# An Instructional Model of Bank Portfolio Credit Risk and Return

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**Abstract**

The measurement of bank portfolio credit risk and risk-adjusted return is increasingly important to banking institutions. Banking curriculum needs to reflect this trend with instructional models that present and integrate these topics. This paper presents a model that allows students to experiment with the interaction of several components of credit risk, risk-adjusted return, and capital risk. It further provides a tangible estimate of the benefits of diversification and allocates these gains to the contributing loan division. Finally, the proposed model illustrates several factors affecting credit pricing and shows how bank profits can be allocated to customers (borrowers or depositors), managers, regulators, or shareholders. The model is assessable to upper level undergraduate finance students or MBA finance students in a few class periods.

## 1.0 Introduction

Increasing competition, global markets, and international regulatory bodies are requiring improved accuracy in the measurement of bank portfolio credit risk and risk-adjusted return. Banks have long been experts at estimating credit risk, but slow to implement the portfolio effect of different loans or divisions on the risk of the bank. The Basle Committee, an international regulatory body, has recently introduced new guidelines encouraging the inclusion of diversification gains in estimating portfolio credit risk. Banks have actively responded and generated numerous models. Several models for portfolio credit risk developed by banks or consulting firms have become focal points of development. Accurate implementation of these models requires estimates based on historical data, precise economic forecasts, relevant loan data and/or other proprietary data. Most models are very complicated including a variety of statistical processes, some of which are proprietary often beyond the scope of undergraduates. The practitioner models estimate model inputs and then estimate portfolio credit risk. The results of these models focus on credit risk and usually do not provide estimates of required capital or return. Estimation of inputs for these models is critical and often defines these practitioner models.

Students, as future managers, need to understand a variety of risks, their interaction, and their impact on return. In academics, risk-adjusted return, credit risk, capital risk, and portfolio diversification are often treated as separate topics. Koch (2006) and Saunders (2006) are excellent texts that treat these topics primarily as silos. Discussions in these texts suggest interaction of risk factors and return, but stay intuitive. Examples demonstrating integration of topics require numerous assumptions about bank credit data and statistical modeling. Several 'bank games' have done a good to excellent

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job integrating topics, but require very large instructional fixed costs. Whereas practitioner models estimate credit risk, academic models often use assumptions about credit risk as an input and compute the portfolio effect. The portfolio credit risk results are then included to estimate risk-adjusted return and capital risk. Finally, academic examples are used to illustrate relationships; the results are not necessarily implemented at a bank.

This paper presents a simple academic model, integrating credit risk and risk-adjusted return and generating results that are intuitively and practically reasonable. The model assumes credit data characteristics regarding failure rate and correlations of failure rates. It uses standard academic models to estimate the benefits of diversification and capital risk. The results of the model are tangible estimates of diversification gains, required capital, risk-adjusted return and how they are correlated. The model demonstrates the benefits of diversification through a decrease in standard deviation, but also through a specific estimate for the reduced capital requirement and higher risk-adjusted return.

Following is the literature or curriculum review. The third section provides a detailed explanation of the model. The fourth section outlines the learning objectives the model facilitates. The final section concludes.

## 2.0 Literature Review

There are practitioner, regulator, and academic models that estimate portfolio credit risk and the related risk-adjusted return. The review of practitioner models has led to several models as discussion points of development. These may include proprietary data and/or methods, and generate estimates of portfolio credit risk used by the bank. Basle II provides guidelines for implementation of complex portfolio credit risk models and provides a basic model for smaller banks. Academics has a variety of models to estimate portfolio credit risk and risk-adjusted return. Generally, the inputs of these academic models, credit performance data, are assumed and then statistically processed to develop a hypothetical value for portfolio credit risk, risk-adjusted return and capital risk. However the models provide useful intuition.

The most popular practitioner portfolio credit risk models include KMV Portfolio Manager, CreditMetrics, CreditRisk, and CreditPortfolioView. The strategies of these models are widely available, but because of their proprietary nature, usually some critical details are omitted. These models incorporate several common risk factors: expected default frequency (EDF), loss given default (LGD), and loss correlation. Estimation of these variables is critical to every portfolio credit risk model. Typically, credit risk is proxied by EDF. EDF is the probability that a loan defaults. LGD assumes the loan has defaulted and measures what percent of the loan is non-recoverable. LGD is correlated to loan seniority, but beyond that is very difficult to forecast. Many models use a random variable from a non-normal distribution to approximate LGD. Loss correlation or default correlation is difficult to estimate because defaults are rare and finding a correlation between rare events leads to weak statistics. Typically, debt default correlation is about 0.04. These practitioner models focus on estimation of portfolio credit risk.

The practitioner models are briefly summarized. KMV estimates credit risk by estimating the expected default frequency, EDF, or probability of default. KMV uses the

Merton model to forecast a firm's asset and equity value. They use these estimates in conjunction with the capital structure to develop a proprietary ranking called the Distance to Default, DtD. The DtD is a ratio measuring the equity coverage of asset volatility. KMV has a proprietary matching of DtD to EDF. CreditMetrics measures downgrade risk as well as default risk. Both risks are measured using an asset-value distribution. CreditMetrics uses credit ratings and historic migration rates incorporated with asset values to generate a distribution of expected asset value based on the migration rates. The model is well-known, but inputs are proprietary to each bank. Credit Suisse's CreditRisk is an actuarial model forecasting the portfolio loss distribution as opposed to the asset-value distribution. It considers only loan default, as opposed to downgrades, in forecasting losses. Therefore, the model forecasts a loss distribution as opposed to an asset-value distribution. It is the simplest model, but still requires proprietary inputs. McKinsey's CreditPortfolioView (CPV), developed in 1997 by Tom Wilson, is referred to as both an econometric model and a migration model. Using econometric modeling, CPV develops a distribution of asset values that are designed to model and vary with the economic cycle. The default estimate is a function of economic variables as well as variables for industry and country. The inclusion of macroeconomic data is unique to this model.

Basle II takes a less sophisticated, more general, and more practical approach. Basle II models loss correlation as a function of loan concentration in the loan portfolio. The greater percent a loan is of the portfolio, the greater the correlation of losses to the portfolio.

Academic models include portfolio credit risk, risk-adjusted return, and capital at risk. Bessis (2002) develops a distribution of credit losses defined by the expected or average loss and the unexpected losses or loss volatility. Loss volatility may then be used in the standard portfolio risk model.

Portfolio diversification for equities has been taught for years and demonstrates the reduction in risk resulting from diversification. The same principle may be applied to credits. Although the distribution of returns is not normal, the model provides a good benchmark and good intuition. Generally, the benefits of diversification may be measured by the reduction in portfolio standard deviation. Also, there has been some debate about how to allocate the benefits of diversification to each portfolio asset.

Capital at Risk, CAR, models the capital a financial institution may lose during a low performing, low probability time period. Bessis (2002) presents the development of CAR as an application of Value at Risk, VAR, to bank capital. Simons (1996) provides an excellent introduction to Value at Risk. CAR has been used in academic settings and serves as a benchmark for comparing regulatory risk based capital, RBC. CAR computes the distribution of possible capital values in any time period. It also allows banks to compute the probability of large losses, which we use in the denominator of our risk adjusted return.

Risk adjusted return, RAR, is typically an ROE measure adjusted for risk. A popular model is to include expected losses in the ROE numerator and unexpected losses and loss correlation in the denominator. The ROE denominator is estimated using CAR. CAR assumes the realization of extreme credit risk and estimates the required capital necessary for the bank to survive. The unexpected losses and correlation of losses are used to estimate divisional loss volatility and portfolio loss volatility, included in CAR.

## 3.0 Bank Portfolio Model

The proposed model uses hypothetical simplified financial statements to compute CAR and the RAR for an undiversified and diversified bank portfolio. The simplified balance sheet includes loans, debt, and capital. The loans represent three divisions: business, consumer and agriculture. The simplified income statement includes contracted loan income, expected loan losses for each loan division, a single operating expense item (measuring both deposit interest expense and non-interest overhead expense), and taxes to compute net income. In addition to the balance sheet and income statement data, the model uses the unexpected loan losses (standard deviation of expected returns/losses) and the correlation of unexpected loan losses between divisions. The correlations between each loan division are assumed and used with the correlation between each division and the bank portfolio (computed in the Appendix) to compute loan division betas. These betas are used to assign portfolio risk to each division. The book value of each loan is given and the loan division financing components of deposits and capital are computed based on loan risk. Riskier loans require more capital. More detail on the model is provided under the following sections: credit risk, capital risk, portfolio diversification, allocation of diversification gains to each product line, risk-adjusted return on equity, and the allocation of profits between shareholders, management, regulators, and customers.

### 3.1 Credit Risk

The credit risk of each division and of the portfolio, results from the non-payment of interest and/or principal. The model includes both a stand-alone and portfolio measure of credit risk. On a stand-alone basis, loan losses are categorized as expected and unexpected. Expected losses are the average historical loss; unexpected losses are the standard deviation of historical losses. The model assumes the return distribution for each loan division is normal with the mean equal to expected returns (contracted returns minus expected losses) and the standard deviation equal to unexpected losses. Many security return distributions deviate from the normal. For example, the distributions often demonstrate kurtosis and negative skewness. These tendencies decrease the accuracy of the normal distribution as an approximation to the real distribution. In particular, they cause the normal to underestimate the left-hand tail, which in this model, is very important. Additionally, loan returns generally have a truncated upward return, further decreasing the exactness of the normal distribution. Regardless, the intuition of the model is not affected by the use of the normal and the normal provides a good benchmark.

For the industrial product line, the contracted loan rate is 7.5%, the expected loss is 1.20%, and, therefore, the expected income is 6.30% or $12.6 million (6.30% * $200 million). A portfolio measure of credit risk is subsequently developed and presented in section 3.4.

### 3.2 Capital Risk

Capital is used as a 'cushion' against losses and the capital risk is inversely related to the amount of capital. Capital risk is the composite of all risks the bank faces that may decrease capital. In this model we only include credit risk, so capital risk is directly related to the credit risk of the loan portfolio. This paper uses Capital at Risk

(CAR), which is similar to parametric Value at Risk, and computes the capital risk as the product of the asset value and a multiple of divisional loss volatility. In this model, we use a negative asset return associated with a probability of 1%, implying a z-score of -2.33 from the normal distribution. This implies a 1% failure rate for a one year time horizon and almost a 5% failure rate for a five year time horizon. Using the industrial line of business as an example, the market value is $200 million and the divisional loss volatility is 0.8% implying a low probability return/loss of –1.864% (-2.33 * 0.8%) and the CAR is -$3.72 million (-1.864% * $200 million). If the bank allocates $3.72 million in capital to the industrial line, the division will fail (lose all its capital) 1% of the time periods. This estimate assumes no diversification between loan divisions.

The portfolio CAR includes portfolio value and a multiple of portfolio loss volatility, having only a 1% probability of occurring. The CAR for the portfolio and the diversified CAR for each division are shown in section 3.3 and 3.4 respectively.

### 3.3 Portfolio Diversification

Diversification is demonstrated by the CAR of the portfolio, $CAR_p$, being less than the sum of divisional CAR, $CAR_{1\text{-}3}$. The $CAR_p$ for the entire portfolio is computed as the product of bank assets and a multiple of portfolio loss volatility. To determine the low probability loss, we use the standard deviation of portfolio returns. The equation below computes the bank unexpected loss or portfolio loss volatility as 1.55%.

$$\sigma_p = \left[w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + w_3^2\sigma_3^2 + 2w_1w_2\sigma_1\sigma_2\rho_{1,2} + 2w_1w_3\sigma_1\sigma_3\rho_{1,3} + 2w_2w_3\sigma_2\sigma_3\rho_{2,3}\right]^{1/2}$$

*where: $w_i$ – market value weight of each loan division, i=1-3,*

*$\sigma_i$ – standard deviation of unexpected returns/losses, i=1-3, and*

*$\rho_{i,j}$ – correlation between divisional losses, i,j=1-3, but $i \neq j$.*

The low probability portfolio return is -3.612% (-2.33 * 1.55%) and the $CAR_p$ is $21.58 million (-3.612% * $600 million). This is less than the sum of the product line CARs at $24.66 million. This difference results from the risk reduction of diversification and improves the risk-adjusted returns of the portfolio.

### 3.4 Allocation of Diversification Gains

The risk reduction needs to be allocated between the three product lines to accurately calculate divisional risk-adjusted returns. The best way to allocate the gains of diversification should include a measure of the product line's contribution to overall firm risk. An appropriate and familiar model is CAPM's beta, except the model, in this paper, calculates the beta between the product line and the bank portfolio. The mathematics is demonstrated in Appendix A. The product line CAR' including the gains from diversification is the product of $CAR_p$, the product line market value weight, and the product line beta. For the industrial portfolio, CAR' is $2.41 million ($21.58 million*0.333*0.335). Without diversification, this divisional CAR is $3.72 million. The industrial line beta is the lowest divisional beta at 0.335. The significant CAR reduction for this product line suggests that the benefits of diversification are assigned to the product line with the lowest correlation to the bank portfolio.

### 3.5 Risk Adjusted Return, RAR

The return on equity measure may include a risk adjustment in the numerator, denominator or both. In this model, the numerator, net income, includes expected losses and the denominator, CAR, includes unexpected losses. For the industrial line, the net income is $0.74 million and the stand alone CAR is $3.72 million generating a RAR' of 19.97%. The diversified CAR' is $2.41 generating a RAR' of 30.82%. In this case, diversification improved risk adjusted returns by almost 11%.

### 3.6 Allocation of Excess Profits

Excess profits are the difference between $RAR_p$' and the corporate hurdle rate. The primary goal of the financial manager is to reward shareholders with these excess profits. However, if we set the $RAR_p$' equal to the corporate hurdle rate, other stakeholders may benefit. We may benefit bank management, loan customers, and /or other stakeholders by increasing personnel expenses, decreasing the contracted loan rate, or a variety of other changes. For example, setting the RAR for the industrial line equal to a suggested corporate hurdle rate, 15%, allows expenses to increase to 5.95%, This provides a 20 bp (5.95% - 5.75%) or a $400,000 (0.2% * $200 million) increase in non-interest expenses to allocate to management or depositors. Alternatively, if we hold expenses constant, the excess may be passed on to our loan customers (loan recipients) through a reduced contract rate of 7.30%, 20 bp below the initial contract rate. This strategy may increase market share.

## 4.0 Questions on the Model for Student Inquiry

The following questions are designed to lead students to a deeper understanding of the concepts demonstrated by the model.

*4.1 Questions on Credit Risk*

1. What is the impact of increasing expected losses on the bank's $RAR_p$'? Do expected losses affect the numerator or denominator? What is the impact of increasing unexpected losses on the bank's $RAR_p$'?
2. If $RAR_p$' is greater than the corporate hurdle rate, 15%, where is the income allocated: shareholders, borrowers/creditors, employees, or regulators? If we set $RAR_p$' equal to the bank's hurdle rate, what is the impact on customer prices, loan rates? What variables did we include to calculate loan rates?
3. Set the bank's RAR equal to the hurdle rate, 15%; what is the breakeven loan contract rate for the industrial loan with and without diversification? How much does diversification reduce the contract rate? Explain the advantages of contract rate reduction.

*4.2 Questions on Capital Risk*

4. The model suggests a failure rate of 1% and generates a low CAR relative to risk based capital regulatory standards. What are the implications of a lower failure rate, i.e. 0.5%, on CAR, $RAR_p$', and other spreadsheet accounts? Is 1% a reasonable expectation of bank failure by managers and /or regulators?

4b. What arguments would the CEO, regulator, management, and customer raise concerning failure rate?

5. The CAR for a specific product line/division is determined by the standard deviation of loss and may vary from the RBC regulatory requirement for each product line. How might this difference between regulatory capital requirements and capital at risk requirements affect allocation of capital and assets to each product line?

6. What are the problems with the assumption of losses or returns being normally distributed? Will the normal distribution over or underestimate CAR?

*4.4 Questions on Allocating the gains of diversification*

7. Explain how the different product lines are diversified. Which two lines are the most diversified? How do you know?

8. What is the impact of diversification on $CAR_p$, the CAR for each division, $RAR_p$, and the RAR for each division? Who benefits from the diversification gains: shareholders, borrowers, employees, or regulators?

9. As the result of diversification, $CAR_p$ is reduced. How is the reduction of CAR allocated to each product line? In this model, why did the industrial product line receive a larger share of diversification gains?

10. How do shareholders, customers, and managers potentially benefit from a diversified portfolio? Explain the connection between portfolio benefits and various stakeholders? Are there any detriments to diversification?

11. What other methods, besides a portfolio beta, could be used to allocate gains from diversification to each product line?

*4.5 Questions on Risk-Adjusted Return*

12. Assuming allocation between product lines can be altered, such that the total market value is still $600 million, what is the allocation of assets that generates the highest RAR? (Use Solver add-in.) Which loan division received more assets? Why?

## 5.0 Summary

The model integrates credit risk, capital risk, and portfolio theory as they apply to a hypothetical three-division bank. The model incorporates current risk estimation methodology and clearly demonstrates the interaction between different risk factors and return. The model is assessable to the students within a few class periods. The insightfulness of the model and the Questions for Student Inquiry allow for profound student learning.

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**Appendix A:** Bank Portfolio Beta

CAPM Market Beta: $\beta_{i,m} = Cov_{i,m}/Var_m$

Bank Portfolio Beta: $\beta_{i,p} = Cov_{i,p}/Var_p$

Where, i represents a specific loan division.

$$\beta_{1,p} = \frac{Cov_{1,p}}{Var_p} = \frac{Cov_{1,p}}{\sigma_p^2} = \frac{Cov_{1,p}}{\sigma_p \sigma_1}\frac{\sigma_1}{\sigma_p} = \rho_{1,p}\frac{\sigma_1}{\sigma_p}$$

where, $\rho_{1,p} = w_1\,\sigma_1/\sigma_p + w_2\rho_{1,2}\,\sigma_2/\sigma_p + w_3\rho_{1,3}\,\sigma_3/\sigma_p$

Assuming the portfolio beta equals one because in this model it is the benchmark, we have:

$\beta_p = 1 = w_1\beta_{1,p} + w_2\beta_{2,p} + w_3\beta_{3,p}$

If we multiply the equation by the $CAR_p$, then the product line allocation of diversified $CAR_p$ is the product of $CAR_p$, the market value weight and the product beta.

$CAR_p = CAR_p w_1\beta_{1,p} + CAR_p w_2\beta_{2,p} + CAR_p w_3\beta_{3,p}$
$CAR_1 = CAR_p w_1\beta_{1,p}$

Appendix B: Simplification
Ignore math in App A and use simple weights to allocate capital between divisions.

Figure 1: Bank Portfolio Risk & Return

*Given Information (Italics)*

| | *Industrial* | *Agricultural* | *Consumer* | *Portfolio* |
|---|---|---|---|---|
| *Contract Loan Rate* | *7.50%* | *9.00%* | *10.50%* | *9.00%* |
| *Expected Loss Rate* | *1.20%* | *2.50%* | *3.50%* | *2.40%* |
| *Unexpected Loss* | *0.8%* | *2%* | *2.50%* | *1.55%* |
| *Market Value (Millions)* | *$ 200* | *$ 200* | *$ 200* | *$ 600* |
| *Asset Weight* | *0.3333* | *0.3333* | *0.3333* | |

| | | | |
|---|---|---|---|
| *Total Expenses* | *5.75%* | *(Interest + Non-interest)* | |
| *Probability of Failure* | *1.00%* | *Implies a z-value of* | *-2.33* |

Income Statement (Millions)

| | Industrial | Agricultural | Consumer | Portfolio |
|---|---|---|---|---|
| Contracted Loan Income | $ 15.00 | $ 18.00 | $ 21.00 | $ 54.00 |
| Expected Loss | $ 2.40 | $ 5.00 | $ 7.00 | $ 14.40 |
| Expected Income | $ 12.60 | $ 13.00 | $ 14.00 | $ 39.60 |
| Total Expenses | $ 11.36 | $ 11.01 | $ 10.89 | $ 33.26 |
| Earnings Before Tax | $ 1.24 | $ 1.99 | $ 3.11 | $ 6.34 |
| Taxes (40%) | $ 0.50 | $ 0.80 | $ 1.24 | $ 2.54 |
| Net Income | $ 0.74 | $ 1.19 | $ 1.87 | $ 3.80 |

Risk Adjusted Return (RAR)

| | | | | |
|---|---|---|---|---|
| Standard Dev. Of Returns | 0.80% | 2.00% | 2.50% | 1.55% |
| CAR w/o diversification | $ 3.72 | $ 9.31 | $ 11.63 | $ 24.66 |
| CAR' w/ diversification | $ 2.41 | $ 8.53 | $ 10.64 | $ 21.58 |
| RAR (w/CAR) | 19.97% | 12.83% | 16.05% | 15.43% |
| RAR' (w/ CAR') | 30.82% | 14.00% | 17.55% | 17.63% |

Figure 1: Bank Portfolio Risk & Return (cont.)

*Given Information (Italics)*

*Correlation Coefficients*

| | *Individual* | *Agricultural* | *Consumer* | *Portfolio* |
|---|---|---|---|---|
| *Individual* | *1* | | | |
| *Agricultural* | *0.597* | *1* | | |
| *Consumer* | *0.404* | *0.709* | *1* | |
| Portfolio | 0.6478 | 0.9165 | 0.9145 | |
| Beta a,p | 0.335 | 1.186 | 1.479 | 1 |

Balance Sheet (Millions)

| | Industrial | Agricultural | Consumer | Portfolio |
|---|---|---|---|---|
| Assets | $ 200 | $ 200 | $ 200 | $ 600 |
| CAR' | $ 2.41 | $ 8.53 | $ 10.64 | $ 21.58 |
| Debt | $ 197.59 | $ 191.47 | $ 189.36 | $578.42 |