

A Contingent-Claim Interpretation of Leases on Real Assets

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Abstract

Real estate leases often contain contingent rights that may accrue to either the lessee or the lessor. This paper looks at how these contingent claims can be viewed as the equivalent of exchange traded, and over-the-counter, derivative instruments. It is shown that with this equivalence the potential for derivatives written on real estate represents a major market opportunity for these contracts. The paper also briefly examines how the underlying asset price may be measured, which has been the primary impediment to the development of a real estate derivatives market.

Introduction

Real estate leases have been used for decades to diminish the business risk of property owners and insure extended stability for tenants. Features have been added over time that significantly extends these initial purposes to cover a variety of contingencies for both parties to the real estate lease contract. In most cases these features are option like claims that transfer a valuable right from one party to the other. The inclusion of these claims alters the value of the lease and the change in value must necessarily be included in the contracted rental rate as a decrease to standard market rent if in favor of the lessor or an increase if in favor of the lessee. Unfortunately, the added (or decreased) rent is not the result of explicit pricing of the feature attached to the standard lease; rather it is most often the result of a best guess of its value on the part of the parties to the lease.

This paper, examines the contingent relationships that may exist between lessors and lessees and frames these relationships in the context of derivative contracts. Specifically, the real estate lease is viewed as a derivative contract and the applicability of the tools and concepts of contingent claim valuation to real estate leasing is examined.

The first section of the paper discusses the purpose of the lease contract and identifies several lease situations which are commonly encountered in commercial leasing. The second section demonstrates how these arrangements can be viewed as a commodity swap and evaluated in a similar fashion. In the third section applications of option pricing to lease evaluation is presented. In the final section a discussion of applications of financial innovation to lease negotiations is offered.

The Lease as a Risk Management Tool

The traditional real estate lease was designed to reduce the business or market risk of the property owner and to guarantee the use of the space at a predetermined rent for the

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lessee. In this sense it is an instrument which smoothes the cash flows to both parties and insulates both to adverse changes in the market. The traditional lease then is a simple, straightforward approach to managing financial price risk of both parties. However, few commercial leases today are traditional leases where the lessee agrees to pay the lessor a fixed rent for some time period with the lessor providing and maintaining the leased space. The modern lease has been extended to shift, or share, more diverse risk than market risk and has also become an instrument with a variety of contingent claims attached.

An initial extension to the basic lease was the shifting of operating expenses from the lessor to the lessee in the form of a net lease. This did not represent a shifting of a fixed cost but, rather a variable cost that would fluctuate with the prices of the individual cost items. Though the lessee has some control over operating expenses by varying the intensity of use of the space (quantity of operating inputs), the price of operating inputs is largely beyond the lessee's control. With the net lease the lessee assumes greater financial price risk than in the case of the traditional lease.

With the traditional lease the property owner is insulated against market rent declines while the lease is operative but, bears the risk that inflation induced increases in market rent can not be realized. To address this inflation risk the traditional gross rent lease was modified to allow rent increases which were indexed to an inflation index such as the consumer price index (CPI). Purchasing power risk is then shifted to the lessee in the form of an indexed lease. With an indexed net lease much of the price risk faced by the lessor is transferred to the lessee.

Obviously, when any type of risk is shifted from one party in the lease to the other party, adjustments to the initial base rent is required. Albert and McIntosh [3] have presented a framework which allows identification of the indifference rents for leases with differing risk profiles.

In recent years more interesting extensions to the basic lease have developed in the form of contingent claims attached to the lease. These claims give one party an option to invoke a claim against the other party should the claim prove attractive. For example, it is common in commercial leases to include an option for the lessee to renew the lease at some formula defined rent. The renewal rent could simply be defined at market, in which case the option has little value, or it can be defined as a predetermined amount or as the initial contract rent grossed up by the change in an index, typically the CPI. It is also not uncommon for tenants using a large portion of the total property to be granted an option to purchase the property at some formula defined price. The optional purchase price is typically defined in the same manner as renewal rent. It is also not unusual to have both an option to renew and an option to purchase embedded in the lease.

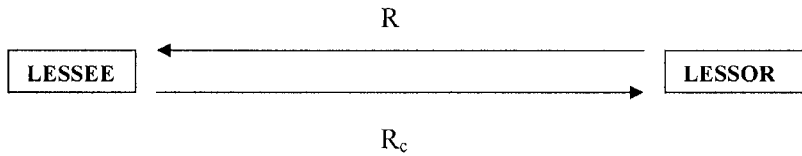
These extensions to the basic lease make evaluation of the arrangement more difficult than is the case in a traditional fixed gross lease contract. The initial rent pricing and the valuing of the position of the two parties at interim periods is not as straightforward as it is with the traditional lease. In the next section the lease is viewed as a derivative contract, in the form of a commodity swap, and these alternative lease structures are examined in this context.

The Lease as a Commodity Swap

The traditional fixed rent gross lease is the equivalent of a commodity swap where a

fixed stream of cash flows is swapped for a cash flow stream representing the market price of a commodity. The commodity is the space being leased and the fixed cash flow stream is the fixed rent paid by the lessee. Though the lessee does not receive a cash flow of market rents, real estate worth market rents are received. If the lessee has the right to sublet the property, a sublease where rent was variable based on changes in market rent would exactly duplicate a commodity swap. For this reason the lease can be viewed as a commodity swap and the cash flows could be diagrammed as in Exhibit 1.

Exhibit 1



Market rent is denoted as R and the fixed contract rent as R_c .

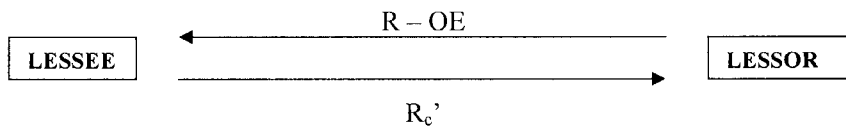
At the origination of the swap the value of the swap position for each party is zero or more formally:

$$V_s = \sum_{t=0}^T \frac{R_c}{(1+k)^t} - \sum_{t=0}^T \frac{E(R_t)}{(1+k')^t} = 0 \quad (1)$$

where t is the rent interval (i.e. annual, monthly), V_s is the value of the swap position and k and k' are the appropriate risk adjusted discount rates. At interim periods $0 < t < T$ it is possible for the value of the swap position to become positive for one party and negative for the other depending on the behavior of R . If R changes by more or less than expectations as reflected by k' then the swap takes on a positive value to one party and negative for the other.

When operating expenses are shifted to the lessee the basic structure of the swap is not altered, there is simply an adjustment to the cash flow streams as shown in Exhibit 2.

Exhibit 2

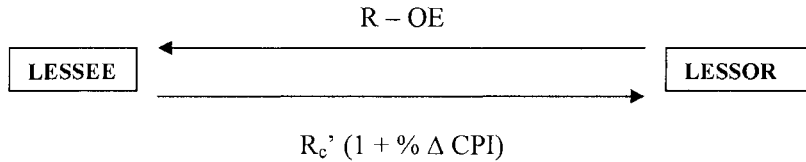


Where OE reflects the market price of operating expenses and R_c' is the adjusted fixed rent reflecting the shifting of operating expenses to the lessee. The conditions for the swap taking on value remain the same.

When the lease is written as an indexed rent net lease the likelihood that market rent and contract rent will differ over the life of the lease is reduced because of the removal of the general inflationary effect. Real estate market fluctuations, and the subsequent impact on market rents remain. With an indexed lease there is a floating for floating swap of cash

flows. One floating leg of the swap is tied to an index (CPI) and the other to the market price of leased space as shown in Exhibit 3.

Exhibit 3



In the three lease structures examined it is straightforward to view the lease as a commodity swap. The value of the swap during interim periods can be determined during the framework presented in Albert and McIntosh and will not be presented here. Greater complexity to the basic swap arrangement is encountered when contingent claims are embedded in the lease. These situations are examined in the next section.

The Lease With Embedded Options

As indicated previously, options to renew a lease or purchase the leased property are not uncommon in commercial leases. This is particularly so for the option to renew the lease and it is this option on which this paper focuses.

When an option to renew is embedded in the lease, valuing the swap position becomes significantly more difficult. The condition that the swap must have an initial value of zero is not altered with the presence of the option because the value of the option will be offset by higher contract rents or formally:

$$V_s = \sum_{t=0}^T \frac{R_c^*}{(1+K)^t} - \sum_{t=0}^T \frac{E(R_t)}{(1+K)^t} - O = 0 \tag{2}$$

Where O is the value of the option to renew and the remaining variables are as defined in equation 1. The Value of R_c* in equation 2 will be greater than R_c, in equation 1 since the option price is annuitized in the contract rent. At interim periods 0 < t < T the value of the swap position is a function of both market rent R and the value of the option to renew, which will itself be a function of market rent and other variables depending on how the option is defined.

Typically, renewal options define renewal rent in one of three ways. In many cases the option to renew calls for a renewal rent to be the prevalent market rent at time of renewal. Though the option binds the lessor to releasing the space to the lessee, it does not have a measurable economic value, since the parties will recontract at market and the option will by definition expire at-the-money. While the option does allow the lessee to forego relocation cost there is no advantage to be gained in terms of a more favorable rent. Renewal rents can also be defined to be some percentage of prevailing market rents with 90% and 95% being the most commonly used percentages. Such an option does have economic value attached since the lessee gains the right to re-lease at less than market. Finally, renewal rent could be defined to be the initial contract rent grossed up by the change in an index such as the CPI.

Valuing the swap position at some interim period $0 < t < T$ requires that the renewal option attached to the lease be priced at time t . It is therefore, necessary to identify appropriate models for pricing renewal options.

An option to renew a lease is a call option, held by the lessee, on the underlying space. A typical call option pricing model requires the determination of the probability that the market price (market rent) will exceed the option's exercise price (renewal rent) at the time the option expires (renewal date) and also identify the expected market price given that it is greater than the exercise price. For both the probability and the expectation, it is necessary to define the stochastic process which the underlying market price follows over time. In this paper, market rent for commercial space is assumed to follow Geometric Brownian Motion (GBM), the process typically assumed for stock price movements overtime. Since real estate, like stock, is a tradable asset and the value of real estate is a direct function of market rent, the assumption that market rent follows a GBM process is reasonable.

When a renewal rent is defined to be some percentage of current market rent at time of renewal, the probability that the option will be in-the-money (i.e. market rent greater than renewal rent) is, by definition, one and the pricing of the renewal option requires only the identification of the expected market rent at renewal. The assumption of GBM implies that market rents have a lognormal distribution and by the properties of the lognormal distributions the price of this type of renewal option is given by:

$$O = (1-p) R_t e^{(\mu-r)(T-t) + \sigma^2 T/2} \quad (3)$$

where p is the fractional proportion of market rent, μ is the constant expected rate of change in rents over time period T , σ is the standard deviation of rent changes and r is the risk free interest rate. When the right hand side of equation 3 is substituted into equation 2 for O , the value of the swap can be determined for any time period $0 < t < T$.

Valuing the renewal option when the renewal rent is tied to the CPI is a significantly more difficult task. Because renewal rent is a function of the CPI, which itself is a random variable, two stochastic processes must be modeled. It is reasonable to assume that the CPI like market rents also follow GBM. It should be noted that other stochastic processes can be assumed if the underlying economics suggest an alternative. For example, a mean reverting process (MRP) of one or both of the stochastic variables could as easily be modeled as a GBM. However, there is, no theory that would suggest an MRP would most closely approximate the stochastic properties of real estate rents and the CPI than would GBM. For this reason GBM is used for both stochastic variables.

Using no arbitrage conditions and a variant of the riskless-hedge portfolio the partial differential equation (PDE), when both rent and the CPI (denoted as C) follow GBM, which models the renewal options price is expressed as:

$$O = \frac{\sigma_R^2 R^2}{2} O_{RR} + \frac{\sigma_C^2 C^2}{2} O_{CC} + \rho_{RC} \sigma_R \sigma_C R C O_{RC} + r R O_R + r C O_C - O_{(T-t)} - r O \quad (4)$$

where the subscripts on O represent partial derivatives and ρ_{RC} is the correlation coefficient of market rents and the CPI.

Unfortunately equation 4 does not have a known analytic solution and must be approximated with a numerical process. While approximating the solution to equation 4 is beyond the scope of this paper, Buetow and Albert [4] demonstrate the application of the Finite Difference Method (FDM) with Absorbing Boundaries (A-B) technique to approximate the PDE and interested readers are referred to that work.

As before, substituting the right hand side of equation 4 into equation 2 for O and approximating the solution of O will provide the value of the swap position at any time period $0 < t < T$.

Potential Applications In Leasing

In the previous two sections, the real estate lease has been framed in the context of a derivative contract and the basic equations for evaluating a position in a lease have been identified. While this is an interesting exercise, in and of itself, there must be potential applications for the framework to be useful. As indicated in the introduction, the marketplace for financial innovation and the supply of structured derivatives has shown phenomenal growth and adaptability in recent years. It is only a matter of identifying the demand for real estate derivatives for the market to respond with the supply of such contracts. The lease arrangement discussed here provides a glimpse at potential needs, of lessors and lessees, for lease oriented derivatives.

In a classic commodity swap, one of the parties may find it advantageous to terminate (unwind) or alter the swap position prior to the expiration of the contractual period. To do so only requires that party to take an offsetting position in another commodity swap of duration equal to the remaining contractual period on the original swap. Analogously, a lessor who has entered into a fixed rent lease may determine that market rents are more likely to increase over the life of the lease than was anticipated at the time the lease was struck. For example, assume a lessor has entered into a fixed rent lease for 5,000 sq. ft. of office space at \$20/sq.ft. for a five year period. At the end of the second year, the lessor believes that market rent will soon exceed the \$20/sq.ft. contract rent. To take advantage of this rent outlook, the lessor could enter into a three year commodity swap to receive 5,000 times the market rent per square foot with market rent defined by the quarterly rents reported in the National Real Estate Index (NREI) for the metropolitan area in which the space is located. Alternatively, a lessee who thought market rents may decline, or not increase as much as originally anticipated, may wish to enter into a commodity swap to pay a fixed cash flow in return for a market rent indexed cash flow.

These offsetting positions only require that a commodity swap with real estate market rents as the floating price leg be available. It is not unreasonable to expect that such a swap would be offered by intermediaries if the demand is identified.

Currently, when leases have options attached, such as the option to renew, it is not possible for the lessor to unwind the position should market rents move against the lessors position. For example, assume a lessor has struck a five year lease at \$20/sq.ft. with a five year renewal option where the renewal rent is \$20 $(1 + \% \Delta \text{CPI})$. At some point during the initial five year period the lessor decides that market rents are likely to increase at a greater rate than the CPI and would like to redefine the renewal rent. While this is contractually impossible, the same economic result could be duplicated if the lessor could buy a call on market rent thereby, offsetting the short call position. To do so would require that indexed

options be available where the underlying price is simply the indexed rent reported by the NREI.

Though details for structuring the kinds of contracts referred to above are complex, it is not an insurmountable task, and given the pace of financial innovation it is reasonable to expect that real estate based derivatives will soon be readily available in the marketplace.

Conclusions

While the above discussion of real estate leasing identifies the activity as fertile ground for the application of financial innovation, it is by no means the only real estate area where derivatives would find wide and useful application. The management of real estate and mixed asset portfolios is probably an even richer area to apply the tools of financial innovation. Portfolio managers wishing to alter their price risk exposure could do so with options or forward contracts indexed to a real estate price series such as reported in the NREI. Mixed asset portfolio managers may wish to either increase or decrease their exposure to real estate prices and could do so without altering the content of the portfolio by using the same type of instruments. Indeed, the applications for real estate based derivatives is no more constrained than in any other area of investment or business activity.

This paper has hopefully, made clear the need for real estate derivatives. Given the pace of financial innovation to other applications, the advent of real estate derivatives in the marketplace is simply a matter of time.

Bibliography

1. Albert, J. and G. Buetow, 2000, "Explicit Valuation of Real Estate Lease Options", *Real Estate Finance Journal*, Vol 15 No. 3, Winter 2000.
2. Ambrose, B., P. Hendershott and M. Klosek, 2002, "Pricing Upward-Only Adjusting Leases", *The Journal of Real Estate Finance and Economics*, Vol. 25 No. 1.
3. Albert, J. and W. McIntosh, 1989, "Identifying Risk Adjusted Indifference Rents for Alternative Operating Leases", *Journal of Real Estate Research*, Vol. 4 No. 3, 22-36.
4. Buetow, G. and J. Albert, 1998, "Pricing Embedded Options In Real Estate Lease Contracts", *Journal of Real Estate Research*, Vol. 3 No. 15.
5. Case, K. E., R. J. Shiller and A. N. Weiss, 1993, "Index-Based Futures and Options Markets in Real Estate", *The Journal of Portfolio Management*, Winter 1993, 83-92.
6. Grenadier S.R., 1995, "Valuing Lease Contracts: A Real Options Approach", *Journal of Financial Economics*, # 38, 297-331
7. Hull, J., 1993, *Options, Futures, and Other Derivative Securities*, Prentice Hall, New York.

8. Newman J., 1991, "The Option Period - Is It a One-Way Street Called Tenant Avenue?", *Real Estate Review*, Fall 91, 77-81.
9. Posner K., 1992, "Hedging Against Changes in Market Rent Levels", *Real Estate Review*, Fall 92, 21-26.