Detection of Multiple Beta Shifts in Monthly Returns Data

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Introduction

For over three decades, studies have used the single index market model (Sharpe 1963) to analyze risk or performance of stocks, often in response to an information-containing event. The market model is

$$R_{it} = a_i + b_i R_{mt} + u_{it} \tag{1}$$

where R denotes return, i and t denote the asset and time period, respectively, m denotes the market, b is the beta, that is, the sensitivity of the asset returns to the market returns, a is the intercept, and u is an error term.

Early research into the consistency of the beta of common stocks over time indicates that the betas of individual assets are nonstationary (Levy 1971). Recent studies which acknowledge this nonstationarity include Graddy, Kyle, Strickland, and Bass (2004) and Kaplanski (2004). One parameter stability model is the shifting regimes model (Mehta and Beranek 1982; Bey 1983; McDonald 1983; Hays and Upton 1986), in which the return-generating process is assumed to follow a stationary regime for n_1 periods, then shift to another stationary regime for n_2 periods, and so on. The shifts are considered to be infrequent; that is, for any given k, n_k is large enough for reliable estimation of the parameters.

Major difficulties in applying the shifting regimes model are the detection and location of regime shifts. Possible techniques include the Chow (1960), Farley-Hinich (1970), recursive residual (RR) (Brown, Durbin, and Evans 1975), and dummy variable (Harvey 1976) tests, and variable parameter regression (Garbade 1977) for the detection of a change, and the Quandt log likelihood ratio (LR) (1958,1960) for both detection and location of the change. The RR and LR are often used in combination, the first to detect the presence of a shift and the second to estimate the location of the shift point. Garbade (1977) and Farley, Hinich, and McGuire (1975), however, raise questions about the suitability of the RR and LR techniques.

To the extent that the methodology is inadequate, the previous research is called into question. For example, Howe and Upton (1992) suggest that an individual stock's beta shift is difficult to detect reliably. This calls into question the results of studies such as Hays and Upton (1986). In addition, Howe and Pope (2005) suggest that attempts to detect multiple beta shifts in daily stock return data are fruitless.

Other than the fact that Howe and Upton use simulated returns while Hays and Upton use actual returns, there are two major differences between the studies. First, Howe and Upton use daily returns while Hays and Upton use monthly returns. Second, Howe and Upton assume only one beta shift while Hays and Upton allow for multiple beta shifts. Howe and Pope allow for multiple beta shifts using daily returns.

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For greater comparability with Hays and Upton, this study uses monthly simulated stock returns to examine the ability of techniques to detect multiple beta shifts. Techniques this study uses include partition regression (Guthery 1974; Miller and Gressis 1980) and sequential variations of the Brown-Durbin-Evans (BDE) cumulative sum of the squared recursive residuals (Brown, Durbin, and Evans 1975; Hays and Upton 1986), stabilogram (Ashley 1984), and variable parameter regression (Garbade 1977) techniques. As in Howe and Upton (1992), this study improves over previous studies by examining conditions which more realistically approximate those found in security returns. Also as in Howe and Upton, this study uses a range of conditions which depart from the nominal assumed conditions in directions indicated from empirical studies. This range of conditions allows an examination of the robustness of the techniques. A class of the Stable Paretian distribution is used to generate residuals to assess the effect of non-normality and outliers.

This study finds single beta changes of 25 percent nearly impossible to detect. It finds the cumulative sum of the squared recursive residuals procedure to be highly sensitive to extreme observations and to have no ability to detect a beta change, even when there are three 25 percent beta changes within 180 months. In addition, the partition regression procedure and the cumulative sum of the squared recursive residuals procedure have type I error rates far greater than the significance level of the tests. While the variable parameter and stabilogram tests do not show such high type I error rates, the percentage of the cases in which they detect even one beta shift is low. None of the methods shows much ability to detect any beta changes beyond the first.

Methodology

Techniques for Detection of Parameter Shifts

Variable Parameter Regression (VPR)

Variable parameter regression assumes that beta follows a random walk. If we express this as $b_t = b_{t-1} + P$, where b_t is the beta for period t and P is the drift parameter, the null hypothesis is that P=0. The maximum likelihood estimate of P is asymptotically chi-squared with one degree of freedom. A full discussion of variable parameter regression can be found in Garbade (1977).

Stabilogram (STAB)

The stabilogram procedure divides the period to be studied into a number of subperiods of approximately equal length. Dummy and interactive variables are used to obtain ordinary least squares estimates of the market model parameters for the subperiods. The stabilogram procedure considers the market model parameters to be unstable if the null hypothesis that the market model parameters for all subperiods are equal is rejected.

Cumulative Sum of the Squared Recursive Residuals (RR)

Recursive residuals are obtained by recursively computing the standardized prediction error of r_{it} when the market model parameters are estimated from the preceding t-1 observations. Under the null hypothesis of no parameter change, the errors are independent and distributed $N(0,s^2)$. A shift in either a_i, b_i, or the error variance is indicated if a residual shows significant departure from zero. BDE present tests on the cumulative sum (cusum) of the recursive residuals and on the cusum of the squared recursive residuals. This study uses the latter test because Garbade (1977) found it to be much more powerful than the former. A full discussion of the technique and can be found in Brown, Durbin, and Evans (1975).

One problem with the RR technique is that the distribution of the test statistic is known only under the null hypothesis, and will be violated in the presence of multiple shifts. Additionally, it has been suggested that the RR technique is sensitive to outliers or other departures from the assumed conditions. In response to these objections, several papers (for example, Hays and Upton (1986), Johnson (1989)) have used a sequential approach for the RR technique, in which the RR technique is applied to progressively longer periods to avoid the possibility of multiple shifts. In this approach, shift indications are accepted as valid only if consistently observed, and the analysis is restarted after a shift has been detected. A full discussion of this approach can be found in Hays and Upton (1986).

Howe and Upton (1992) suggest that the VPR technique does not show this outlier sensitivity. Also, one can infer from their Chow test results that the STAB technique would likely be insensitive to outliers. However, the VPR and STAB techniques, as originally presented, test for general instability rather than multiple shifts. To attempt to detect multiple beta shifts, this study modifies these techniques by using a sequential approach.

Garbade (1997) compared the ability of RR and VPR to detect instability in the coefficients of a linear regression model, and found that the VPR dominated. In addition to comparing a larger number of techniques, this study differs from Garbade in using conditions which are more realistic for event-study applications, and by including a number of conditions not investigated by Garbade. Additionally, Garbade used only totally simulated data, while the proposed studies use simulations based on observed market data to include the possible effects of instabilities in the market process.

Partition Regression (PR)

The partition regression approach presented by Guthery (1974) and applied by Miller and Gressis (1980) differs from the RR, VPR, and STAB approaches in that it was originally designed to attempt to identify multiple parameter shifts rather than a single shift. Partition regression employs the following algorithm.

1. Divide the period into subperiods (initially two) so as to minimize the error sum of squares.
2. Use Chow (1960) tests to test the null hypothesis that the market model parameters of every two neighboring subperiods are equal.
3. If the null hypothesis is rejected for all neighboring subperiods, increase the number of subperiods by one and go to step 1.
4. If any of the Chow tests on neighboring subperiods are not rejected, stop the procedure. The number of parameter shifts is considered to be two less than the number of subperiods most recently examined.

Technique for Shift Point Locations

The log likelihood ratio (LR) can be calculated as follows:

$$LR = \ln \left[\frac{\text{likelihood of the observations under } H_0 \text{ (no parameter change)}}{\text{likelihood of the observations under } H_1} \right]$$

It can be used to detect the presence of a shift by examining the size of the minimum value over the period of interest. The distribution of the ratio has not yet been specified, however, and application of the technique to the detection of a shift itself is judgmental in nature. Because of the qualitative nature of the technique, it is sometimes used in conjunction with other techniques (such as RR) as a method of locating the shift point. That is, given that a shift has been detected within a series of observations, the likely location of the shift is at the minimum of the ratio.

Return Simulations

Market-based simulated daily security return series are generated using the following form of the single index model:

$$r_{it} = r_{ft} + b_{1it}(r_{mt} - r_{ft}) + u_{it}, \tag{2}$$

where:

b_{1it} = the beta coefficient for "security" i in period t
u_{it} = error term
r_{mt} = observed monthly returns to the S&P 500
r_{ft} = observed monthly rate on 90 day T-bills.

For comparability with Hays and Upton (1986), each series contains 180 monthly returns. These simulated returns are based on the S&P 500 and T-bill returns over the period from January 1962 through July 1995.

Two hundred simulated return series (corresponding to 200 "securities") are created, each using a beta drawn from a normal distribution with a mean of 1 and a variance of 0.16. Examination of a cross-sectional distribution of betas suggests that such a distribution is realistic. In order to provide a variety of conditions under which the techniques can be tested, the parameters in (2) are varied as follows:

b_{1it}: Eight conditions of change: (a) constant beta (no change), (b) a 25% increase in beta, (c) two 25% increases in beta, (d) three 25% increases in beta, (e) a 25% increase in beta, followed by a 20% decrease in beta, so the beta after the second shift is the same as the beta before the first shift, (f) a 25% increase in beta, followed by a 20% decrease in beta, followed by a 25% increase in beta, (g) two 20% decreases in beta followed by a 25% increase in beta, and (h) two 25% increases in beta followed by a 20% decrease in beta For each simulated security, three beta change dates are selected at random from a uniform distribution consisting of days 3 through 177.

u_{it}: Two distributions of error terms: a) Normal (Stable Paretian with $\alpha = 2.0$) and b) Stable Paretian with $\alpha = 1.95$. These values are consistent with those observed by Fama (1965). The mean of the distributions is zero. The variance (scale factor in the case of the Stable Paretian 1.95 error terms) is estimated from a randomly chosen set of 200 securities.

This results in eight conditions and two error term distributions, or 16 (8×2) data sets. Each data set is composed of 180 "observations" for each of 200 "securities."

Procedure

The studies are to apply the detection techniques to the sixteen data sets at the 0.05 significance level. The RR test is performed on the first six observations. After that, we increased the interval tested six observations at a time between RR tests. In this study, for a shift indication to be considered consistent and persistent one of the following conditions has to be met:

1. three consecutive indications of significance, all with the same LR estimate of the location of the shift
2. four consecutive indications of significant, with three identical LR estimates of the location of the shift

This is consistent with the procedure used by Hays and Upton (1986).

In the STAB procedure, there is a tradeoff regarding the number of subperiods. The fewer the subperiods, the more degrees of freedom there are, but using fewer, but longer, subperiods reduces the ability of the procedure to identify relatively short regimes. This study uses six subperiods in all stabilogram runs.

As in Howe and Pope(2005), this study uses the minimum of the LR as the estimate of the beta change date in the sequential application of the VPR and STAB procedures. After obtaining this estimate, the study performs the VPR or STAB procedures on the two regimes that are immediately before and after the LR minimum.

This method of locating beta shifts has the weakness of indicating a disproportionate number of the shifts in the first few or last few observations of the return series. Many of these shift indications are likely spurious. At the same time, if one treats indications of shifts in the first few or last few observations to be spurious, one may miss actual beta shifts. As a compromise, this study constrains the beta shift indications to between approximately the $N/8^{th}$ and $7N/8^{th}$ observation, where N is the number of returns in the series being tested for beta changes.

The results from the data sets with no beta change indicate the Type I error rate. The various techniques are then compared on the basis of their ability to detect shifts in parameters by examining the frequency of detection in data sets with one or more beta changes. The robustness of the techniques is investigated by comparing the changes in Type I error and detection frequency for the data sets with normally-distributed error terms with the Type I error and detection frequency for the data sets with Stable Paretian 1.95 error terms.

Expectations

Given the results of Ashley (1984), one would expect the VPR test to be conservative; Ashley attributes this at least partly to the fact that the test statistic is only approximately chi-squared. On the other hand, because the Chow tests involved in the stabilogram procedure are exact, one would expect the Type I error rate of the stabilogram to approximately equal the significance level, in this study 5 percent.

The partition regression procedure involves calculating the SSE for all possible parameter shift points and choosing the shift point(s) which minimize the SSE. Only after that is a Chow test run to test for parameter instability. Since this amounts to running all possible Chow tests and choosing the one which produces the highest F-statistic, the expectation is that the Type I error rate is greater than the significance level.

It is not clear *a priori* whether the sequential RR procedure employed in this study has a Type I error rate greater than or less than the significance level of the test. One would expect the requirement that the RR procedure's indication of parameter change be persistent and consistent to weaken the test. On the other hand, the fact that the RR procedure is repeated a number of times over progressively longer subsets of the return series increases the probability of finding three or more consecutive indications of significance within any given simulated security return series. Given the extreme sensitivity to outliers documented in Howe and Upton (1992), one would expect more indications of significance when the error terms have a Stable Paretian distribution than when they are normally distributed.

Results

Table I presents the first-pass rejection frequencies, that is, the frequencies with which the null hypothesis that there is no beta change is rejected. Regardless of the method used, a single 25% beta change is practically undetectable. Also, the more beta increases there are the more likely it becomes that there will be an indication of beta instability. Also, there is more likely to be an indication of a beta change in cases in which there are two consecutive beta changes in the same direction than when beta increases, then decreases, and then increases.

While studies such as Howe and Upton (1992) and Howe, Upton, and Pope (1997) find the RR procedure has a slight ability to detect beta changes using daily stock returns, this study suggests the method has no ability to detect beta changes in monthly stock returns; the method gives no more indications of a beta change in the three-beta-increase cases than in the no-beta-change cases. Also, the results from the samples with Stable Paretian error terms indicate that the procedure is highly sensitive to extreme observations. Finally, the rejection frequency of the null hypothesis of no beta change when there is in fact no beta change and the error terms are normally distributed (12.0%) is substantially greater than the 5 percent one would expect. A binomial test finds this difference to be significant at much less than the 0.01 level (p-value = 2.61×10^{-5}).

The PR procedure does appear to show some ability to detect beta changes, at least in the cases with two or more consecutive beta changes in the same direction Also, the PR procedure appears to be largely unaffected by extreme observations, such as those found in the Stable Paretian samples. However, as expected, the type I error rate of the PR procedure in the no beta change samples is significantly higher than the 5 percent one would expect (p-value = 7.66×10^{-15}). The ability to detect beta changes is comparable to what Howe and Pope (2005) report for daily returns.

Consistent with the findings of Ashley (1984), the VPR test is conservative, with type I error rates of 1 to 4 percent when the VPR test is run at the 5 percent level. In all of the samples in this study the VPR procedure detects a beta change less than half of the time, and except for the samples with three beta increases, no more than 15 percent of the time. A paired t-test on the rejection frequencies on the normally-distributed samples versus the Stable Paretian 1.95 samples suggests that the VPR test is sensitive to extreme observations (two-tailed p-value = 0.018). For the most part, the ability to detect beta changes appears to be slightly less than that reported by Howe and Pope (2005).

The stabilogram test appears to have power at least comparable to that of VPR, especially in the cases in which there is either only one beta change or a mixture of beta increases and decreases. Also, the STAB test appears to be robust to extreme observations. Similar to VPR, the ability to detect beta changes appears to be slightly less than that reported by Howe and Pope (2005).

Table II presents frequency distributions of the number of beta shifts indicated by the RR procedure. In the samples with normally distributed error terms, there were no cases in which three beta shifts were indicated, and only a few cases in which two beta shifts were indicated. There appears to be no relationship between the number of cases in which two beta changes were indicated by the procedure and the actual beta change pattern. Although the Stable Paretian error term samples showed indication of more beta changes than the normally distributed error term samples, the incidence of these indications appears to be unrelated to the actual beta change pattern. The number of beta changes detected is less than the number Howe and Pope (2005) find when using daily stock returns.

Table III presents frequency distributions of the number of shifts indicated by the partition regression procedure. In the samples with two or more beta changes, the procedure gives an indication of two or more beta changes no more than 9 percent of the time. Furthermore, there appears to be no correlation between the number of actual beta changes and the number of beta changes indicated by partition regression. This study finds fewer indications of two or more beta changes but slightly more indications of four or more beta changes than Howe and Pope (2005).

Table IV presents frequency distributions of the number of beta shifts indicated by the VPR procedure. Consistent with the results in Table I, the samples with more actual beta changes generally yield more indications of any given number of beta changes. However, in the samples with two beta changes, the second one is rarely detected, and in samples with three beta changes the third one is detected in less than 2 percent of the cases. There appear to be more indications of market model parameter changes per security in the samples with Stable Paretian error terms than in the samples with normal error terms. This suggests that when applied sequentially the VPR procedure is somewhat sensitive to extreme observations. The number of beta change indications is considerably less than when using series of 600 daily simulated stock returns (Howe and Pope 2005).

Table V presents frequency distributions of the number of beta shifts indicated by the STAB procedure. Consistent with the results in Table I, the samples with more actual beta changes generally yield more indications of any given number of beta changes. However, the procedure detects a third beta change in no more than 1.5 percent of the securities in any of the samples. Furthermore, in the samples with exactly two beta changes, the STAB procedure finds two or more beta changes less than 3 percent of the time. While it is conceivable that the sequential application of the STAB procedure is less sensitive to extreme observations than the sequential application of the VPR procedure is, it is also possible that the STAB procedure weakens considerably as the subperiods become shorter. The number of beta change indications is somewhat less than when using series of 600 daily simulated stock returns (Howe and Pope 2005).

Summary and Conclusion

This study has examined the power and type I error rate of four methods of testing for regression parameter changes when applied to detecting beta changes in monthly stock return series. The study used simulated stock return series with known betas, error variances, beta change dates, and error term distributions.

The study found a single 25% beta change to be nearly impossible to detect, regardless of the method used to attempt to detect the changes. The partition regression and sequential recursive residual procedures were found to have type I error rates far greater than the 5 percent significance level used in this study, a finding consistent with expectations. Furthermore, the recursive residuals procedure showed no ability to detect beta changes and was found to be very sensitive to extreme observations.

On the other hand, variable parameter regression and the stabilogram procedure showed some ability to detect market model parameter changes when there were two or more beta shifts of 20 or 25 percent in the same direction. Still, even in the case of three 25 percent increases in beta, which amounts to almost a doubling of beta during a 180-month period, the change in beta was detected in no more than 32 percent of the cases at the 5 percent significance level.

None of the methods showed ability to detect a second or third beta change in more than 9 percent of the cases. This could be due to the high level of residual risk in monthly stock return data or, in the case of the recursive residuals procedure and possibly the variable parameter regression procedure, the low power of the tests.

In most cases the number of beta change indications is less than those in Howe and Pope (2005). This suggests that the lower level of residual variance in monthly stock returns compared to daily stock returns is more than offset by smaller number of observations.

In summary, it appears to be nearly impossible to detect or find the location of small or moderate beta changes in monthly stock return series. This suggests that the market model parameter changes reported by Hays and Upton (1986) are most likely not beta changes. However, they find of market model nonstationarity in almost all of the stocks in their sample—far more than this study finds. This suggests that if non-normality of stock returns accounts for the results obtained by Hays and Upton, the Stable Paretian 1.95 distribution does not adequately explain monthly stock returns.

There are certainly other tests that could be employed, such as central Chow tests, Farley-Hinich tests, and dummy variable tests specifically aimed at detecting beta changes rather than market model parameter changes in general. However, the ability of these tests to detect multiple beta shifts will still be limited by the poor performance of methods used to locate the beta changes. Thus, even though failure to account for beta changes could bias risk-adjusted return measurement and the results of abnormal returns tests, the techniques examined in this study appear to be of almost no help in correcting for beta changes.

Table I. First Pass Rejection Frequencies of Null Hypothesis of No Market Model Parameter Change (N=200)

Beta Change Pattern*	RR**	PR	VPR	STAB
		Normal Error Terms		
OOO	12.0%	43.5%	1.5%	6.0%
UOO	10.0%	47.5%	4.5%	8.0%
UUO	10.5%	67.0%	14.0%	17.0%
UUU	11.5%	84.0%	32.0%	33.0%
UDO	10.5%	49.5%	3.0%	8.5%
UDU	11.0%	48.0%	3.0%	8.0%
DDU	11.5%	55.0%	3.5%	9.5%
UUD	11.5%	58.0%	9.0%	13.5%
		Stable Paretian 1.95 Error Terms		
OOO	33.5%	43.0%	4.0%	7.5%
UOO	33.5%	47.5%	7.0%	8.5%
UUO	32.0%	63.0%	14.5%	17.0%
UUU	33.0%	81.5%	31.5%	33.0%
UDO	32.0%	47.0%	5.5%	10.0%
UDU	33.0%	46.5%	5.0%	9.0%
DDU	34.5%	54.5%	5.5%	11.0%
UUD	32.5%	54.5%	10.5%	14.0%

* O, U, and D denote no change, an increase, and a decrease, respectively. Therefore, for example, UDO refers to a beta increase, followed by a beta decrease, followed by no beta change.

** RR, PR, VPR, and STAB refer to the recursive residuals, partition regression, variable parameter regression, and stabilogram procedures, respectively.

Table II. Cumulative Frequency Distributions of Number of Parameter Shifts Detected by the Recursive Residuals Procedure (N=200)

	Number of Shifts			
Beta Change Pattern*	1	2	3	4
	Normal Error Terms			
OOO	24	6		
UOO	20	4		
UUO	21	5		
UUU	23	3		
UDO	21	5		
UDU	22	6		
DDU	23	5		
UUD	23	5		
	Stable Paretian 1.95 Error Terms			
OOO	67	30	8	
UOO	67	33	6	
UUO	64	33	6	
UUU	66	35	8	1
UDO	64	31	5	
UDU	66	32	10	
DDU	69	33	6	
UUD	65	36	6	1

* O, U, and D denote no change, an increase, and a decrease, respectively. Therefore, for example, UDO refers to a beta increase, followed by a beta decrease, followed by no beta change.

Table III. Cumulative Frequency Distributions of Number of Parameter Shifts Detected by Partition Regression (N=200)

	Number of Shifts			
Beta Change Pattern*	1	2	3	4+
		Normal Error Terms		
OOO	87	16	7	5
UOO	95	11	5	3
UUO	134	15	7	6
UUU	168	14	7	6
UDO	99	16	11	6
UDU	96	10	6	5
DDU	110	17	8	6
UUD	116	14	6	4
		Stable Paretian 1.95 Error Terms		
OOO	86	13	7	6
UOO	95	12	5	5
UUO	126	15	6	0
UUU	163	14	6	6
UDO	94	18	10	6
UDU	93	13	7	6
DDU	109	17	7	5
UUD	109	14	5	4

* O, U, and D denote no change, an increase, and a decrease, respectively. Therefore, for example, UDO refers to a beta increase, followed by a beta decrease, followed by no beta change.

Table IV. Cumulative Frequency Distributions of Number of Parameter Shifts Detected by Variable Parameter Regression (N=200)

	Number of Shifts			
Beta Change Pattern	1	2	3	4
		Normal Error Terms		
OOO	3	1	1	1
UOO	9			
UUO	28	1		
UUU	64	5		
UDO	6	2		
UDU	6	1	1	1
DDU	7	2	2	1
UUD	18	3		
		Stable Paretian 1.95 Error Terms		
OOO	8	4	2	1
UOO	14	5	3	
UUO	29	4	2	
UUU	61	8	3	
UDO	11	6	3	
UDU	10	4	2	1
DDU	11	3	2	2
UUD	21	7	2	

* O, U, and D denote no change, an increase, and a decrease, respectively. Therefore, for example, UDO refers to a beta increase, followed by a beta decrease, followed by no beta change.

Table V. Cumulative Frequency Distributions of Number of Parameter Shifts Detected by the Stabilogram Procedure (N=200)

	Number of Shifts			
Beta Change Pattern	1	2	3	4
		Normal Error Terms		
OOO	12	3	1	
UOO	16	3	1	
UUO	34	5	1	
UUU	72	8		
UDO	17	5	1	
UDU	16	4	2	
DDU	19	5	2	
UUD	27	3	0	
		Stable Paretian 1.95 Error Terms		
OOO	15	4	3	
UOO	17	4	2	
UUO	34	4	2	1
UUU	66	11	2	2
UDO	20	5	2	
UDU	18	5	3	
DDU	22	5	1	
UUD	28	3	1	

* O, U, and D denote no change, an increase, and a decrease, respectively. Therefore, for example, UDO refers to a beta increase, followed by a beta decrease, followed by no beta change.

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