Indications of Risk: Standard Deviation of Returns or VaR of Terminal Wealth?

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Abstract

Investors with long time horizons are more concerned with the dispersion of their terminal wealth than with within-period volatility of asset returns. The risk of terminal wealth will indicate whether the liquidated value of the portfolio will be sufficient to finance liabilities. In this paper we apply three different strategies to construct portfolios using the historical returns of five asset classes and report their performance over several time horizons. These three investment strategies are: mean-variance optimized portfolios, Roy's safety first rule, and Value at Risk, VaR to minimize the impact of downsize risk when selecting portfolios. The distributions of terminal wealth generated from these portfolios are used to compare these investment strategies.

I. Introduction

A well known benefit of diversification is reduction in the standard deviation of portfolio returns across time. In analyzing portfolio performance, researchers have mainly focused on the probability distribution of annual returns, which is often assumed to be a normal distribution that is described by the expected annual returns and the standard deviation of annual returns. Under this assumption, investor's risk from variation in returns corresponds with the standard deviation of annual returns. As actual returns do not meet this assumption, investors need a more appropriate and useful indicator of risk for constructing investment portfolios.

Even though the focus in investing has been on volatility of returns, Arrow (1964) and Pratt (1964) consider the source of risk being the random terminal wealth or cash payment, not the dispersion of rates of return. Terminal wealth is measured by the expected portfolio value at the end of investment horizon value. Risk is the possibility that terminal wealth will be less than expected. The issue for investors is how to incorporate the impact of this future risk on their current investment decisions.

Over a one year time horizon, the dispersion of terminal wealth and the dispersion of rates of return are similar. Copeland and Weston (1988) using what they call the mean-variance paradox state that investors with specific long term goals and institutions with predefined liabilities such as pension funds and insurance companies are more concerned with the dispersion of terminal wealth from which benefits will be paid. Investors/individuals will rely on the terminal value of their portfolio to convert into a steady stream of income. Therefore, the possibility of shortfall in terminal wealth is a better measure of risk than the time series standard deviation of returns for investors with long-term financial goals. This is not to say that the value of terminal wealth is not affected by the volatility of returns over the investment horizon.

Individual investors with specific long-term and intermediate investment goals such as retirement and college savings are concerned with the expected terminal wealth and its shortfall

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risk, the possibility that the terminal value of their investment will fail to meet its longer term financial goals, rather than in the volatility of the rates of return over the holding period. The risks that the amount accumulated by a retiree is not enough to buy an adequate retirement annuity, the risk that college fund is below the desired level to cover tuition fees at the chosen college. Institutional investors such as pension funds and insurance companies use the terminal value of their portfolio to pay promised benefits. As these investors contemplate that terminal wealth may be less than expected they need a measure of risk that is appropriate and intuitively simple to interpret.

In this paper we use three different strategies to construct portfolios using the historical returns of five asset classes over a seventy-four year time horizon from the 1926-1999 and report their simulated performance over several time horizons. The first investment strategy involves finding mean-variance optimized portfolios with constant quadratic utility. Then use these portfolios in Monte Carlo simulations to forecast the distribution of expected future values from $1 invested today. The second approach involves choosing portfolios based on Roy's safety first rule. The third approach uses Value at Risk, VaR to minimize the impact of downsize risk when selecting portfolios. The distributions of terminal wealth generated from these portfolios are used to compare these investment strategies.

II. Literature Review

The conventional measure of financial risk has been the standard deviation of returns as stated by Markowitz (1952) "the investor does (or should) consider expected return a desirable thing *and* variance of return an undesirable thing." Markowitz was the first to provide a quantitative framework to determine portfolio risk. The investor has to make a tradeoff between risk and return based on utility of wealth along the efficient frontier.

Another influential paper on portfolio theory was by Roy (1952). Roy's purpose was to find the best risk return tradeoff without having to specify a utility function. Roy's safety-first criterion, given by the following expression:

$$Roy\ Safety = \frac{E(R_p) - R_l}{\sigma_p}$$

Where $E(R_p)$ is the expected portfolio return, R_l is the lowest return required by the investor, σ_p is the standard deviation of portfolio returns. In a minimum-variance framework, for each portfolio, calculate the Roy safety first ratio then pick the portfolio with the highest ratio. The investor will choose the portfolio with the lowest probability of going below the lowest return tolerated. In essence, Roy's measure is the first recognition of shortfall risk. In implementing the Roy's safety-first measure, a decision has to be made about the minimum acceptable rate of return. If risk-free rate is substituted for minimum acceptable rate, then the Roy's measure becomes the Sharpe ratio. For the purpose of our analysis we apply this rule to the change in terminal value. The investor seeks to at least preserve the principal invested.

Alternatively, to measure the shortfall risk, the risk that the value of the portfolio will fall below some minimum acceptable level at the end of the planning horizon, a new measure was adopted, the value at risk or VaR. VaR was first developed by financial institutions as a measure of total risk for internal control in the 1980s. This new measure of risk has gained increasing acceptance, and has been adopted by banks, security firms, and asset managers as a tool for risk assessment. VaR assumes that we have a portfolio that generates a random return over a chosen horizon. Essentially, VaR measures the minimum loss on a portfolio given small probability over a specific time horizon. Though VaR does not require any assumptions about the distribution of returns, its implementation requires choosing a horizon period and a confidence level. These values are usually arbitrary. It is a common practice to set the confidence level at 95%. Jorion (2001) says "… VaR systems will bring about better control and management of financial risks". Further, Jorion states "…VaR provides a simple, transparent, and consistent measure of overall risk." VaR has become widely used as it allows comparison of risk across different portfolios and measures "lost money" the easiest measurement unit.

Despite its popularity, VaR has its critics. A loss more than VaR, even though its probability may be very small, is not captured by VaR, i.e., there is no indication what that loss maybe. Basak and Shapiro (2001) show that in the worse states of the world, the VaR users may take on more risk and consequently increase the stock market volatility. The disparity of these claims calls for an empirical exploration.

III. Data and Simulation

To estimate the value of terminal wealth and its distribution, we consider three investment strategies using historical returns of five asset classes: small stocks, large stocks, long-term T-bonds, intermediate T-bonds, and T-bills over a seventy-four year time horizon from the 1926 to the 1999. Mean-variance efficient portfolios for alternative levels of risk tolerance are created from the five asset classes. Efficient portfolios have maximum expected return at every level of standard deviation of returns. The allocation of $1 invested among the five asset classes is determined by the mean-variance framework. Selection among efficient portfolios is determined by maximizing a quadratic utility function defined by the following: $U= E[r]-0.5A\sigma^2$. The coefficient A measures the degree of risk aversion taking the following values: 1, 2, 4, 8, and 16. The strongest risk aversion is associated with the largest value of A. This gives us a historical perspective of diversified portfolios over varying time horizons (t=1, 3, 5, 10, 20).

From these efficient portfolios, for chosen levels of investor risk aversion, we use Monte Carlo simulation of optimal rates of return to estimate the distributions of expected terminal wealth for each time horizon. The expected value of terminal wealth is calculated as the average of the 10,000 simulations. Investors' goal is to achieve high terminal wealth, with reduced risk measured by standard deviation of annual rates of return.

The above simulation is based on the returns generated from optimized portfolios using the mean- variance framework with the underlying assumption that the returns are normally distributed. To conduct an analysis that is distribution free, we performed a bootstrap procedure for constructing portfolios to reduce downside risk by using Roy's safety first ratio and the VaR measure. Boostrapping generates a probability distribution of returns under the assumption that returns are equally likely. For example, we simulate 20-year sample of possible returns by sampling 20 randomly selected returns from the 74-year historical returns. The twenty-year returns are compounded to obtain a terminal wealth for the 20 year holding period return. This procedure was repeated 10,000 times to generate a probability distribution of returns for t years (t=1, 3, 5, 10, 20). We use two methods of selecting optimum portfolio weights for each asset class for each time horizon that minimize downside risk of terminal wealth.

The first method uses Roy's safety first rule applied to the change in terminal wealth. We assume the reference level is the initial wealth level. The change in terminal wealth is divided by the standard deviation of terminal wealth. By maximizing this ratio, we minimize the probability that the expected terminal wealth will be lower than initial wealth.

The second method uses VaR measured at two probability levels, 1 percent and 5 percent. VaR is the percentile of the distribution of change in terminal value which represents the change in terminal wealth. For example, 1 percent VaR of $10 is interpreted as the decrease in terminal wealth will be by no more than $10 with a 99 percent confidence level. By choosing optimum portfolios based on VaR, investors minimize the downside risk to terminal wealth.

IV. Empirical Results

The purpose of this study is to compare indications of risk from standard deviation of annual returns and value of VaR by estimating the terminal value of wealth from different investment strategies. Panel A Table 1 reports summary statistics of the historical rates of return for the five asset classes from 1926-1999 used to generate simulated mean-variance efficient investment portfolios as shown in figure 1 and optimum portfolios that minimize downside risk using Roy's safety first rule and VaR measure.

The five asset classes are small stocks, large stocks, long term T-bonds, intermediate T-bonds, and T-bills. Small stocks have the highest rate of return and highest standard deviation and T-bills have the lowest rate of return and lowest risk supporting the notion that investors require higher rates of return to bear risk. The difference between average return on small stocks and average return on large stocks is equal to 5.7 % yet the standard deviation of small stock returns is almost double the standard deviation of large stock returns. There is a strong positive correlation between the returns of small stock returns and large stock returns.

Panel B Table 1 shows portfolio annual rates of return for several levels of risk aversion including the minimum variance portfolio. These mean-variance efficient portfolios have expected returns varying from 4.19% to 15.92% with respective standard deviations of 3.14% to 28%. The minimum variance portfolio has the lowest Sharpe ratio as expected. For higher values of A (more risk averse investors), the Sharpe ratio is lower. The Roy safety-first ratio uses zero as reference rate, and is increasing with the degree of risk aversion. Figure 1 illustrates the risk return tradeoff of the mean-variance efficient portfolios.

Table 2 reports the distribution of terminal wealth simulated from the minimum variance portfolios and optimal mean variance for a coefficient of risk aversion set at 16 over several time horizons (t= 1, 3, 5, 10, 20). Among all possible mean variance optimal portfolios, we selected these two conservative investment strategies as they have risk return tradeoffs similar to the other portfolio strategies. The minimum variance portfolio is the most conservative strategy with low return and low risk. The mean variance optimal portfolio with a coefficient of risk aversion of 16 has a large penalty of risk as measured by the standard deviation of annual returns. This portfolio has a small standard deviation of change in terminal value of wealth.

The third panel on Table 2 presents the distribution of terminal wealth for the optimal portfolios using Roy's safety rule applied to the change in terminal wealth. Roy's safety first uses initial wealth as the reference value. This preservation of wealth is a conservative goal which results in a conservative investment strategy. These portfolios have risk-return tradeoffs that fall between those of the minimum variance and conservative investment strategies shown in the first two panels. Compared to the other strategies, the Roy's safety first rule has the portfolios with lowest terminal wealth in the last time horizon.

The fourth panel reports the optimal portfolios that minimize the downside risk using a 1 percent VaR for each time horizon. This procedure selects portfolios that reduce the probability that the change in terminal wealth will be less than expected which makes a very conservative investment strategy. The 1 percent VaR portfolios have a risk return tradeoff similar to the minimum variance portfolios in the time horizon t=1, 3, and 5 years. For the other time horizons, the 1 percent VaR portfolios have increasing standard deviation a substantial increase in skewness which is reflected in a much higher change in terminal wealth by restricting downside risk but allowing large gains.

The fifth panel shows the optimal portfolios that minimize the downside risk using a 5 percent VaR for each time horizon. By comparison to the 1 percent VaR, these optimal portfolios are more conservative. For the time horizons t=1, 3, and 5 the optimal portfolios using 1 percent VaR and 5 percent VaR are fairly similar. On the 10 and 20 year horizons the 5 percent VaR portfolios depict an increasing skewness which adds to increase in terminal wealth.

V. Conclusion

Sufficiently reducing the dispersion of terminal wealth is of direct relevance to investors with fixed long-term obligations and/or savings goals. This study uses three investment strategies to estimate terminal wealth over five different time horizons. Investment strategies focused on reducing downside risk result in conservative portfolios. Over short time horizons, all three investment strategies simulated in this paper yield similar conservative low risk and low return performance. Over the long time horizons such as 10 and 20 years, the investment strategy based on value at risk, VaR, allow positive skewness which increases expected terminal wealth. Overall, we find the use of VaR of terminal wealth to minimize downside risk results in similar portfolio performance as a risk averse investment strategy minimizing the standard deviation of annual returns.

Figure 1 Efficient Frontier

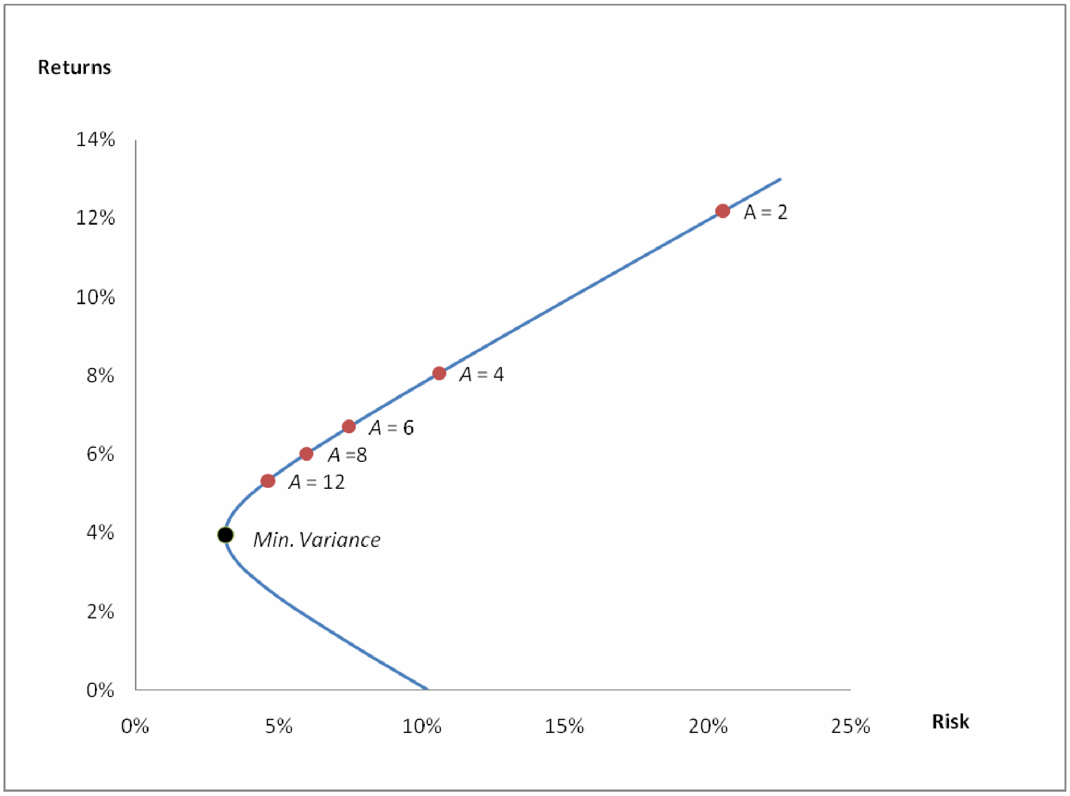

Table 1
Panel A

Sample Statistics 1930-1999

	Small Stocks	Large Stocks	Long-Term T-Bonds	Intermediate-Term T-Bonds	T-Bills
E[*r*]	18.81%	13.11%	5.36%	5.19%	3.81%
St Dev	39.41%	20.08%	8.06%	6.33%	3.27%
Minimum	-52.71%	-45.56%	-8.74%	-5.81%	-1.59%
Maximum	187.82%	54.56%	32.68%	33.39%	14.95%
Correlations					
Small Stock	1.00	0.78	0.01	-0.02	-0.17
Large Stock		1.00	0.20	0.15	-0.03
L-T T-Bonds			1.00	0.78	0.26
I-T T-Bonds				1.00	0.43
T-Bills					1.00

Panel B
Rates of Returns Risk Measures

Portfolio	Expected Rate of Return	Standard Deviation of Return	Expected Utility	Sharpe's Ratio	Roy's Safety First Ratio
Minimum Variance	4.19%	3.14%	*N/A*	0.119	1.335
A = 16	5.60	4.35	4.08%	0.410	1.287
A = 8	7.05	6.80	5.21	0.476	1.038
A = 4	9.54	11.69	6.81	0.490	0.816
A = 2	13.46	20.74	9.15	0.465	0.649
A = 1	15.92	28.08	11.97	0.431	0.567

Table 2
Statistics of Portfolio Terminal Wealth

Portfolios created by minimizing standard deviation of annual rates of return.

	1 year	3 year	5 year	10 year	20 year
E[ΔV]	0.042	0.131	0.228	0.507	1.271
σ of ΔV	0.032	0.059	0.083	0.143	0.308
First Quartile of ΔV	0.021	0.091	0.171	0.409	1.060
Second Quartile of ΔV	0.042	0.130	0.225	0.500	1.255
Third Quartile of ΔV	0.063	0.170	0.284	0.599	1.468
Skewness of ΔV	0.026	0.114	0.131	0.269	0.391

Portfolios created by minimizing standard deviation of annual rates of return with risk aversion coefficient of 16 and expected utility of 4.08%.

	1 year	3 year	5 year	10 year	20 year
E[ΔV]	0.056	0.178	0.313	0.724	1.973
σ of ΔV	0.043	0.084	0.122	0.227	0.557
First Quartile of ΔV	0.027	0.121	0.229	0.567	1.576
Second Quartile of ΔV	0.057	0.176	0.309	0.714	1.928
Third Quartile of ΔV	0.086	0.233	0.395	0.868	2.320
Skewness of ΔV	-0.053	0.172	0.216	0.368	0.610

Portfolios created by maximizing Roy's Safety First Rule applied to change in terminal wealth.

	1 year	3 year	5 year	10 year	20 year
E[ΔV]	0.048	0.147	0.248	0.526	1.208
σ of ΔV	0.034	0.065	0.090	0.157	0.312
First Quartile of ΔV	0.027	0.100	0.183	0.413	0.987
Second Quartile of ΔV	0.045	0.141	0.241	0.515	1.177
Third Quartile of ΔV	0.066	0.188	0.307	0.628	1.397
Skewness of ΔV	0.431	0.390	0.393	0.447	0.535

Portfolios created by using 1% VaR to reduce downside risk to change in terminal wealth.

	1 year	3 year	5 year	10 year	20 year
E[ΔV]	0.043	0.136	0.249	0.654	3.512
σ of ΔV	0.032	0.062	0.094	0.244	4.035
First Quartile of ΔV	0.019	0.091	0.182	0.482	1.662
Second Quartile of ΔV	0.038	0.130	0.241	0.624	2.413
Third Quartile of ΔV	0.063	0.175	0.308	0.784	3.868
Skewness of ΔV	0.841	0.564	0.538	1.058	6.392

Portfolios created by using 5% VaR to reduce downside risk to change in terminal wealth.

	1 year	3 year	5 year	10 year	20 year
E[ΔV]	0.044	0.152	0.281	0.789	8.367
σ of ΔV	0.039	0.069	0.116	0.398	9.307
First Quartile of ΔV	0.014	0.104	0.199	0.533	3.131
Second Quartile of ΔV	0.038	0.146	0.270	0.716	5.622
Third Quartile of ΔV	0.065	0.195	0.351	0.951	10.305
Skewness of ΔV	1.382	0.477	0.690	2.575	4.154

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