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A Queuing-Based Supply Chain Model With Fuzzy Theory For Multiproduct Items Under Steady-State Probability Distribution

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KEYWORDS

Optimization,

carbon emission, Wetland, Multiproduct items, supply chain, queuing theory, backorder, steady-state probability, fuzzy environment

ABSTRACT: We proposed a sustainable queuing-based supply chain model with a steady-state probability distribution for multiproduct items in wetland area under a fuzzy environment. In this proposed model, the manufacturer produces multiproduct items as demanded by several retailers, and after completing the production process, different items are placed in other warehouses. The rent of warehouses varies occasionally and is temporary. Some warehouses are built with advanced facilities and have a low steady-state probability distribution. Due to varying holding costs, we considered the holding cost imprecise and treated it as a triangular fuzzy number. The total fuzzy cost is defuzzified with the help of the centroid method. Each product has a different backorder cost due to needing more items. We created a linear fuzzy cost function by adding the costs associated with the shipment, back ordering, and fuzzy holding. The theory of the proposed model is managed with FCFS, GI/G/1 queue because, during manufacturing, the manufacturer sets a setup time for each product, and multiple products do not overlap during the production of items and switch to the output of another. Finally, we develop the proposed model to analyse the logistic process to a three-echelon inventory model and compare the total fuzzy cost with the total cost without a fuzzy environment, termed a crisp model. Lastly, we have taken a numerical example to justify the proposed model. We aim to know about the effect of fuzzy on the total fuzzy cost as per assumptions, and we included the sensitivity analysis part

Introduction: Our proposed model solves some problems during manufacturing and production under fuzzy environment for multi-product where backorders are allowed. Some problems are;

(i) What is the minimum inventory cost during production and manufacturing for multi-product? Where the holding cost varies according to variation of demand. Present study tried to nullify imprecise nature of the holding cost by using centroid method.

(ii) When the rate of demand of products increases or decreases the companies of production or manufacturing for the multi-product may be loss or profit and in this situation, our proposed model is very helpful to control the variations of costs by using Steady-state probability distribution.

Almost, queuing-based supply chain model for multi-product are considered fixed holding cost and calculated inventory cost but our proposed model calculated fuzzy minimum total inventory cost when the holding cost is taken as fuzzy holding cost.

Objectives: The highlights of this proposed model are : (i) Impact of inventory inputs on the total

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inventory cost and the decision variables. (ii)Impact of fuzzy inputs on the decision variables and total inventory cost for the supply chain. (iii) Comparison of total inventory cost with and without a fuzzy environment. (iv) Our proposed model is more applicable and usable and is presented using mathematical effects. (v) The conclusion section is also presented, in which more results are explained, and future work is presented to improve.

Methods: The task of the proposed work is presented in Figure 1. The rest part of this manuscript is structured in section/subsections. Subsection 1.2 covers the research gap and contributions of the proposed study. Section 2 presents the basic definitions for the development of the proposed study. Section 3 explains the formulation of proposed model and numerical examples with discussion. Section 4 reveals that about the sensitivity analysis and results discussion. Section 5 shows the managerial application and social implication. Further, Section 6 explains the conclusions and future scope of the proposed study.

Results: we analysed that when applying fuzzy concepts to the holding cost of the inventory in such cases, the total cost is affected, and we get the best-optimised values through fuzzy concepts compared to crisp modelling concepts. From the above discussion, it is clear that the total cost calculated using fuzzy holding cost is better than the cost calculated without fuzzy holding.

Conclusions: we concluded that we get the optimum results when we take the holding cost as imprecise, treat it as a fuzzy triangular number, and then find the total fuzzy cost. In the proposed model, we obtained the warehouses' lead time, fill rate, and stock-out probability performance.

1. Introduction

The supply chain is an intricate web of businesses, individuals, information, and resources used to develop and provide goods and services to final customers. It includes every process step, from obtaining raw materials to producing, distributing, and delivering goods or services to the customer. A well-functioning supply chain aims to optimise resource utilisation, reduce expenses, and increase customer value. Although trade routes were established to exchange goods in ancient civilisations, supply chain management has its roots in those same societies. However, the Industrial Revolution has significantly changed manufacturing and transportation over the last two to three centuries. The twenty-first century is crucial for streamlining the supply chain and connecting ideas like material requirement planning and other cutting-edge methods for improving production and inventory control. As markets became more globalised, supply chains became more complex. Businesses implemented optimised manufacturing concepts to lower inventory levels and enhance customer demand responsiveness.

A sustainable supply chain model is a strategy that companies use to reduce their negative impacts on the environment, society, and stakeholders by incorporating socially and environmentally conscious practices into their supply chain operations. A sustainable supply chain aims to achieve equilibrium between environmental, social, and economic aspects to maximise long-term value while reducing environmental impact and fostering moral behaviour. As a major worldwide issue, sustainability encourages stakeholders to concentrate on enlarging the social, ecological and economic dimensions (Negri et al. [1]). To achieve this, industries must adopt sustainable practices to solve social issues, drastically cut emissions, and conserve resources (Mashud et al. [2], Taghikhah et al.[3]). Many customers actively seek goods and services from businesses with a sustainability track record. Because of this, companies hoping to succeed over the long term in a more competitive and aware market must strategically implement a sustainable supply chain model.

In some seasons or on some occasions, the manufacturer produces multiproduct items as demanded by retailers, and after completing the production process, items are placed in other warehouses homogeneously. In real-life situations, it is seen that the honours of the rented warehouses changed the rent charges occasionally/seasonally and did not permanently fix them. Some warehouses are built with advanced facilities and add extra charges, like waterproof compartments, AC facilities, freezing facilities and deep freezers. In this situation, finding an estimated inventory holding cost directly affecting the total price is difficult. This proposed model includes the related problem with additional effects and evaluates the total cost. We simultaneously developed the crisp and fuzzy inventory models, evaluated the total inventory cost and analysed it using an example. We have used previous numerical data, which is appropriately cited, and the results are finally concluded.

We studied many literature reviews based on EOQ, supply chain, queuing theory, backorder, steady-state probability, fuzzy concepts, etc. The forthcoming section presents some selected literature reviews for our proposed plan.

1.1 Literature review

Harris [4] first presented the Economic Order Quantity (EOQ) model in an article titled "How Many Parts to Make at Once" that appeared in the magazine "Factory: The Magazine of Management." The EOQ model is a traditional inventory management technique that assists businesses in determining the ideal order quantity to minimise total inventory costs. In inventory management, Harris, a management consultant, sought to address the trade-off between ordering and holding costs. Harris established the groundwork, but the EOQ model was independently rediscovered and improved upon by Chicago-based consultant R. H. Wilson [5] of The Scientific Management Service. Later, several scholars and researchers developed the EOQ model. Businesses can use the EOQ model as a helpful tool to maximise inventory management and balance the costs of placing repeated orders with those of holding excess inventory, despite its assumptions and limitations like constant demand, fixed order and carrying costs, etc. Furthermore, variants of the model have been developed to address various scenarios and complexity in inventory management, such as the Economic Production Quantity (EPQ) model. Utama et al. [6] developed a sustainable production-inventory model incorporating probabilistic demand, quality degradation, and multi-material.

The authors generated the base of this model by analysing the sustainable production-inventory problem using bibliometric literature.

Maity et al. [7] worked on an EOQ model of green item's carbon emissions. The developed fuzzy model considers the demand rate a pentagonal intuitionistic dense fuzzy number. A novel defuzzification method and a solution algorithm have been designed to solve the suggested model. An analysis of the difference between crisp and other fuzzy environments is also provided for fruitfulness, and some numerical experiences are provided in the illustration using LINGO 18.0 software. Karmakar et al. [8] developed an EOQ model in which the author examines different fuzzy triangular norms in depth and applies them for the first time, in which the demand is contingent upon time and fluctuating numbers of travellers. The paradigm shift in traditional economic theory addresses objective factors like income and price that impact demand. Some authors improved the EOQ and EPQ for the supply chain by using queuing theory with some realistic situations.

Queuing theory helps model sustainable supply chains because it illuminates the effectiveness, resource usage, and environmental effects of different processes. There are several effects and applications of queuing theory in the context of sustainable supply chain modelling in closed-loop supply chains, green logistics planning, waste reduction and recycling, energy efficiency analysis, emission reduction strategies, transportation network optimisation, and carbon footprint reduction. However, in the proposed work, we use its effects on sustainable supply chain modelling for multiproducts.

Teimoury et al. [9] developed an inventory model with queuing applications for multiproduct items for different production item schemes. They created a linear cost function to total all the holdings, back ordering, and ordering expenses. Every time, the manufacturer must spend time setting it up. In the developed model, authors considered finite production rate and stochastic production times, which alternate between producing different product types, optimising the cost and finding valuable outcomes. Liu et al.[10] adopted two-phase queuing theory and an optimisation technique for solving location-inventory problems with stochastic supply and random supply disruptions. The authors exploited a tailored hybrid genetic algorithm, which is embedded with a direct search method, and derived some interesting insights in the sensitivity analysis part; for the performance evaluation, the authors also provided some examples. In addition, Nazari-Ghanbarloo and Ghodratnama [11] optimised a robust, tri-objective, multi-period supply chain system and created a model utilising a queuing system. The authors converted the tri-objective model to a robust mathematical model using the Ben-Tal method, also known as the ρ-Robust method, and in the process, they discovered a few novel solutions.

Sharma and Pandey [12] analysed a B2C model in supply chain systems using M|M|1 and M|M|C queuing models and concluded some fruitful performance measures related to waiting and service time of jobs in the supply chain system. Rostami et al. [13] modelled a

cross-docking in a three-level supply chain system. The authors used the G/G/M queueing model, non-dominated genetic sorting, and a simulated annealing algorithm to find the optimum solutions. Finally, they analysed their algorithms using a statistical test and performance method. Jahani and Gholizadeh [14] used queuing and inventory optimisation techniques to model a flexible closed-loop supply chain for multi-stage manufacturing. Robust genetic algorithms were used to solve the non-linear problems. The researchers concluded that their findings provided important insights for supply chain flexibility design. Nozari and Rahmaty [15] created a model based on a Make-to-Order problem, in which the authors considered an order queuing system in an uncertain environment. The authors also did a case study based on the developed model and analysed and concluded the results by reducing costs. Pandey and Sharma [16] developed a production inventory model with queuing and a learning effect both in the supply chain management system and in optimising the total inventory cost.

For multiple conflicts in sustainable green supply chain systems, an optimisation approach was developed by Asha et al. [17], and it simultaneously considered economic, environmental, and some other social objectives during strategic decision-making. Dey et al. [18] also developed a sustainable supply chain model involving automated production, inspection, and intelligent transportation schemes. They concluded that the model maximises profit during the supply chain by optimising the decision variables of manufacturers and retailers. Maheshwari et al. [19] developed a sustainable inventory model using optimal waste management for a three-layer supply chain focused on creating a resource-efficient rework and remanufacturing system. The authors determined the system's minimal cost with the optimal planning horizon using the analytical optimisation technique and the software programs Lingo and Mathematica. Sharifi et al. [20] developed a novel sustainable soybean supply chain model with a two-stage multi-objective optimisation approach in an uncertain environment. They concluded it using a scenario-based analysis method.

A case study for the coffee industry was done by Torabzadeh et al. [21], which analysed the design of a dynamic, multi-objective green supply chain network for perishable goods in an uncertain environment and found some valuable outcomes. The overall model worked around reducing carbon emissions; the authors used fuzzy for parametric evaluation. Soon et al. [22]) examined a multi-objective closed-loop supply chain system with different quality levels and found that When managing uncertainties, the robust possibilistic programming method performs better than the possibilistic programming method. Alinezhad et al. [23] investigated an inventory model for a sustainable supply chain network. They presented the network as a fuzzy demand and return rate, a bi-objective linear programming model that proposes a multi-product, multi-period obstacle. The authors employed the model's uncertainty, which is then addressed using fuzzy linear programming and the Lp-metric approach. Karthick [24] used a genetic algorithm to model a sustainable supply chain with variable production and an ambiguous carbon emission factor. With this specific model, the vendor plans to manufacture the product in compliance with the carbon emission level, resulting in the vendor paying the carbon emission tax. The author considers the carbon emission-conversion factor a triangular intuitionistic fuzzy number during modelling because of its uncertainty. Panja and Mondal [25] developed a sustainable production inventory model through bi-level greening performance for supply chain management systems and generated fruitful outcomes in the field. In their study, the authors assumed a type-2 fuzzy application to deal with non-random uncertainty. The model is advantageous for industries that use an inventory production system. Ruidas et al. [26] considered an EOQ model with imperfect items where the incoming lot is re-workable item types. Imperfect quality items under different imprecise environments such as interval number [27] and rough interval [28]. Some authors moved to the theory of learning to minimise the total inventory fuzzy cost. The learning was first suggested by Wright [29], who gave a quantitative teaching shape with some authentic data. When demand becomes imprecise, the order quantity is not finite. There may be some problems for sellers and buyers during the supply chain, and queuing is affected by the nature of demands. The learning phenomenon observes the repetition of tasks during the multiproduct transaction. In this way, we consider the learning effect involved in the holding and ordering costs related to the number of shipments. Huang and Wang [30] developed a model using a supply chain with a learning effect and some advanced technologies and found some impacts on pricing decisions. Yang et al. [31] also developed a supply chain model for various constructs using learning concepts and a conceptual framework. Li et al. [32] developed a model and discussed organisational learning with service innovation performance. Chen et al. [33] also developed a supply chain model including the learning effect and some other parameters and found fruitful outcomes. Wang et al. [34] developed a dynamic inventory model, combining reinforcement learning and other terms and applied the approach in the aerospace field. Ganesan and Uthayakumar [35] used the learning effect in their supply chain inventory

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control method to find the optimal order quantity of retailers. An EOQ model of deteriorating items with learning in fuzziness was developed by Mahapatra et al. [36] in their development of a Fuzzy Economic Order Quantity Model of Deteriorating Items with Promotional Effort and Learning in Fuzziness with a Finite Time Horizon. The authors look into deteriorating items with ambiguous demand, which they consider fully backordered for a specific period. Additionally included is the learning effect within the fuzzy environment. Alamri et al. [37] compared the buyer's overall profit in a crisp and fuzzy environment, and the authors concluded that the buyer made more money in the fuzzy environment. They also developed a model with a learning effect and other policies for imperfect types of items under a fuzzy environment. Rani et al. [38] developed a fuzzy-based green supply chain inventory model for deteriorating items, and in the model, authors considered the demand to be carbon dependent; authors also developed a crisp model to minimise the total cost. Singh et al. [39] developed an inventory model with multi-stage EOQ and backlogging under a fuzzy environment to evaluate the steady-state probability distribution produced by the inventory's holding and transportation costs. Finally, they demonstrated that the carbon-emitting model is preferable for cost-cutting and environmental awareness. A fuzzy-based EOQ model with credit financing and back ordering was developed by Jayaswal et al. [40]. The authors fuzzified the fuzzy triangular numbers, and learning was done in the fuzzy numbers and finally minimised the fuzzy cost function. Alamri et al. [41] also developed an EOQ model using steady-state probability distribution under the learning effect. The authors also discussed the observations and managerial insights. Kumar et al. [42] developed a sustainable inventory in a fuzzy environment for deteriorating items. The authors also include partial back ordering with social responsibilities and the effect of learning and conclude some optimised costs in the form of final results. A sustainable greening product with back-ordering and a steady-state probability distribution model with ordering cost treated as a variable order cost was created by Pervin [43]. With both full and partial backorders in the system, the authors employed green technology and preservation technology.

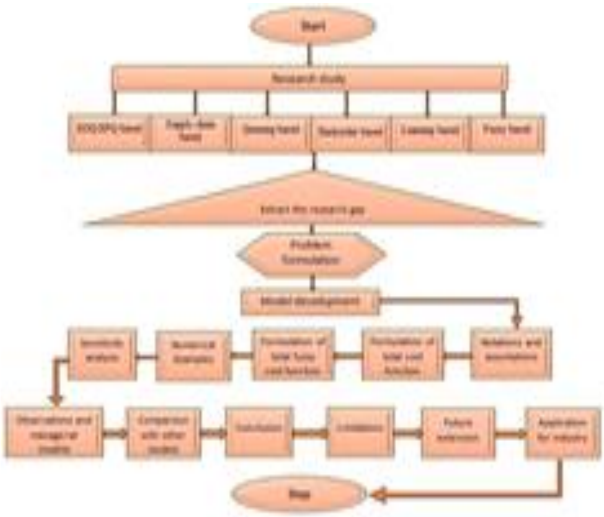

Figure 1: Flow chart for the proposed plan

2. Research gap and our contribution

We discussed our work and research gaps, which benefit industries, firms, and industrial sectors where the holding cost could be imprecise during the supply chain. Our proposed work combined the supply chain situation with realistic situations like steady-state probability distribution, queuing concepts, fuzzy holding and back-ordering

costs, which still need to be discussed in the literature review. Our research gap and contribution are presented at the bottom of Table 1. All the attributes mentioned above and factors are essential in multi-production inventory management and supply chain systems. However, in the literature discussed above, a model has yet to be developed that considers all these factors together, as we have proposed. Considering the proposed model, we regarded a steady-state probability distribution for multiproduct items under a fuzzy environment. In this proposed plan, the manufacturer produces multiproduct items as demanded by several retailers, and after completing the production process, different items are placed in other warehouses. The rent of warehouses varies occasionally/seasonally and is not permanently fixed. Some warehouses are built with advanced facilities and have a low steady-state probability distribution. Due to varying holding costs, we considered the holding cost imprecise and treated it as a triangular fuzzy number. Each product has a different backorder cost due to needing more items. We created a linear fuzzy cost function by adding the costs associated with the shipment, back ordering, and fuzzy holding. The theory of the proposed model is managed with $FCFS$, $GI/G/1$queue because, during manufacturing, the manufacturer sets a setup time for

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each product, and multiple products do not overlap during the production of items and switch to the output another. Finally, we develop the proposed model to analyse the logistic process to a three-echelon inventory model. The Centroid method is used to find the total fuzzy cost.

Table 1: Author's contributions and proposed work

Authors	Items	Queuing theory	Backorder	Fuzzy Concept	Steady-state probability distribution
Daroudi et al. [44]	Perishable multiproduct	No	No	No	No
Liu et al. [10]	Inventory product	Yes	No	No	No
Azizi and Hu [45]	Multi-product	No	No	No	No
Aliahmadi et al. [46]	Inventory product	Yes	No	Yes	No
Karim and Nakade [47]	Finished Inventory product	Yes	No	No	No
Omar et al. [48]	Multi-product	Yes	No	No	No
Ghalehkhondabi and Suer [49]	Stock/Inventory	Yes	No	No	No
Wang et al. [50]	Inventory product	No	No	No	No
Hum et al. [51]	Inventory product	Yes	No	No	No
Dehghani and Taki [52]	Inventory product	Yes	No	No	No
Keshavarz et al. [53]	Inventory product	No	No	Yes	No
Ventura et al. [54]	Multiple products	No	No	No	No
Masruroh et al. [55]	Green items	No	No	No	No
Aziziankohan et al. [56]	Green inventory products	Yes	No	No	No
Alinezhad et al. [57]	Bakery items	No	No	Yes	No
Nozari and Rahmati [15]	Ideal inventory product	No	No	Yes	No
Dey et al. [18]	Single Inventory product	No	No	No	No

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Current study	Multi Product	Yes	Yes	Yes	Yes

3. Basic definitions

Fuzzy set theory is an essential tool that can sometimes control and mathematically represent decision-making. We can use fuzzy sets to deal with imprecision by making the input parameter values functions of trapezoidal or triangular shapes (Kahraman et al. [57]). Some other primary definitions related to fuzzy are taken from Kazemi et al. [58] and are given below. These definitions also improve the methodology of the proposed work.

➢ **Fuzzy set:** A set with the condition: $\widetilde{A} = \{\alpha, \mu_{\widetilde{A}}(\alpha) | \alpha \in U\}$, where $\mu_{\widetilde{A}}(\alpha)|: U \to [0,1]$, as α in $\widetilde{A}$ ranging from 0 to 1 is called a fuzzy set.

➢ **Normalised fuzzy set:** If a set $\widetilde{A}$ in U has a property: $\exists\ \alpha \in U$ such that $\mu_{\widetilde{A}}(\alpha) = 1$, then it is known as a normalised fuzzy set.

➢ **Convex fuzzy Set:** A fuzzy set $\widetilde{A}$ in U with property

$$\mu_{\widetilde{A}}(\lambda\alpha_1 + (1 - \lambda\alpha_2)\,) \geq \min\{\mu_{\widetilde{A}}(\alpha_1), \mu_{\widetilde{A}}(\alpha_2)\}$$

where α_1 lies in [0,1] is known as a Convex fuzzy set.

➢ **Fuzzy number:** A fuzzy number desires to comply with the following requirements: The member function of a convex, normalised fuzzy set must be piecewise continuous. For that membership function, a fuzzy number is nothing more than an extension of α −cut in interval form, i.e. $\widetilde{A}_\alpha = [\alpha_1, \alpha_2]$.

➢ **Triangular Fuzzy Number (TFN):** A fuzzy number$\widetilde{A} = (a, b, c)$, with the condition $a < b < c$ and is defined on R, is termed as a triangular fuzzy number, conditionally if its membership function is

$$\mu_{\tilde{A}} = \begin{cases} \dfrac{x-a}{b-a} & a \leq x \leq b \\ 1 & x = b \\ \dfrac{c-x}{c-b} & b \leq x \leq c \\ 0 & otherwise \end{cases}$$

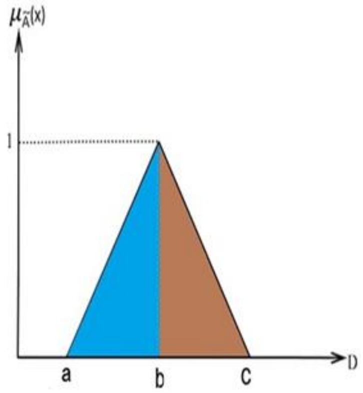

Figure: 2 Pictorial view of triangular fuzzy number

4. Assumptions and Notations

4.1 Assumptions: We have the following assumptions for this proposed plan:

- Consider that the retailer's$i$is an autonomous renewal process with a fixed rate $\gamma_i \geq 0$ and Squared Coefficient of Variation (SCV) D_i^2. The customer needed each type of product at retailer$i$and is represented by the probability vectors, $y_i = (y_{i1}, y_{i2}, \dots, y_{iL})\left(\sum_{j=1}^{L} y_{ij} = 1, 0 \leq y_{ij} \leq 1; i = 1,2, \dots.., n\right)$. Consequently, the orders in the warehouse j are processed steadily.

In SCV, $\gamma_i \geq 0, D_{aj}^2 = \frac{1}{\gamma_{aj}\sum_{i=1}^{n}\gamma_i y_{ij} D_i^2 and}\gamma_{aj} = \sum_{i=1}^{n}\gamma_i y_{ij}$. Put differently, γ_{aj} and D_{aj}^2 represent the average arrival rates and standard deviation of the combined product input streams j, respectively.

Our problem assumes that every warehouse holds a single product in batch size Y_j, up to a maximum of O_j batches. As a result, $X_j = O_j \times Y_j$ is the maximum inventory level of the product j warehouse. For every product type, we use the production authorisation system. It is assumed that products are stored in batches in the inventory, with a production authorisation card affixed to all batches. In the present plan, we examine the scenario where the number of batches and production authorisation cards are equal. When Y_j units in an inventory batch are exhausted, the PA system functions as follows:

- The matching PA card is sent to the manufacturing unit and serves as fresh production orders, starting the plant's production cycle. To fulfil these orders, the manufacturing unit typically employs an FCFS discipline. When the manufacturing unit produces Y_j units, the completed units and the PA card are sent to the warehouse. If a customer orders without production inventory, we assume they will wait until the product becomes available.

- Production policy is a make-to-stock strategy that follows the $(O_j - 1, O_j)$ control rule for the product j warehouse and is forecast-based.

Based on the assumptions mentioned above, the manufacturer's batch arrival streams follow a Poisson process, where rate $=\gamma_a(B_L) = \sum_{j=1}^{L} \frac{\gamma_{aj}}{Y_j}$. The SCV of batch arrival streams in a manufacturer, using the asymptotic method proposed by Whitt [57], is:

$$D_a^2(B_L) = \frac{\sum_{j=1}^{L} \frac{\gamma_{aj}}{Y_j} D_{aj}^2}{\gamma_a(B_L)}.$$

The likelihood that a manufacturer will arrive for product j is defined as follows:

- Every time a manufacturer changes from producing one type of product to another, it must set everything up.

- Assumed to represent the process time and setup time a batch experiences, respectively, are P_b and S_U, random variables. Since the likelihood of a product j batch experiencing a setup (a random variable including mean τ_j and variance χ_j) is $1 - P_j$, the following formula can be used to determine the setup time's mean and standard deviation for any batch:

$$E[S_U] = \sum_{j=1}^{L} P_j(1 - P_j)\tau_j \qquad (1)$$

$$D_U^2 = \frac{Var\,[S_U]}{E^2[S_U]} = \frac{\sum_{j=1}^{L} P_j(1-P_j)\xi_j}{\left(\sum_{j=1}^{L} P_j(1-P_j)\tau_j\right)^2} \qquad (2)$$

For product j, we considered unit production times at the manufacturer by using distributed random variables, represented by $1 / \mu_{S_j} = E(B_{L_j})$ and SCV, D_U^2. As a result, Y_j / μ_{S_j} and D_U^2 / Y_j are the mean production times and coefficients of variation for batch product j. Similarly, for any batch, we can get the mean and SCV of processing time as follows:

$$E[P_b] = \sum_{j=1}^{L} P_j \frac{Y_j}{\mu_{S_j}} \qquad (3)$$

$$D_{P_b}^2 = \sum_{j=1}^{L} Y_j D_j^2 \qquad (4)$$

After determining the effective batch service time (S), which is $S = S_U + P_b$ for any given batch, the mean and coefficient of variation can be calculated as follows:

$$E[S] = E[S_U] + E[P_b] \qquad (5)$$

$$D_S^2 = D_{S_U}^2 D_{P_b}^2 \qquad (6)$$

The manufacturing plant's utilisation is given by

$$\Phi = \sum_{j=1}^{L} \frac{\gamma_{aj}}{Y_j} E[S] = \gamma_a(B_L)E[S] \qquad (7)$$

The system incurs the following expenses: back ordering costs b_{c_j} per unit of product j per unit time; holding costs H_j (in \$) per unit of inventory of product j per unit time; and D_{s_j} order setup cost for product j (\$ / set up). The modelling of such a supply network attempts to minimise the overall cost of the supply chain to ascertain the optimal values of O_j and Y_j.

Cost consists of order setup costs D_{s_j}, back ordering costs B_{L_j}, and inventory fuzzy holding costs $\widetilde{(H_j)}$. The representation of the graded mean integration of

$$P(\tilde{A}) = \frac{\int_0^{EwA} H\left(\frac{L^{-1}(H) + R^{-1}(H)}{2}\right) dH}{\int_0^{EwA} H dH}$$

4.2 Notations:

The following notations are used in this proposed plan:

Y_j: Quantity of units in a single product bucket$j$, $j = 1, 2, \ldots, L$.

O_j: Total number of buckets in the product warehouse j.

H_j: Inventory holding cost (\$/unit time /year).

$\widetilde{H_j}$: Inventory fuzzy holding cost(\$/unit time /year).

$\widetilde{\widetilde{TC}}$: Total fuzzy inventory cost (in \$).

TC**:** Total inventory cost (in \$).

(H_{1j}, H_{2j}, H_{3j}) : Triangular fuzzy number for the holding cost.

X_j: Maximum product inventory at the warehouse j; $O_j Y_j$ (units).

γ_{aj}: Retailers' demand for product arrival rate $j\left(\sum_{i=1}^{n} \gamma_i y_{ij}\right)$ (units/year).

D_{aj}^2: SCV of retailers' product arrival rate demand $j\left(\frac{1}{\gamma_{aj}}\sum_{i=1}^{n}\gamma_i y_{ij}\, D\right)$.

N_{1_j}: Quantity of product j orders that reached the manufacturer (units).

μ_{S_j}: In the manufacturing plant, the product j's service rate (units/unit time).

D_j^2: In a manufacturing plant, the product j's SCV of service rate (units/unit time).

P_j: The likelihood that a product will arrive j.

τ_j: Average product setup time j.

χ_j: Variations in the product's setup j.

S_U: Setup time.

P_b: Batch processing time.

I_{L_j}: Inventory level for product j.

N_{2_j}: Total quantity of product j orders in a $GI\ /\ (G\ /\ 1)$ queue at the manufacturing facility(in units).

B_{L_j}: Product j's backorder level at the warehouse (in units).

N_{3_j}: Numerous orders were received by product$j$'s warehouse; however, H_j's inventory holding costs ($ per unit/unit time) increased after the final batch was released for processing.

b_{c_j}: Costs associated with product backorders j ($ /unit/unit time).

S_{c_j}: Order setup expenses for goods j ($ / set up).

ρ: The intensity of the manufacturing plant.

L_{ji}: Lead time for retailer i to get merchandise from product j's warehouse.

ω_{ji}: Expected order quantity for product j at retailer i, in the steady-state queue $M^{D_j}/M/\infty$.

Φ: The logistics process's service rate.

ρ_j': Strength of the logistics centre $\gamma_{aj}/\Phi\ < 1$.

E_{W_j}: Anticipated waiting period at product$j$'s warehouse solely as a result of backorder.

M_{ji}: Average lead time (including back ordering delay) for a retailer's order that will be fulfilled from the product warehouse$j$, $M_{j1} = M_{j2} = \ ...\ = M_{jn} = M_j$.

$E_{D_{ji}}$: Demand for product j that is expected at the retailer during each item's lead time $i(\gamma_{a,j}M_{ji})$.

We are developing the proposed model in the forthcoming subsection 3.2 using basic definitions, assumptions and notations.

5. Mathematical formulation

In this part, the mathematical formulation covers theoretical and mathematical interpretation, problem formulation and other concepts used during modelling, which are given below:

Our proposed model assumes holding and ordering costs as triangular fuzzy numbers. We defuzzified the total cost function using GMI and the centroid method. We compared the total inventory cost in a fuzzy environment with and without a fuzzy environment. We designed the delivery of the lot as ordered, which is more beneficial for the supply chain members (seller, buyer, and customer). The fuzzy theory improves the efficiency of the queuing-based supply model and minimises the total inventory cost, where holding and ordering costs are imprecise. The fuzzy and learning theory affects the queue, where some inventory inputs become imprecise. In this way, we considered fuzzy theory for managing the queue and minimising the system's total inventory cost.

5.1 Mathematical interpretation

In our proposed plan, the manufacturer produces multiproduct items as demanded by several retailers, and after completing the production process, items are placed in other warehouses homogenously. The rent of warehouses varies occasionally and is not permanently fixed. Some warehouses are built with advanced facilities and have a low steady-state probability distribution. Due to varying holding costs, we considered the holding cost to be imprecise. During manufacturing, the manufacturer sets a setup time for each product. Multiple products do not overlap during production and switch to the output of another; we managed it with FCFS and $GI/G/1$ queuing concepts.

Figure 3 shows a manufacturing unit plant with a three-echelon supply chain network system. We assume there are L warehouses, retailers, and a manufacturing facility. When consumers arrive at the retailer's node, this network provides L different product types. Consumer requests are received by retailers, who then

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forward the total demand for each product to the product's warehouses. Then, a production authorisation (PA) is sent to manufacturing work, keeping in mind that each product should have Y_j total orders for each of its warehouses 1, 2,.... batches of orders are sent from the warehouse to the manufacturing plant for each product. We look at a production-inventory system with multiple items, where a manufacturing facility produces L different kinds of goods and keeps distinct inventory buffers for each product. A buffer stock order is filled if it is available. The demand is backordered if it isn't. It is also assumed that the manufacturing facility provides GI/G/1 queue service to the producers of each product. The product batch is delivered to the warehouse following the manufacturing process to meet retailer demand.

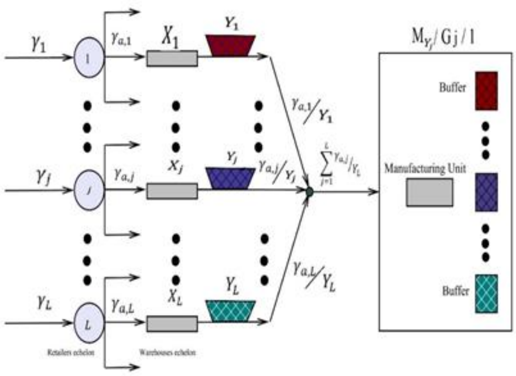

Figure 3: A supply chain network with three echelons in a manufacturing plant.

5.2 Problem formulation

This paper aims to reduce the anticipated overall warehouse cost by minimising total cost. Mathematically, it can be written as:

$$Min \sum_{j=1}^{M} TC\,(O_j, Y_j) = \sum_{j=1}^{M}\left(H_j E\left[I_{L_j}\right] + b_{c_j} E\left[B_{L_j}\right] + S_c\left(\frac{\gamma_{aj}}{Y_j}\right)\right) \quad (8)$$

Such that $O_j, Y_j \in X^+$.

We developed the fuzzy environment using the crisp model and the abovementioned definitions. Hence, when holding costs are imprecise, the minimum expected total fuzzy cost can be written using (8) as :

$$Min \sum_{j=1}^{M} \widetilde{TC}\,(O_j, Y_j) = \sum_{j=1}^{M}\left(\widetilde{H_j} E\left[I_{L_j}\right] + b_{c_j} E\left[B_{L_j}\right] + S_c\left(\frac{\gamma_{aj}}{Y_j}\right)\right) \quad (9)$$

Such that $O_j, Y_j \in X^+$.

The total expected fuzzy cost is defuzzified by using a graded mean index (GMI), and then, from equation(9), we have:

$$Min \sum_{j=1}^{M} \widetilde{\widetilde{TC}}(O_j, Y_j) = \sum_{j=1}^{M}\left(\frac{\int_0^{WA} H\left(\frac{L^{-1}(H)+R^{-1}(H)}{2}\right)}{\int_0^{WA} H dH} E\left[I_{L_j}\right] + b_{c_j} E\left[B_{L_j}\right] + S_c\left(\frac{\gamma_{aj}}{Y_j}\right)\right) \quad (10)$$

where $\widetilde{H}$is a triangular fuzzy number(H_{1j}, H_{2j}, H_{3j}), now from equation (10), after simplification, we can write

$$Min \sum_{j=1}^{M} \widetilde{\widetilde{TC}}(O_j, Y_j) = \sum_{j=1}^{M}\left(\frac{H_{1j}+4H_{2j}+H_{3j}}{6} E\left[I_{L_j}\right] + b_{c_j} E\left[B_{L_j}\right] + S_c\left(\frac{\gamma_{aj}}{Y_j}\right)\right) \quad (11)$$

The stochastic equations, having characteristics of the system mentioned in equation (12), to compute inventory and backorders are:

$$N_{3_j} = N_{1_j} - \left\lfloor \frac{N_{1j}}{Y_j} \right\rfloor Y_j, j = 1, 2, \dots \dots, L \quad (12)$$

$$B_{L_j} = Max\left\lfloor N_{2_j} Y_j + N_{3_j} - K_j Y_j, 0 \right\rfloor, \quad j = 1,2, \dots \dots, L \quad (13)$$

$$I_{L_j} = Max\left\lfloor K_j Y_j - N_{2_j} Y_j - N_{3_j}, 0 \right\rfloor, \quad j = 1,2, \dots \dots, L \quad (14)$$

For N_{3_j}, N_{2_j}, and i_j, the corresponding steady-state probability distribution is given below as follows:

The distribution of $N_{3_j}$is uniform from 0 to $Y_j - 1$. Thus,

$$P\left\{N_{3_j} = m\right\} = \frac{1}{Y}, m = 0, 1, \dots, \; Y_j - 1 \quad (15)$$

We employ a geometric distribution of the following form to find the probability distribution of batches in the queue $GI\ /\ (G\ /\ 1)$ using (15) as follows:

$$P\{N_2 = m\} \approx \begin{cases} 1 - \rho m = 0 \\ \rho(1-\sigma)\sigma^{m-1} m = 1,2, \dots \end{cases} \quad (16)$$

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where $\sigma = (\widehat{N}_2 - \rho/\widehat{N}_2)$, $\widehat{N}_2 = \gamma_a(B_L)w_0 + \rho$ and

$$W \approx \left\{\frac{\rho^2(1+D_s^2)}{1+\rho^2 D_s^2}\right\}\left\{\frac{D_a^2+\rho^2 D_s^2}{2\gamma_a(1-\rho)}\right\}.$$

From equation (16), the approximated order quantity in the queue $GI\ /\ (G\ /\ 1)$ of product j is:

$$P_j\left\{N_{2_j} = m_j\right\} \approx \begin{cases} 1-\left(\frac{\rho}{\sigma}\right)r_j m_j = 0 \\ \left(\frac{\rho}{\sigma}\right)(1-r_j)r_j^{m_j} m_j = 1,2,\dots \end{cases}$$

(17)

Where $r_j = \frac{P_j\sigma}{1-\sigma(1-p_j)}$, $p_j = \frac{\gamma_{a,j}}{Y_j\gamma_a(B_L)}$ and steady-state probability distributions I_{L_j}, $B_{L_j}$are as follow:

$$P\left\{I_{L_j} = m\right\} = \frac{1}{Y_j}P_{N_{2_j}}\left(\left\lceil\frac{X_j-m}{Y_j}\right\rceil\right); m = 1,2,\dots,O_jY_j$$

(18)

$$P\left\{B_{L_j} = m\right\} = \frac{1}{Y_j}P_{N_{2_j}}\left(\left\lceil\frac{X_j+m}{Y_j}\right\rceil\right); m = 1,2,\dots,O_jY_j$$

(19)

We can obtain $E\left[I_{L_j}\right]$ and $E\left[B_{L_j}\right]$ as:

$$E[j] = \sum_{i=1}^{X_j}\frac{X_j-i}{Y_j}P_{N_{2_j}}(i)$$

(20)

Total $E\left[B_{L_i}\right] = \sum_{i=0}^{\infty}\frac{i}{Y_j}P_{N_{2_j}}\left(\left\lceil\frac{X_j+i}{Y_j}\right\rceil\right)$(21)

$$E\left[B_{L_i}\right] = \sum_{i=0}^{\infty}\frac{i}{Y_j}P_{N_{2_j}}\left(\left\lceil\frac{X_j+i}{Y_j}\right\rceil\right) = \frac{1}{2}\left(\frac{\rho}{\sigma}\right)(1-r_j)r_j^{O_j}\left[\frac{Y_j-1}{1-r_j}+\frac{2Y_j}{1-r_j}+\frac{r_j}{1-r_j}\right]$$

$$= \frac{1}{2}\left(\frac{\rho}{\sigma}\right)r_j^{O_j}\left[(Y_j-1)+\frac{2Y_j}{1-r_j}\right]$$

(22)

Now, we can find optimised inventory levels for each type of product. (By using (8) and (9))

5.3 Performance measure of warehouses:

The stock-out probability is the proportion of time that the product j warehouse's on-hand inventory is zero. It can be calculated as follows:

$$P\left\{I_{L_j} = 0\right\} = P\left\{X_j \le N_{2_j}(t)Y_j + N_{3_j}(t)\right\} = P\left\{\frac{X_j - N_{3_j}(t)}{Y(j)} \le N_{2_j}(t)\right\}$$

$$= P\left\{O_j - \frac{N_{3_j}(t)}{Y(j)} \le N_{2_j}(t)\right\}$$

$$= \frac{1}{Y_j}P\left\{O_j \le N_{2_j}(t)\right\} + \frac{Y_j-1}{Y_j}P\{O_j - \frac{N_{3_j}(t)}{Y_j} \le N_{2_j}(t)\}$$

$$= \frac{1}{Y}\left(\frac{\rho}{\sigma}\right)r_j^{O_j} + \frac{Y_j-1}{Y_j}\left(\frac{\rho}{\sigma}\right)r_j^{O_j-1} \quad (23)$$

Additionally, the fill rate at product j's warehouse is the percentage of time that the warehouse's on-hand inventory is higher than zero:

$$P\left\{I_{L_j} > 0\right\} = P\left\{X_j > N_{2_j}Y_j + N_{3_j}\right\} = 1 - P\{I_{L_j} - 0\})$$

$$= 1 - \frac{1}{Y_j}\left(\frac{\rho}{\sigma}\right)r_j^{O_j} - \frac{Y_j-1}{Y_j}\left(\frac{\rho}{\sigma}\right)r_j^{O_j-1} \quad (24)$$

Additionally, the lead time for product j at its production facility is provided by

$$P_{b_{s_j}} = \frac{(Y_j-1)}{2}(1/\gamma_{a,j}) + w_0 + (Y_j/\mu_{s_j})$$

(25)

where batch forming time = $\frac{(Y_j-1)}{2}(1/\gamma_{a,j})$ and mean production time = Y_j/μ_{s_j} for product j batch.

5.4 The inter-departure times' squared coefficient of variation generated by the warehouses

In this segment, we demonstrate the process of obtaining the properties of batching departure streams from the production facility using known $\gamma_d(B_L)$, $D_d^2(B_L)$, which can be expressed as follows:

$$\gamma_d(B_L) = \gamma_a(B_L) = \sum_{j=1}^{L}\frac{\gamma_{a,j}}{Y_j}$$

(26)

Using batches in the $(GI\ /\ G\ /\ 1)$queue, which is provided in equation (12) and discussed in (26) for the batches from the manufacturing plant, we use the approximate SCV of the inter-departure times:

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$$D_d^2(B_L) = (1-\rho)^2 \left\{\frac{D_a^2(B_L)+\rho^2 D_s^2}{(1+\rho^2 D_s^2)}\right\} + \rho^2 D_s^2$$

Additionally, we can use these equations below to get the properties of the product$j$'s batching departure stream from the manufacturing plant:

$$\gamma_{d,j} = \gamma_{a,j} = \sum_{i=1}^{L} y_{ij}\gamma_i$$

$$D_{d,j}^2(B_L) = p_j D_a^2(B_L) + 1 - p_j$$

And also used the following approximation of the SCV of the individual departures from the warehouse of product j:

$$D_{d,j}^2(I_L) = Y_j D_a^2(B_L) + Y_j - 1$$

Where $Y_j$shows the size of batches of product type j. The probability, y_{ij}, that a product will be routed to retailer i after it leaves the warehouse determines the mean inter-arrival time and SCV for arrivals to retailer i.

$$\gamma_{a,ji} = \gamma_i y_{ij} \qquad (27)$$

$$D_{a,ji} = y_{ij} D_{d,j}^2(I_L) + 1 - y_{ij} \qquad (28)$$

5.5 Logistics Process

We add logistics processes to the model in continue. We anticipate that the logistics of getting goods from warehouses to retailers will take some time. We use a $(M \,/\, M^c \,/\, \infty)$ queue in continuous time to model the logistics process of product j, where c_j the deterministic vehicle capacity with exponential logistics time. It is considered that the logistics procedure depends upon the customers' demands during the arrival process.

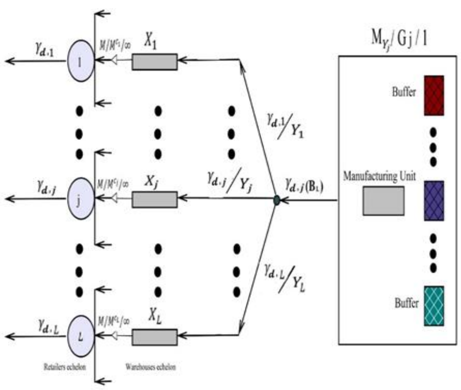

Figure 4: Supply chain network having a logistics hub and retailers

We employ the findings of (15) and (16) for analysing the performance of $M \,/\, (M^c \,/\, \infty)$ queue.

Using Little's law, we can calculate the mean lead time of product j from its warehouse at retailer i, $M_{ji} = L_{ji} + E_{W_j}$. This gives us:

$$E_{W_j} = \frac{E\lfloor B_{Lj} \rfloor}{\gamma_{a,j}}$$

$$M_{ji} = L_{ji} + E_{W_j}$$

$$\omega_{ij} = \frac{y_{ij}\gamma_{a,j}}{\Phi} D_j = \frac{1}{\Phi}(q_{ij}\gamma_{a,j} D_j)$$

$$L_{ji} = \frac{\omega_{ij}}{y_{ij}\gamma_{a,j}} = \frac{D_j}{\Phi}$$

During the lead time for replenishment, we can calculate the expected demand for product j at retailer i as

$$E_{D_{ji}} = \gamma_{a,j,i} M_{ji} \qquad (29)$$

5.6 Numerical example and discussion

In this section, we use an example to analyse the model. We look at a network of supply chains that generates three different kinds of products. The inventory parameters have been taken from Teimoury et al. [9]. The product demands in the supply chain, which consists of a manufacturing facility, three warehouses, and two retailers, are defined as follows: $\gamma_1 = 0.6$, $\gamma_2 = 0.8$, and $D_{a,1}^2 = D_{a,2}^2 = 1$. The likelihood vectors $y_1 = (0.2, 0.3, 0.5)$ and $y_2 = (0.4,0.3,0.3)$ characterise consumers' desire for three products at two stores. Table 2 details the manufacturing facility (in units) and the expenses of the three warehouses (in \$), while Table 3 details the logistics procedures and findings of total cost. Based on the above decision variable table values, we are shown a pictorial representation to show the effects clearly. Figure 5 shows the effect of the decision variable under product type 1; Figure 6 shows the impact of the decision variable under product type 2; Figure 7 shows the effect under product type 3. With the help of tabular values of Table 2 for all products, we find the parameter values of our proposed plan, presented in Table 3. We used the MATHEMATICA 12.0.1 software to solve the problem and find the total cost of both models, i.e., the fuzzy and crisp models. For all three products, we use the variety of batch sizing to determine the optimal valueO_j. We raiseY_j, the optimised maximum inventory

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level, and the total cost of the products under the condition that

$\frac{b_{cj}}{H_j} = 1$. The findings suggest that the system tends to hold more inventories if backorder costs are higher than holding costs (Table 3). Sensitivity analysis and result discussion

Table 2. Information on decision variables and their values.

Product Type	μ_{s_j}	D_j^2	τ_j	H_j	χ_j	b_{c_j}	D_{s_j}
1	0.5	0.7	0.1	1	10	100	6
2	0.6	0.8	0.3	0.9	12	120	10
3	0.8	0.6	0.15	0.95	14	140	12

Here in Table 3, we take three types of products and find the total cost by using fuzzy modelling and crisp modelling, both with some input parameters, where input parameters are taken from Teimoury et al. [9] as discussed below:

In product type 1, on taking the values of $Y_j = 2$(units), $O_j^* = 1$and $\rho_j = 0.6478$, we get the estimated values of inventory level $E\left[I_{L_j}\right] = 45.0047$(units), back ordering level as $E\left[B_{L_j}\right] = 45.0047$(units), total fuzzy cost $\widetilde{\widetilde{TC}}$=1.009932 e^{004}($) and total crisp modelling cost TC=1.0880 $e^{003(}$)$. Similarly, when $Y_j = 4$(units), $O_j^* = 2$ and $\rho_j = 0.1830$, we get estimated values of inventory level $E\left[I_{L_j}\right] = 408.4627$(units), back ordering level

$E\left[B_{L_j}\right] = 64.3846$(units), total fuzzy cost $\widetilde{\widetilde{TC}} =$ 1.49326 e^{004}($)and total crisp modellingcost

TC=1.49326 e^{004}($). On taking the values of $Y_j = 6$(units), $O_j^* = 3, \rho_j = 0.0907$, we get the estimated values of inventory level $E\left[I_{L_j}\right] = 1.462838$ (units) and back ordering level $E\left[B_{L_j}\right] = 46.3148$(units), total fuzzy cost $\widetilde{\widetilde{TC}} = 1.02643$ e^{004}($) and total crisp modelling cost TC =1.1699 e^{003}($)and when we put $Y_j = 10$(units), $O_j^* = 5, \rho_j = 0.0394$we get estimated values of inventory level $E\left[I_{L_j}\right] = 2.4926$(units), back ordering level $E\left[B_{L_j}\right] = 19.4337$(units), total fuzzy cost $\widetilde{\widetilde{TC}} =$ 1.025321 e^{004}($) and total crisp modelling cost TC = 2.8384 e^{003}($).

In product type 2, when values of $Y_j = 3$(units), $O_j^* = 1, \rho_j = 0.3066$,the estimated values of inventory level $E\left[I_{L_j}\right] = 165.4426$(units), back ordering level as $E\left[B_{L_j}\right] = 74.1743$(units),total fuzzy cost $\widetilde{\widetilde{TC}} =$ 2.4273 e^{004}($), and total crisp modelling cost TC = 1.4273 e^{003} ($) on taking the values of $Y_j = 6$(units), $O_j^* = 3, \rho_j = 0.0907$we get, $E\left[I_{L_j}\right] =$ 732.4936(units), $E\left[B_{L_j}\right] = 63.4236$(units), total fuzzy cost $\widetilde{\widetilde{TC}} = 2.3562$ e^{004}($) and total crisp modelling cost TC = 1.4192 e^{003}($).Similarly, on taking the values of $Y_j = 9$(units), $O_j^* = 4, \rho_j = 0.0465$we get estimated values of inventory level $E\left[I_{L_j}\right] = 2.9787$(units), back ordering level $E\left[B_{L_j}\right] =$ 48.4060(units),total fuzzy cost $\widetilde{\widetilde{TC}} =$ 3.928 e^{004}($) and total crisp modelling cost TC = 2.6978 e^{003}($). On putting values of $Y_j = 18$(units), $O_j^* = 9, \rho_j = 0.0163$ we get estimated values of inventory level $E\left[I_{L_j}\right] =$ 5.6877 e^{004}(units), back ordering level $E\left[B_{L_j}\right] =$ 26.4316(units), total fuzzy cost $\widetilde{\widetilde{TC}} = 2.0340$ e^{004} ($) and total crisp modelling cost TC=1.7453 e^{003}($). In

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product type 3, on putting the values of $Y_j = 5(\text{units}), O_j^* = 2, \rho_j = 0.1238$, we get the estimated values of inventory level $E\left[I_{L_j}\right] = 2.7320\ e^{004}(\text{units})$, back ordering level $E\left[B_{L_j}\right] = 32.132(\text{units})$, total fuzzy cost $\widetilde{\widetilde{TC}} = 2.4632\ e^{004}(\$)$and total crisp modelling cost TC=1.3949 $e^{003}(\$)$. When $Y_j = 10(\text{units}), O_j^* = 5, \rho_j = 0.0394$ we get $E\left[I_{L_j}\right] = 3.8246\ e^{004}(\text{units})$, $E\left[B_{L_j}\right] = 28.6246(\text{units})$, $\widetilde{\widetilde{TC}}$=3.6248 $e^{004}(\$)$ and TC=3.9171 $e^{003}(\$)$. On taking the values of $Y_j = 15(\text{units}), O_j^* = 7,\ \rho_j = 0.0212$, we get the estimated values of inventory level $E\left[I_{L_j}\right] = 8.3268\ e^{004}(\text{units})$ and back ordering level $E\left[B_{L_j}\right] = 21.6242(\text{units})$, total fuzzy cost $\widetilde{\widetilde{TC}} = 2.1936\ e^{004}(\$)$ and total crisp modelling total cost TC=1.1948 $e^{003}(\$)$. Similarly, on taking the values of $Y_j = 25(\text{units}),\ O_j^* = 12,\ \rho_j = 0.0103$, we have the estimated values of inventory level $E\left[B_{L_j}\right]$=2.1632(units), back ordering level $E\left[B_{L_j}\right] = 8.9632(\text{units})$, total fuzzy cost $\widetilde{\widetilde{TC}} = 4.4132\ e^{004}(\$)$ and total crisp modelling cost TC=5.4166 $e^{003}(\$)$

From the above observations, we analysed that when applying fuzzy concepts to the holding cost of the inventory in such cases, the total cost is affected, and we get the best-optimised values through fuzzy concepts compared to crisp modelling concepts. From the above discussion, it is clear that the total cost calculated using fuzzy holding cost is better than the cost calculated without fuzzy holding (crisp modelling).

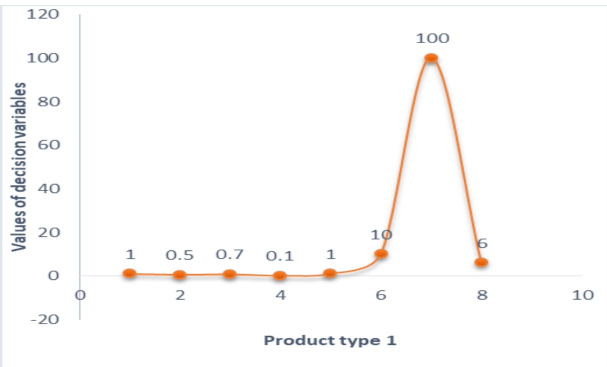

Figure 5: Representation of decision variables under product type 1

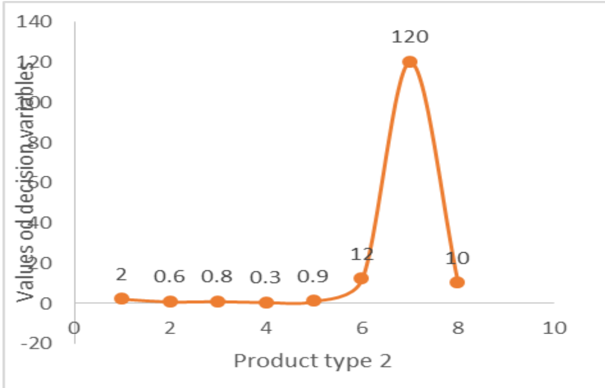

Figure 6: Representation of decision variables under product type 2

4.1 Comparison of the proposed model to other models.

We have compared our proposed model with other selected models based on the total cost/profit with the help of Table 4, which is given below, in which we provided the keywords and costs/profit calculated in the models. This comparative study positively promotes the model we have developed. The comparative table is given below. From Table 4, after comparing the total cost /profit to some selected contributions of the same field but independent from each other, i.e. no one is an earlier/extended version of each other, we observed that we have some optimised novelty in our finding which is very beneficial with seasonal or occasional inventory management systems in the supply chain management system. This is good managerial insights for the decision maker when the demand rate is imprecise in nature during supply chain.

In Figure 8, we compare the optimum value of the total fuzzy modelling cost of our proposed model using Table 3 and three other models that are independent of the proposed model and have optimum total inventory cost using Table 4 (see S. no. 2, 4, and 5 of Table 4). It can be easily seen that the total fuzzy inventory cost of the proposed plan is less than that of the other three.

Compared results are taken from the individual models and are independent of each other, and the proposed model is also independent of all, i.e. our proposed plan and findings are not the extended version of any of the compared models. This comparision is used to visualise our unique and optimised useful findings only for decision maker. The data values above the bars represent the total inventory cost in $.

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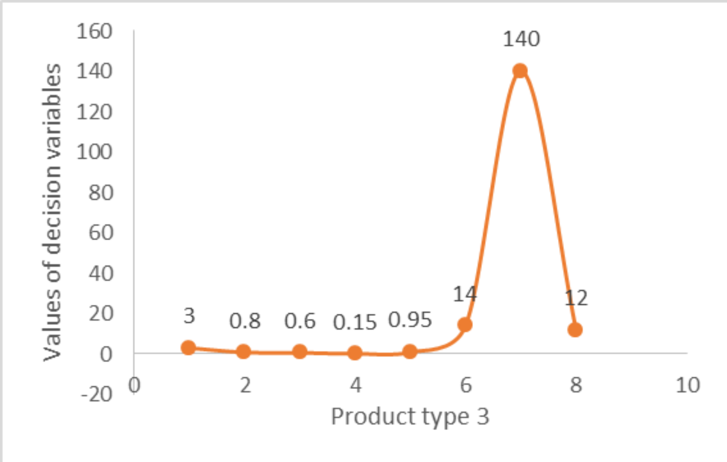

Figure 7: Representation of decision variables under product type 3

Table 3. Total cost variation by increasing Y_j if $\frac{b_{cj}}{H_j} = 1$

Product Type	Y_j (units)	O_j^*	ρ_j	$E\left[I_{L_j}\right]$ (units)	$E\left[B_{L_j}\right]$ (units)	$\widetilde{\widetilde{TC}}$ $(in\ \$)$	TC (in \$)
1	2	1	0.6478	45.0047	98.4932	$1.09932e^{004}$	$1.0880e^{003}$
	4	2	0.1830	408.4627	64.3846	$1.49326e^{004}$	$1.056e^{003}$
	6	3	0.0907	1.462838	46.3148	$1.02643e^{004}$	$1.1699e^{003}$
	10	5	0.0394	2.4926	19.4337	$1.025321e^{004}$	$2.8384e^{003}$
2	3	1	0.3066	165.4426	74.1743	$2.4273\ e^{004}$	$1.4273e^{003}$
	6	3	0.0907	732.4936	63.4236	$2.3562\ e^{004}$	$1.4192e^{003}$
	9	4	0.0465	2.9787	48.4060	$3.928\ e^{004}$	$2.6978e^{003}$
	18	9	0.0163	$5.6877\ e^{004}$	26.4316	$2.0340\ e^{004}$	$1.7453e^{004}$
3	5	2	0.1238	$2.7320e^{004}$	32.132	$2.4632e^{004}$	$1.3949e^{003}$
	10	5	0.0394	$3.8246e^{004}$	28.6246	$3.6248e^{004}$	$3.9171e^{003}$
	15	7	0.0212	$8.3268e^{004}$	21.6242	$2.1936e^{004}$	$1.1948e^{004}$
	25	12	0.0103	$2.1632e^{004}$	8.9632	$4.4132e^{004}$	$5.4166e^{004}$

5.7 Managerial application and Social implication

The managerial application of our proposed model is more and more applicable where the rate of demand is not constant and also sometimes more increases or decreases as well as decision maker (DM) can get more profit by apply of our strategies during seasonal period. This proposed work can be applied to seasonal factories like ice cream, umbrellas, sugar, etc., and is also more

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applicable to some production and manufacturing companies. In subsequent studies, we can consider a central warehouse where customers' demands are met when each warehouse is stock-out (adding transport cost). Furthermore, the pricing idea can be incorporated into our model as a compelling feature of further study. This study is useful for retailer in domestic business for the selling of seasonal items. The retailer may be loss when the some seasonal items are not sale during seasonal period and also some items are not useful as well as damages after seasonal period because retailer has less lacks of knowledge about the variation of the demand. Our proposed study helps in this situation for such retailers and advice to include the waste management policy for the balancing of profit. The waste management policy prevents the global warming and cleans the environment.

Table 4: Comparison of total profit/ cost with some other models

S.No.	Authors	Keywords	Particulars	Optimised total Cost/Profit (in $)
1.	Liu et al. [10]	Queuing theory, Supply chain, Disruption, Genetic algorithm, Facility location, Inventory system.	General products	41873.401$, approximately
2.	Aghsami et al. [59]	Inventory theory, Queuing theory, Imperfect inspection and production, Misclassifications, Cost optimisation, etc.	For defective and fresh items after screening.	1767.823 $ where $\eta_1 = \eta_2 = 20.924$
3.	Ahmadini et al. [60]	Fractional programming, fuzzy methods with goal programming, green supply chain, and inventory with production management.	For Multi-items	The evaluated lot size for inventory under the supply chain.
4.	Zhang et al. [61]	Queuing theory, Inventory theory, Mixed sales.	Inventory items	The total inventory cost is 408.336 $ when the service rate is 22.
5.	Pandey et al. [16]	Stochastic nature demand, Queuing theory, Learning with supply chain management.	For product-1 and product-2	cost for item-1 = 2639$ and for item-2=4313 $
7.	Proposed model	Multiproduct items, Supply chain, Queuing theory, Backorder, Steady-state probability distribution, Fuzzy theory	Multiproduct items	Its total fuzzy cost has been calculated in Table 3.

8. Conclusions

The proposed study deals with a queuing-based supply chain model with fuzzy theory for multiproduct items under steady-state probability distribution, and it is very suitable for the three-layer supply chain model. We compared our results with and without fuzzy environments under the same inventory input parameters. We analysed that the total inventory cost is minimal under a fuzzy environment and more without a fuzzy environment. The application of this proposed work can be applied in the performance of the warehouse for $(GI / G / 1)$ queue operating where the inventory control policy O_{j-1}, O_j is allowed and holding cost follows the effect of fuzzy environment. Our findings can be easily seen in Table 3. This work is individual and is not any extended form of other models. In Table 4, we discussed some renowned

authors' contributions regarding calculating total costs under different approaches, and our contribution has been presented in Table 3. Finally, we concluded that we get the optimum results when we take the holding cost as imprecise, treat it as a fuzzy triangular number, and then find the total fuzzy cost. In the proposed model, we obtained the warehouses' lead time, fill rate, and stock-out probability performance. To analyse the logistics process, we used the $(M / Mc_j / \infty)$queue. This paper examines the impact of order batching and fuzzy concepts on holding cost in multiproduct, multi-echelon supply chains. Proposed study has some limitations (i) the values of fuzzy hilding cost should be according to our proposed model and rest parameters may be change (ii) proposed study should be supply chain for multi-product and bot valid for ordering policies (iii) the quene type $(M / Mc_j / \infty)$ should be and not valid other type. This proposed model can also extend for imperfect quality items under different fuzzy environments.

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