Modeling Banks' Risk Taking Behavior in a Presence of the Long-Tail Risk Guarantor with the Truncated Version of the St. Petersburg Coin Flip Model

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Abstract

This paper draws on the sequential coin toss gamble, used in the St. Petersburg paradox, to model investments with long-tailed risks– those that result in very rare but huge losses. It applies the model to the depository institutions' assets with long-tail risk, where depositors serve as guarantors of tail risk, and also to other investments with long-tails, such as collateralized debt obligations. The model simulates the financial institutions risk taking behavior, where agents with limited liability are able to shift their tail risk onto external parties who have either imperfect information or act with disaster myopia (i.e. "too big to fail" scenario). A simulation shows that such self-interested agents benefit by increasing leverage and increasing the chance of bankruptcy.

I. Introduction

Self-interested agents may be expected to seek investments with long-tail risk if they are sheltered by limited liability and if the long-tail risk is underpriced, hidden, or ignored by their creditors. The long-tail risk investments are those with outcomes that have extremely low chances of very large losses. Hidden tail risk appears to play a part in recent financial events. Some examples include:

- collapse of the hedge fund Long-Term Capital Management
- implosion of investment banks, Bear Sterns, Lehman Brothers
- failure of many structured finance vehicles
- ruin of AIG in the insurance of credit default swaps
- bankruptcy of Fannie Mae and Freddie Mac
- insolvency of U.S. and European banks
- recent failure of the futures broker dealer, MF Global

Indirect costs of the failure of these financial institutions include the general deterioration of world wide economic activity as subsequent credit squeeze impact many kinds of businesses. The events of the financial crises of 2008 might have appeared catastrophic and random due to a rear and extreme nature of the losses. The US Government has stepped in as a guarantor to bail out a number of institutions at the time to prevent the economy from further collapse. In this paper we use the truncated version of the St. Petersburg gamble to argue that extreme and rear losses are actually part of the probability distribution of the financial institutions' risk taking behavior. Further, these long-tailed risks appear to be known to the investors and priced accordingly. Investors appear to both realize the limited ability of the financial institutions to make full payment in case of extremely rear and huge losses and count on the implicit government bail out of the "too large to fail" institutions. The nature and the value of these risks, though, appears to be either ignored or underpriced by the guarantor.

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A vivid example of such behavior is a bankruptcy case of the Lehman Brothers. Lehman Brothers was the fourth largest US investment bank at the time of the collapse with $639 billion in assets and $619 billion in debt. It was the largest bankruptcy in the US history. Lehman Brothers filed for bankruptcy on 15, 2008. The news that the bank will not be bailed out by the government lead to a massive erosion of global equity with the estimated loss close to $10 trillion in the market capital over the following month. We would argue that the implicit government guarantee was priced in by the investors and the news of its absence lead to the market's re-pricing of the expected values of the payouts. The investors truncated the value of possible payouts at a lower limit available just from the financial institutions. The investments were re-prices based on the payouts without the additional capital that was implicitly available from the guarantor prior to the collapse of Lehman Brothers. This re-pricing of the value of the possible payouts lead to the huge ripple effect in the global equity markets post-bankruptcy of the Lehman Brothers.

In this paper we build a truncated version of the St. Petersburg gamble to demonstrate that the long-tail risk of the financial institutions is in fact priced in by both sellers and investors who extract profits from the underpricing of the explicit insurance of the large losses by a guarantor. Understanding the non-random and non-catastrophic nature of the long-tailed risk taking behavior would allow for better pricing of the insurance premiums of the guarantor's services and would be a step toward more stable financial markets.

Alan Greenspan famously testified to an error in his thinking, stating, "I made a mistake in presuming that the self-interests of organizations, specifically banks and others, were such as that they were best capable of protecting their own shareholders and their equity in the firms." However, self-interested organizations with tail risk would be less interested in avoiding those losses not borne by their shareholders. Income can be captured by seeking tail risk limited liability firms that can pass the infrequent but large losses onto their unsuspecting creditors and the remaining society.

A post crisis example of a self-interested organization with hidden tail risk is the futures broker dealer, MF Global. In 2010 it made heavily levered investments in Italian, Spanish and other European sovereign bonds maturing in 2012. The firm attempted to profit on the spread between the yield on the sovereign bonds and its payment to creditors. Whether or not the risk was objectively a long-tail risk, MF Global's executives certainly perceived that these European sovereign credits would be extremely unlikely to default. MF Global's creditors in the interbank market appeared to be uninformed about the risk that they bore, were uninsured and ultimately bore potential tail losses. Indeed, even the clients' brokerage accounts, supposedly separated, were ultimately at risk. When a rating agency downgrade revealed extent of the risk to all, the creditors realized the nature of the risk. Their reaction precipitated the firm's demise. If the true risk had never been revealed to its creditors, the profitable arbitrage between the low cost of funds in the inter-bank market and the European sovereign debt market may likely have succeeded. Only the insiders would know the nature of the arbitrage. Indeed, as of the time of the MF Global's bankruptcy, none of the European bonds had defaulted (Azam, Protess, and Craig, 2011).

A famous coin flip model, based on the process underlying the St. Petersburg Paradox gamble, could be used to build intuition about processes that result in long-tailed outcomes. St. Petersburg Paradox gamble is a model based on sequential coin flipping where payouts to the gambler grow exponentially, i.e. the gambler receives $\$2^N$ as payout based on the number (N) of the sequential "head" results of flipping a coin. Such game results in highly skewed outcomes, with frequent small payouts to the gamblers, a few medium payouts, and extremely rare, but enormous payouts. Relative to standard normal models, St. Petersburg model outcomes have long tails. The probability density function for a St. Petersburg type game is a power law distribution where the probability of a next sequential "head" outcome is 2^{-N}, (Liebovitch, Larry S. and Daniela Scheurle, 2000). The sample mean and variance of the data drawn from power law distributions are not good predictors of the mean and variance.

Coin flip models, in spite of their limitations, inform by their simplicity, since they are easily grasped. This paper uses the St. Petersburg paradox coin toss gamble as a model to illustrate how self-interested agents could generate fake alpha and illusory risk-adjusted profits. The agents seem to create real excess returns, but do so by not pricing in the long-tail risk and transferring it to the less informed creditors or to their guarantors instead.

II. The Classical and Bounded St. Petersburg Gamble

In the classical St. Petersburg gamble the outcomes of a game are determined by tossing a fair coin until a "tail" occurs. The customer (buyer) pays the lottery operator (seller) a premium to play. In return the buyer receives a payout based upon the sequence of coin tosses. If the first tail occurs on the N^{th} toss, the payout (Q) equals a stakes parameter [X] times 2^N. Through the rest of the paper we will assume X=\$1 for simplicity.

$$Q = [X] \cdot 2^N = \$1 \cdot 2^N = 2^N \qquad [1]$$

Thus, if the tail occurs on the 1st toss, the buyer receives 2^1 = \$1, if the tail occurs on the 2nd toss, then the buyer receives 2^2 = \$4, if the tail occurs on the third toss, the buyer receives 2^3 = \$8, etc. A St. Petersburg gamble has a high probability of small payouts, a low probability of medium sized payouts, and a vanishingly small probability of extremely large ones. For a fair coin, the probability of a head or a tail outcome for each toss is the same and is equal to ½. The probability of the tail on the 1st toss is ½ , probability of the tail occurring on the second toss is ½ · ½ = ¼ , probability of the tail occurring on the third toss is ½ · ½ · ½ = 1/8, etc. Probability of a "tail" outcome occurring on the N^{th} toss is therefore given by 2^{-N}.

The expected probability weighted value of such a gamble to the customer (buyer) is arbitrarily large and is given by the following formula:

$$E(V) = \sum_{N=1}^{\infty} P \cdot 2^N = \sum_{N=1}^{\infty} \frac{1}{2^N} \cdot 2^N = \infty \qquad [2]$$

Since potential customers (buyers) would be willing to pay only a limited sum to play the lottery even with such a large expected payoff value on one hand, and no sellers could credibly offer to pay unlimited sums, on the other hand, the classical St. Petersburg gamble cannot exist. It has been suggested as a popular solution to the St. Petersburg paradox that both the sellers and the buyer realize the limited nature of the potential payoffs for the gamble and tend to equate extremely small probabilities to zero, the players would price the gamble accordingly, i.e. the truncated form of the St. Petersburg coin flipping model. Practical experiments have shown that players would be willing to pay, on average, about \$15 per game. There is a vast literature on the St. Petersburg paradox, reviewed elsewhere (Martin, 2008; Samuelson, 1977). This paper appropriates the coin flip model in a truncated form.

In a truncated version of the St. Petersburg gamble used in this paper the number of coin tosses is limited to a maximum number N^{max}, that is determined based on the maximum amount of funds available for each player from the firm conducting the gamble. Each gamble ends on the toss in which a first "tail" occurs. If no toss results in a tail the truncated version of the game ends on N^{max} toss of the coin. The payout to each player, Q, equals 2^N, where $N \leq N^{max}$. The maximum payout the player can receive from the gamble is therefore $Q^{max} = \$1 \cdot 2^{Nmax}$. If the buyer knows that the seller can pay a maximum of \$1,024, the buyer realizes that the game will end at the maximum of 10 consecutive head tosses even if the tail outcome never occurs (maximum payout = $\$1 \cdot 2^{Nmax} = \$1 \cdot 2^{10} = \$1{,}024$) . The expected value of the gamble to the buyer in this case would be \$10 and more generally given by the following formula:

$$E(V_{truncated\_gamble}) = \sum_{N=1}^{N^{\max}} \frac{1}{2^{N^{\max}}} \cdot 2^{N^{\max}} = N^{\max} \qquad [3]$$

In this paper we are applying the bounded St. Petersburg gamble to the firm that has implicit assurance from the independent entity to make payout to the players in case of extremely rare but high values of consecutive "head" coin tosses. The maximum payout offered by the firm, $Q^{firm} = \$1 \cdot 2^{Nfirm}$, is less that $Q^{max} = \$1 \cdot 2^{Nmax}$, that is offered to the buyer. A guarantor with deeper pockets covers the very rare, but huge tail losses. That is a guarantor covers payouts exceeding Q^{firm} up to a maximum of Q^{max}. An example of such guarantor would be implicit government bailout of "too big to fail" banks. The expected value of the gamble with limited resources available to the seller-firm and with explicit or implicit guarantor stepping in for the rear extreme payouts would be given by:

$$E(V_{truncated\_gamble\_with\_guarantor}) = \sum_{N=1}^{N^{firm}} \frac{1}{2^N} \cdot 2^N + \sum_{N^{firm}+1}^{N^{\max}} \frac{1}{2^N} \cdot 2^N = N^{\max} \qquad [4]$$

The game described is a zero sum game among three parties if the gamble is priced based on its full expected value with the firm and the guarantor sharing the premium paid by the players. In such a case there would be no real excess returns to be earned. However, the gamble can potentially generate false alpha for both the seller-firm and buyer-players, at the expense of the guarantor, if the guarantor underprices the value of the tail losses. The guarantor would be willing to accept the lower premium for its role if the actual probability of tail losses is hidden or ignored. The rare losses, when they do occur, can appear to the guarantor to be like being hit by a random bolt of lightning.

If the guarantor undercharges for the tail risk, the seller-firm can offer gambles to the buyer-players at discount compared to the actual expected value of the gamble but above the seller-firm's expected losses. The maximum number of consecutive tosses in the lottery equals N^{max}, with the maximum payout, $Q^{max} = \$1 \cdot 2^{Nmax}$. N^{firm} equals the maximum number of consecutive tosses before the seller-firm's funds, Q^{firm}, are exhausted. $Q^{firm} = \$1 * 2^{Nfirm}$.

The expected value of payout for the seller-firm equals ($\$1 \cdot N^{firm}$). The expected value of the seller-firm payout to the players, in dollars, equals:

$$E(V_{of\_payout\_by\_seller-firm}) = \sum_{N=1}^{N^{firm}} \frac{1}{2^N} \cdot 2^N = N^{firm} \qquad [5]$$

The payout occurs on the N^{firm} toss whether it is a tail, in which case the seller's funds were just exhausted, or a head, in which case the gamble continues and the guarantor must make up for the further additional losses.

The guarantors' expected loss for each game equals $\$1 \cdot (N^{max} - N^{firm})$, the difference between the buyers' expected gain and the sellers' expected loss. The game creates false alpha for both the seller-firm and the buyer-players if the guarantor charges an insurance premium less than $\$1 \cdot (N^{max} - N^{firm})$.

$$E(V_{payout\_by\_guarantor}) = \sum_{N^{firm}+1}^{N^{max}} \frac{1}{2^N} \cdot 2^N - \sum_{N=1}^{N^{firm}} \frac{1}{2^N} \cdot 2^N = N^{max} - N^{firm} \qquad [6]$$

Why would guarantors underprice their exposure to long-tailed losses? The chance that the number of consecutive head tosses would require a payout greater than the seller-firm's funds is very small, less than $^{1}/_{\mathrm{Nfirm}}$. Continuing with the example of maximum funds available to the seller-firm of $2^{10} = \$1{,}024$, the probability of the guarantor having to step in is $1/2^{10} = 0.00098$. If we assume that a new gamble takes place on monthly bases, the probability of the guarantor having to step in is once in about 85 years. In strictly private markets, the seller-firm's guarantor may underprice tail risk because of information asymmetry about the chance of the extremely rare losses. They may also exhibit disaster myopia - the human tendency to act as if extremely rare outcomes have a zero chance of occurring (Kahneman and Tversky, 1979). The guarantee may be either implicit or explicit. In the presence of substantial negative externalities associated with failure of seller-firm, the too-big-to-fail model, government may socialize extremely rare but huge losses of the private market with long-tail risk.

The following Table I outlines the numerical example with stake parameter X = \$1. It shows the probability of the "tail" outcome occurring on the N^{th} coin toss, the payout that will be made in each of the scenarios and the frequency of that outcome. We assume that a new gamble takes place on monthly bases and that the seller-firm has resources to make a maximum payout of \$1,024 per gamble. This means that the seller-firm exhausts its resources if the tail outcome does not occur at least until the 10th coin toss. If the tail outcome does not occur on the 10th coin toss, the guarantor will have to step in and make the payout to the player. Later in the paper we also assume that the guarantor can make a maximum payout of \$32,768 ($=2^{15}$), i.e. the gamble continues only to a maximum of 15 coin tosses. This scenario implies E(V) to the player is \$15 for each month gamble, and E(V) of payouts that can be made by the seller-firm each month is

$10. E(V) of the guarantor's payouts is, therefore, $5 for each month gamble. We use an example with maximum payout from the guarantor of $Q=2^{15}$ (i.e. E(V) = $15) because this is in line with empirical studies of St. Petersburg gamble where players are willing to pay, on average, $15 to $20 to enter the game.

Table I

Summary of the Probability Distribution and the Payout Amounts for the Coin Flip Model for up to the Assumed Maximum Capital Available from the Seller-Firm $Q^{firm} = 2^{10}$ = $1,024 and Assumed Maximum Capital Available from the Guarantor $Q^{max} = 2^{15}$ = $32,768

1st tail occures on a toss # N	Probability of the 1st tail outcome occuring on a toss # N = $1/2^N$	Payout amount, $\$1*2^N$	Expected friquency of occurance assuming new gamble starts on anual bases; once in ...	
1	0.50000	$ 2	1 year	Payouts that can be covered by the seller -firm
2	0.25000	$ 4	2 years	
3	0.12500	$ 8	4 years	
4	0.06250	$ 16	8 years	
5	0.03125	$ 32	16 years	
6	0.01563	$ 64	32 years	
7	0.00781	$ 128	64 years	
8	0.00391	$ 256	128 years	
9	0.00195	$ 512	256 years	
10	***0.00098***	***$ 1,024***	***512 years***	***max # of tosses and $ payout available from the seller-firm***
11	0.00049	$ 2,048	1,024 years	Guarantor will have to step in to make these payouts (and higher)
12	0.00024	$ 4,096	2,048 years	
13	0.00012	$ 8,192	4,096 years	
14	0.00006	$ 16,384	8,192 years	
15	0.00003	$ 32,768	16,384 years	

III. Example: Bank Loans with Long-Tailed Risks

In this section we use the described above bounded St. Petersburg gamble with implicit or explicit guarantor (i.e. FDIC and the government) to create a simple models of a rudimentary bank. We show that the agents capture false alpha by externalizing the cost of tail risk to the guarantor when highly probable small gains are combined with rare enormous losses.

A rudimentary bank that invests in risky loans with long-tailed outcomes can be modeled by the above bounded version of the St. Petersburg gamble. The bank finances these risky loans with a combination of owners' equity capital and depositors' funds. The bank's equity owners are tantamount to the seller-firm. Their maximum funds for making a payout equal their invested equity, Q^{firm}. The depositors serve as guarantors for losses exceeding the equity of Q^{firm}, up to a maximum of the bank's loans, equal to Q^{max}. The bank's loan portfolio serves as an aggregate St. Petersburg gamble buyer. The loan portfolio pays a default premium to the equity owners. The defaults are the gamble payouts.

The bank's portfolio of loans experiences losses according to the St. Petersburg distribution. The portfolio experiences defaults in the rare event of extremely adverse economic conditions (Coval, Joshua, Jurek, Stafford, 2009). Diversification of extremely long-tailed gambles can increase risk, such as St. Petersburg paradox (Ibragim, 2005). Each period a single series of coin flips takes place. The total loss each period on the portfolio equals $\$1 \cdot 2^N$ (stake parameter set to equal $1). Thus the default or total payout equals $1·2 for a tail on the first toss, $1·4 on the second, $1·8 on the third, etc. The bank's loan portfolio yields risk-free rate of interest plus the default premiums charged by the bank.

The depositors guarantee the tail risk. Their funds cover any extreme losses not covered by the equity owners' capital. In a world of perfect information, the depositors would charge a correct premium for the potential default on their deposits. However, if the bank's depositors know less about the tail risk than the bank's risk management, they may accept an incorrect risk premium. The following model considers the case in which depositors, either due to informational asymmetry or to disaster myopia, charge too low a risk premium. Depositors effectively subsidize tail risk loans if they receive a risk premium insufficient for the risk beard.

Depositors accept a risk-free rate of return on their deposits. With perfect information, the depositors could accurately assess and price the total risk. Here, the bank's retail depositors rely on the bank's misleadingly superior judgment of the risk. The bank's wholesale depositors, such as money market funds, rely on credit ratings that underestimate tail risk as well. Given the relative infrequency of banks' bankruptcies, both types of depositors may find it difficult to imagine true extent of potential losses and so feel safe (Slovic, Fischhoff, Lichtenstein, Corrigan, 1977).

The bank's equity owners receive the risk-free rate of return and the entire default premiums on the bank's loan portfolio. The bank's equity owners bear only those portfolio losses that do not exceed size of their equity investment, while the bank's depositors bear those losses that do.

In an illustrative numerical example, consider the following bank balance sheet. To simplify the example, we use numbers proportional to $\$2^N$, i.e. equity holder contributes a capital of $\$2^{10}$ = $1,024, total assets equal $32,768 (or $\$2^{15}$). Deposits equal the difference between assets and equity. In this example, the bank's equity covers losses up to 10 tosses ($N^{firm} = 10$ tosses) since the equity owners' liability is limited to their equity investment. The depositors cover the very rare losses that occur if there are 11 to 15 tosses ($N^{max} = 15$).

Table II

Example of Bank's Balance Sheet
Based on Truncated St. Petersburg Gamble with Guarantor, where
Bank is referred to as bank-seller, equity capital represents investment made by equity-buyers, depositors are insured by FDIC and act as guarantors.

Balance Sheet						
Assets				Liabilites + Equity		
Loan Portfolio	$ 32,768	$= \$2^{15}$		Deposits	$ 31,744	
				Equity	$ 1,024	$= \$2^{10}$
Total	$ 32,768	$= \$2^{15}$		Total	$ 32,768	$= \$2^{15}$

If the bank's assets lose \$2,048 million in value (\$$2^{11}$ = \$2,048, equivalent to 11 tosses), the bank exhausts \$1,024 million equity, and the depositors lose the difference, \$1,024 million. The bank ceases to exist. In the simulation example, we assume that failed bank's equity owners can continue by capitalizing a new bank with an equity investment of \$1,024 and attracting new depositors and borrowers.

The bank maintains a constant equity capital ratio (equity/total assets). In this example it equals (\$$2^{15}$) ÷ (\$$2^{10}$) or 3.125%. The bank distributes all gains, either as dividends or as repurchases, to its owners. Once paid, these funds are off the table and not available to cover losses. As long as the bank's equity capital remains positive, it replaces all losses by raising new equity capital. In the case of insolvency, the depositors cover all losses exceeding the bank's equity.

Each period, the defaults are determined by a random process modeled by the coin toss. Each period, the bank receives a premium, Π, on its loan portfolio. The premium is greater than its expected loss of \$10 (N^{firm}) so that it earns a profit. At the same time the premium is less than the expected loss on the entire portfolio of \$15 (N^{max}).

The model concentrates on insolvency issues. There are no bank runs. The tosses each period occur instantaneously, so depositors cannot anticipate their outcomes. Everyone is in the dark concerning the outcome of the coin tosses in any given period until the sequence is completed. In addition, depositors do not use a negative outcome in the previous period as a signal to liquidate. The underlying random process does not change from period to period and probabilities of default do not change.

The bank's owners generate false alpha from the imperfect information on the part of the depositors and their limited liability for losses. Only if the cost of the deposits to the bank includes a premium sufficient to compensate for the tail risk would the owners' expected profits vanish.

Governments may socialize depositors' losses to mitigate the external costs resulting from financial failure by providing explicit guarantees, such as FDIC insurance. Depositors, who are unable to distinguish between institutions with and without tail risk, would demand high risk premiums on all deposit. This can create a "lemons problem" for the banking system where all banks, those with or without tail risk, would need to pay higher risk premiums to attract uninsured depositors (Akerlof, 1970).

Deposit insurance shifts the task of pricing tail risk to the guarantor. Moral hazard occurs when agents do not bear the full cost of their decisions. In this model, moral hazard arises because of asymmetric information about the tail risk of the assets, not because of deposit insurance. The moral hazard occurs whether the deposits are uninsured, implicitly insured, or explicitly insured. Either correct deposit insurance premiums or a correct depositor risk premiums would eliminate the moral hazard (Chan, Shee, Greenbaum, and Thakor, 1992). The insurer, although better informed than most depositors, might still be informatively disadvantaged, unable to see the true risk in the haze of financial reports. In addition, deposit insurers may be subject to political influences and regulatory capture that can also lead to the underprice deposit insurance (Kane, 2007; Kane, 1995). Alternatively, depositors who understand the risks and who lack explicit guarantees may expect to exert political pressure to force taxpayers guarantee their deposits ex-post. These expectations generate moral hazard as well (Demirgüç-Kunt and Kane, 2002).

The above model agrees with Nicholas Taleb's observation about bankers.

> "If they look conservative, it's only because their loans go bust on rare, very rare occasions. There is no way to gauge the effectiveness of their lending activity by observing it over a day, a week, a month, or … even a century! . . . They are not conservative at all; just phenomenally skilled at self-deception by burying the possibility of a large, devastating loss under the rug." (Taleb, 2007).

If higher risk investments yield higher expected returns, raising low risk money and investing it at higher risk yields positive expected spreads. Creating profitable spreads from investments with long-tailed negative payouts distributions takes advantage of imperfect information about very small probability outcomes (Acharya, Thomas Cooley, Matthew Richardson, and Ingo Walter, 2009). For example, suppose agents in a bank discover an investing opportunity with an attractive spread that is in fact based upon unknown tail risks. The agents do not need to personally believe they are accepting the long-tailed risk. They merely need to spot the spread between their cost of funds and the expected return on apparently safe assets with tail risk.

The bank's risk managers might underestimate the expected losses of the long-tailed risks. They may use measures based upon past historical events and underestimate long-tail risk by applying normal distribution models (Powell, 2008). Moreover, they may lack internal authority. As a result, risk management may charge the unit an insufficient risk premium. The unit appears to earn a spread. Its employees receive generous compensation based upon the units' apparent performance. Internally, firms over allocate capital to units with the underpriced risk. Moreover, such compensation systems can motivate trading units to create new long-tailed contracts to trade. A Gresham's law-like outcome results. Risky long-tailed trading replaces trading with more normal distribution of losses.

IV. Example: Credit Default Swaps

As another example the coin flip model can represent a derivatives trading unit that sells credit default swaps. The trading unit sells credit default swaps, receives the premiums, and covers all losses on the swaps up to a point. Since selling long-tailed gambles can be profitable for extended time periods, tying trader compensation to current income encourages taking risks of enormous losses that have very small probability. These losses are likely to occur at very long time intervals. The firm in which the trading unit operates serves as the guarantor. Back up guarantors are the firm's creditors, if the unit's losses exceed the entire firm's equity, and the greater society for too-big-to-fail firms.

The unit's creditors may understand the exposure to long-tailed outcomes on their debt and willingly insure the risk in exchange for a premium. Alternatively, certain investors will pay the premiums for positive exposure to long-tailed payouts and advocate hedging or speculating on tail risk (Pia, 2009). These customers credibly anticipate very large payments to be guaranteed by the larger firm or by the larger society, if bankruptcy of credit default trading unit would impose severe external social costs (Taleb, 2007).

V. Simulation of St. Petersburg Gambles

A bounded St. Petersburg type lottery can simulate the above examples. We will present simulation using bank as an example. Depositors and FDIC insurance acting as explicit guarantors. The limit covered by the seller-bank, Q^{firm}, equals the bank's equity capital. The upper limit of the gamble, Q^{max}, equals the bank's total assets and the amount covered by the FDIC insurance. A guarantor covers the portion of any payouts exceeding Q^{firm}.

The following assumptions are made for simplicity of presentation.

- New gamble takes place over one time period. Each time period is set to be one year. Each annual gamble is independent of the previous one.
- Q^{firm} is constant for each time period. At the end of each period, all required payouts are made, all premiums for the next period are collected from the equity holders.
- Whenever the premiums collected exceeds the payouts, the difference is distributed as a dividend.
- Whenever the payouts exceed the premiums collected, the guarantor covers the difference above Q^{firm} up to Q^{max}, and bank recapitalizes.
- The guarantor's risk premium is zero. Assuming a zero premium does not affect the qualitative results of the simulation. Insufficient premium for the level of risk taken will generate similar outcomes of the simulation. Insufficient premiums charged by the guarantors permit false alpha to exist for the equity investors.
- The risk free rate is set to zero, so that returns in the simulation are in excess of the risk free rate.
- Recapitalization and bankruptcy are costless. The sellers can raise new capital, form a new bank, and resume. The equity investors' outside capital is shielded by limited liability.
- The bank's equity covers losses up to 10 tosses (N^{firm} = 10 tosses). The guarantor cover the very rare losses that occur, up to 15 tosses (N^{max} = 15).

Each year a fair coin is tossed until a "tail' occurs up to a maximum of 15 tosses. The stakes parameter 'X' equals $1. Each year the sellers' payout, or loss, equals $Q = \$1 \cdot 2^N$ dollars.

N equals the number of consecutive coin tosses until the first "tail" occurs. The seller's maximum loss occurs at N^{firm} =10 tosses, for a maximum amount that will be paid by bank's equity holders, Q^{firm}, equal to $\$2^{10}$ (\$1,024). The maximum total number of tosses is limited N^{max}=15 tosses for a maximum payout that can be made by the guarantor, Q^{max}, equal to $\$2^{15}$ (\$32,768).

The guarantor's maximum payment equals $Q^{max} – Q^{firm}$ ($\$2^{Nmax}$ - $\$2^{Nfirm}$). If the first head occurs on the 10th toss, the bank-seller makes its maximum payout of $\$2^{10}$ and recapitalizes, but the guarantor suffers no losses. If the first head occurs on the 11th or higher toss, the bank-seller also pays $\$2^{10}$ and recapitalizes. The guarantor makes any additional payouts up to its limit of $\$2^{15}$. The guarantor's maximum payout equals \$31,744 ($\$2^{15}$-$\$2^{10}$). This payout occurs whenever the first tail does not occur on the first 14 tosses. The maximum is paid if the 15th toss is either the first tail or another head, the game always ends on the 15th coin toss because the guarantor's funds are exhausted as well.

The probability of N heads occurring in a row is given by $2^{-(N-1)}$. For N=10, this probability is 0.195%. Its reciprocal, 2^{10}, equals the expected time to occurrence, which in our example would be equal to 512years. Thus, the probability that the guarantor must make any payment is less than the risk of a 100-year flood.

The expected value, $E(V^{firm})$ of each annual payout for the sellers equals \$10, but for the buyer the expected value of the gamble equals $E(V^{max})$, or \$15, using equations [2] and [3]. While the payout covered by the guarantor might seem unforeseeable by its rarity, the expected value of the premium for the covered losses, $E(V^{max})$ - $E(V^{firm})$, is \$5 each year in this example. Only if the guarantor charges an insurance premium [Φ] of \$5 per year, there would be no false alpha. In our analysis the insurance premium charged by the guarantor is set to zero. The main interest of this paper is to investigate the excess returns that market participant can capture when the guarantor fails to charge the sufficient insurance premium for acting as such. Setting the insurance premium to zero leads to no loss of generality of the simulation outcomes.

If the bank-seller and the equity-buyers do understand the nature and the level of the risk insured by the guarantor, they would be willing to split the amount of the full insurance premium of the gamble, [Φ] = \$5 per month. The actual premium, P, paid by the equity-buyers above what the value of the gamble that the bank would be able to pay out, $E(V^{firm})$, would determine the relative share of the false alpha captured by the equity-buyers and bank-seller. The higher the P, the greater the share captured by the bank-seller. For the simulation example, we assume that the bank and equity holders split the full insurance premium of the gamble, [Φ], equally. The monthly premium, [Π] paid by the equity-buyers to bank-seller above the $E(V^{firm})$ that the bank is able to payout, is arbitrarily set at \$2.50 - half way between the \$15 (N^{max}) and \$10 (N^{firm}). Hence, the total bank-seller would charge equity-buyers to enter the gamble would be

$$\text{Total Charge} = E(V^{firm}) + \Pi = \$10 + \$2.50 = \$12.50 \qquad [7]$$

VI. Results of the Simulation

The following reports the results of coin toss simulations using the geometric distribution. The probability of a head on each coin toss equals 0.5. The results of 10 simulations are shown in Table 3. Each simulation consists of 1 independent gamble per bank for 100 banks over 100-year time period (10,000 gambles per simulation). Total of 10 simulations are ran.

In the simulations both bank-sellers and their equity-buyers experience positive earnings as a group over a long period. The buyers have much greater variability of outcomes than the firms. Out of 10 century-long simulations the bank-sellers have profited in every one of the simulations, while the buyers profited in 7 out of 10 simulations. Over the 10 century-long simulations, though, both bank-seller and equity-buyers has positive capital gain (average of $1.64 per gamble for the banks and $1.89 per gamble for the equity-buyers).

In all 10 century-long simulations, the guarantor had to step in with the bailout. On average, there are 9 banks requiring bail out per century out of 100 banks. The infrequency of failure maintains the appearance of low risk. The average amount of each bailout by the guarantor was $3,883. The average payout per gamble to the equity-buyers was $14.39, which is close to the E(V) of $15 per gamble that is truncated at a maximum of 15 coin tosses.

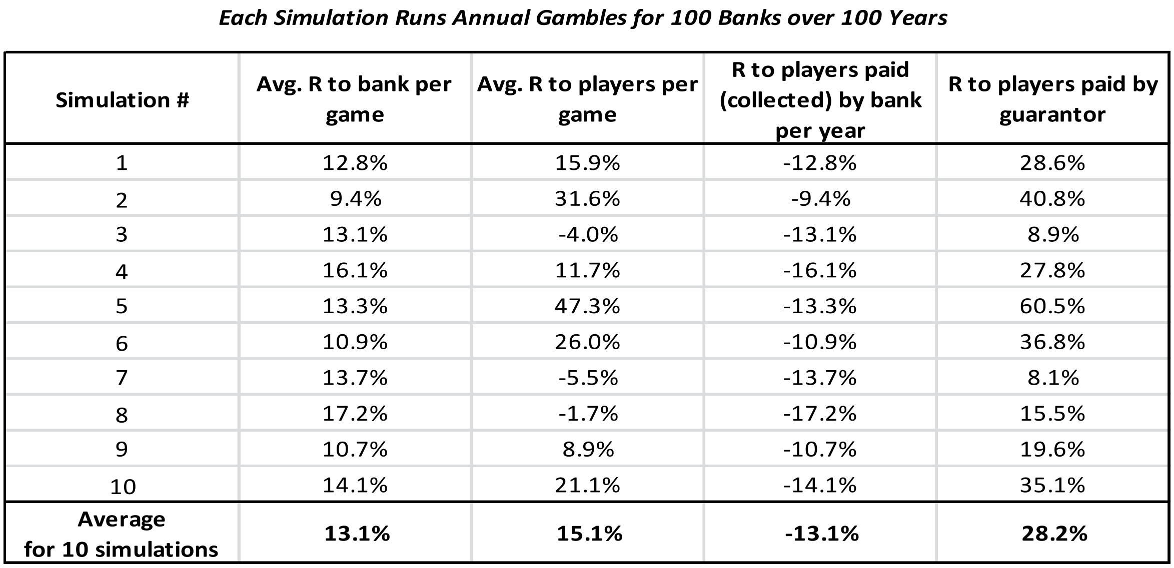
Table III. Summary of the Payouts, Gains and Losses, in dollars
10 Simulations, each Simulation Runs Annual Gambles for 100 Banks over 100 Years

Each Simulation Runs Annual Gambles for 100 Banks over 100 Years

Simulation #	Avg. payout per game, per player	# of bailouts per simulation	Avg. Paid by Guarantor per bailout	Avg. Net Gains (Losses) by Players per game	Avg. Net Gaines (Losses) by Bank per game
1	$ 14.49	7	$ 5,120	$ 1.99	$ 1.60
2	$ 16.44	18	$ 2,844	$ 3.94	$ 1.18
3	$ 11.99	7	$ 1,609	$ (0.51)	$ 1.63
4	$ 13.97	10	$ 3,482	$ 1.47	$ 2.01
5	$ 18.42	10	$ 7,578	$ 5.92	$ 1.66
6	$ 15.75	9	$ 5,120	$ 3.25	$ 1.36
7	$ 11.81	6	$ 1,707	$ (0.69)	$ 1.71
8	$ 12.29	5	$ 3,891	$ (0.21)	$ 2.15
9	$ 13.62	6	$ 4,096	$ 1.12	$ 1.34
10	$ 15.14	13	$ 3,387	$ 2.64	$ 1.76
Average for 10 simulations	**$ 14.39**	**9**	**$ 3,883**	**$ 1.89**	**$ 1.64**

The returns to both bank-sellers and equity-buyers were covered by the guarantor and were about evenly split between the two (13.1% and 15.1%, respectively, for the total of 28.2% that came from the guarantor's losses), see Table IV for the summary of all 10 simulations.

Table IV. Summary of the Returns, Gains and Losses, in percent
10 Simulations, each Simulation Runs Annual Gambles for 100 Banks over 100 Years



Each Simulation Runs Annual Gambles for 100 Banks over 100 Years

Simulation #	Avg. R to bank per game	Avg. R to players per game	R to players paid (collected) by bank per year	R to players paid by guarantor
1	12.8%	15.9%	-12.8%	28.6%
2	9.4%	31.6%	-9.4%	40.8%
3	13.1%	-4.0%	-13.1%	8.9%
4	16.1%	11.7%	-16.1%	27.8%
5	13.3%	47.3%	-13.3%	60.5%
6	10.9%	26.0%	-10.9%	36.8%
7	13.7%	-5.5%	-13.7%	8.1%
8	17.2%	-1.7%	-17.2%	15.5%
9	10.7%	8.9%	-10.7%	19.6%
10	14.1%	21.1%	-14.1%	35.1%
Average for 10 simulations	**13.1%**	**15.1%**	**-13.1%**	**28.2%**

The bank-seller's profit equals the difference between total charged from equity-buyers, income from investment assets, payouts to equity-buyers, payments to depositors and insurance premium to the guarantor.

$$\textit{Bank-Seller's Profit} = E(V^{firm}) + \Pi + Assets \cdot I - Payouts - Deposits \cdot R_f - \Phi \qquad [8]$$

For the purpose of the simulation and without loss for general outcomes, we have set I (interest earned on invested assets by the bank) and R_f (risk free rate paid to depositors) equal to zero. The main interest of this analysis is to investigate the results of the scenario where the guarantor fails to charge sufficient insurance premium Φ for bearing the long-tailed risk. The bank-seller and the equity-buyers split the insurance premium Φ and capture the false alphas resulting from it. The bank-seller's profit becomes total charged from equity-buyers less payouts made to them.

$$\textit{Bank-Seller's Profit} = E(V^{firm}) + \Pi - Payouts \qquad [9]$$

The gains to the bank-seller and their equity-buyers equal the guarantor's losses. Bank-seller expected value of the profits is equal to the premium, Π, charged above the $E(V^{firm})$, the maximum amount that the bank-seller can payout (the expected value of the Payouts made by the bank over the log-run is equal to $E(V^{firm})$). The equity-buyers capture the profits above the expected value of the gamble, $E(V^{max})$, equal to $(\Phi - \Pi)$, the value of the insurance premium provided but not charged by the guarantor, less the premium paid to the bank-seller. The equity-buyers' earnings are much more volatile. The division of earnings between the buyer and the seller depends upon Π, the premium charged by the bank-seller above the value of the payouts that it can make. It would be set by the competitive conditions at the market. In the simulation Π is assumed to equal \$2.50 for expositional purposes.

Tables V and VI report selective gambles results from the simulations for two random banks, #1 and #2, over 100 years of annual gambles. Bank #1 has not experienced bailout scenario, while Bank #2 had been bailed out in Year #13. In both examples Banks suffered Net Losses. For Bank #1, Average Payout to the equity-buyers is $14.16, which is equal to the average Profit of $1.66 ($14.16 - $12.50). Banks report the average Net Loss of $1.66, i.e. the gamble is a zero-sum game between the bank-seller and the equity-buyer when guarantor does not step in with the bailout. For Bank # 2, one bailout has occurred in year 13. For that year Total Payout was $4,096, Bank #2 has covered $1,024 of it, and the guarantor has covered the rest of it, $3,072. The averages over 100 years reflect that the gamble was a zero sum game between the bank-seller, equity-buyers and the guarantor. Equity-buyers have received a total of $6,420 over the century ($64.20 average payout * 100 years), they have made the total payment of $1,250 ($12.50 total charge per gamble * 100 annual gambles), which leaves the equity-buyers with the profit of $5,170. Bank #2 has paid $2,098 of it ($20.98 average bank loss * 100 years), and the rest was covered by the guarantor, $3,072.

Table V

Simulation for Bank #1 over 100 years (no bailout has occured)

Year #	Random Probability that N consequtive toss has 1st tail	1st tail is on the toss # N	Payout per Game, $	Bailout: Y or N	Bailout Amount = Payout - $1,024 Paid by Bank	Net Gaines (Losses) by Bank = $12.50 - Payout
Average		***2.2***	***$ 14.16***	***0***	***$ -***	***$ (1.66)***
1	0.5000	1	2	N	0	10.5
2	0.2500	2	4	N	0	8.5
…	…	…	…	…	…	…
13	0.5000	1	2	N	0	10.5
14	0.2500	2	4	N	0	8.5
…	…	…	…	…	…	…
94	0.1250	3	8	N	0	4.5
95	0.1250	3	8	N	0	4.5
…	…	…	…	…	…	…
99	0.5000	1	2	N	0	10.5
100	0.5000	1	2	N	0	10.5

Table VI

Simulation for Bank #2 over 100 years (bailout has occurred in Year # 13)

Year #	Random Probability that N consequtive toss has 1st tail	1st tail is on the toss # N	Payout per Game, $	Bailout: Y or N	Bailout Amount = Payout - $1,024 Paid by Bank	Net Gaines (Losses) by Bank = $12.50 - Payout
Average		***2.1***	***$ 64.20***	***1***	***$ 3,072***	***$ (20.98)***
1	0.5000	1	2	N	0	10.5
2	0.2500	2	4	N	0	8.5
…	…	…	…	…	…	…
13	0.0002	12	4096	Y	3072	-1011.5
14	0.2500	2	4	N	0	8.5
…	…	…	…	…	…	…
94	0.5000	1	2	N	0	10.5
95	0.0010	10	1024	N	0	-1011.5
…	…	…	…	…	…	…
99	0.5000	1	2	N	0	10.5
100	0.5000	1	2	N	0	10.5

While each individual lottery appears risky, diversification radically reduces the risk. A portfolio diversified across all gambles and including both the buyers and the sellers yields positive returns with little variation. This can be inferred from Tables III and IV above. Such an investment strategy consists of owning all seller-banks and investing all the equity for those banks. The appropriately diversified investors can easily capture the close to the average false alpha with minimal risk.

Hedging strategies could be an alternative method to reduce the risk of collecting the false alpha. Investors with sufficiently precise information about the market could invest in equal shares of each seller and each buyer. In the case of the banks' loan portfolio, a simple hedge consists of being long the bank's stock and being short the bank's portfolio of securities by using a credit default swap.

For each gamble, if the investor is long in both the seller and the buyer positions, all negative returns would be eliminated, since the buyer's gains equal the seller's losses plus payouts by the guarantor. There are some games in which the hedge results in zero returns over the century, but on average, equity-buyers losses are eliminated through diversification. Table VII reports summary results of diversification strategies for the 10 simulations. Average returns on the hedging strategies that invest into both bank-sellers and equity-buyers are higher than for investment into banks or equity holders separately. The hedged investor is better positioned to capture the gains from the guarantor.

Table VII. Returns on Hedging Strategies for 10 Simulations
Each simulation is the average for 100 banks over 100 years

Simulation #	R on 50/50 Bank/Equity Hedge	R on 40/60 Bank/Equity Hedge	R on 60/40 Bank/Equity Hedge
1	22.5%	23.9%	21.1%
2	34.0%	38.6%	29.4%
3	9.0%	7.2%	10.9%
4	59.0%	70.5%	47.4%
5	30.3%	34.3%	26.4%
6	16.8%	16.8%	16.8%
7	19.7%	21.9%	17.4%
8	26.6%	29.7%	23.6%
9	16.0%	16.6%	15.4%
10	25.4%	28.4%	22.4%
Average for 10 simulations	**25.9%**	**28.8%**	**23.1%**

VII. Discussion

The model of a zero-sum game based on this gamble is obviously an unrealistic depiction of actual investment behavior in complex environments. As with any model, it utilizes simplifications and assumptions not descriptive of real markets. As is common in financial models, there are no transactions costs in the model. The participants face a constant and independent probability distribution for each gamble and a constant real interest rate. The herd behavior by depositors that creates a liquidity crisis through bank runs and illiquidity is absent. There are no fire sales, creating unexpected losses. Insolvency is caused by a random collapse in the external value of the assets, not bank runs. Further, raising new debt and new equity in the market is frictionless and always possible, even after the experience of large losses.

A. Leverage

The greater the seller's leverage, the more likely it is that the guarantor, whether private or public, will eventually have to make payouts. More leverage creates greater "false alpha" to be captured. The equity ratio, total equity divided by total assets, measures the bank-seller's leverage. For example, the equity ratio in the above simulation equals 3.125%[$2^{10}/2^{15}$]. This is comparable to the 4% Tier 1 capital requirement in 2003, before the risk-weighted reduction for AAA debt.

Increasing the equity ratio decreases the expected value of the guarantor's payout in each period. In the above example, the expected value of the guarantor's payout is $N^{max} – N^{firm}$ and equals $5 per game per player. Leverage can be reduced by increasing equity capital or by decreasing assets. If equity capital increases to 2^{12}, holding total assets the same, the equity ratio becomes $2^{12}/2^{15}$ = 12.5%. Likewise, decreasing total assets to 2^{13}, and holding equity the same, results in the same equity ratio, $2^{10}/2^{13}$ = 12.5%. In both cases the expected value of the guarantor's payout, $N^{max} – N^{firm}$, is reduced to $3 per game per player. With less leverage, false alpha is decreased, but not eliminated.

Bank-seller's agents have strong incentives to allocate a great deal of effort and talent to disguising the actual level of the expected value of the guarantor's payout, $N^{max} - N^{firm}$. In complex regulatory environments, they may disguise their true leverage from auditors, rating agencies, and regulators. One example would be the use of off-balance sheet assets and liabilities of banks. Another is the innovative use of repo transactions to disguise the risk. Accounting practices that overstate value of assets or understate value of liabilities ultimately falsely inflate the equity and disguise the true degree of leverage.

Lowering the protection afforded by limited liability for firms or demanding claw backs of compensation from units within firms, reduces the expected value of the guarantor's payout, $N^{max} - N^{firm}$, by effectively increasing N^{firm}. From the end of the Civil War up to the introduction of deposit insurance during the 1930s most U.S. banks operated with double liability provisions. Owners of failing banks not only lost their investment in the banks, but were liable for additional assessments up to the par value of the bank's equity. Evidence suggests that double liability reduced risk taking among banks in the late nineteenth and early twentieth centuries, but was not very effective in reducing the risk of failure during the financial collapse of 1930s. Such provisions would increase effective equity to 2^{Nfirm} in this model and so would have the effect of decreasing leverage (Grossman, 2001). Double liability was discarded with the advent of deposit insurance.

The guarantor's liability for tail risk can be reduced by requiring such institutions to be financed with contingent long-term debt or preferred stock. A regulator can mandate the conversion of these bonds to equity. Presumably long-term investors in these contingent claims could better assess and monitor tail risk than most depositors. The rates demanded by creditors could reflect the portion of the tail risk they assume. The solvency of the institution can be thus maintained without liquidation of payments by the deposit guarantor. Financial fragility and moral hazard is reduced. N^{firm} is increased. A severe enough shock, modeled here by a very long sequence of "heads" before a "tail", would exhaust the contingent liabilities. This implies that even more tail risk would be priced internally, therefore reducing, but not eliminating, incentives that capture false alpha.

B. Size

The model presented here is scalable. Size, per se, is not an issue in the model. However, when a firm is considered to be "too-big-to-fail", effective leverage and relative amount of implicit guarantee are increased. As a result of the bail out of the "too-big-to-fail" firms some of their equity capital is not at risk, but is preserved. This confers a competitive advantage over their smaller rivals. The "too-big-to-fail" firms are able to pass on more of the tail risk to guarantors or governments. In addition to "too-big-to-fail", politically well-connected firms are more likely to be bailed out than their non-connected peers as well. In our model, being "too-big-to-fail", is similar to having a lower equity capital ratio (Grossman, 2001).

C. Pricing Risk

Bank-sellers and depositors, or their guarantor, can set the insurance premium Φ. A correctly priced Φ eliminates the opportunity for false alpha. To correctly price Φ, the guarantor must be aware of both the actual tail risk and the degree of financial leverage. In the coin flip model, correctly priced risk means that $\Phi = N^{max} - N^{firm}$. Low interest rates on deposits, or low deposit insurance premiums, result in the underpricing of Φ in the bank model. Underpricing of risk occurs if Φ is less than $N^{max} - N^{firm}$.

In our model, the premium, Π, charged by the bank-sellers to the equity-buyers determines the relative share of the false alpha captured by buyers and the sellers of the zero sum game, but does not affect its total gains. The size of Π is not relevant to a diversified investor who invests proportionately in both the seller and the buyer, unless it affects the volume of the activity.

Basing unit and management bonuses on a long-term performance could mitigate some of the incentive to seek investments with long-tail risk. However, truly rare events are unlikely to occur during a reasonable measurement period for compensation. Claw-back provisions over a five year period are unlikely to affect the truly long-tailed bets. The expected time to the occurrence of truly small probability events will likely exceed the length of any meaningful measurement period of performance.

VIII. Conclusions

The St. Petersburg coin-flip gamble is an accessible example of a power law distribution. Its simplicity allows us to focus on long-tail risk, ignoring other types of risks. It shows that self-interested agents can extract illusory positive returns when long-tail risk and limited liability are present. Self-interested sellers have little incentive to avoid the risk of tail losses that exceed their own limited liability, but can capture the long-tail risk when less informed creditors, or their guarantors, fail to price it correctly. With long-tail risk the huge losses occur so rarely that short term creditors may act as if their assets are risk free and ignore their exposure to tail risk. This creates an arbitrage opportunity to capture the mispriced value of the long-tail risk. Even though short-term losses are still possible for the outside investors, they should be able reduce any risk with diversification and hedging.

The simple sequential coin toss gamble can serve as an intuitive model of investments with cascade-like risks that result in long-tailed outcomes. The maximum payout of a gamble is limited by the sum of the resources of the game's seller and the resources of any guarantor, explicit or implicit. A seller receives premiums from buyers in exchange for frequent small payouts, rare large payouts, and extremely rare, enormous ones. Even in a zero sum game, buyers and sellers can both extract gains if tail risk is either hidden from or underestimated by the guarantors. The St. Petersburg coin-flip game models aspects of depository institutions' risk taking behavior where depositors and government serve as guarantors of tail risk. Positive expected returns result when agents with limited liability shift tail risk onto external parties due to imperfect information or disaster myopia. The model shows that it is actually in self-interest of the sellers with limited liability to increase leverage and the chance of bankruptcy. It also shows a possibility of creating a hedge by investing into both the bank-sellers and equity-buyers that could, in principle, capture the full value of the external guarantee.

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