



# Evaluating BOD and DO models in context of the River Yamuna: A Critical Assessment

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## KEYWORDS

Biological oxygen demand; dissolved oxygen; deoxygenation rate constant; re-aeration rate coefficient, benthic oxygen demand

## ABSTRACT:

**Introduction:** Dissolved Oxygen (DO) and Biochemical Oxygen Demand (BOD) are important parameters for evaluating the health of water bodies like rivers. Over time, many scientific models have been developed to foresee BOD and DO levels, and these models have been applied to rivers worldwide, including the Yamuna River in Delhi, India to test their pertinence.

**Objectives:** The research investigates different models for biochemical oxygen demand (BOD) and Dissolved oxygen (DO) established in the past and were compared hence to gain insight into the existing water quality models for BOD and DO simulations in the River Yamuna of India. Our research addresses a critical gap in improved accuracy in stimulating BOD-DO dynamics, under varying environmental and climatic changes and pollution load.

**Methods:** A total of 384 field data sets were collected for the years 2013-2022. The key processes in the Yamuna River in Delhi are advection, dispersion, decay, settling, and pollutant loading. However, due to continuous wastewater disposal from different drains in the River Yamuna and unsteady-state flow conditions, the dispersion effects are significant. However, due to the disposal of huge quantity of pollution without prior treatment its effect is insignificant. For the study purpose and to make sure that the optimization process converges efficiently, the model parameters were obtained using the Newton-Raphson technique. For evaluating BOD and DO values, correlation statistics ( $r^2$ ), standard error, and mean multiplicative error, which measures the proportion of variance, precision, and the average deviation among observed and projected values respectively were used.

**Results:** The importance of this work lies in critical advancement of the different models developed and to assess their applicability for the river Yamuna in India with the Real time data. We have been able to summarize that the results obtained by Jha et al. (2007) provided the best results

**Conclusions:** The conclusions indicate that there is a strong correlation between the observed data and the BOD-DO models created by Bhargava and Camp. Furthermore, the best results were found following Jha et al. (2007).

## 1. Introduction

Most Indian river's flora, fauna, ecology, ecosystem, and aquatic life are in danger because of the influx of industrial and municipal pollutants as point source pollution and the influx of non-point source pollution from urban and agricultural areas. Many municipal and industrial drains in Yamuna River (Delhi stretch) are dumping pollutants into the Ganga River. Furthermore, the floating population contributes significantly to non-point source pollution. It is appropriate to apply a generic model to simulating the water quality of these types of watercourses. Developing such models is also crucial in to develop scenarios with different BOD and DO water quality values and to reflect the conditions that have been observed. The primary mechanism influencing water's oxygen concentration is the microorganism's consumption of oxygen as it breaks down biodegradable organic debris. The typical DO-BOD

model of Streeter & Phelps (1925) takes these two opposing processes into account in mathematical form. Adding to it, various models have been established in the past to provide a background to the development of BOD and DO models at different reaches of any river system. Initial effort on quality of water, focussing on vital aspects of pollution load (Theriault, 1927), contributed to early water quality modelling (Fair, 1939); concentrated on natural water bodies, concerning to the hydrodynamics for developing models (Thomas, 1948), mathematical modelling was introduced for pollution load, including BOD-DO (Li, 1962, 1972), model incorporates factors like the rate of deoxygenation and reaeration to predict DO levels in rivers/streams (Camp, 1963), contributed to the improvement and advancement of models for water quality assessment (Gundelach & Castillo, 1976); their emphasis was on solute transport modelling (van

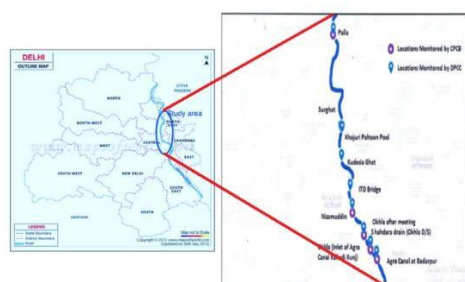


Genuchten & Alves, 1982), worked on many rivers of India, including river Yamuna and proposed tailored modelling method (Bhargava, 1983) provided an elaborative text and framework for BOD-DO calculation (Thomann & Müller, 1987), improvement in water quality modelling (Jolankai, 1997), focused on the specific pollutants in water quality modelling (Yu et al., 1991), applicability of model concerning the real world water bodies (Adrain et al., 1994; Jha et al., 2007). The goal of this study is to learn more about the current models (Bhargava, 1983; Jolankai, 1997; Camp, 1963; Thomann & Müller, 1987; and Jha et al., 2007) of water quality model suitable for BOD-DO analysis in the River Yamuna at Delhi. These models accounted for the effects of benthic oxygen, non-point load, respiration, photosynthesis, settling of organic matter, and mass flux longitudinal fluctuation (the product of flow and concentration) based on the first-order decay kinetics of organic matter.

## 2. Study Area and Data Collection

Yamunotri glacier is the origin of River Yamuna in the Himalayas and is the largest tributary of River Ganges in India (Figure 1). While travelling the length of approximately 1380 km, it crosses many states such as Himachal Pradesh, Uttar Pradesh,

Uttarakhand, Haryana and Delhi. Yamuna river has multiple uses for community, it is used for water supply, bathing, irrigation, industrial water supply, and disposal of industrial effluents as well as for disposal of sewage.



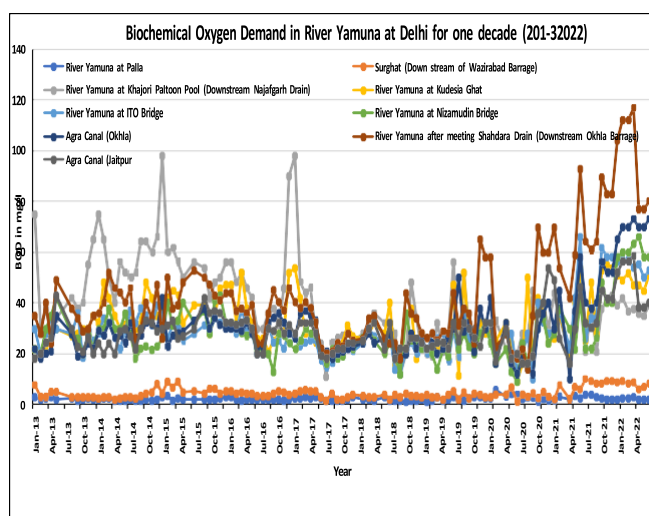
**Figure 1: Location map of the study area in River Yamuna at Delhi, India**

Yamuna is the only natural resource to sustain the different forms of life in Delhi, but the rise in population and urban activities in Delhi are putting great pressure and demands on this natural resource. River Yamuna is the only source of water supply and sanitation in Delhi. Yamuna turns through Palla village in Delhi. From Palla

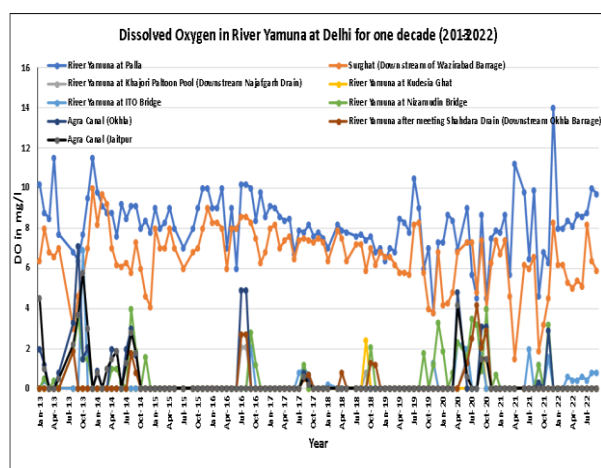
to Okhla barrage, the total stretch of the Yamuna River in Delhi is about 48 km, however, it is the 22 km-long urban stretch from Wazirabad barrage (15 Km D/S to Palla) to Okhla barrage that has been identified as one of the most polluted stretches of the river.

The Delhi region experiences three distinct seasons, namely winter, summer, and monsoon, with a semi-arid environment. Throughout the course of seventy years, the Meteorological Department has maintained data, and the average annual temperature reported in Delhi is typically between 31.5°C. Delhi receives about 611.8 mm of rain annually, with the monsoon months of June through September accounting for 87% of the total.

The quality of river water is determined by multiple interconnected factors that are influenced by discharge rates and water flow volume, as well as local and temporal fluctuations. The Delhi Pollution Control Committee (DPCC) is responsible for monitoring the water quality in Delhi's water bodies. Every month, the DPCC analyses the Yamuna River's water quality at a number of locations, including Palla, Wazirabad, the ISBT and ITO bridges, Nizamuddin, the Agra Canal, and the downstream Okhla barrage. These observations are based on physicochemical characteristics. 27 drains are monitored by DPCC which fall in river Yamuna, based on physicochemical parameters. Najafgarh drain and Shahdra drain (Thomann & Müller (1987) contribute about 74% (61.44 TPD) and 81% (133.82 TPD) of total BOD load respectively of total flow in Delhi. Barapulla drain (10.46 TPD), Sarita Vihar drain (7.73 TPD), and Sen Nursing Home (4.69 TPD) also contribute BOD load to the river. As per, Chandra & B.S. Sajwan, 2020, In year 2019, the total BOD load from 22 drains was 264.31 TPD. The BOD and DO values of river Yamuna are shown in Figures 2 and 3.



**Figure 2: BOD values at different sampling stations in Delhi for the year 2013-2022**



**Figure 3: DO values at different sampling stations in Delhi for the year 2013-2022**

### 3. Method

The detailed water quality data collection from the Delhi Pollution Control Board (DPCB) and the monthly analysis was done, along with the latest sampling between November 2022 and June 2023, provide a robust dataset for understanding and modelling the BOD-DO dynamics for Yamuna River. Water samples were taken at each sampling point along the Yamuna River in Delhi, at a depth of roughly 15 cm (to prevent floating debris).

Temperature plays a significant role in the rates of biological reactions that follow variations in the concentration of organic substances, as well as in the ion and phase equilibrium. Significant temperature fluctuations were observed in the Yamuna River.

To calculate the temporal variations in the river, the cross-sectional area, water depth, and velocity were observed at all the sampling sites for a period of one complete annual cycle.

The Winkler method was used to determine DO by using reagents (a) divalent manganese solution and, (b) alkali-iodide-azide. After five days of incubation, the unpreserved samples gave the BOD<sub>5</sub> values. With the "light and dark" bottle approach, photosynthesis and respiration rates were ascertained. It was discovered that the small amount of algae in river water does not affect the DO as a result of photosynthesis or respiration.

The deoxygenation rate constant,  $K_1$  ( $L d^{-1}$ ) was calculated by the method of Texas Water Development Board, (1971). BOD<sub>5</sub> at different reaches is plotted (log scale) with the travel time between two river reaches (normal scale). The resultant slope represents the deoxygenation coefficient ( $K_1$ ).

The re-aeration coefficient ( $K_2$ ) for DO modelling was estimated using equation developed by Jha et al. (2004) as follows:

$$K_2 = 0.603286V^{0.4}S^{-1.0}H^{0.154} \quad (\text{MME} = 1.76) \quad Fr \leq 1 \quad (1)$$

$$K_2 = 866.307V^{1.393}S^{-0.173}H^{0.8} \quad (\text{MME} = 1.2) \quad Fr > 1 \quad (2)$$

Here  $V$  denotes the velocity of flow in  $m s^{-1}$ ,  $H$  denotes the depth of flow in  $m$ ,  $Fr$  represents Froude number and  $S$  is the bed slope ( $m m^{-1}$ ).

It is very challenging task to estimate the non-point source BOD influx ( $L_d$ ). The BOD levels of the groundwater near Yamuna River were utilised in this study as a proxy for  $L_d$  estimation. To compute  $L_d$ , a mass balance method was used, accounting for the different BOD inputs and outputs in the river system.

A general equation for mass balance is :

Input–Output+ Production–Consumption=Accumulation.

As recommended by Bhargava (1983), the BOD (settleable),  $p$ , values were used for the analysis. The fraction of settleable BOD,  $p$ , and consequently the optimal  $p$  values for the Yamuna River were found in



order to evaluate the sensitivity of the parameter.  $K_3$ , the BOD rate constant

elimination through deposit, is thought to range from 0.4 to 0.6. 1-4 g d<sup>-1</sup> was the benthic oxygen demand, or B.

#### 4. Scientific Formulations

Using the dynamics and pollution load of the Yamuna River, an overview of these governing equations from models by various experts in this field is as under.

**Streeter and Phelps model:** The Streeter and Phelps model focuses on deoxygenation

and reaeration processes. In the year 1925, Streeter & Phelps related the decay of an organic waste measured by the BOD and the DO sag model. The equations are as follows:

$$\text{BOD: } L = L_0 \exp(-K_1 t) \quad (3)$$

Here  $L$  is the BOD at any location of the river in g O<sub>2</sub> m<sup>-3</sup> represents initial BOD,  $K_1$  denotes biochemical decomposition rate coefficient of organic matter in d<sup>-1</sup>, and  $t$  is the time of travel in the river ( $t = x/v$ , here  $x$  is distance downstream of the point of effluent discharge and  $v$  is the velocity).

$$\text{DO: } D = \frac{K_1 L_0}{K_2 - K_1} (\exp(-K_1 t) - \exp(-K_2 t)) + D_0 \exp(-K_2 t) \quad (4)$$

Here  $D$  denotes oxygen deficit of water in g O<sub>2</sub> m<sup>-3</sup> and  $K_2$  represents re-aeration rate coefficient as T<sup>-1</sup>.

**Camp model:** The Camp model further boosts water quality modeling by incorporating additional parameters that capture key processes affecting biochemical oxygen demand (BOD) and dissolved oxygen (DO) dynamics in water bodies. These processes include sedimentation, benthic oxygen demand, and the contributions of photosynthesis (P) and respiration (R).

Four more parameters are added by the Camp model:

(1) Benthic Oxygen Demand (B): The oxygen used up by organic materials in the sediment is taken into consideration by this parameter.

(2) The Sedimentation Rate Constant ( $K_3$ ) denotes the rate at which organic matter separates from the water column and settles onto the bed.

(3) Photosynthesis Input of DO (P): The oxygen produced during photosynthesis by aquatic plants and algae is represented by the photosynthetic input of DO (P).

(4) Respiration Input of DO (R): The amount of oxygen that aquatic organisms use for respiration is taken into consideration by the respiration input of DO (R).

The equations are as under for BOD and DO-

BOD:

$$L = L_0 \exp[-(K_1 + K_3)t] + \frac{D}{(K_1 + K_3)} \{1 - \exp[-(K_1 + K_3)t]\} \quad (5)$$

DO:

$$D = D_0 \exp(-K_2 t) + \frac{K_1 L_0}{K_2 - (K_1 + K_3)} \{ \exp[-(K_1 + K_3)t] - \exp(-K_2 t) \} + \frac{K_1 B}{K_2 (K_1 + K_3)} [1 - \exp(-K_2 t)] - \frac{K_1 B}{[K_2 - (K_1 + K_3)](K_1 + K_3)} \cdot \{ \exp[-(K_1 + K_3)t] - \exp(-K_2 t) \} - \frac{P - R}{K_2} [1 - \exp(-K_2 t)] \quad (6)$$

**Bhargava model:** The Bhargava model adds more processes, like bioflocculation and organic matter settling, to the traditional Streeter and Phelps (1925) model. These procedures are essential for a more precise estimation of the dynamics of BOD and DO in water bodies, particularly in situations where biological and physical interactions have a major impact on water quality.

The equations are:

BOD:

$$L = L_0 \exp(-K_1 t) + (1 - p) \left\{ L_1 \exp(-K_1 t) + L_2 \exp(-K_1 t) \exp(-K_1 t_{1-t}) \right. \\ \left. + L_3 \exp(-K_1 t_{1-t}) + \dots + L_n \exp(-K_1 t_{(n-1)-t}) \right\} \\ + p \left[ L_1 \left( 1 - \left( \frac{1}{T} \right)^{t_{0-T}} \right) + L_2 \left( 1 - \left( \frac{1}{T} \right)^{t_{1-(1+T)}} \right) + L_3 \left( 1 - \left( \frac{1}{T} \right)^{t_{2-(2+T)}} \right) \right. \\ \left. + \dots + L_n \left( 1 - \left( \frac{1}{T} \right)^{t_{n-1-(n-1+T)}} \right) \right] \quad (7)$$

Here  $p$  gives the fraction of settleable BOD;  $L_1$ ,  $L_2$ , and so on.  $L_n$  represents influx of BOD

at different river sections and,  $t_{1-t}$ ,  $t_{2-t}$ , ...,  $t_{(n-1)-t}$  is the travel time at various reaches of river.



DO:

$$\frac{dD}{dt} = -K_1 \left\{ \begin{array}{l} L_0 e^{-K_1 t} + (1-p) \left[ L_1 \exp(-K_1 t) + L_2 \exp(-K_1 t) \exp(-K_1 t_{1-T}) \right] \\ + L_3 \exp(-K_1 t_{1+T}) + \dots + L_n \exp(-K_1 t_{(n-1)+T}) \end{array} \right\} + p \left\{ \begin{array}{l} L_1 \left( 1 - \left( \frac{1}{T} \right) t_{0-T} \right) + L_2 \left( 1 - \left( \frac{1}{T} \right) t_{1-(1+T)} \right) + L_3 \left( 1 - \left( \frac{1}{T} \right) t_{2-(2+T)} \right) \\ + \dots + L_n \left( 1 - \left( \frac{1}{T} \right) t_{m-1-(m-1+T)} \right) \end{array} \right\} \quad (8)$$

**Thomann & Müller model:** This model can be represented by a set of differential equations that describe the variation in BOD/DO concentrations lengthway of the stream or river. These equations typically include terms for advection, dispersion, and various source and sink processes. The equations are:

BOD:

$$L = L_0 \exp(-K_1 t) + \frac{L_d}{K_1} (1 - \exp(-K_1 t)) \quad (9)$$

Here  $L_d$  represents non-point source BOD in  $\text{mg L}^{-1}$ .

DO:

$$D = D_0 \exp(-K_2 t) + \frac{K_1 L_0}{(K_2 - K_1)} [\exp(-K_1 t) - \exp(-K_2 t)] + \frac{K_1 L_d}{(K_1 K_2)} [1 - \exp(-K_2 t)] - \frac{K_1 L_d}{(K_2 - K_1) K_1} [\exp(-K_1 t) - \exp(-K_2 t)] \quad (10)$$

**Jolankai model:** A generalized model created by Jolankai (1997). Nonpoint source input loads and benthic oxygen demand, photosynthesis, respiration, and longitudinal fluctuation of mass flux are among the greatest number of parameters that are taken into account. But unlike the commonly used Streeter and Phelps 1925 model, this model has a different functional form. By combining the concepts of first-order decay of organic matters provided by Streeter and Phelps (1925) and Jolankai (1997) in terms of mass flux while taking into account the nonpoint source pollution load, benthic oxygen demand, photosynthesis, respiration, settling of organic matter, and loading function, Jha et al. created BOD and DO models by combining the ideas of Streeter and Phelps (1925) and Jolankai (1997) regarding the mass flux of first-order decay of organic matter, taking into account the nonpoint source pollution load, benthic oxygen demand,

photosynthesis, respiration, settling of organic matter, and loading function.

BOD:

$$L(l) = L_0 F^{\beta_1} + \frac{(L_d + B\phi(l))}{\beta_1} [1 - F^{\beta_1}] \quad (11)$$

$K_1$  is the rate coefficient of biochemical decomposition of organic

$$\text{in which } F = \frac{Q_u}{Q_u + q_l l}, \beta_1 = 1 + K_1 \phi(l), \text{ and } \phi(l) = \frac{Q_u}{v} + l$$

matter measured in  $\text{d}^{-1}$ ,  $L_d$  is later concentration of BOD in  $\text{mg L}^{-1}$ ,  $Q$  denotes the rate of flow ( $\text{m}^3 \text{ s}^{-1}$ ),  $Q_u$  designates the rate of flow at the start of the river reach measured in  $\text{m}^3 \text{ s}^{-1}$ ,  $q$  is lateral inflow rate in  $\text{m}^3 \text{ s}^{-1}$ ,  $B$  is benthic oxygen demand in  $\text{mg m}^{-1} \text{ d}^{-1}$ ,  $l$  is distance downstream along the river in  $\text{m}$ , and  $v$  gives mean flow velocity in  $\text{m s}^{-1}$ ,  $A$  designates cross-sectional area in  $\text{m}^2$ .

DO:

$$C_{\text{ox}}(l) = \left[ \frac{L_d + B\phi(l)}{\beta_1} - L_0 \right] \frac{K_1}{(K_2 - K_1)} (F^{\beta_1} - F^{\beta_2}) + C_{\text{ox}_0} F^{\beta_1} + \left[ C_{\text{ox}_{\text{sat}}} (\beta_2 - 1) - \frac{L_d + B\phi(l)}{\beta_1} (\beta_1 - 1) + \phi(l)(P - R) + C_{\text{ox}_d} \right] \quad (12)$$

here  $\beta_2 = 1 + K_2 \phi(l)$  and  $F$ ,  $\beta_1$  and  $\phi$  are as defined above.

Above equations denotes,  $C_{\text{ox}}$  as DO concentration in  $\text{mg L}^{-1}$ ,  $C_{\text{ox}_0}$  gives the initial DO concentration downstream of the waste water discharge measured in  $\text{mg L}^{-1}$ ,  $C_{\text{ox}_{\text{sat}}}$  denotes saturated DO concentration expressed in  $\text{mg L}^{-1}$ ,  $C_{\text{ox}_d}$  represents the DO concentration in the lateral inflow in  $\text{mg L}^{-1}$ ,  $K_2$  denotes re-aeration rate coefficient in  $\text{d}^{-1}$ ,  $(P - R)$  gives net difference between oxygen production by the photosynthesis ( $P$ ) and respiration ( $R$ ) of aquatic plants in  $\text{mg L}^{-1} \text{ d}^{-1}$ .

However, this model becomes more complex and does not change to the widely used Streeter and Phelps 1925



model.

A model combining the ideas of Streeter and Phelps (1925) and Jolankai (1997) was created by Jha et al. in 2007. This model combines the mass flux technique with the first-order decay of biological matter, accounting for nonpoint source pollution load, just like Jolankai's model, Benthic oxygen demand, organic matter that settles and leaves the water column, photosynthesis, and respiration.

With the addition of the previously discussed extra factors and processes, the combined model of Jha et al. 2007 may be expressed as a system of differential equations that characterise the variations in BOD and DO concentrations across time and space.

$$L = L_0 e^{-(K_1+K_3)t} + \frac{L_d q l (1 - e^{-(K_1+K_3)t})}{(K_1 + K_3)(Q_u + ql)} + \frac{B Q_u (1 - e^{-(K_1+K_3)t})}{(K_1 + K_3)(Q_u + ql)} \quad (13)$$

$$D = D_0 e^{-K_2 t} + \frac{K_1 Q_u L_0 (e^{-(K_1+K_3)t} - e^{-K_2 t})}{(K_2 - (K_1 + K_3))(Q_u + ql)} + \frac{K_1 Q_u L_d q l (1 - e^{-K_2 t})}{K_2 (K_1 + K_3)(Q_u + ql)^2} + \frac{K_1 Q_u (e^{-(K_1+K_3)t} - e^{-K_2 t}) L_d q l}{(K_2 - (K_1 + K_3))(K_1 + K_3)(Q_u + ql)^2} + \frac{K_1 Q_u^2 B (1 - e^{-K_2 t})}{K_2 (K_1 + K_3)(Q_u + ql)^2} - \frac{K_1 Q_u^2 B (e^{-(K_1+K_3)t} - e^{-K_2 t})}{(K_2 - (K_1 + K_3))(K_1 + K_3)(Q_u + ql)^2} + \frac{D_d q l (1 - e^{-K_2 t})}{K_2 (Q_u + ql)} - \frac{(P - R) Q_u (1 - e^{-K_2 t})}{K_2 (Q_u + ql)} \quad (14)$$

## 5. Performance Appraisal

Model evaluation was done by the iterative method of Newton Raphson, for better approximation to the zeros of real-valued function, providing a fine tuning to the parameters of the model to better fit the real-world data. Measured by standard error (SE), mean multiplicative error (MME), and coefficient of determination ( $r^2$ ). The standard error measures the accurateness with a sample representing population and is evaluated as:

$$SE = \left( \sum_{i=1}^N \frac{(K_P - K_M)_i^2}{N} \right)^{1/2} \quad (15)$$

$K^P$  and  $K^M$  are the projected and measured values respectively, in the above equation. N denotes the number of measurements

MME is a metric assessment that gives the effect of errors or average deviation in modelling between observed and predicted values, this is as per Moog and Jirka (1998).

$$MME = \exp \left[ \frac{\sum_{i=1}^N \left| \ln \left( \frac{K_P}{K_M} \right)_i \right|}{N} \right] \quad (16)$$

Correlation Analysis helps in determining the proportion of the variance in the correlation between two or more random water quality indicators. Presently, Pearson's Product Moment correlation equation is used to calculate the values of this coefficient or  $r^2$ .

## 6. Results and Discussion

By incorporating the above parameters and equations, the model aims to simulate the BOD and DO dynamics in the Yamuna River more accurately, taking into account the non-point source pollution, sedimentation, and biological processes. Table 1. shows the input parameters during the analysis. On the other hand, equations (4) and (5) were used to obtain the re-aeration coefficients for the present study.

The total BOD for the Indian rivers Yamuna and Ganga is separated into two sections, according to Bhargava (1983): the first part has a fast rate of BOD removal (known as the "transition time"), while the second part has a slower rate of BOD removal for subsequent reaches. The results of the study show that the sewage's colloidal material coagulates and settles down fast once it enters the rivers, and that these rivers similarly show a rapid decline in BOD.

The Bhargava model was used at Delhi in the Yamuna River, with the proportion of settleable BOD (p) in transition period being maintained in the range of 0.1 to 0.90. The result is displayed in Figure 4 and suggests that the Yamuna River can be used with the Bhargava (1083) model, but not the section of the river that runs through Delhi. In particular, DO values do not match since, in the majority of instances, their value is zero.



Table 1: Various Parameters considered for the models

Palla	Wazirabad	ISBT Bridge	ITO Bridge	Nizamudin Bridge	Okhla Barrage	Agra Canal at Okhla Barrage	River Yamuna at Asgarpur
$K_1$	0.15	0.15	0.15	0.15	0.15	0.15	0.15
$K_2$	27	27	27	27	27	27	27
$K_3$	0.02315	0.02315	0.02315	0.02315	0.02315	0.02315	0.02315
$L_d$	0.02315	0.02315	0.02315	0.02315	0.02315	0.02315	0.02315
$D_d$	0.02315	0.02315	0.02315	0.02315	0.02315	0.02315	0.02315
$P$	0.02315	0.02315	0.02315	0.02315	0.02315	0.02315	0.02315
$R$	0.02315	0.02315	0.02315	0.02315	0.02315	0.02315	0.02315
Time	(Days)0.02315	0.02315	0.02315	0.02315	0.02315	0.02315	0.02315

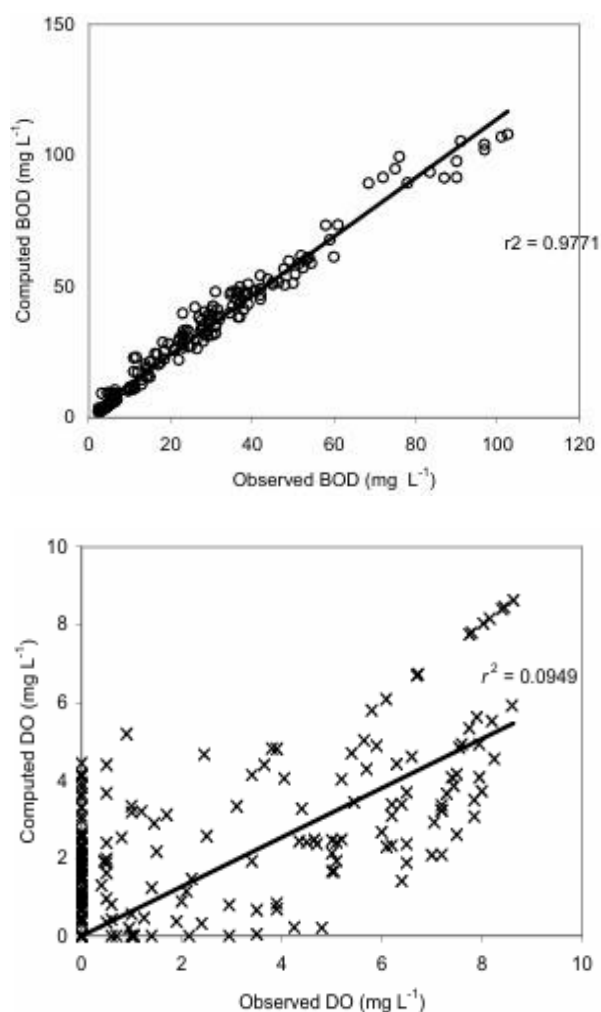


Figure 4: Observed Vs computed BOD and DO values by Bhargava (1983) approach

An advancement of Streeter & Phelps model (1925) is the Camp model (1962) shown in Figure 5. Camp model provides improved results for the for BOD and DO values.

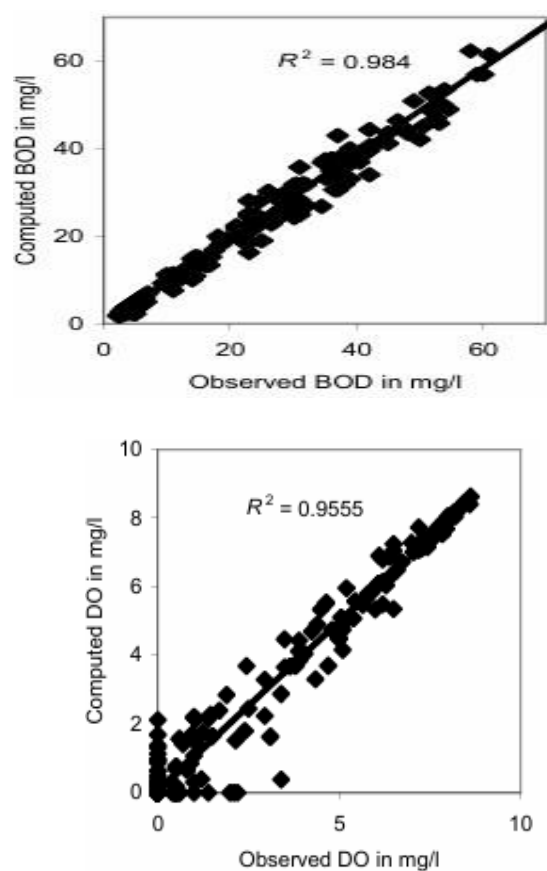
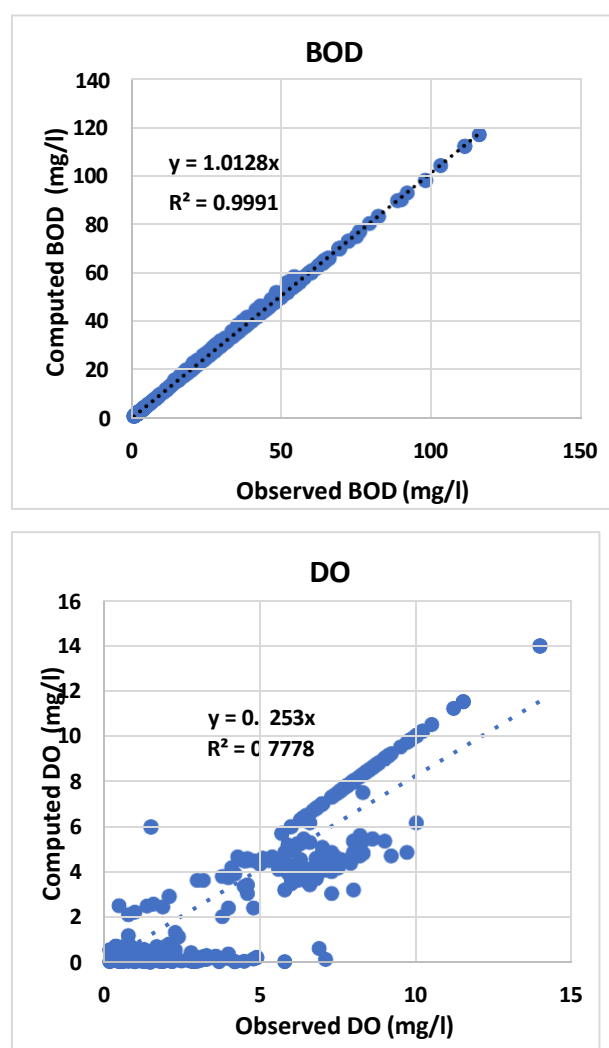


Figure 5: Computed Vs Observed BOD and DO values by Camp (1962) Model



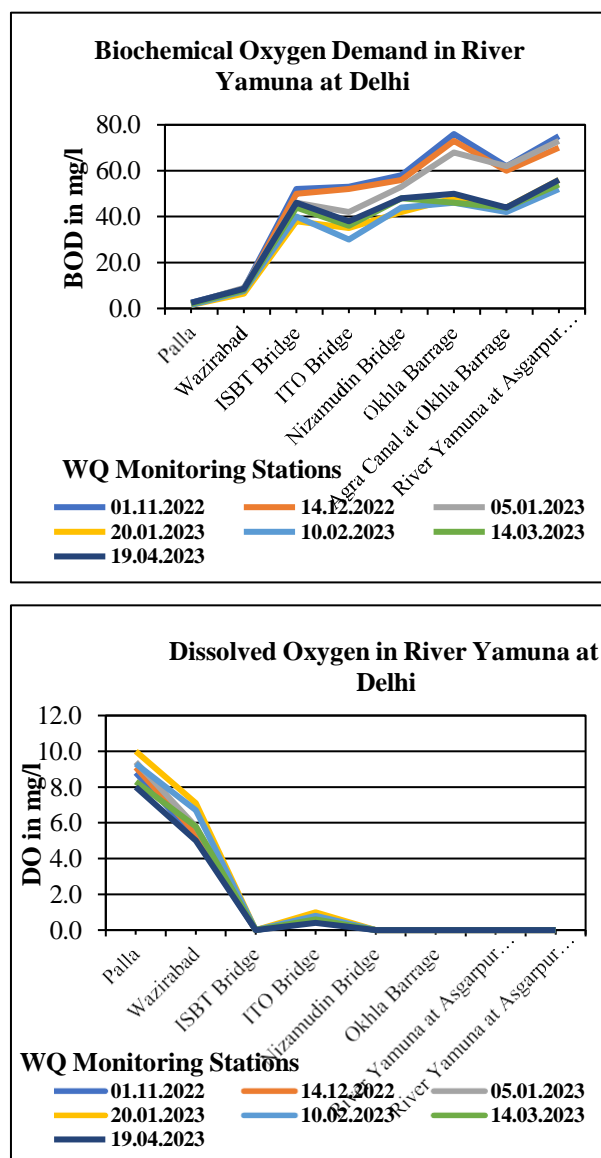
As was previously said, the Yamuna River in Delhi is extremely contaminated, hence it was determined that more advanced BOD and DO models with flexible physically based parameters and input variables were needed. Currently, the suitability of the Jha et al. (2007) model for simulating BOD and DO in the River Yamuna is being examined. Figure 6 displays the outcomes derived from information gathered from the Delhi Pollution Control Board for the years 2013–2022.



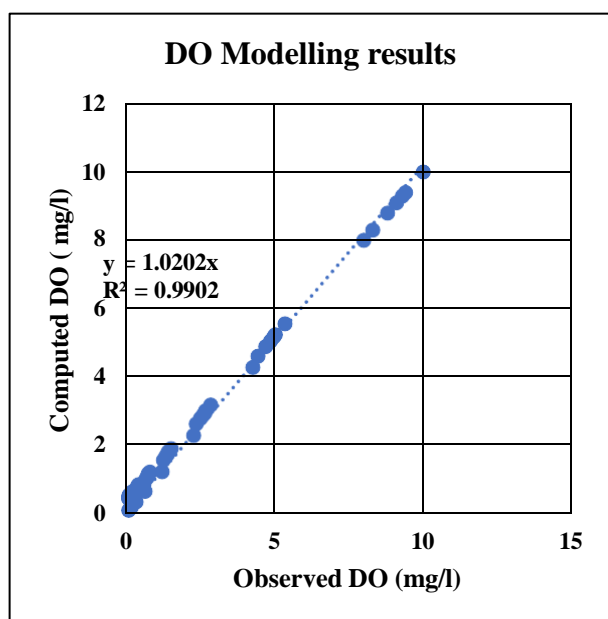
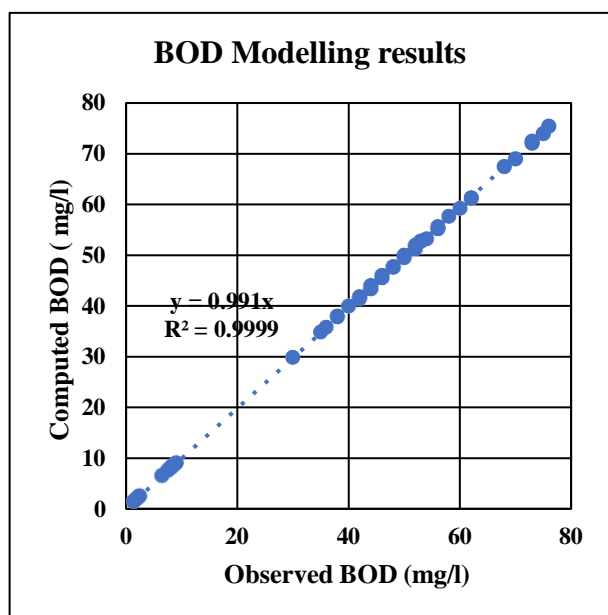
**Figure 6: Calculated Vs Observed BOD and DO values by Jha et al. (2007) Model**

It is to mention that the outcomes based on the collected data for post monsoon period of the year 2022-2023 in River Yamuna (Figure 7). The BOD and DO values computed at different stations of river Yamuna in Delhi are shown in Figure 8. The results are better than the

results obtained using other models for BOD and DO for the Yamuna river at different sampling stations in Delhi within a stretch of 48 km.



**Figure 7: BOD and DO Values observed during post monsoon period of the year 2022-2023**



**Figure 8: BOD and DO Values observed during post monsoon period of the year 2022-2023**

Equations (15) and (16) were used for evaluating performance, for SE and MME respectively. Outcomes for SE, MME are  $r^2$  are shown in Table 2 below.

**Table 2: Performance Evaluation of different BOD-DO models**

Error Statistics	Camp (1962) Model		Bhargava (1983) Model		Jha et al. (2007) Model	
	BOD	DO	BOD	DO	BOD	DO
SE	2.11	0.59	2.61	6.10	1.9	0.80
MME	1.15	1.46	1.50	1.20	1.10	1.34
$R^2$	0.98	0.95	0.97	0.09	0.99	0.91

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## 7. Discussion

When it comes to the first-order degradation processes, benthic BOD, sedimentation, and lean periods, the River Yamuna actually operates in a steady-state manner. Upon incorporating all of these factors into the Bhargava, Camp, and Jha models, the outcomes exhibit a high degree of agreement with the observed values. Because of the high BOD levels and negligible amount of algae present in the River Yamuna, photosynthesis and respiration are not significant processes. In a similar vein, it is also discovered that the outcomes of the DO model are improved when it is estimated sequentially using the Bhargava, Camp, and Jha models.

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