness

The relative closeness degree $R(A_i)$ is calculated as $R(A_i) = \frac{\overline{R}\,(A_i)}{R^{+}(A_i)+R^{-}(A_i)}$

The alternative with maximum relative closeness degree is taken as the best alternative among all other alternatives.

5.1Example: Consider the parent wants to choose the best private college in Trichy among some well-known private sector colleges. Let us take there are three well-known available private colleges.

D_1: Private colleges P_1

D_2: Private college P_2

D_3: Private college P_3. To choose the best one, we need to consult some educational expert and their opinions are represented by three criteria as;

T_1: Highly qualified and experienced faculty

T_2: Discipline and co-curricular activities

T_3: Timely assessment and parents teacher meeting.

For the selection of best alternatives, we apply our proposed entropy measure equation (3) to equation(5).

Step-1: First we construct the beal's fuzzy decision matrix as follows

$$\begin{matrix} & T_1 & T_2 & T_3 \\ \begin{matrix} D_1 \\ D_2 \\ D_3 \end{matrix} & \left[\begin{matrix} (0.6252, 0.7855, 0.6201) \\ (0.4320, 0.9324, 0.7740) \\ (0.1190, 0.9930, 0.2351) \end{matrix}\right. & \begin{matrix} (0.5520, 0.9152, 0.4026) \\ (0.7310, 0.8143, 0.4111) \\ (0.4265, 0.9715, 0.1765) \end{matrix} & \begin{matrix} (0.4320, 0.9324, 0.7740 \\ (0.7310, 0.8143, 0.4111 \\ (0.0265, 0.999, 0.0669) \end{matrix} \end{matrix}$$

Table-2

Step-2: We calculate the weights of each criteria using the three proposed entropy measure form equation (3) to equation (5).

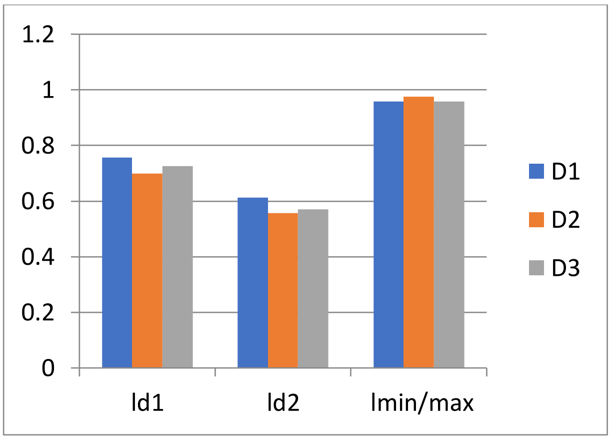
Entropies	$\overline{w}_1$	$\overline{w}_2$	$\overline{w}_3$
l_{d_1}	0.4098	0.4593	0.1306
l_{d_2}	0.4799	0.4406	0.0792
$l_{min/max}$	0.7147	0.1843	0.1027

Table-3 Entropy values and weight of criteria

Step-3 : we calculate the distance alternative of beal's fuzzy positive and negative ideal solutions

l_{d_1}	$R^{+}(\overline{A_i})$	$R^{-}(\overline{A_i})$
$\overline{D_1}$	0.7517	0.3417
$\overline{D_2}$	0.6129	0.5576
$\overline{D_3}$	0.9576	0.0625

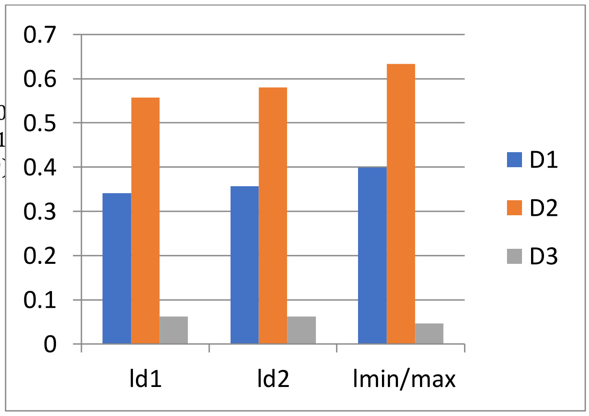
Table-4 Distance of alternative l_{d_1}



Beal's fuzzy positive ideal solution

Table-5

Distance of alternative l_{d_2}



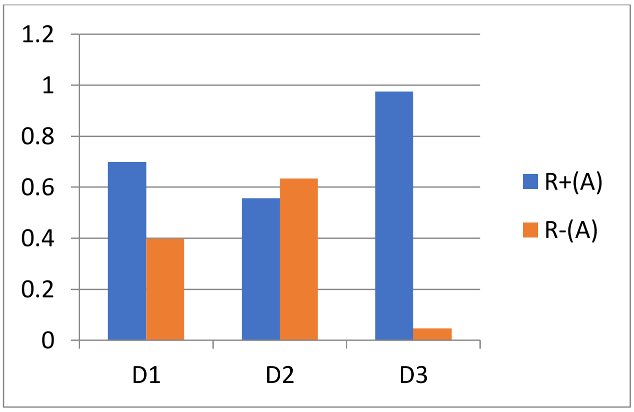
Beal's fuzzy negative ideal

$l_{min/max}$	$R^{+}(\overline{A_i})$	$R^{-}(\overline{A_i})$
$\overline{D_1}$	0.6988	0.3986

Journal of Chemical Health Risks
www.jchr.org
JCHR (2025) 15(6), 1963-1974 | ISSN:2251-6727

$\overline{D_2}$	0.5563	0.6333
$\overline{D_3}$	0.9757	0.0462

Table-6 Distance of alternative $l_{min/max}$ ideal



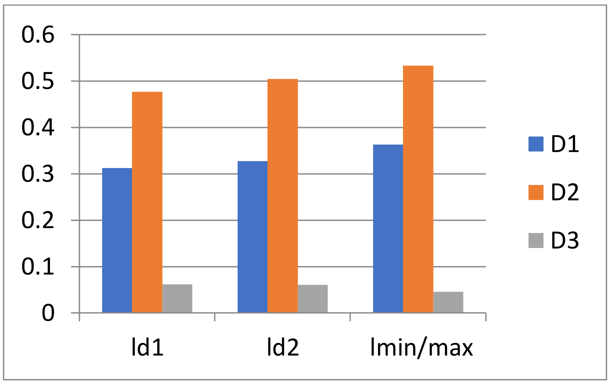
Beal's fuzzy $l_{min/max}$

Step-4: The degree of relative closeness is calculated

l_{d_1}	$R(\overline{A_i})$	l_{d_2}	$R(\overline{A_i})$	$l_{min/max}$	$R(\overline{A_i})$
$\overline{D_1}$	0.3123	$\overline{D_1}$	0.3267	$\overline{D_1}$	0.3632
$\overline{D_2}$	0.4762	$\overline{D_2}$	0.5042	$\overline{D_2}$	0.5327
$\overline{D_3}$	0.0610	$\overline{D_3}$	0.0602	$\overline{D_3}$	0.0452

l_{d_2}	$R^+(\overline{A_i})$	$R^-(\overline{A_i})$
$\overline{D_1}$	0.7362	0.3569
$\overline{D_2}$	0.5705	0.5807
$\overline{D_3}$	0.9587	0.0616

Table-7 Degree of relative closeness

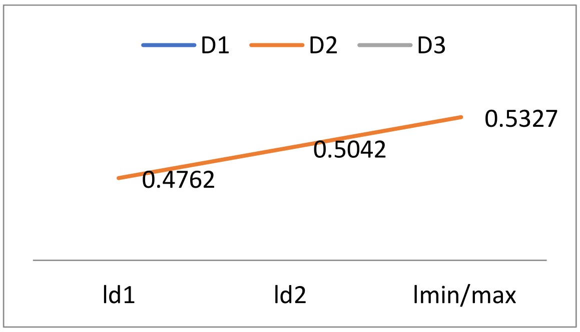


Composite of relative closeness

Step-5: The ranking alternatives is calculated by

Entropy	Rank	Best alternative
l_{d_1}	$\widetilde{D_2} > \widetilde{D_1} > \widetilde{D_3}$	$\widetilde{D_2}$
l_{d_2}	$\widetilde{D_2} > \widetilde{D_1} > \widetilde{D_3}$	$\widetilde{D_2}$
$l_{min/max}$	$\widetilde{D_2} > \widetilde{D_1} > \widetilde{D_3}$	$\widetilde{D_2}$

Table-8 Ranking of alternatives.



Comparison of ranking alternative

It is clearly shown in table-8. there is no conflict in ranking the alteranatives using suggested entropy measures equation(3) to equation (5). Hence $\widetilde{D_2}$ is the alternative from table-8. (i.e) Private college $\widetilde{D_2}$ is the best chosen among $\widetilde{D_1}, \widetilde{D_2}, \widetilde{D_3}$.

References

[1] M. A. Alghamdi, N. O. Alshehri and M. Akram, Multi- criteria decision-making methods in bipolar Fuzzy environment, Int. J. Fuzzy Syst., 20 (2018), 2057-2064.

[2] T. M. Al-shami, H. Z. Ibrahim, A. Mhemdi and R. Abu- Gdairi, n^{th} power root Fuzzy sets and its topology, International Journal of Fuzzy Logic and Intelligent Systems, 22 (2022), 350-365.
[3] K. T. Atanassov, Intuitionistic Fuzzy relations, in: L. Antonov, Ed.,III International School Automation and Scientific Instrumentation, Varna (Oct.1984), 56-57.
[4] K.T. Atanassov, Intuitionistic Fuzzy Sets, Fuzzy Sets Syst. 20 (1986), 87–96. https://doi.org/10.1016/S0165-0114(86) 80034-3.
[5] P. Burillo and H. Bustince, Intuitionistic fuzzy relations (Part I), Mathware and Soft Computing., 2 (1995), 5-38.
[6] P. Burillo and H. Bustince, Intuitionistic fuzzy relations (Part II), Eect of Atanassov's operators on the properties of the intuitionistic fuzzy relations, Mathware & Soft computing, 2 (1995), 117-148.
[7] G. Deschrijver and E. E. Kerre, On the composition of intuitionistic fuzzy relations, Fuzzy Sets and Systems, 136 (2003), 333-361.
[8] H. Z. Ibrahim, T. M. Al-shami and A. Mhemdi, Applications of nth power root Fuzzy sets in multicriteria decision making, Journal of Mathematics, 2023, Article ID 1487724, 14 pp.
[9] J. Jakel, R.Mikut and G. Bretthauer, Fuzzy control systems. institute of applied computer science, Forschungszentrum Karlsruhe GmbH, Germany, (2004), 1-31.
[10] E. Kim, M. Park, S. Ji and M. Park, A new approach to fuzzy modeling, IEEE Transactions on Fuzzy Systems, 5 (1997), 328-337.
[11] S. Kumar and C. Gangwal, A study of fuzzy relation and its application in medical diagnosis, Asian Research Journal of Mathematics, 17 (2021), 6-11.
[12] S. Kupongsak and J. Tan, Application of fuzzy set and neural network techniques in determining food process control set points, Fuzzy Sets and Systems, 157 (2006), 1169-1178.
[13] V. Murali, Fuzzy equivalence relations, Fuzzy Sets and Systems, 30 (1989), 155-163.
[14] G. Sun, X. Guan, X. Yi and Z. Zhou, Grey relational analysis between hesitant fuzzy sets with applications to pattern recognition, Expert Systems with Applications, 92 (2018), 521-532.
[151] C.N. Wang, N.A.T. Nguyen, T.T. Dang and C.M. Lu, A compromised decision-making approach to third party logistics selection in sustainable supply chain using fuzzy AHP and fuzzy VIKOR Methods, Mathematics, 9 (886), (2021), 1-27.
[16] X. Wang, D. Ruan and E. E. Kerre, Mathematics of Fuzziness-Basic Issues, Studies in Fuzziness and Soft Computing, Springer, 245, 2009, 1-227.
[17] Y. Xie, J. Liu, L.Wang, H1 Filtering Design for Takagi-Sugeno Fuzzy Model with Immeasurable Premise Variables by Applying a Switching Method, ICIC Express Lett. 14 (2020), 257–264. https://doi.org/10.24507/icicel.14.03.257.
[18] R. R. Yager and L. A. Zadeh, An Introduction to Fuzzy Logic Applications in Intelligent Systems, Springer Science & Business Media, 165, 2012.
[19] R.R. Yager, Pythagorean Fuzzy Subsets, in: 2013 Joint IFSA World Congress and NAFIPS Annual Meeting, IEEE, Edmonton, AB, Canada, 2013: pp. 57–61. https://doi.org/10.1109/IFSA-NAFIPS.2013.6608375.
[208] R.R. Yager, Pythagorean Membership Grades in Multi criteria Decision Making, IEEE Trans. Fuzzy Syst. 22 (2014), 958–965. https://doi.org/10.1109/TFUZZ.2013.2278989.
[21] R.R. Yager, A.M. Abbasov, Pythagorean Membership Grades, Complex Numbers, and Decision Making: Pythagorean Membership Grades and Fuzzy Subsets, Int. J. Intell. Syst. 28 (2013), 436–452. https://doi.org/10. 1002/int.21584
[22] C. Zhang, D. Li and R. Ren, Pythagorean fuzzy rough set over two universes and its applications in merger and acquisition, International Journal of Intelligent Systems, (2016), 1-23.
[23] L.A. Zadeh, Fuzzy Sets, Inf. Control 8 (1965), 338–353. https://doi.org/10.1016/S0019-9958, (65) 90241-X.