Detection of Multiple Beta Shifts in Mutual Fund Returns Data

Thomas S. Howe and Ralph A. Pope

Abstract

Research suggests that mutual fund betas are not stationary. However, the performance of the nonstationarity tests has been called into question. This study examines the ability of four such tests. None of the tests shows good ability to detect sudden 25 percent beta shifts, and two of the tests have inflated, one of them grossly inflated, Type I error rates.

I. Introduction

For nearly four decades, studies have used the single index market model (Sharpe 1963) to analyze risk or performance of stocks, often in response to an information-containing event. The market model is

$$R_{it} = a_i + b_i R_{mt} + u_{it} \quad (1)$$

where R denotes return, i and t denote the asset and time period, m denotes the market, b is the beta, that is, the sensitivity of the asset returns to the market returns, a is the intercept, and u is an error term. Despite studies which question the appropriateness of this model, for example, Fama and French (1992), beta continues to be widely reported by providers of data used by individual and institutional investors.

Early research into the consistency of the beta of common stocks over time indicates that the betas of individual assets are nonstationary (Levy 1971). Recent studies which acknowledge this nonstationarity include Graddy, Kyle, Strickland, and Bass (2004) and Kaplanski (2004). One parameter stability model is the shifting regimes model (Mehta and Beranek 1982; Bey 1983; McDonald 1983; Hays and Upton 1986; Bauer Hays and Upton 1987), in which the return-generating process is assumed to follow a stationary regime for n_1 periods, then shift to another stationary regime for n_2 periods, and so on. The shifts are considered to be infrequent; that is, for any given k, n_k is large enough for reliable estimation of the parameters.

Major difficulties in applying the shifting regimes model are the detection and location of regime shifts. Possible techniques include the Chow (1960), Farley-Hinich (1970), recursive residual (RR) (Brown, Durbin, and Evans 1975), and dummy variable (Harvey 1976) tests, and variable parameter regression (Garbade 1977) for the detection of a change, and the Quandt log likelihood ratio (LR) (1958,1960) for both detection and location of the change. The RR and LR are often used in combination, the first to detect the presence of a shift and the second to estimate the shift point. Garbade (1977) and Farley, Hinich, and McGuire (1975), however, raise questions about the suitability of the RR and LR techniques.

To the extent that the methodology is inadequate, the previous research is called into question. For example, Howe and Upton (1992), suggest that an individual stock's beta shift is difficult to detect reliably, thus calling into question the results of studies such as Hays and Upton (1986). In addition, Howe and Pope (2005, 2006) suggest that attempts to detect multiple beta shifts in daily stock return data are fruitless.

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Because mutual funds are portfolios, in which normally no one security has a weight of more than five percent, one would expect the error variance of market model regressions on mutual fund returns to be lower than the error variance of market model regressions on returns for individual stocks. Thus, the results of Howe and Pope (2005, 2006) may not apply to mutual fund returns data. If a procedure intended to detect beta changes has any ability to detect beta changes this ability should be greater for mutual fund returns than for stock returns.

The study evaluates the ability to detect beta changes in mutual funds. Miller and Gressis (1980) and Bauer, Hays, and Upton (1987) apply beta change detection methods to actual returns on mutual funds. However, because one does not know whether a beta change actually occurred, one cannot draw conclusions regarding the power or Type I error rate of their tests. The study applies the methodology of Howe and Pope (2005, 2006) to simulated mutual fund return data to examine the power, Type I error rate, and outlier sensitivity of the partition regression (Guthery 1974; Miller and Gressis 1980) and sequential variations of the Brown-Durbin-Evans (BDE) cumulative sum of the squared recursive residuals (Brown, Durbin, and Evans 1975; Hays and Upton 1986), stabilogram (Ashley 1984), and variable parameter regression (Garbade 1977) techniques.

This study finds both procedures used by Miller and Gressis (1980) and Bauer, Hays, and Upton (1987) to be unsatisfactory—the recursive residual procedure primarily because of its low power and the partition regression procedure because of its extremely high Type I error rate. It finds the cumulative sum of the squared recursive residuals procedure to be sensitive to extreme observations, though less so than in Howe and Upton (1992) and Howe and Pope (2005, 2006). On the other hand, the variable parameter and stabilogram procedures do not show such high Type I error rates, and show much more power than the recursive residuals test. Only the variable parameter and stabilogram procedures show any appreciable ability to detect a second beta shift; even they fail to detect a second beta shift more than 22.5 percent of the time. None of the methods shows any appreciable ability to detect any beta changes beyond the second. Still, the procedures show better detection ability when applied to simulated mutual fund returns than when applied to simulated returns on individual stocks.

II. Methodology

Techniques for Detection of Parameter Shifts

Variable Parameter Regression (VPR)

Variable parameter regression assumes that beta follows a random walk. If we express this as $b_t = b_{t-1} + P$, where b_t is the beta for period t and P is the drift parameter, the null hypothesis is that P=0. The maximum likelihood estimate of P is asymptotically chi-squared with one degree of freedom. A full discussion of variable parameter regression can be found in Garbade (1977).

Stabilogram (STAB)

The stabilogram procedure divides the period to be studied into a number of subperiods of approximately equal length. Dummy and interactive variables are used to obtain ordinary least squares estimates of the market model parameters for the subperiods. The stabilogram procedure considers the market model parameters to be unstable if the null hypothesis that the market model parameters for all subperiods are equal is rejected.

Cumulative Sum of the Squared Recursive Residuals (RR)

Recursive residuals are obtained by recursively computing the standardized prediction error of r_{it} when the market model parameters are estimated from the preceding t-1 observations. Under the null hypothesis of no parameter change, the errors are independent and distributed $N(0,s^2)$. A shift in a_i, b_i, or the error variance is indicated if a residual shows significant departure from zero. BDE present tests on the cumulative sum (cusum) of the recursive residuals and on the cusum of the squared recursive residuals. This study uses the latter test because Garbade (1977) found it to be much more powerful than the former. A full discussion of the technique can be found in Brown, Durbin, and Evans (1975).

One problem with the RR technique is that the distribution of the test statistic is known only under the null hypothesis, and will be violated in the presence of multiple shifts. Additionally, it has been suggested that the RR technique is sensitive to outliers or other departures from the assumed conditions. In response to these objections, several papers (for example, Hays and Upton (1986) and Johnson (1989)) have used a sequential approach for the RR technique, in which the RR technique is applied to progressively longer periods to avoid the possibility of multiple shifts. In this approach, shift indications are accepted as valid only if consistently observed, and the analysis is restarted after a shift has been detected. A full discussion of this approach can be found in Hays and Upton (1986).

Howe and Upton (1992) suggest that the VPR technique does not show this outlier sensitivity. Also, one can infer from their Chow test results that the STAB technique would likely be insensitive to outliers. However, the VPR and STAB techniques, as originally presented, test for general instability rather than multiple shifts. To attempt to detect multiple beta shifts, this study modifies these techniques by using a sequential approach.

Garbade (1977) compared the ability of RR and VPR to detect instability in the coefficients of a linear regression model, and found that the VPR dominated. In addition to comparing a larger number of techniques, this study differs from Garbade in using conditions which are more realistic for event-study applications, and by including a number of conditions not investigated by Garbade. Additionally, Garbade used only totally simulated data, while this study uses simulations based on observed market data to include the possible effects of instabilities in the market process.

Partition Regression (PR)

The partition regression approach presented by Guthery (1974) and applied by Miller and Gressis (1980) differs from the RR, VPR, and STAB approaches in that it was originally designed to attempt to identify multiple parameter shifts rather than a single shift. Partition regression employs the following algorithm.

1. Divide the period into subperiods (initially two) so as to minimize the error sum of squares.

2. For each set of neighboring subperiods, use Chow (1960) tests to test the null hypothesis that the market model parameters of the two subperiods are equal.

3. If the null hypothesis is rejected for all neighboring subperiods, increase the number of subperiods by one and go to step 1.

4. If any of the Chow tests on neighboring subperiods are not rejected, stop the procedure. The number of parameter shifts is considered to be two fewer than the number of subperiods most recently examined.

<u>Technique for Shift Point Locations</u>

The log likelihood ratio (LR) can be calculated as follows:

$$LR = \ln \left[\frac{\text{likelihood of the observations under } H_0 \text{ (no parameter change)}}{\text{likelihood of the observations under } H_1} \right]$$

It can be used to detect the presence of a shift by examining the size of the minimum value over the period of interest. The distribution of the ratio has not yet been specified, however, and application of the technique to the detection of a shift itself is judgmental in nature. Because of the qualitative nature of the technique, it is sometimes used in conjunction with other techniques (such as the RR) as a method of locating the shift point. That is, given that a shift has been detected within a series of observations, the likely location of the shift is at the minimum of the ratio.

III. Return Simulations

Market-based simulated mutual fund return series are generated using the following form

of the single index model:

$$r_{it} = r_{ft} + b_{1it}(r_{mt} - r_{ft}) + u_{it}\,, \tag{2}$$

where:

r_{it} = the return of simulated mutual fund i in period t
b_{1it} = the beta coefficient for mutual fund i in period t
u_{it} = error term
r_{mt} = observed weekly returns to the S&P 500
r_{ft} = weekly yield on 90-day Treasury bills.

The choice of which differencing interval (daily vs. weekly vs. monthly) to use is not clear. On one hand, increasing the differencing interval improves the fit of the market model. On the other hand, increasing the differencing interval reduces the number of observations. For comparability with Miller and Gressis (1980) and Bauer, Hays and Upton (1987), each series contains 104 weekly returns. These simulated returns are based on the S&P 500 and Treasury bill returns over the period from January 3, 2001 through March 31, 2007, with the week of September 11, 2001 and the following week deleted because of the closing of markets and unusual market volatility during those weeks.

Two hundred simulated return series (corresponding to 200 funds) are created. The betas are estimated from a set of 200 growth, growth/income, equity income, aggressive growth, small cap, and mid cap ("traditional equity") funds that is randomly chosen except that funds with beta estimates of less than 0.0 or more than 2.0 are excluded.

In order to provide a variety of conditions under which the techniques can be tested, the parameters in (2) are varied as follows:

b_{1it}: Eight conditions of change: (a) constant beta (no change), (b) a 0.25 increase in beta, (c) two 0.25 increases in beta, (d) three 0.25 increases in beta, (e) a 0.25 increase in beta, followed by a 0.25 decrease in beta, so the beta after the second shift is the same as the beta before the first shift, (f) a 0.25 increase in beta, followed by a 0.25 decrease in beta, followed by a 0.25 increase in beta, (g) two 0.25 decreases in beta followed by a 0.25 increase in beta, and (h) two 0.25 increases in beta followed by a 0.25 decrease in beta For each simulated fund, three beta change dates are selected at random from a uniform distribution consisting of weeks 3 through 101.

u_{it}: Two distributions of error terms: a) Normal (Stable Paretian with $\alpha = 2.0$) and b) Stable Paretian with $\alpha = 1.95$. These values are consistent with those observed by Fama (1965). The mean of the distributions is zero. The variance (scale factor in the case of the Stable Paretian 1.95 error terms) is estimated from a randomly chosen set of 200 traditional equity funds.

This results in eight beta change conditions and two error term distributions, or 16 (8×2) data sets. Each data set is composed of 104 observations for each of 200 funds.

<u>Procedure</u>

This study applies the detection techniques to the sixteen data sets at the 0.05 significance level. The RR test is performed on the first four observations. After that, we increased the interval tested four observations at a time between RR tests. In this study, for a shift indication to be considered consistent and persistent one of the following conditions has to be met:

1. three consecutive indications of significance, all with the same LR estimate of the location of the shift

2. four consecutive indications of significant, with three identical LR estimates of the location of the shift

This is consistent with (though not identical to) the procedure used by Bauer, Hays and Upton (1987).

In the STAB procedure, there is a tradeoff regarding the number of subperiods. The fewer the subperiods, the more degrees of freedom there are, but using fewer, but longer, subperiods reduces the ability of the procedure to identify relatively short regimes. This study uses four subperiods in all stabilogram runs.

As in Howe and Pope (2005, 2006), this study uses the minimum of the LR as the estimate of the beta change date in the sequential application of the VPR and STAB procedures. After obtaining this estimate, the study performs the VPR or STAB procedures on the two regimes that are immediately before and after the LR minimum.

This method of locating beta shifts has the weakness of indicating a disproportionate number of the shifts in the first few or last few observations of the return series. Many of these shift indications are likely spurious. At the same time, if one treats indications of shifts in the first few or last few observations to be spurious, one may miss actual beta shifts. As a compromise, this study constrains the beta shift indications to between approximately the $N/8^{th}$ and $7N/8^{th}$ observation, where N is the number of returns in the series being tested for beta changes.

The results from the data sets with no beta change indicate the Type I error rate. The techniques are then compared on the basis of their ability to detect shifts in beta by examining the frequency of detection in data sets with one or more beta changes. The robustness of the techniques is investigated by comparing the changes in Type I error and detection frequency for the data sets with normally-distributed error terms with the Type I error and detection frequency for the data sets with Stable Paretian 1.95 error terms.

IV. Expectations

Given the results of Ashley (1984), one would expect the VPR test to be conservative; Ashley attributes this at least partly to the fact that the test statistic is only approximately chi-squared. On the other hand, because the Chow tests involved in the stabilogram procedure are exact, one would expect the Type I error rate of the stabilogram to approximately equal the significance level, in this study 5 percent.

The partition regression procedure involves calculating the SSE for all possible parameter shift points and choosing the shift point(s) which minimize the SSE. Only after that is a Chow test run to test for parameter instability. Since this amounts to running all possible Chow tests and choosing the one which produces the highest F-statistic, the expectation is that the Type I error rate is greater than the significance level.

It is not clear *a priori* whether the sequential RR procedure employed in this study has a Type I error rate greater than or less than the significance level of the test. One would expect the requirement that the RR procedure's indication of parameter change be persistent and consistent to weaken the test. On the other hand, the fact that the RR procedure is repeated a number of times over progressively longer subsets of the return series increases the probability of finding three or more consecutive indications of significance within any given simulated security return series. Given the extreme sensitivity to outliers documented in Howe and Upton (1992), one would expect more indications of significance when the error terms have a Stable Paretian distribution than when they are normally distributed.

In theory, it is not clear whether one would expect the tests to be more powerful than in Howe and Pope (2006). On one hand, there are fewer observations (104 in this study versus 180 in Howe and Pope (2006)) and this study uses simulated weekly returns rather than simulated monthly returns. On the other hand, mutual funds are portfolios rather than individual stocks, and thus have much higher market model R^2s than individual stocks do. For example, the funds in the Bauer-Hays-Upton study have an average R^2 of approximately 0.8. This compares with a typical market model R^2 of about 0.3 when using monthly returns. One would expect the effect of the difference in R^2 to be much stronger than the effect of the difference in degrees of freedom. Therefore, in the samples with one or more beta changes, we would expect more ability to detect the beta changes than in Howe and Pope (2006).

V. Results

Table I presents the first-pass rejection frequencies, that is, the frequencies with which the null hypothesis of no beta change is rejected. Only the partition regression procedure shows a detection rate of greater than 22 percent in the normally distributed sample with one beta change, and when there is no change it reports a beta change in at least 36.5 percent of the cases. The more beta increases there are the more likely it becomes that there will be an indication of beta instability. Also, in most cases there is more likely to be an indication of a beta change when there are two or more consecutive beta changes in the same direction than when there are not two or more consecutive beta changes in the same direction.

This study suggests the RR method has a slight ability to detect beta changes in weekly mutual fund returns; but not as much as the other methods. In fact, in two of the samples the number of cases is which a beta change was detected is not significantly greater than that of the corresponding no beta change sample at the 0.05 level. Also, the results from the samples with Stable Paretian error terms indicate that the procedure is sensitive to extreme observations, though not as sensitive as when the method is applied to simulated stock return data (Howe and Upton (1992); Howe and Pope (2005, 2006)). Finally, the rejection frequency of the null hypothesis of no beta change when there is in fact no beta change and the error terms are normally distributed (8.5%) is substantially greater than the 5 percent one would expect. A binomial test finds this difference to be significant with a p-value of 0.0121.

The PR procedure does appear to show some ability to detect beta changes. Also, the PR procedure appears to be largely unaffected by extreme observations, such as those found in the Stable Paretian samples. However, as expected, the Type I error rate of the PR procedure in the no beta change samples is significantly higher than the 5 percent one would expect (p-value = 1.01×10^{-42}). As expected, the ability to detect beta changes is greater than what Howe and Pope (2005, 2006) report for returns on individual stocks.

Consistent with the findings of Ashley (1984), the VPR test is conservative, with Type I error rates of 0.5 to 3 percent when the VPR test is run at the 5 percent level. In all of the samples in this study except the sample with three beta increases the VPR procedure detects a beta change less than half of the time. A McNemar (1962) test on the rejection frequencies on the normally-distributed no beta change sample versus the Stable Paretian 1.95 no beta change sample does not suggest that the VPR test is sensitive to extreme observations (two-tailed p-value = 0.119). The ability to detect beta changes is considerably greater than that for individual stocks reported by Howe and Pope (2005, 2006).

The stabilogram test shows more power than the VPR test in virtually all of the samples. Also, the STAB test appears to be robust to extreme observations. As is the case with the other tests, the stabilogram test shows much more power than when applied to individual stock returns.

Table II presents cumulative frequency distributions of the number of beta shifts indicated by the RR procedure. In the samples with normally distributed error terms, there were no cases in which three beta shifts were indicated, and only a few cases in which two beta shifts were indicated. There appears to be no relationship between the number of cases in which two beta changes were indicated by the procedure and the actual beta change pattern. Although the Stable Paretian error term samples showed indication of more beta changes than the normally distributed error term samples, the incidence of these indications appears to be unrelated to the actual beta change pattern. The number of beta changes detected is less than the number Howe and Pope (2005, 2006) find when using stock returns.

Table III presents cumulative frequency distributions of the number of shifts indicated by the partition regression procedure. In the samples with two or more beta changes, the procedure gives an indication of two or more beta changes no more than 4 percent of the time—much less often than Howe and Pope (2005, 2006) find in individual stock returns. Furthermore, there appears to be no correlation between the number of actual beta changes and the number of beta changes indicated by partition regression.

Table IV presents cumulative frequency distributions of the number of beta shifts indicated by the VPR procedure. Consistent with the results in Table I, the samples with more actual beta changes generally yield more indications of any given number of beta changes. However, in the samples with two beta changes, the second one is rarely detected, and in samples with three beta changes the third one is detected in less than 2 percent of the cases. For the most part, there appear to be indications of more market model parameter changes per security in the samples with Stable Paretian error terms than in the samples with normal error terms; however, the difference is not large This suggests that when applied sequentially the VPR procedure shows little sensitivity to extreme observations. The number of beta change indications is considerably less than when using series of 600 daily or 180 monthly simulated stock returns (Howe and Pope 2005, 2006).

Table V presents cumulative frequency distributions of the number of beta shifts indicated by the STAB procedure. Consistent with the results in Table I, the samples with more actual beta changes generally yield more indications of any given number of beta changes. However, the procedure detects a third beta change in no more than 3.5 percent of the securities in any of the samples. The number of beta change indications is slightly greater than when using series of 180 monthly simulated stock returns (Howe and Pope 2006) but less than when using series of 600 daily simulated stock returns (Howe and Pope 2005) .

VI. Summary and Conclusion

This study has examined the power and Type I error rate of four methods of testing for regression parameter changes when applied to detecting beta changes in weekly traditional equity mutual fund returns. The study used simulated mutual fund return series with known betas, error variances, beta change dates, and error term distributions.

The study found the two methods used by Bauer, Hays, and Upton (1987) not to work well in detecting beta changes in simulated weekly mutual fund returns. The recursive residual procedure showed a slightly inflated Type I error rate and only a slight ability to detect a single 25% beta change, and was somewhat sensitive to extreme observations. While using a Chow test to verify the regime shifts, as done in Bauer, Hays, and Upton (1987) would most likely have reduced the Type I error rate, it also would most likely have reduced the already poor ability of the recursive residuals procedure to detect a single beta change. As expected, the partition regression procedure had a grossly inflated Type I error rate. . This calls into question the results of Miller and Gressis (1980) and Bauer, Hays, and Upton (1987). On the other hand, variable parameter regression and, in particular, the stabilogram procedure showed some ability to detect market model parameter changes. Still, only in the case of three 25 percent increases in beta was the change in beta was detected in more than half of the cases at the 5 percent significance level.

For the recursive residuals and partition regression procedures the number of beta change indications is less than those in Miller and Gressis (1980) and Bauer, Hays, and Upton (1987) Furthermore, none of the tests showed much ability to detect more than one known beta change. This suggests that many of the apparent indications of beta shifts indicated by the recursive residuals and partition regression procedures are false signals. The number of second and subsequent beta changes indicated by the variable parameter regression and stabilogram procedures is generally higher than that reported in Howe and Pope (2005, 2006), a finding which is not surprising, considering the lower error variance in mutual fund returns as compared to returns of individual stocks. It is not clear whether the stabilogram procedure is better than the variable parameter regression procedure.

On one hand, the stabilogram procedure generally seems to have better ability to detect at least the first beta change. On the other hand, depending on the length of the series of the series of returns the number of degrees of freedom may become more problematic for the stabilogram procedure than for the variable parameter regression procedure. Beta is a fundamental variable in two measures of risk-adjusted performance, the Treynor (1965) measure and Jensen's (1968) alpha. Because failure to allow for beta changes can make beta estimates biased and inefficient (Kon and Lau, 1979), it would be ideal if one could allow for beta changes when measuring risk-adjusted performance. Unfortunately, even in the case of traditional equity mutual funds, which fit the market model much better than individual common stocks do, the ability to detect beta changes appears to be very limited.

There are at least two general areas for further study. First, the variable parameter regression and stabilogram procedures could be applied to the data used by Bauer, Hays, and Upton (1987). Second, there are other tests that could be employed, such as sequential versions of central Chow tests, Farley-Hinich tests, and dummy variable tests specifically aimed at detecting beta changes rather than market model parameter changes in general.

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Table I. First Pass Rejection Frequencies of Null Hypothesis of No Market Model Parameter Change (N=200)

Beta Change Pattern*	RR**	PR	VPR	STAB
		Normal Error Terms		
OOO	8.5%	36.5%	0.5%	6.5%
UOO	10.0%	63.0%	10.5%	22.0%
UUO	12.5%	80.5%	38.0%	45.0%
UUU	19.0%	90.0%	55.0%	69.0%
UDO	10.5%	54.5%	5.0%	15.0%
UDU	9.0%	57.0%	8.5%	22.0%
DDU	9.5%	72.0%	24.5%	34.5%
UUD	13.5%	70.0%	25.0%	33.5%
		Stable Paretian 1.95 Error Terms		
OOO	19.5%	42.0%	3.0%	6.0%
UOO	24.0%	59.0%	14.0%	21.0%
UUO	25.0%	79.0%	35.5%	40.5%
UUU	34.0%	87.5%	54.0%	64.5%
UDO	23.0%	53.0%	9.0%	12.5%
UDU	24.0%	54.5%	10.5%	20.5%
DDU	26.0%	70.5%	22.5%	21.0%
UUD	28.0%	65.5%	21.5%	31.0%

* O, U, and D denote no change, an increase, and a decrease, respectively. Therefore, for example, UDO refers to a beta increase, followed by a beta decrease, followed by no beta change.

** RR, PR, VPR, and STAB refer to the recursive residuals, partition regression, variable parameter regression, and stabilogram procedures, respectively.

Table II. Cumulative Frequency Distributions of Number of Parameter Shifts Detected by the Recursive Residuals Procedure (N=200)

Beta Change Pattern	Number of Shifts 1	2	3
		Normal Error Terms	
OOO	17	3	
UOO	20	3	
UUO	25	4	
UUU	38	3	
UDO	21	2	
UDU	18	1	
DDU	19	4	
UUD	27	2	
		Stable Paretian 1.95 Error Terms	
OOO	39	15	2
UOO	48	17	3
UUO	50	20	5
UUU	68	19	3
UDO	46	18	1
UDU	48	18	1
DDU	52	19	2
UUD	56	19	4

Table III. Cumulative Frequency Distributions of Number of Parameter Shifts Detected by Partition Regression (N=200)

Beta Change Pattern	Number of Shifts 1	2	3	4+
	Normal Error Terms			
OOO	73			
UOO	126	2	1	1
UUO	161	3	2	
UUU	180	4	3	1
UDO	109	2	1	
UDU	114	2	1	1
DDU	144	8	5	2
UUD	140			
	Stable Paretian 1.95 Error Terms			
OOO	84	2		
UOO	118	4	1	
UUO	158	4	1	
UUU	175	5	2	1
UDO	106	4	1	
UDU	109	2		
DDU	141	8	3	1
UUD	131	1		

Table IV. Cumulative Frequency Distributions of Number of Parameter Shifts Detected by Variable Parameter Regression (N=200)

	Number of Shifts			
Beta Change Pattern	1	2	3	4
	Normal Error Terms			
OOO	1			
UOO	21	9		
UUO	77	23		
UUU	110	44	2	
UDO	10	6		
UDU	17	6	1	
DDU	49	34	1	1
UUD	50	32	1	
	Stable Paretian 1.95 Error Terms			
OOO	6	2	1	
UOO	28	12	1	
UUO	71	24	1	
UUU	108	45	3	1
UDO	18	8	2	
UDU	21	8	2	
DDU	45	30	4	
UUD	43	27	3	1

Table V. Cumulative Frequency Distributions of Number of Parameter Shifts Detected by the Stabilogram Procedure (N=200)

	Number of Shifts			
Beta Change Pattern	1	2	3	4+
	Normal Error Terms			
OOO	13	3		
UOO	44	8	1	
UUO	90	18	3	1
UUU	138	30	3	
UDO	30	8	4	
UDU	44	8	1	
DDU	69	19	7	1
UUD	67	14	4	1
	Stable Paretian 1.95 Error Terms			
OOO	12	2		
UOO	42	6	1	
UUO	81	16	2	1
UUU	129	26	4	1
UDO	25	6	4	
UDU	41	7	3	
DDU	64	16	4	1
UUD	62	12	4	1

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