How Long Is Too Long?
Volatility-Based Holding Strategies for Leveraged Bull and Bear ETFs

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Abstract

Since their favorable introduction in the U.S. in 2006, leveraged bull and bear exchange-traded funds (ETFs) have provided short-term investors with the opportunity to express their directional views regarding a wide variety of indexes. However, unlike traditional unleveraged ETFs, leveraged ETFs are not intended to be used as long-term trading instruments. Instead, leveraged ETFs are designed to return a multiple of their benchmark index on a daily basis. Leveraged ETFs are structured only for short-term investors because these funds must be rebalanced each day to prevent leverage from becoming too excessive. This daily rebalancing complicates predicting long-term returns for leveraged ETFs due to both compounding and volatility. Using Morningstar return data and Chicago Board Options Exchange volatility index data, we investigate the effects of compounding and expected market volatility on specific long-term holding strategies for leveraged bull and bear ETF returns. We show that compounded leveraged returns over these holding periods are comparable to compounding the respective multiple of their underlying benchmark return with tracking error increasing over time and with leverage. Our results also show that expected market volatility has a significant effect on tracking error, after adjusting for expenses, and that this effect increases over time and with leverage. These results suggest that volatility indexes may be used by sophisticated investors to devise trading rules for long-term holding strategies for leveraged bull and bear ETFs.

I. Introduction

Exchange-traded funds (ETFs) are investment products traded on major exchanges much like stocks. However, unlike stocks, ETFs are usually designed to mirror the holdings and performance of an underlying index such as the S&P 500 index. Over the last two decades, the ETF market has quickly grown from one broad-based domestic equity fund tracking the S&P 500 index in 1993 to over 1,000 ETFs with total net assets surpassing the trillion dollar mark by year-end 2011 (Investment Company Institute, 2012). As this demand for ETF products increased, more investment options became available. In 2006, the first leveraged bull and bear ETFs were introduced to the market. Like traditional ETFs, leveraged bull and bear ETFs quickly carved out their own niche with investors. As of May 2009, there were over 125 leveraged bull and bear ETFs with over $35 billion in assets listed on North American exchanges.

As leveraged ETFs play an increasingly larger role in the market, more long-term investors have also considered investing in these leveraged funds. Although several long-term holding strategies for traditional, unleveraged ETFs have been researched such as end-of-month trading strategies (Rayhorn, Janson, and Drosen, 2010), there is little discussion about long-term holding strategies for leveraged ETFs. For example, would the same long-term holding strategies for unleveraged ETFs apply to leveraged ETFs? The short answer is "no." Leveraged ETFs were not originally designed for long-term investors and the mechanics of leveraged ETFs

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are far more complicated than their unleveraged counterparts. One major problem with these leveraged investments is that investors do not clearly understand the effects of compounding and volatility on tracking error, especially over time. Although recent research suggests that the effects of compounding are negligible for holding periods less than one day (Shum and Kang, 2013; and Tang and Xu, 2013), our research is unique in that we focus on holding periods ranging from one day to one year. Thus, our motivation for this paper is to address the following question: "How long can an investor reasonably hold leveraged bull and bear ETFs in their portfolio?" – or more simply, "How long is too long?"

II. Advantages and Disadvantages of ETFs

To answer the previous question, we must have a firm understanding of the main advantages and disadvantage of passively managed ETFs over actively managed mutual funds. The primary advantage is that passive funds usually have fewer fees. These fees can include front-end loads, redemption fees, 12b-1 fees, and deferred sales charges (Kostovetsky, 2003). Not surprisingly, the vast majority of research shows that lower fees lead to higher returns. Bogle (1998) finds that average costs of mutual funds from 1992 to 1996 have an inverse effect on average returns. In addition, Frino and Gallagher (2001) find that S&P 500 index funds, after controlling for expenses, outperformed actively managed funds.

However, all index funds, including ETFs, have one Achilles heel or primary disadvantage – tracking error. In other words, ETFs do not always perfectly replicate the performance of the underlying index. Several proposed causes of tracking error for index funds in general include the volatility of benchmarks, corporate decisions such as dividend policies that impact index weights, cash flow effects like cash drag, and transaction and liquidity costs regarding bid-ask spreads. Although similar to other index funds, ETFs are especially affected by corporate behavior like dividend decisions, which can quickly change the asset allocation of an index (Frino and Gallagher, 2001). Moreover, Kostovetsky (2003) states that the fund liquidity costs and transactions costs are almost negligible for ETFs because they operate through a process called creation/redemption-in-kind. In other words, ETFs can only be created (redeemed) by purchasing (selling) large numbers of shares that match the target index in both share weights and share values. Gastineau (2001) goes further and explains that two extremely important advantages of the redemption-in-kind process are that it enhances the tax efficiency of the fund and it also provides an arbitrage mechanism to assure a market price close to net asset value. Gastineau (2004) states that an ETF's performance should closely replicate the performance of its underlying index, and that the fund's tracking error should equal its expense ratio. This is hardly a secret. One can find similar wording in the prospectus of almost every ETF.

III. Leveraged ETFs

However, not all ETFs are created alike. Calculating the tracking error for leveraged bull and bear ETFs is far more complicated than for unleveraged ETFs, especially over the long-term. In fact, the Financial Industry Regulatory Authority (2009) recently issued a regulatory notice to all investors warning investors that leveraged ETFs are specifically designed for short-term holding periods. Moreover, the long-term performance of these funds cannot be guaranteed because of the effects of compounding and volatility. As evidence, some companies are getting out of the leveraged ETF market. For example, Stein and Condon (2009) reported in the Bloomberg News that the UBS AG US brokerage firm stopped selling leverage ETFs. The brokerage firm's primary reason was because their leveraged ETF products contradicted the firm's emphasis on long-term investing.

On the contrary, other firms have expressed a desire to hold leveraged ETFs long-term. For example, Direxion has announced plans to introduce a leveraged ETF that seeks rebalancing every 30 days instead of on a daily basis. Furthermore, Hill and Foster (2009) list several reasons why investors may want to employ holding strategies for longer than one day. First, investors may invest long-term to express their directional view on the economy or segments of the market. In addition, investors could overweight or underweight an index to change positions within their overall portfolio. Investors may also want to manage long-term risk through hedging. Finally, investors can bet on one index outperforming another index by creating an index-spread strategy to capture the relative returns of two indexes.

Thus, there are contradictory views on utilizing long-term holding periods for leveraged bull and bear ETFs. The purpose of this paper is to explore these contradictory views. In particular, we mathematically examine the link between compounding and long-term leveraged ETF returns. We also explore the effects of expected market volatility on long-term holding strategies for leveraged bull and bear ETFs using regression analysis on the dataset detailed in the next section. Specific predictions for variables are addressed in the linear regression model section, which is followed by the results and conclusion.

IV. Morningstar and Chicago Board Options Exchange Dataset

We create our dataset by matching daily Morningstar ETF and index return data to daily Chicago Board Options Exchange (CBOE) volatility index data. The data range is from the inception of the first leveraged ETFs by ProShares on June 19, 2006, through the CBOE data cutoff on September 22, 2009. We first form a dataset of 219 ETFs by combining the 145 ETFs listed as leveraged bull, leveraged bear, and unleveraged bear ETFs in the Morningstar database with the 74 unleveraged bull ETFs that also tracked the same benchmark index.

These 219 ETFs are then reduced to 129 ETFs for the following three reasons. First, we exclude all ETFs that lack sufficient data for the benchmark index. Second, we discard all ETFs with an inception date after the (September 22, 2009) data limitations for the CBOE volatility index data. Third, we remove all exchange-traded notes (ETNs). Unlike traditional ETFs, ETNs more closely resemble a type of debt security in that they can be held until maturity and are partially valued on the credit rating of the issuer.

Table I lists the 129 ETFs in this study based on their leverage and benchmark index, respectively. The fund family for each ETF is also identified. The majority of the 39 unleveraged bull ETFs are primarily offered by iShares (29) followed by State-Street (7), Vanguard (2), and PowerShares (1). The majority of the 90 leveraged bull, leveraged bear, and unleveraged bear ETFs are primarily offered by ProShares (66). However, Rydex offers seven double-bull and seven double-bear ETFs, some of which track the same indexes as some of the

ETFs offered by ProShares. Direxion offers only five triple-bull and five triple-bear ETFs that are used in this study.

We match each stock ticker for our return data to the four CBOE volatility indexes (VIX, VXO, VXN, and VXD), which measure the market's expectation of volatility over the next 30-day period. The four volatility indexes are based on the S&P 500 index (VIX), the S&P 100 index options (VXO), the NASDAQ 100 (VXN), and the Dow Jones Industrial Average (VXD). We use the open daily measure (instead of the close, high, or low daily measures) for each CBOE volatility index because it captures the entire day of the first observation in our holding period strategies.

Of the four volatility indexes, our study primarily uses the VIX. First introduced by Whaley (1993), the VIX is easily the most well-known volatility index among investors today. As evidence of the popularity of the VIX, the CBOE began offering VIX futures in 2004 and VIX options in 2006. In addition, Whaley (2000) states that the VIX is often referred to as the "fear index" or "the investor fear gauge" because the price of options increases with the volatility of the market. Whaley (2009) puts the VIX to the test. Whaley examines 274 monthly trials using the VIX to predict future volatility in the S&P 500 Index. Whaley finds that 34.7% of the trials fell outside of the 50% range, 7.3% of the trials fell outside of the 75% range, and only 1.1% of the trials fell outside of the 95% range. Whaley concludes that the VIX works reasonably well as a predictor of expected stock index movements.

Like the VIX, there are six differently-leveraged ETFs in Table I that track the S&P 500 index (SPXU, SDS, SH, SPY, SSO, and UPRO). Since the VIX and these six ETFs all track the same index, we further isolate this sample of observations for additional testing in our results section. Note that for some leverage groups there is more than one ETF that tracks the S&P 500 index. In each case, we selected the fund with the earlier inception date in order to maximize observations. For example, both SSO and RSU are listed in Table 1 as double-bull ETFs that track the S&P 500. We selected SSO over RSU for additional testing in our results section because SSO had more observations.

Throughout our entire results section, we control for the expenses of all 129 ETFs by matching their respective expense ratio data and the risk-free rate of interest to the return and volatility data. Specifically, we use the respective prospectus for each ETF listed on the firm website to hand-collect each expense ratio. Consistent with Avellaneda and Zhang (2010), we include the daily 3-month LIBOR rate as the risk-free rate. Avellaneda and Zhang maintain that the risk-free rate is an important variable to include because a fund manager needs access to money market accounts and/or swap counterparties to maintain a constant leverage ratio with the fund's benchmark index. In addition, as a robustness check, we use the 3-month Treasury bill rate as the risk-free rate and yield similar results. The following section addresses all of our variables including expenses using a linear regression model.

Table I
Summary of ETFs and benchmark indexes

The 129 ETFs in Table I are separated by their leverage multiplier and benchmark index, respectively. Each ETF is also listed by its specific ticker followed by a one-letter abbreviation for its respective fund family (D = Direxion, I = iShares, P = ProShares, Q = PowerShares, R = Rydex, S = State-Street, and V = Vanguard). Note that more than one fund family offers similarly-leveraged ETFs tracking the Russell 2000, S&P 500, and S&P MidCap 400.

Benchmark Index	Leverage Multiple for ETFs					
	-3	-2	-1	1	2	3
BarCap US Tsy 20+ Yr		TBT_P	TBF_P	TLT_I		
BarCap US Tsy 7-10 Yr		PST_P		IEF_I		
DJ Industrial Avg		DXD_P	DOG_P	DIA_S	DDM_P	
DJ US Basic Materials		SMN_P		IYM_I	UYM_P	
DJ US Cons Goods		SZK_P		IYK_I	UGE_P	
DJ US Cons Services		SCC_P		IYC_I	UCC_P	
DJ US Financial		SKF_P	SEF_P	IYF_I	UYG_P	
DJ US Health Care		RXD_P		IYH_I	RXL_P	
DJ US Industrials		UXI_P		IYJ_I	SIJ_P	
DJ US Real Estate		SRS_P		IYR_I	URE_P	
DJ US Technology		REW_P		IYW_I	ROM_P	
DJ US Telecom		TLL_P		IYZ_I	LTL_P	
DJ US Utilities		SDP_P		IDU_I	UPW_P	
London Fix Silver		ZSL_P		SLV_I	AGQ_P	
MSCI Brazil		BZQ_P		EWZ_I		
MSCI EAFE	DPK_D	EFU_P	EFZ_P	EFA_I	EFO_P	DZK_D
MSCI Europe		EPV_P		VGK_V		
MSCI Pacific ex Japan		JPX_P		EPP_I		
MSCI US REIT GR	DRV_D			VNQ_V		DRN_D
NASDAQ 100		QID_P	PSQ_P	QQQQ_Q	QLD_P	
Russell 1000	BGZ_D			IWB_I		BGU_D
Russell 1000 Growth		SFK_P		IWF_I	UKF_P	
Russell 1000 Value		SJF_P		IWD_I	UVG_P	
Russell 2000	TZA_D	TWM_P & RRZ_R	RWM_P	IWM_I	UWM_P & RRY_R	TNA_D
Russell 2000 Growth		SKK_P		IWO_I	UKK_P	
Russell 2000 Value		SJH_P		IWN_I	UVT_P	
Russell 3000		TWQ_P		IWV_I	UWC_P	
Russell Mid Cap	MWN_D			IWR_I		MWJ_D
Russell Mid Cap Growth		SDK_P		IWP_I	UKW_P	
Russell Mid Cap Value		SJL_P		IWS_I	UVU_P	
S&P 500	SPXU_P	SDS_P & RSW_R	SH_P	SPY_S & IVV_I	SSO_P & RSU_R	UPRO_P
S&P 500 Energy		REC_R		XLE_S	REA_R	
S&P 500 Financials		RFN_R		XLF_S	RFL_R	
S&P 500 Health Care		RHO_R		XLV_S	RHM_R	
S&P 500 Info Tech		RTW_R		XLK_S	RTG_R	
S&P MidCap 400		MZZ_P & RMS_R	MYY_P	MDY_S & IJH_I	MVV_P &RMM_R	
S&P SmallCap 600		SDD_P	SBB_P	IJR_I	SAA_P	

V. Linear regression model

The primary focus for our regression models is to calculate tracking error for leveraged bull and bear ETFs over long-term holding periods. Holzhauer, Lu, McLeod, and Mehran (2013) define tracking error as the difference in return between the actual return for an ETF and the leverage multiple of the underlying index as stated in each leveraged ETF's respective prospectus. In order to calculate tracking error over long-term holding periods, we adapt the model developed by Holzhauer et al. for daily holding periods. Using equation (1), we calculate each ETF's compounded return by accounting for its respective leverage multiplier:

$$R_{L,i} = I_{L,i}M_L + \beta_1 E_{L,i} + \beta_2 V_{I,i} + \varepsilon \text{ where } E_{L,i} = (((1\text{-}M_L)r_i) - f_L) \div 252 \quad (1)$$

and where R = return of ETF, L = ticker specific ETF, i = the specific date of the start of the holding period, t = the holding period in trading days, I = return of benchmark index, M = multiplier or leverage group associated with ETF (i.e. -3, -2, -1, 1, 2, or 3), E = expenses, V = expected volatility of the index or market, ε = error term, r = annual interest rate on 3-month LIBOR, and f = annual expense ratio. It should be noted that we have only included two beta coefficients (instead of three) for our independent variables for continuity purposes with equation (2), which is a simplified version of equation (1). It should also be noted that Holzhauer et al. include ΔV in a second linear model when analyzing daily returns. However, there are currently not enough observations for many of the leveraged ETFs (especially the triple-leveraged ETFs) to appropriately analyze the impact of this variable. We plan to incorporate this variable into future studies with datasets that contain additional observations.

Consistent with Whaley (2000) and Lu, Wang, and Zhang (2009), we form our holding periods using trading days instead of calendar days. Specifically, we calculate compounded returns for the following holding periods: 2 days, 3 days, 4 days, 5 days (weekly), 10 days (bi-weekly), 21 days (monthly), 42 days (bi-monthly), 63 days (quarterly), 126 days (semiannually), and 252 days (annually). To maximize our number of observations, we drop the prior daily return and add the subsequent daily return to our calculations. Mathematically, we lose only t-1 observations by using this method to compound. However, even though our volatility index data is limited to September 22, 2009, our Morningstar return data was not. Thus, we used Morningstar return data through January 2010, and lost far less observations than t-1. To compound the daily returns for the ETFs and their benchmark indexes, we use the following models from Lu et al. (2009):

$$R_{L,t} = -1 + \prod_{i=1}^{i+t}\left(1 + R_{L,i}\right)$$

$$(I_L M_L)_t = -1 + \prod_{i=1}^{i+t}\left(1 + M_L I_{L,i}\right)$$

Notice that the daily index returns are multiplied by the leverage multiplier before compounding. For example, for t=2, $R_{L,2} = (I_L M_L)_2 = -1 + ((1 + M_L I_{L,1})(1 + M_L I_{L,2}))$. By factoring out this equation, we can see the effects of compounding on tracking error for the following six types of unleveraged and leveraged ETFs:

- 1X Unleveraged (Single-Bull) ETF = $R_{L,t} = 1I_{L,t} + 0\,(I_{L,1})\,(I_{L,2})$
- -1X Unleveraged (Single-Bear) ETF = $R_{L,t} = -1I_{L,t} + 2\,(I_{L,1})\,(I_{L,2})$
- 2X Leveraged (Double-Bull) ETF = $R_{L,t} = 2I_{L,t} + 2\,(I_{L,1})\,(I_{L,2})$

- -2X Leveraged (Double-Bear) ETF $= R_{L,t} = -2I_{L,t} + 6\,(I_{L,1})\,(I_{L,2})$
- 3X Leveraged (Triple-Bull) ETF $= R_{L,t} = 3I_{L,t} + 6\,(I_{L,1})\,(I_{L,2})$
- -3X Leveraged (Triple-Bear) ETF $= R_{L,t} = -3I_{L,t} + 12\,(I_{L,1})\,(I_{L,2})$

Notice that the effect of compounding on tracking error (i.e. the second term) is stronger with bear ETFs than with bull ETFs. Thus, we expect to see larger differences in tracking error as the leverage increases, especially for bear funds.

In addition to compounding, we expect that market volatility will also impact tracking error. One of the primary ways that market volatility affects tracking error for leveraged bull and bear ETFs is the constant leverage trap associated with the daily rebalancing requirement. The daily rebalancing requirement ensures that funds have to buy assets when the index has increased and sell assets when the index has decreased. In other words, the fund is trapped by its own leverage because it must constantly buy high and sell low. Moreover, the rebalancing demands of ETFs in general will only increase the volatility towards the market close. Consequently, the effects of daily rebalancing are especially prevalent for funds during periods when returns for assets within the underlying index are highly volatile but overall index returns remain stagnant. As evidence, Haryanto, Rodier, Shum, and Hejazi (2013) use trade data for U.S. blue-chip equities from 2006 to 2011 to examine the rebalancing demand of leveraged ETFs in driving up end-of-day volatility. They show that the impact of leveraged ETFs is statistically significant, and that this impact is also economically significant during volatile periods. Their findings also suggest that tracking error for leveraged ETFs is larger during periods of high volatility due to the daily rebalancing requirement. In addition, Shum and Kang (2013) show that tracking error increases for leveraged ETFs with a more volatile underlying index. Thus, previous research clearly shows that volatility plays a significant role in the tracking error for leveraged bull and bear ETFs.

After considering the effects of compounding and volatility, we developed the following two competing hypotheses. Our null hypothesis, H_0, is that compounding and expected market volatility will have no effect on the long-term tracking error for leveraged bull, leveraged bear, or unleveraged bear ETFs. In contrast, our competing hypothesis, H_1, is that compounding and expected market volatility will have an increasing effect on the long-term tracking error for these funds as both leverage and holding period increases. Furthermore, due to the effects of volatility and compounding, we expect these effects on tracking error to be stronger with bear ETFs than with bull ETFs. Our findings are reported in the following section.

VI. Results

To calculate the effect of expected market volatility on tracking error for leveraged bull and bear ETFs, the linear regression model shown in equation (1) was further simplified as shown below:

$$(D_{L,i})_t = (R_{L,i})_t - (I_{L,i}M_L)_t = \beta_1 E_{L,i} + \beta_2 V_{I,i} + \varepsilon \qquad (2)$$

where D = tracking error (i.e. return differences) between the ETF's actual return and stated return goal in its respective prospectus.

Expected market volatility for six leverage multiplier groups

Table II uses the equation above to provide separate results for the 11 holding period strategies for leveraged bull and bear ETFs. Table II shows that expected market volatility has a significant effect on tracking error for leveraged bull, leveraged bear, and unleveraged bear ETFs. This evidence clearly rejects H_0 and supports H_1. Notice in Panel B that the expected market volatility is significant for all leverage multipliers as long as the time horizon for holding strategies is greater than or equal to 10 trading days. The only exception is for unleveraged ETFs, which is only affected by expected market volatility when the time horizon includes a bi-monthly or longer holding period strategy.

As expected, the β_2, T-test, and F-test are higher for bear funds than similarly leveraged bull funds. The β_2, T-test, and F-test values also increase with leverage and over time. We suspect that the slight drop in β_2, T-test, and F-test values from the semi-annual holding period ($t = 126$) to the annual holding day ($t = 252$) is primarily due to a decrease in observations. For instance, due to data limitations from compounding returns, a total of 149 (23) fewer observations were recorded for each ETF in the annual (semi-annual) holding period strategy. However, the results suggest that the small loss of observations in the semi-annual period did not have a major impact on the results for this strategy.

Table II also shows that expenses had highly significant effects for all leverage multipliers once the time horizon was sufficiently high. In fact, the β_1 values converge to one as the time horizon increases. Once again, the chief exception is the unleveraged ETF, which is interesting considering the only expense included for the unleveraged ETF is the expense ratio. Thus, the expense ratios for the unleveraged ETFs are either inaccurate or insignificant (at least when regressed with expected market volatility).

Finally, one key observation from Table II is that the results for all 11 holding strategies show that expected market volatility positively (negatively) effects the tracking error for leveraged bull (bear) ETFs. The differing effect between leveraged bull and bear funds supports previous literature from Avellaneda and Zhang (2010) and Hill and Foster (2009), which argue that leveraged bear funds have much higher break-even levels than similarly-leveraged bull funds. Avellaneda and Zhang (2010) show algebraically that the underlying index returns and the realized variance of the underlying index determine the break-even levels for both leveraged bull and bear ETFs. They also show that variance has a much stronger effect on the break-even levels for double-bear ETFs than double-bull ETFs. Likewise, Hill and Foster (2009) use the absolute return of the S&P 500 index to define the break-even level for a double-bull ETF, and find that the effect of volatility on double-bull ETFs increases with the magnitude (i.e. absolute return) of the index returns. In other words, when the absolute return of the index was higher (lower) than the break-even level, the tracking error between the double-bull ETF and the twice leveraged S&P 500 index was positive (negative) and increasing with volatility. Thus, we form our second hypothesis, H_2, which states that when the magnitude of the underlying index returns is higher (lower) than the respective volatility-dependent break-even levels, the leveraged bull (bear) ETF return will be greater (less) than the multiple of the index return. We examine H_2 more closely later in this section. However, next we analyze a sub-sample of our overall data.

Table II
Impact of volatility on different holding strategies for leveraged ETFs

Table II uses the daily open value for VIX to test the effect of expected market volatility on long-term tracking error, controlled for expenses, for six different ETF groups based on their respective leverage multiplier (-3, -2, -1, 1, 2, 3). Table II assumes the following equation for long-term tracking error (i.e. return differences):

$$(D_{L,i})_t = (R_{L,i})_t - (I_{L,i}M_L)_t = \beta_1 E_{L,i} + \beta_2 V_{I,i} + \varepsilon$$

where t is equal to the number of trading days the ETF is held in 11 different holding strategies ranging from one trading day to 252 trading days. The total observation values in Table II correspond only to the first nine holding strategies. A total of 23 (149) fewer observations were recorded for each ETF in the 126 (252) trading day holding period strategy due to data limitations. Note that significance for F-test and T-test results are denoted by (***) at the .001 level, (**) at the 0.01 level, and (*) at the 0.05 level.

Panel A: Daily to Weekly Holding Strategies

Holding Period		Leverage Multiple for ETFs					
		-3	**-2**	**-1**	**1**	**2**	**3**
1 day	E	16.06	1.52***	1.50***	-0.34	1.59***	129.80
	V	-6.37E-03	-1.04E-03**	-1.04E-03	2.85E-05	4.78E-04**	4.02E-02
	F-test	1.56	49.18***	13.23***	0.07	14.86***	1.49
2 days	E	8.28	1.19***	1.22***	-0.99	1.11***	72.58*
	V	-7.58E-03*	-1.05E-03**	-1.17E-03***	-8.15E-06	2.43E-04	4.44E-02*
	F-test	2.36	112.90***	37.24***	0.26	39.53***	2.34
3 days	E	6.02	1.16***	1.17***	-0.52	1.15***	47.24*
	V	-8.99E-03*	-1.51E-03***	-1.50E-03*	-1.40E-05	3.87E-04	4.30E-02*
	F-test	2.87	240.38***	88.26***	0.13	92.07***	2.29
4 days	E	4.91	1.16***	1.16***	-0.42	1.14***	39.74*
	V	-1.05E-02**	-2.02E-03***	-1.94E-03**	3.64E-05	5.34E-04	4.78E-02*
	F-test	4.41*	380.22***	141.92***	0.32	148.24***	2.46
5 days	E	4.32	1.14***	1.13***	-0.41	1.11***	32.49*
	V	-1.20E-02**	-2.55E-03***	-2.40E-03***	5.05E-05	6.51E-04	4.87E-02*
	F-test	5.90**	532.56***	201.48***	0.49	224.56***	2.24

Table II (continued)

Panel B: Bi-weekly to Annual Holding Strategies

Holding Period		Leverage Multiple for ETFs					
		-3	-2	-1	1	2	3
10 days	E	-0.02*	1.15***	1.12***	-0.45	1.13***	24.04**
	V	-1.90E-02***	-4.85E-03***	-4.61E-03***	6.10E-05	1.33E-03***	7.01E-02**
	F-test	12.72***	1389.46***	572.15***	1.22	682.47***	5.49**
21 days	E	1.89**	1.15***	1.10***	-0.37	1.12***	15.64***
	V	-2.96E-02***	-1.01E-02***	-9.25E-03***	3.33E-04	3.10E-03***	9.15E-02***
	F-test	33.16***	3130.79***	1569.97***	4.90***	1857.15***	15.39***
42 days	E	1.74***	1.12***	1.05***	-0.47*	1.12***	8.44***
	V	-5.42E-02***	-1.98E-02***	-1.69E-02***	7.12E-04*	7.39E-03***	9.57E-02***
	F-test	92.40***	5129.40***	3346.38***	15.10***	3956.86***	16.66***
63 days	E	0.83**	1.10***	1.02***	-0.57***	1.08***	6.42***
	V	-5.66E-02***	-2.93E-02***	-2.39E-02***	8.19E-04*	1.09E-02***	1.08E-01**
	F-test	126.74***	7772.90***	5699.88***	26.80***	6783.99***	18.38***
126 days	E	0.81***	1.02***	0.93***	-0.46***	0.90***	1.72
	V	-8.13E-02***	-6.22E-02***	-4.79E-02***	2.06E-03***	1.36E-02***	4.54E-02
	F-test	143.97***	13792.03***	11016.01***	46.87***	16729.85***	6.99***
252 days	E	0.41*	0.88***	0.82***	0.16	0.61***	0.18
	V	-7.89E-02***	-1.30E-01***	-1.07E-01***	8.58E-03***	1.63E-02***	-7.04E-02
	F-test	38.25***	11784.82***	6571.90***	57.73***	10047.37***	49.38***
Total Observations		913	18,895	5,267	31,662	17,658	914

Expected market volatility for six differently-leveraged ETFs

Unlike the general leverage multiplier groups found in the previous table, Table III shows the effects of expected market volatility for six differently-leveraged ETFs specifically tracking the S&P 500 (SPXU, SDS, SH, SPY, SSO, and UPRO). As mentioned in the data collection section, to maintain consistency within our results, we include only one ETF for each leverage multiplier even though there are two ETFs to choose from for unleveraged bull, double-bull, and double-bear ETFs. In the case of multiple ETFs, priority was given to the ETF with the earliest inception date because it contained more observations.

As expected, the results in Table III are similar to the results in Table II. Expected market volatility has a significant effect on tracking error. The chief exceptions are the triple-bear ETF (SPXU) and triple-bull ETF (UPRO), which most likely have insignificant results because only 63 observations were recorded for both of these funds.

Table III
Impact of volatility on different holding strategies for differently-leveraged ETFs tracking the S&P 500 index

Table III uses the daily open value for VIX to test the effect of expected market volatility on long-term tracking error, controlled for expenses, for six differently-leveraged ETFs tracking the S&P 500 index. Table III assumes the following equation for long-term tracking error (i.e. return differences):

$$(D_{L,i})_t = (R_{L,i})_t - (I_{L,i}M_L)_t = \beta_1 E_{L,i} + \beta_2 V_{I,i} + \varepsilon$$

where t is equal to the number of trading days the ETF is held in 11 different holding strategies ranging from one trading day to 252 trading days. The total observation values in Table III correspond only to the first nine holding strategies. A total of 23 fewer (zero) observations were recorded for each ETF in the 126 (252) trading day holding period strategy due to data limitations. Note that significance for F-test and T-test results are denoted by (***) at the .001 level, (**) at the 0.01 level, and (*) at the 0.05 level.

Panel A: Daily to Weekly Holding Strategies

Holding Period		ETFs Tracking the S&P 500 Index (Leverage Multiple)					
		SPXU (-3)	SDS (-2)	SH (-1)	SPY (1)	SSO (2)	UPRO (3)
1 day	E	145.61	1.18***	1.14***	2.16	1.34	42.52
	V	-2.42E-02	-5.91E-04	-3.19E-04*	6.21E-06	3.40E-04	1.43E-02
	F-test	1.11	7.41***	13.12***	0.04	1.67*	0.60
2 days	E	5.26	1.02***	1.02***	-0.53	1.00*	-47.78
	V	-1.88E-03	-5.00E-04	-3.00E-04	-3.36E-05	-1.46E-05	-2.72E-02
	F-test	0.06	25.35***	44.61***	0.48	7.20**	0.97
3 days	E	6.05	1.02***	1.02***	0.33	1.01***	-47.90
	V	-3.04E-03	-7.40E-04	-4.49E-04	-1.01E-05	-1.90E-05	-4.14E-02
	F-test	0.02	55.49***	99.02***	1.04	15.57***	0.60
4 days	E	6.07	1.02***	1.01***	0.38	1.01***	-46.14
	V	-3.83E-03	-9.86E-04**	-5.98E-04**	-9.58E-06	-1.81E-05	-5.34E-02
	F-test	0.07	101.97***	173.10***	1.36	30.79***	0.72
5 days	E	6.21	0.99***	0.99***	0.55	1.02***	-31.97
	V	-4.82E-03	-1.17E-03*	-7.28E-04**	7.19E-07	1.13E-05	-4.66E-02
	F-test	0.09	194.26***	355.07***	1.83	42.77***	0.33

Table III (continued)

Panel B: Bi-weekly to Annual Holding Strategies

Holding Period		ETFs Tracking the S&P 500 Index (Leverage Multiple)					
		SPXU (-3)	SDS (-2)	SH (-1)	SPY (1)	SSO (2)	UPRO (3)
10 days	E	8.84	1.00***	1.02***	0.88	1.03***	-5.68
	V	-1.27E-02	-2.33E-03***	-1.53E-03***	5.54E-05	1.40E-04	-1.92E-02
	F-test	0.84	792.29***	1122.82***	4.40*	162.76***	0.23
21 days	E	9.04*	1.01***	1.01***	0.96**	1.04***	12.91
	V	-2.73E-02	-5.02E-03***	-3.29E-03***	1.33E-04	6.31E-04	6.91E-02
	F-test	2.40	2466.94***	4303.12***	9.62***	655.45***	4.79*
42 days	E	5.50*	0.99***	1.00***	0.97***	1.05***	9.90
	V	-3.62E-02*	-1.02E-02***	-6.96E-03***	2.70E-04	2.24E-03***	9.93E-02
	F-test	3.31*	7113.52***	12678.95***	16.30***	2094.82***	3.30*
63 days	E	2.39	0.97***	0.99***	1.19***	1.07***	4.73
	V	-3.22E-02*	-1.53E-02***	-1.09E-02***	6.17E-04**	5.03E-03***	7.09E-02
	F-test	8.50***	10307.00***	20520.15***	28.75***	3849.48***	7.88***
126 days	E	0.24***	0.94***	0.96***	-0.04	1.02***	1.14*
	V	-1.45E-02***	-3.53E-02***	-2.56E-02***	-9.85E-04***	1.06E-02***	-1.18E-02
	F-test	698.46***	14373.67***	24217.86***	19.40***	3300.28***	1126.28***
252 days	E	N/A	0.92***	0.92***	-0.36***	0.88***	N/A
	V	N/A	-9.79E-02***	-6.97E-02***	-1.43E-03***	3.27E-02***	N/A
	F-test	N/A	9161.01***	18133.83***	10.30***	1124.27***	N/A
Total Observations		63	806	821	821	821	63

Average tracking error for six leverage multiplier groups

Table IV shows the average tracking error for each of the 11 holding period strategies for the six different leverage multipliers. Notice that tracking error for each holding period is listed before and after adjusting for expenses. The results for tracking error before adjusting for expenses show that the absolute value for tracking error increases over time for all six leverage multipliers. However, the tracking error does not always increase in the same direction for every multiplier or even for bear funds compared to similarly-leveraged bull funds.

To better gage tracking error, Table IV lists tracking error after adjusting for expenses. Once again, the absolute value for tracking error increases over time for all six leverage multipliers. However, the tracking error after adjusting for expenses decreases for the bear funds and increases for the bull funds. Intuitively, this effect of expenses on tracking error makes sense considering that borrowing expenses are negative (positive) for bull (bear) funds. Thus, in the case of bull (bear) funds, tracking error before adjusting for expenses is less (greater) than tracking error after adjusting for expenses. Furthermore, Table IV shows that tracking error after adjusting for expenses increases (decreases) as the time horizon increases for bear (bull) funds.

Table IV
Tracking error for different holding strategies for leveraged bull and bear ETFs

Table IV lists the average tracking error for six different ETF groups based on their respective leverage multiplier (-3, -2, -1, 1, 2, 3). The tracking error is listed for 11 different holding strategies ranging from one trading day to 252 trading days. Table IV uses the following respective equations for calculating long-term tracking error (i.e. return differences) before and after adjusting for expenses:

$$(D_{L,i})_t = (R_{L,i})_t - (I_{L,i}M_L)_t + \varepsilon$$

$$(D_{L,i} - E_{L,i})_t = (R_{L,i})_t - (I_{L,i}M_L)_t - E_{L,i} + \varepsilon$$

where t is equal to the number of trading days the ETF is held.

Holding Period		**Leverage Multiple for ETFs**					
		-3	**-2**	**-1**	**1**	**2**	**3**
1 day	D	-0.03	0.01	0.01	0.00	-0.01	0.01
	D - E	-0.04	-0.02	-0.01	0.00	0.01	0.02
2 days	D	-0.07	0.04	0.03	0.00	-0.02	-0.01
	D - E	-0.09	-0.02	-0.02	0.00	0.01	0.01
3 days	D	-0.10	0.06	0.04	0.00	-0.04	-0.02
	D - E	-0.13	-0.03	-0.02	0.00	0.01	0.02
4 days	D	-0.14	0.08	0.06	0.00	-0.05	-0.03
	D - E	-0.18	-0.04	-0.03	0.01	0.01	0.01
5 days	D	-0.17	0.10	0.07	0.00	-0.06	-0.03
	D - E	-0.22	-0.06	-0.04	0.01	0.02	0.02
10 days	D	-0.31	0.20	0.13	0.00	-0.13	-0.10
	D - E	-0.42	-0.11	-0.08	0.02	0.03	0.01
21 days	D	-0.64	0.43	0.27	0.01	-0.26	-0.25
	D - E	-0.86	-0.22	-0.18	0.04	0.06	-0.02
42 days	D	-1.17	0.87	0.52	0.03	-0.51	-0.35
	D - E	-1.62	-0.43	-0.39	0.08	0.14	0.11
63 days	D	-1.54	1.27	0.75	0.04	-0.74	-0.39
	D - E	-2.21	-0.67	-0.62	0.11	0.23	0.29
126 days	D	-1.94	2.47	1.32	0.08	-1.44	-0.59
	D - E	-3.48	-1.60	-1.55	0.23	0.56	0.88
252 days	D	-2.07	5.13	2.73	0.08	-2.37	-4.53
	D - E	-7.17	-5.05	-4.21	0.37	2.30	-0.58

Monthly tracking error deciles for double-bull and double-bear ETFs

Table V provides monthly tracking error deciles for double-bull and double-bear ETFs to examine the impact of expenses, expected market volatility, and the magnitude of index returns on tracking error. We specifically define the magnitude of index returns as the absolute value of the multiple of the underlying index return. Only nine deciles are shown in Table V because the values for the lowest and highest five percent of the monthly tracking error were truncated to adjust for extreme observations among outliers.

As expected, Table V clearly shows that as tracking error increases for double-bull (double-bear) ETFs, the monthly expenses increase, and the expected market volatility increases (decreases) for all four CBOE volatility indexes. However, the most interesting result is that the magnitude of index returns has a positive (negative) impact on tracking error for the double-bull (double-bear) ETFs. These results support prior research from Avellaneda and Zhang (2010) and Hill and Foster (2009) by providing additional evidence of different volatility-dependent break-even levels for leveraged bull ETFs compared to similarly-leveraged bear ETFs. The results also clearly support H_2, which states that if the magnitude of the underlying index returns is higher (lower) than the respective volatility-dependent break-even levels, the leveraged bull or bear ETF return will be greater (less) than the multiple of the underlying index return.

Table V
The impact of volatility, expenses, and the magnitude of index returns on monthly tracking error for double-bull and double-bear ETFs

Using monthly tracking error deciles, Table V lists the median values for the following variables respectively: monthly tracking error, monthly magnitude of index returns, monthly expenses, and the expected market volatility of four CBOE volatility indexes. Table V assumes the following equation for long-term tracking error (i.e. return differences):

$$(D_{L,i})_t = (R_{L,i})_t - (I_{L,i}M_L)_t = \beta_1 E_{L,i} + \beta_2 V_{I,i} + \varepsilon$$

where t is equal to 21 trading days (i.e. one month). The monthly magnitude (denoted as MAG in the table below) is equal to the absolute value of $(I_{L,i}M_L)_{t=21}$. Note that only nine deciles are provided below because the values for the lowest and highest five percent of tracking error were truncated to adjust for extreme observations among outliers.

Holding Period		Tracking Error Deciles								
		5% - 15%	15% - 25%	25% - 35%	35% - 45%	45% - 55%	55% - 65%	65% - 75%	75% - 85%	85% - 95%
		Median Values for Double-Bull ETFs								
21 days	D	-0.60	-0.54	-0.47	-0.37	-0.29	-0.23	-0.17	-0.11	-0.01
	MAG	7.32	4.67	6.54	10.52	7.58	12.02	12.00	10.12	13.94
	E	-0.53	-0.53	-0.51	-0.34	-0.31	-0.30	-0.18	-0.14	-0.18
	V=VIX	15.10	15.25	20.25	24.25	24.20	25.48	30.85	29.70	36.50
	V=VXO	15.04	14.79	20.97	26.27	25.69	26.96	30.51	29.24	38.22
	V=VXN	18.67	18.92	22.66	28.43	27.83	28.87	31.46	30.49	37.58
	V=VXD	14.26	14.37	18.74	22.33	22.30	23.42	27.68	26.29	33.32
		Median Values for Double-Bear ETFs								
21 days	D	-0.50	-0.15	-0.05	0.31	0.47	0.76	1.06	1.17	1.33
	MAG	12.33	12.64	11.34	8.40	8.07	8.56	7.45	5.48	7.19
	E	0.23	0.09	0.17	0.61	0.62	0.69	1.26	1.26	1.26
	V=VIX	42.35	31.19	32.97	24.02	22.85	25.58	20.34	15.08	16.50
	V=VXO	42.84	30.64	33.95	25.13	24.27	27.32	20.93	15.04	16.66
	V=VXN	42.41	31.47	34.05	28.14	26.61	28.80	22.83	18.78	18.68
	V=VXD	37.62	27.94	29.26	22.21	21.21	23.18	18.34	14.26	15.59

Expected market volatility deciles for double-bull and double-bear ETFs

Finally, Table VI provides expected market volatility deciles for double-bull and double-bear ETFs to examine the relationship between expected market volatility and tracking error for the 11 holding period strategies. Like Table V, only nine deciles are shown in Table VI because the lowest and highest five percent of the VIX values were truncated to adjust for extreme observations among outliers. The results show that as volatility increases, tracking error for the double-bull (double-bear) ETFs increases (decreases). These results suggest that volatility has a positive (negative) effect on returns for bear (bull) ETFs. The difference in the average tracking error for the lowest and highest deciles also increases as the holding period increases, suggesting that volatility has an increasing effect on tracking error over time. These results support both H_1 and the previous results from our linear regression analysis. Our findings also suggest that volatility indexes may be useful to sophisticated investors in devising trading rules for investing in long-term strategies for leveraged bull and bear ETFs. However, we acknowledge the limitations of this suggestion in our conclusion section.

Table VI
Expected market volatility deciles and long-term tracking error for double-bull and double-bear ETFs

Using the daily open value for VIX to form expected market volatility deciles, Table VI lists the median tracking error for 11 holding periods ranging from one trading day to 252 trading days for both double-bull ETFs and double-bear ETFs. Table VI assumes the following equation for long-term tracking error (i.e. return differences):

$$(D_{L,i})_t = (R_{L,i})_t - (I_{L,i}M_L)_t = \beta_1 E_{L,i} + \beta_2 V_{I,i} + \varepsilon$$

where t is equal to the number of trading days the ETF is held. Note that only nine deciles are provided below because the lowest and highest five percent of expected market volatility values were truncated to adjust for extreme observations among outliers.

Holding Period		Expected Market Volatility Deciles								
		5% - 15%	**15% - 25%**	**25% - 35%**	**35% - 45%**	**45% - 55%**	**55% - 65%**	**65% - 75%**	**75% - 85%**	**85% - 95%**
		Median VIX Values								
		-0.60	-0.54	-0.47	-0.37	-0.29	-0.23	-0.17	-0.11	-0.01
		Median Tracking Error for Double-Bull ETFs								
1 day	D	-0.02	-0.02	-0.02	-0.02	-0.02	-0.01	-0.01	-0.01	-0.01
2 days	D	-0.04	-0.04	-0.04	-0.03	-0.03	-0.02	-0.03	-0.02	-0.02
3 days	D	-0.08	-0.08	-0.08	-0.05	-0.05	-0.04	-0.05	-0.02	-0.03
4 days	D	-0.10	-0.11	-0.10	-0.07	-0.06	-0.05	-0.06	-0.03	-0.04
5 days	D	-0.13	-0.13	-0.12	-0.08	-0.08	-0.06	-0.08	-0.04	-0.05
10 days	D	-0.26	-0.26	-0.24	-0.15	-0.15	-0.12	-0.14	-0.07	-0.09
21 days	D	-0.55	-0.56	-0.50	-0.31	-0.29	-0.23	-0.30	-0.15	-0.17
42 days	D	-1.15	-1.12	-0.99	-0.57	-0.56	-0.39	-0.58	-0.29	-0.34
63 days	D	-1.81	-1.61	-1.57	-0.77	-0.69	-0.47	-0.86	-0.46	-0.52
126 days	D	-3.80	-3.16	-2.62	-0.94	-0.94	-0.95	-1.44	-0.95	-1.10
252 days	D	-6.97	-4.93	-3.56	-1.17	-1.13	-1.18	-1.09	-1.41	-2.33

Table VI (continued)

Holding Period		**Expected Market Volatility Deciles**								
		5% - 15%	**15% - 25%**	**25% - 35%**	**35% - 45%**	**45% - 55%**	**55% - 65%**	**65% - 75%**	**75% - 85%**	**85% - 95%**
		Median VIX Values								
		-0.60	-0.54	-0.47	-0.37	-0.29	-0.23	-0.17	-0.11	-0.01
		Median Tracking Error for Double-Bear ETFs								
1 day	D	0.04	0.04	0.04	0.03	0.03	0.01	0.02	0.00	0.00
2 days	D	0.09	0.09	0.08	0.06	0.05	0.03	0.05	0.00	-0.01
3 days	D	0.18	0.19	0.16	0.08	0.07	0.04	0.08	-0.01	-0.02
4 days	D	0.23	0.24	0.22	0.11	0.10	0.05	0.11	-0.01	-0.02
5 days	D	0.28	0.28	0.26	0.12	0.12	0.07	0.14	-0.02	-0.03
10 days	D	0.55	0.56	0.54	0.25	0.25	0.16	0.26	-0.04	-0.05
21 days	D	1.15	1.17	1.15	0.67	0.54	0.32	0.51	-0.08	-0.12
42 days	D	2.26	2.35	2.29	1.70	1.14	0.44	0.90	-0.14	-0.21
63 days	D	3.38	3.65	3.28	2.62	1.96	0.25	1.32	-0.19	-0.25
126 days	D	6.33	6.80	6.73	5.25	3.99	2.44	2.67	-0.32	-0.32
252 days	D	12.00	12.05	11.68	5.37	4.74	6.60	8.43	1.05	-0.47

VII. Conclusion

We focus on the effects of expected market volatility on long-term returns for leveraged bull and bear ETFs. Using Morningstar return data and CBOE volatility index data, we examine the effects of compounding and expected market volatility on 11 different holding period strategies for leveraged bull and bear ETFs. We find evidence that the compounded leveraged returns over most holding periods are comparable to compounding the multiple of the underlying daily benchmark returns. In other words, our results suggest that, for the most part, the returns for these funds adequately match the prospectus investment objectives.

Controlling for expenses, we show empirical evidence that expected market volatility has a significant effect on the long-term tracking error between the compounded returns for leveraged bull and bear ETF returns and the compounded returns of the multiple of their respective benchmark indexes. We also find that this effect increases with leverage and with the time horizon for long-term holding strategies. Our results also suggest that the effect of expected market volatility is stronger for bear funds than similarly-leveraged bull funds. Finally, we observe a contrasting effect for expected market volatility on the tracking error for leveraged bull ETFs compared to similarly-leveraged bear ETFs. This evidence provides support for different volatility-dependent break-even levels for leveraged bull ETFs versus bear ETFs.

In conclusion, our results suggest that volatility indexes may be used by sophisticated investors to devise trading rules for long-term holding strategies for leveraged bull and bear ETFs. However, we acknowledge that designing any specific holding strategy based on volatility goes beyond the scope of this paper. In the future, we plan to break tracking error and expected market volatility data into deciles for each specific leveraged ETF to explore using volatility indexes to devise trading rules for investing in leveraged ETFs long-term. We would also like to reexamine triple-bull and triple-bear ETFs as more data becomes available with time. With additional data, we also plan to investigate the effects of the "change in expected market volatility" variable similar to Holzhauer et al. (2013). Holzhauer et al. apply this variable to daily holding strategies for leveraged ETFs, whereas we would like to analyze this variable for long-term holding strategies. In particular, we plan to use time series and trend analysis to further explore the effects that the "change in expected market volatility" variable has on specific break-even levels and on long-term tracking error for leveraged bull and bear ETFs. Finally, as more diverse leveraged ETF products become available to investors, it would be interesting to compare the tracking error of traditional, daily-rebalanced funds to any new funds that rebalance at longer-term intervals.

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