How Close is the Implied Volatility Derived from Black Scholes for Individual Stocks to VIX

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Abstract

The implied volatility derived from Black Scholes model represents total volatility which includes both systematic and unsystematic components. The purpose of this paper is to examine the accuracy of the implied volatility derived by Black Scholes (after excluding the unsystematic component) with reference to VIX, which is a widely used proxy for market (systematic) volatility. We utilize CAPM to extract the volatility linked to systematic component for the 30 individual stocks within Dow Jones and compare them to VIX. The systematic volatility for these stocks adjusted for beta turn out to be rather close to the VIX, which implied that CAPM is an effective approach for separating market risk from total risk.

I. Introduction

The two most well-known classical models in finance are CAPM and the Black Scholes. It would be interesting to evaluate these two models by examining how well they can explain real world data. In this paper, we will apply both models to estimate market volatility and see how close this estimate is to the observed volatility as revealed by the VIX. Since the release of Black and Scholes (1973), different kinds of modification on the valuation of option have been proposed by researchers who are enthused as well as those who are critical to Black and Scholes model. One key contribution of the Black and Scholes model is that it can be used for estimating the implied volatility. Volatility is the key to understand and analyze stock price fluctuations. Therefore, it is a common practice for academicians and practitioners to develop models for deriving the implied volatility of individual stocks. For most models, as in Black and Scholes (1973), the calculation for implied volatility requires information on the actual option price, risk free rate, strike price and time-to-maturity as inputs. The implied volatility derived represents total volatility which comprise both systematic and unsystematic components.

In light of CAPM theory and the Black Scholes model, we propose a method to extract

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total risk for each individual stocks into systematic and unsystematic risk; 4) by using the result from 3), we calculate a ratio of unsystematic risk over the total risk and adjust the Black Scholes implied total volatility by multiplying 1 minus the ratio to extract the market volatility. Finally, the estimated market volatility from all 30 stocks are averaged and compared to the actual VIX.

The paper is structured as follows: Section 2 briefly introduces different related existing literature. The hypothesis development and model are included in Section 3. Empirical experiment is conducted and the result is shown in Section 4. The last section will provide the conclusion.

II. Literature Review

There is intense debate in existing literature on option valuation and the individual stock's implied volatility since the release of the classical Black Scholes Model (Black and Scholes, 1973). As a vital factor to understand the stock market and predict stock return, Corrado and Miller (2006), tried to examine the factors affecting the implied volatility. David and Veronesi (2002) and Guidolin and Timmerman (2003) identified that implied volatility changes over time because investors are uncertain about the fundamental economic factors. Gemmill and Kamiyama (2000) suggested that the lagged implied volatility in another market is one of the drivers of the changes in the implied volatility at a specific market. Goyal and Saretto (2007) confirmed and extended the above studies using cross-sectional implied volatilities.

After recognizing the factors of the dynamics of implied volatility, a stream of studies emerged for developing multiple methods to predict the implied volatility. Merton (1980) suggested that the diffusive risk measured by the implied volatility can be identified by quadratic variation of realized stock returns. Dumas et al. (1998), Goncalves and Guidolin (2006) and Fengler et al (2007) tried to predict the implied volatility surface of S&P500 index options across moneyness and expiration dates. Konstantinidi et al. (2008) employed six different models and suggested that there are statistical and economic significance in the predictability of European and U.S indices on the evolution of implied volatility. The results are consistent in both point and interval forecasts. Bernales and Guidolin (2014) conducted a similar research by using a two-stage model to investigate whether there is dynamic and cross-sectional correlation between stock options' and implied volatility surface for index options. In a different direction, Banerjee et. al (2007) investigated the predicting power implied volatility of index option for portfolio returns. Yan (2010) estimated the slope of the implied volatility smile as a proxy to measure the stock jump size, and to quantify the jump risk and predict stock returns.

III. Models Explanation

In this section, we will point out the difficulties in deriving the implied volatility which is comparable to the expected market volatility. An inaccurate estimate on the total implied volatility would make the comparison not meaningful. Conventional models are helpful in pricing options given an assumed level of volatility. However, doing the reversal of using the observed price to impute the implied volatility utilizing these models may lead to the first shortcomings - the observed price might not be an efficient price. Specifically, substituting the quoted option prices into the models to calculate the implied volatility can pose several problems. First, the actual option price might be inaccurate if the option is thinly traded in the CBOE. For any individual stock, there are many types of options with many different strike prices. In addition, there are weekly and quarterly options, index options etc. Choices of different strike prices for investors are so diverse that many of the options are thinly traded. Because of the low volume, the options have rather wide spreads between ask and bid. Under this circumstance, using the observed price of low volume options to estimate the implied volatility can lead to inaccurate result.

In addition, another difficulty we face in deriving volatility comparable to expected market volatility. For example, the stochastic volatility and ARCH model used by Fouque et. al (2000), the Constant Elasticity Model suggested by Davydov and Linetsky (2001) and the jump-diffusion models used by Duffie et. al (2000) may not be easily applied on pricing the path-dependent options such as perpetual American options and lookback options by incorporating the implied volatility smile. In terms of the enormously complicated calculations, Longstaff and Schwartz's (2001) simulation-based model is extremely time-consuming to implement and can only be applied to three-factor affine specification.

Thus, in light with the above difficulties, it is important for us to use options with high volume and a less complicated method to yield accurate estimates for implied volatilities. In the paper, we utilize Black Scholes model to evaluate sample of highly traded options from 30 stocks within DJIA. Also, we utilize CAPM to extract the portion of risk that are unsystematic.

In the following part of this section, we will thoroughly explain how the volatility is derived from (1) Black and Scholes' approach; from (2) CAPM and regression analysis, and how to derive VIX.

Firstly, we use call option valuation as an example. According to the Black and Scholes' (1973), the price of call option is as follow:

$$C_{s,t} = S_t N(d_1) - Ke^{-rt} N(d_2) \qquad (1)$$

where

$$d_1 = \frac{\ln\left(\frac{S_t}{K}\right) + \left(R_f + \frac{S_t^2}{2}\right)t}{\sigma_s\sqrt{t}}$$

and

$$d_2 = d_1 - \sigma_s\sqrt{t}$$

$C_{s,t}$ refers to the call option price of stock s at time t; S_t refers to the stock price at time t; K refers to the exercise price at time t; $N(d_1)$ and $N(d_2)$ are the probability density functions; R_f or r refer to the annualized risk free rate measured by the daily U.S. Treasury bill rate. σ_s represents the annualized implied volatility of individual stocks. It can be derived by plugging into the model the actual option price and other variables.

The following illustrates the method for separating the variation linked to market from the variation linked to unsystematic risk in detail using CAPM. According to CAPM, the regression model is as follow:

$$R_s = R_f + \beta_s(R_m - R_f) + \varepsilon \quad (2)$$

where R_s is the return of individual stock; β_s is the beta of stock; R_m - R_f is the market premium and ε is the error term. We re-write the equation as follow:

$$R_s - R_f = \beta_s(R_m - R_f) + \varepsilon \quad (3)$$

By utilizing the CAPM, we regress the market premium (R_m - R_f) on excess return of stock (R_s - R_f) in order to estimate β_s. Each variable should be prorated according to the length of the sample period. The variance of stock return in CAPM is written as:

$$\sigma_s^2 = (\beta_s)^2(\sigma_m)^2 + \sigma_u^2 \quad (4)$$

where σ_s is the annualized standard deviation of stock return or the volatility of stock; σ_m is the annualized standard deviation S&P market return; σ_u represents the unsystematic risk as measured by the annualized non-market related volatility. This method suggests the concept that risk of individual stock equals to the sum of systematic and unsystematic risk. The unsystematic risk is extracted from the results of the regression (3), which is the variance of residuals. This figure can be altered by using different time periods of data (for example, 30-day or 60-day stock return etc.). It is necessary to annualize σ_u^2 and σ_m^2 to keep all variables in the equation (4) consistent. If we use daily stock and market returns in the regression (3), we should annualize σ_u^2 and σ_m^2 by multiplying 260 (given that there are 260 trading days per year). After deriving the σ_s^2 from equation (4), we square root the results to obtain the annualized volatility of individual stocks.

The following illustrates the derivation of Volatility Index (VIX). VIX is provided by CBOE to represent the 30-day expected volatility for the S&P index based on a set of S&P 500 index options. It is derived from near-the-money call and put index options with more than 23 days but less than 37 days to expiration. The calculation is as follow:

$$VIX = \left[\frac{2}{t}\sum_i \frac{\Delta K_i}{K_i^2} e^{rt} Q(K_i) - \frac{1}{t}\left(\frac{F}{K_0} - 1\right)^2\right]^{0.5} x\ 100 \quad (5)$$

where F is the forward index level derived from the price of index option; K_0 is the first strike price below F; K_i is the strike price of the i[th] out-of-the-money option, a call is out-of-the-money if $K_i > K_0$, a put is out-of-the-money if $K_i < K_0$; ΔK_i is calculated as $(K_{i+1} + K_{i-1})/2$; $Q(K_i)$ is the mid-point of the bid-ask spread for each option with K_i. The market volatility is an annualized figure which can be obtained by dividing VIX by 100.

IV. Empirical estimate of volatility

In this section, we first derive the total volatility from the Black and Scholes Model for the 30 stocks. Afterward, the implied volatility derived from Black and Scholes is separated into systematic and unsystematic portion. As VIX is the benchmark for expected market volatility, it should not reflect the unsystematic risk. Therefore, the unsystematic component should be subtracted from total volatility. Only the systematic component for the implied volatility of 30 stocks are used for comparison.

Table 1

Symbols descriptions and source of data

R_s	The daily percentage change of Return Index of individual stocks in the sample	DataStream
R_m	The daily percentage change of Return Index of S&P 500 Composite	DataStream
R_f	The annualized 30-day U.S. Treasury Bill rate	DataStream
VIX	The annualized volatility index derived from S&P 500 index option	CBOE
σ_m	The annualized volatility derived from VIX/100	CBOE
K	Strike price for the individual stock options	CBOE
S_t	The stock price on the evaluation date; 18 July 2017	CBOE
$C_{s,t}$	Actual option price of stock in time t used for deriving the implied volatility in the Black and Scholes (1973) model	CBOE

We use the options of the 30 stocks listed in the Dow Jones Industrial Average Index (DJIA) which have relatively high trading volume as our sample. Since the sample are drawn from individual options that are highly traded, the estimates should be more accurate.

For our first study, we choose a common expiration date in 2017 for all 30 stock options which is 18th August. As VIX is based on a 30 day window for volatility, the time point for our empirical evaluation occurs 30 days before the common expiration date, which is 18th July. Thus, we estimate the volatility as of 18th July for all 30 stocks using both Black Scholes and CAPM and compare the derived estimates to the VIX on the same date.

To utilize equation (4) for deriving volatility for both systematic and unsystematic, we need to estimate the β_s and σ_u^2 as of 18th July. To estimate these figures, we regress equation (3) on 18th July with a sample period of 30 days from 18th June to 18th July. All R_s, R_f and R_m are prorated to daily basis. The regression (3) for the 30 stocks yields estimate for β_s and σ_u^2, which are summarized on Table (2).

Table 2

Summary of the CAPM regression results –
Beta for individual stocks, daily and unsystematic risk from 18 June to 18 July 2017.

TIC	Name	β_s	Daily σ_u^2	Annualized σ_u^2
AAPL	Apple Inc.	1.64173	3.40E-05	8.84E-03
AXP	American Express Co.	0.691724	2.82E-05	7.33E-03
BA	Boeing Co.	0.815754	4.80E-05	1.25E-02
CAT	Caterpillar Inc.	0.95219	1.09E-04	2.82E-02
CSCO	Cisco Systems Inc.	1.2885	1.95E-05	5.06E-03
CVX	Chevron Corp.	0.339425	6.68E-05	1.74E-02
DD	E I du Pont De Nemours and Co.	1.08666	1.06E-04	2.75E-02
DIS	Walt Disney Co.	1.07868	5.94E-05	1.54E-02
GE	General Electric Co.	1.10983	1.23E-04	3.19E-02
GS	Goldman Sachs Group Inc.	0.565026	1.41E-04	3.67E-02
HD	Home Depot Inc.	0.613699	7.91E-05	2.06E-02
IBM	International Business Machines Corp.	0.454618	3.12E-05	8.12E-03
INTC	Intel Corp.	1.79561	6.24E-05	1.62E-02
JNJ	Johnson & Johnson	0.409476	4.38E-05	1.14E-02
JPM	Coca-Cola Co.	0.456906	9.08E-05	2.36E-02
KO	JPMorgan Chase & Co.	0.621831	1.52E-05	3.95E-03
MCD	McDonald's Corp.	0.571233	4.24E-05	1.10E-02
MMM	3M Co.	0.88765	1.29E-05	3.36E-03
MRK	Merck & Co Inc.	0.587117	5.74E-05	1.49E-02
MSFT	Microsoft Corp.	1.73813	4.85E-05	1.26E-02
NKE	Nike Inc.	1.18995	6.62E-04	1.72E-01
PFE	Pfizer Inc.	0.49314	3.08E-05	8.00E-03
PG	Procter & Gamble Co.	0.597309	2.61E-05	6.80E-03
TRV	Travelers Companies Inc.	0.248992	3.12E-05	8.11E-03

UNH	UnitedHealth Group Inc.	0.562679	2.59E-05	6.73E-03
UTX	United Technologies Corp.	0.70218	1.15E-05	2.98E-03
V	Visa Inc.	1.1765	4.26E-05	1.11E-02
VZ	Verizon Communications Inc.	1.17726	5.72E-05	1.49E-02
WMT	Wal-Mart	0.29086	9.18E-05	2.39E-02
XOM	Exxon Mobil Corp.	0.510307	4.99E-05	1.30E-02

Variance of S&P annualized return: 0.00712
Mean of β_s: 0.8218322

We then utilized the estimated values of β_s, unsystematic risk σ_u^2 from the regression (3) for the 30 stocks as summarized in Table (2), and the standard deviation of S&P daily return (σ_m), which is 0.084, to calculate the total volatility of individual stock using equation (4). Since both σ_m^2 and σ_u^2 in Table (2) are in daily terms, we annualized them by multiplying 260 as there are 260 trading days in a year. For estimating implied volatility, we utilized the standard Black Scholes approach where the observed option price is used to estimate the standard deviation of annual return. Results from both approaches are shown in Table (3).

Total variance and volatility Equation (4) and Black Scholes approach

Ticker	σ_s^2 from Equation (4)	σ_s from Equation (4)	σ_s^2 from Black and Scholes	σ_s from Black and Scholes
AAPL	0.02804	0.16744	0.05611	0.236879
AXP	0.01074	0.10364	0.03430	0.185214
BA	0.01722	0.13123	0.04029	0.200714
CAT	0.03467	0.18620	0.03935	0.198363
CSCO	0.01688	0.12994	0.03611	0.190019
CVX	0.01819	0.13488	0.02374	0.15409
DD	0.03595	0.18961	0.03225	0.179582
DIS	0.02373	0.15405	0.04597	0.214416
GE	0.04068	0.20170	0.03575	0.18908
GS	0.03901	0.19750	0.04452	0.211003
HD	0.02325	0.15249	0.04063	0.201567
IBM	0.00959	0.09793	0.04112	0.202788
INTC	0.03918	0.19793	0.03566	0.188832
JNJ	0.01257	0.11212	0.01369	0.117014
JPM	0.02510	0.15843	0.02650	0.162802
KO	0.00670	0.08185	0.01750	0.132297
MCD	0.01336	0.11557	0.03106	0.176252

MMM	0.00897	0.09470	0.02660	0.163103
MRK	0.01738	0.13185	0.03304	0.181773
MSFT	0.03412	0.18472	0.04598	0.21444
NKE	0.18208	0.42671	0.02582	0.16068
PFE	0.00973	0.09866	0.01246	0.111644
PG	0.00934	0.09663	0.01095	0.104662
TRV	0.00855	0.09246	0.03160	0.177754
UNH	0.00899	0.09479	0.02033	0.142578
UTX	0.00649	0.08058	0.01868	0.13669
V	0.02093	0.14467	0.03272	0.180874
VZ	0.02475	0.15733	0.03063	0.175008
WMT	0.02447	0.15642	0.00964	0.098175
XOM	0.01482	0.12173	0.01705	0.130573
Mean		0.14646		0.1706289

Afterward, in order to isolate the unsystematic risk from total volatility, we use the result from Equation (4) to calculate the ratio $\frac{\sigma_u^2}{\sigma_s^2}$, denoted as h, which is the portion of variance linked to unsystematic risk. Then the σ_s^2 derived from Black and Scholes (BS) approach is multiplied by $(1-\mathrm{h})$ and square-rooted to obtain the pure systematic risk. The result is shown in Table (4).

Table 4. Systematic risk derived from Black Scholes' result

Systematic risk derived from Black Scholes' result			
Ticker	h= $\left(\frac{\sigma_u^2}{\sigma_s^2}\right)$	Systematic variance from Black and Scholes = (1-h)	Systematic standard deviation from Black and Scholes
AAPL	0.31531	0.03842	0.19601
AXP	0.68271	0.01088	0.10433
BA	0.72477	0.01109	0.10530
CAT	0.81374	0.00733	0.08561
CSCO	0.29965	0.02529	0.15902
CVX	0.95489	0.00107	0.03273
DD	0.76606	0.00754	0.08686
DIS	0.65078	0.01606	0.12671
GE	0.78437	0.00771	0.08780
GS	0.94170	0.00260	0.05095
HD	0.88464	0.00469	0.06846

IBM	0.84651	0.00631	0.07945
INTC	0.41383	0.02090	0.14457
JNJ	0.90500	0.00130	0.03607
JPM	0.94076	0.00157	0.03963
KO	0.58890	0.00720	0.08483
MCD	0.82598	0.00541	0.07352
MMM	0.37424	0.01665	0.12902
MRK	0.85877	0.00467	0.06831
MSFT	0.36940	0.02900	0.17029
NKE	0.94461	0.00143	0.03782
PFE	0.82204	0.00222	0.04710
PG	0.72782	0.00298	0.05460
TRV	0.94835	0.00163	0.04040
UNH	0.74905	0.00510	0.07142
UTX	0.45920	0.01010	0.10052
V	0.52898	0.01541	0.12414
VZ	0.60121	0.01221	0.11052
WMT	0.97537	0.00024	0.01541
XOM	0.87482	0.00213	0.04620
Mean			0.08592

According to the result in Table (4), the mean value for implied systematic volatility derived from Black Scholes and CAPM is 0.08592. Given that the average β_s for the 30 stocks is 0.8218 when regressed against the S&P return, we adjust the VIX by multiplying it by 0.8218 since the value of β_s are derived from regressing individual stock return against the S&P return whose beta is implicitly assumed to be 1. The VIX adjusted for beta is 0.08506, which is extremely close to the above Black and Scholes estimate of 0.8218. In addition to this 2017 study, we repeated this procedure for two recent samples in 2019. This provides a good contrast given that the volatility in 2019 is much greater than that in 2017. The result for all three samples are summarized in Table 5 below.

Table 5

Comparison of VIX to Estimated Volatilities for all Three Sample Periods.

Sample Period	18 June 2017 to 18 July 2017.	10th July2019 to 9th Aug 2019	7th Aug 2019 to 6th Sept 2019

VIX (adjusted for beta)	0.08506	0.1605	0.1518
Volatility Estimated by BS and CAPM	0.08592	0.153	0.1596
Difference in percentage	1.01%	-4.67%	5.14%

The last row in Table 5 displays the difference between VIX and the volatility as estimated by CAPM and BS, which is Volatility Estimated by BS and CAPM minus VIX divided by VIX. As seen, the maximum difference is only about 5.14%, which suggests that the accuracy of estimates derived from using Black Scholes combining with CAPM is relatively high. Note that the volatility in Summer 2019 is almost twice as high as in Summer 2017. This is due to the erratic fluctuation of the market caused by the ups and downs of news about the trade war and other world events in 2019. It is interesting to note that the BS and CAPM method seems to work quite well in approximating VIX during both high and low volatility periods.

V. Conclusion

Utilizing both the Black Scholes model and CAPM, we developed an approach of deriving systematic risk from total implied volatility of individual stocks and hence estimate the market expected volatility. We think that simply using the implied volatility from Black Scholes to estimate market expected volatility is biased as there are systematic and unsystematic risk within. Therefore, we use CAPM theory to estimate the portion of systematic volatility in total volatility for individual stocks and use it to derive the implied systematic volatility. Since our estimated implied systematic volatility is rather close to VIX, which is regarded as a proxy for expected market volatility, we conclude that CAPM can be used with Black Scholes to yield both expected systematic and unsystematic risk.

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