

# On the Equivalent Annual Cost Method

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## Abstract

The equivalent annual cost (EAC) method is a useful tool in asset pricing, lease financing and corporate investment. It deals with projects with different economic lives and has wide applications in many fields outside the finance or even business. However, the important EAC concept and its applications are not always very well understood even within the finance academia. There are a few misunderstandings surrounding <sup>1</sup>this widely taught and practiced methodology as evidenced by our experience with finance professionals over the past decades and the lack of clarity in all the mainstream textbooks we have surveyed. One of them is the misconception that the EAC method requires infinite time horizon and infinite repetitions of the identical project or equipment. Given its importance and wide application, the finance profession has an obligation to serve well its gatekeeper role by not overlooking these misconceptions. Thus far, no one has raised and addressed these misconceptions. We fill in this void by explicitly revealing, explaining and addressing these misconceptions. Specifically, we argue that infinite horizon is a convenience but not a necessity by showing how the method can be used for projects or assets with finite horizons and what cautions one has to take. We also provide threads for future studies along this line. Our work makes a marginal contribution the finance theory, pedagogy and practice.

## I. Background

The equivalent annual cost (EAC) method is a very useful corporate investment tool when we deal with mutually exclusive assets with different economic lives. The notion of EAC is an alternative to the traditional NPV rule but with its own additional benefits. For example, it helps us determine the costs in an average sense and identify alternative financing schemes such as debt arrangement, and lease financing. The EAC method may be called differently under different circumstances, such as the equivalent annuity, equivalent annual benefit, equivalent annual cash flows, etc. The essence of these variations is identical – to smooth out an uneven stream of cash flows for a finite period on the time line thereby allowing us to be able to make better comparisons among available options. In this study, we focus on the cost analysis, hence we use the acronym EAC, and refer to it as the EAC method. The points we make in this paper about the EAC method apply to all equivalent cash flow method.

The EAC methodology has long been part of standard finance education at many business schools and it also has wide applications outside the finance profession. However, surveying the literature in finance pedagogy, research and practice, we find misunderstandings and potential traps in this seemingly straightforward method. From our interactions with the academia, practitioners and educators, we are more convinced that these misunderstandings are deeply rooted and it appears increasingly necessary for the finance profession and educators, as

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the gate-keeper, to clarify this important concept. In this study, we fill this gap by specifically making two clarifications, (1) infinite horizon is *not* required for the EAC method to be valid or applicable; (2) the matching EAC stream should not start at any point in time.

By helping to make the EAC methodology more accurate, we make a marginal, yet long over-due, contribution to the finance education, research and practice.

## II. Literature Review

In corporate investment and many other areas, we frequently encounter situations where we need to compare two assets with different economic lives. For this purpose, the well-known NPV rule has limitations as mentioned in Brealey, Myers and Allen (2013) because it does not consider the length of the asset's life span and uses the aggregate value. An asset may cost very little annually to maintain, but if its economic life is very long, the total accumulative cost over its entire life span can be astounding. Having very low maintenance costs and long economic life normally would be two desirable advantages, but once we combine them, the NPV method tends to yield an undesirable result because it is large in aggregate which overlooks the true benefit of lower annual costs and thereby distorting the correct decision making process and the non-optimal final result may be obtained. For example, Brealey, Myers and Allen (2011) shows such an example. For detailed discussion, see pages 155-160 of it. The EAC method helps us avoid this potential fallacy by using a different measure that is more intuitive – evenly spreading the total costs (or NPV) on an annual basis. There is another benefit to the EAC method in addition to the intuitive facility, that is, it can help us determine the equivalent leasing financing. The latter would have both theoretical and practical value because in some real-world situations, the firm may not be able to come up with enough cash to cover some cash flow liabilities even though the entire project is profitable. In this case, arranging leasing financing or some debt arrangement with a bank would be of vital practical value. The EAC method becomes very convenient to suit these goals. Welch (2009) mentions lease financing in this context, and shows that there exist many equivalent leases with payments starting at any point as one desires. While in Brealey and Myers (1996), Brealey, Myers and Allen (2011, 2013), etc. infinite horizon is the default assumption, i.e., once an asset is chosen, the same type of asset will be forever used and new replacement comes when the old one draws close to the end of its economic life. In some other finance literature, infinite horizon is taken explicitly as a necessity for the EAC method (see, e.g., Besley and Brigham, 2007). Similar EAC treatment is found in other mainstream texts, such as Berk, DeMarzo and Harford (2009) which calls the method the Equivalent Annual Annuity (EAA) method. Berk and DeMarzo (2007) very briefly introduce the same idea to measure project's profitability and call it "the equivalent annual benefit method" and show how to use it to pick a better project when multiple positive NPV projects have different economic lives. These, among many mainstream textbooks, do not explicitly clarify the methodology regarding its applicability and the difference in handling infinite and finite horizons.

Outside the finance area, the EAC method has also found many applications. For example, Kahraman and Kaya (2008) mention its application in a study of fuzziness and soft computing. As early as 1960 Ronald Coase applied the idea in his studies on legal issues and property right (Coase, 1960), and as well as other social science studies (e.g., Caulkins, 2010; Schmitt, 2013). Drummon, Stoddart and Torrance (1987) use the EAC method in their studies of

healthcare. Stamuli et al. (2012) apply the principle in their medical research. In the area of environmental research, the method is used in Ranjan and Kayshik (2013) evaluate solar distillation and environment degradation, and in Uihlein and Eder (2009) to policies to improve the environmental performance of building. Venkat (2011) uses the EAC method to investigate food wastes in the United States. Perman et al. (2011) apply the method to resource allocation in dealing with hypertension management for elderly population. Collins and Singhal (1995) show how to apply the EAC concept in the mining industry. Hopp and Darby (1980) use the concept to analyze indirect energy savings for household water conservation. As the movement of green and alternative energy is gaining popularity globally, the EAC method finds its new application as well, such as Krohn, Morthorst and Awerbuch (2009) which analyze the economics of wind energy for the European Wind Energy Association.

Therefore, from application viewpoint, the EAC method already has wide applications in many areas even completely outside the traditional finance area, which means that our finance academicians especially finance educators need to make an effort to ensure that this fundamental concept is accurate in theory and appropriate in practice.

### III. The Traditional EAC Method

Now we use an example to illustrate the EAC method and its application to the infinite horizon. Suppose that a firm is facing two choices in picking machines to invest. We also assume, for simplicity, that the two machines only differ in annual cost and economic life. They produce identical consumer goods without any physical or quality differences. In fact, we can relax this assumption of consumer goods because this method is independent of what actual product we are facing. What ultimately matters is the associated cash flows. By using “identical consumer goods”, we really intend to mean that the cash flows of revenues are identical in terms of amount and risks. Therefore, instead of focusing on the net cash flows, we may just focus on the costs only. Machine A lasts for 4 years and machine B for 6 years. We wish to emphasize that given this context, the physical production is assumed to be not affected by the type of the machine that is chosen in the sense that, on an annual basis, identical products will be produced regardless of the machine type, in terms of quality and annual quantity. Assume that the appropriate discount rate for both is the same,  $r = 10\%$ . The cost cash flows from maintaining and running them are listed in Table I.

**Table I. Cost cash flows from two mutually exclusive machines, A and B.**

Year	0	1	2	3	4	5
A	\$10	\$7	\$12	\$18		
B	\$5	\$6	\$15	\$12	\$9	\$17

It is easy to find out that machine A would cost \$39.81 in present value to run over a 4-year life span, while machine B costs \$48.57 in present value. In terms of the total present value, machine A is superior because it costs less. But once we notice that machine A will have to be replaced (2 years) sooner than machine B, then the conclusion based on the total cost become problematic. To address this flaw, we spread the total cost evenly over the life span of the machine via the following relationship,

$$EAC = PV \times \frac{r}{1+r} \left( 1 + \frac{1}{(1+r)^N} \right)^{-1}$$

which is obtained using the  $N$ -year annuity-due formula, where  $EAC$  is the equivalent annual cost,  $N$  is the number of years within the cycle of the cost cash flow stream. Here we define the cycle of the cost cash flows to be the repeating group of cash flows. Take machine A for example, if the first cash flow of \$10 would reoccur in year 4 hence leaving neither empty years nor overlap between the streams of costs associated with two machine A's, one replacing the other, then  $N$  would be 4. If machine B is treated in the same way, then  $N$  would be 6 for B. If there is overlap, then  $N$  will be reduced. Given this understanding, we find  $EAC = \$11.42$  for machine A and  $EAC = \$10.14$  for machine B, therefore, B is better than A because on average, it costs less to operate.

Thus far, everything makes sense and the implementation is straightforward. However, if the EAC method is only valid for such a simple environment where once a machine is chosen, it will repeat forever, then this method would not have much of a use in reality. This is so-called infinite horizon requirement. Indeed, many in the finance profession critique the EAC method for having little value exactly because of this overly restrictive assumption. Therefore, we raise the question whether this is indeed true or it is a misconception.

#### IV. Finite Horizon and the EAC Method

The EAC method actually can handle finite horizon situations even though some additional issues may arise accordingly. Now we use some examples to illustrate. In the following examples where machines C and D are under consideration, we stick to the same assumptions that they only differ in type, annual costs, and their expected economic lives. The annual production is completely independent of the machine type that we choose. This vital assumption guarantees the annual cash flows of revenues are not affected hence not part of the consideration. This allows us to focus on the annual costs and the expected economic lives, and investigate how these two factors would impact our decision making.

##### 4.1 A special case of rigid finite horizon

Consider a firm that is facing a choice between machine C and D. Machine C lasts for 4 years, and D for 2 years. Suppose after 4 years, a new machine will be used regardless of whether C or D is chosen now. Machine C's and D's costs are given in Table II.

**Table II. Cost cash flows from two mutually exclusive machines, C and D.**

Year	0	1	2	3
C	\$14	\$12	\$10	\$30
D	\$8	\$20		

In this case, we have finite horizon, that is, 4 years. Within the first 4 years, we have some flexibility in our decision. Notice, this is a special case where machine D covers exactly 4 years, and machine B can be repeated once to make its operation exactly 4 years long. The

situation becomes what is described in Table III.

**Table III. Cost cash flows from machines, C and D with D repeated once.**

Year	0	1	2	3
C	\$14	\$12	\$10	\$30
D	\$8	\$20	\$8	\$20

We find the results are  $EAC = \$15.98$  for machine C, and  $EAC = \$13.71$  for machine D. Notice, for machine D, we may consider it to have a two-year cash flow cycle and one repetition or a four-year cycle with no repetition as if were a new machine that has a 4-year economic life. Either way, the EAC method nicely handles a finite horizon situation. Indeed, we acknowledge that this special situation might be too restrictive. Next, we relax these assumptions even more and show how the EAC method can be further extended.

#### 4.2 A case of flexible finite horizon

Now assume the choice is between machine E and F. Machine E lasts for 3 years, and machine F for 2 years. As assumed earlier, the annual physical production is completely independent of the specific machine type that is implemented. This assumption is to guarantee that the revenue cash flows on an annual basis remains completely unaffected by the machine type and therefore the optimality decision making process can 100 percent concentrate on the annual and total costs and the expected economic life of the machine. Appropriate discount rate is  $r = 10\%$ . Table IV shows their annual costs.

**Table IV. Cost cash flows from machines, E and F.**

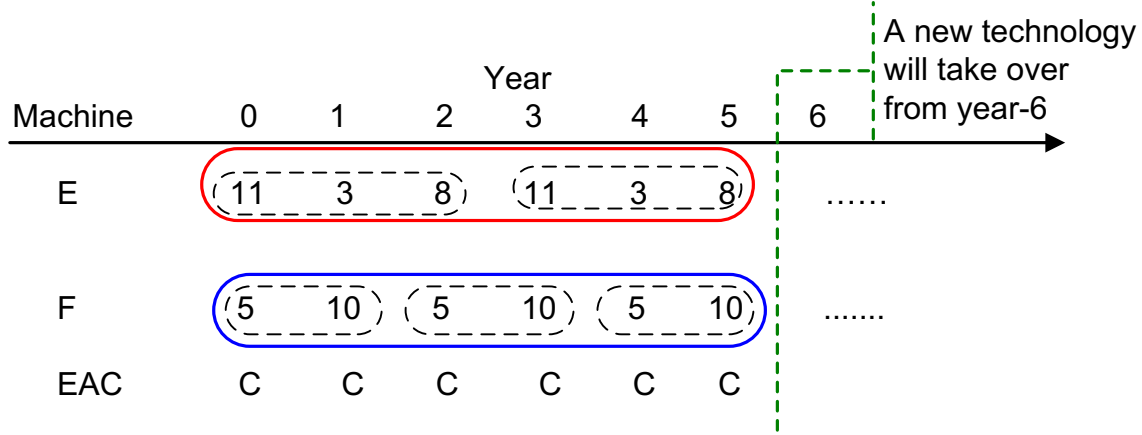
Year	0	1	2
E	\$11	\$3	\$8
F	\$5	\$10	

Suppose that the firm will choose a new technology regardless of whether E or F is chosen at the present time, and when to adopt the new technology has some flexibility – it can choose any time to adopt it after year 5. In this case, we can create a situation where machine E and F can be repeated different numbers of times before a common horizon. In other words, if we can choose a horizon  $T$  for both E and F such that it is divisible by the life span of the both machines, we would be able to create something similar to the example in the previous section, i.e., the two machines' horizons can match each other. Specifically, say, to meet a 6-year horizon, we can repeat machine E twice and machine F 3 times as shown in Figure 1.

Thus, we may consider what we have created is a 6-year quasi-cycle that contains using E twice and F three times. Setting  $N = 6$ , and equating the PV of the costs encircled in the quasi-cycle, at  $r = 10\%$ , we find  $EAC = \$7.38$  for machine E and  $EAC = \$7.44$  for machine F. Therefore, machine E costs less on an annual average basis.

We note that these results remain the same if we treat machine as a 3-year cycle and machine F as a 2-year cycle, respectively. This means that the EAC method can perfectly apply

to some finite horizon situations as long as we are allowed the flexibility to match the horizons as we have just done. In fact, if we have enough flexibility as such, the EAC method can be applied as long as we create a horizon that contains multiples of life spans of the machines.



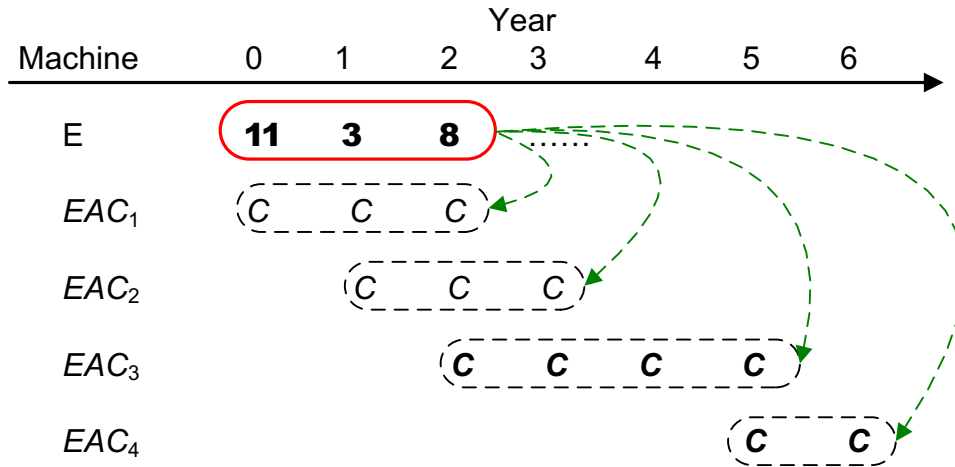
**Figure 1.** Using E twice and F three times creates the horizons that perfectly match.

Nevertheless, the flexibility we enjoy in this section is still quite restrictive because of the requirement of matching the horizons by creating a quasi-cycle. Can we relax this requirement by allowing horizons to be unmatched? In other words, if we completely remove any requirement on the horizon, can we still have a valid EAC framework? The answer is yes. However, with arbitrary horizons, more complicated issues will have to be addressed in order to restore the validity of the EAC method. These issues include incidental cash flows and incremental cash flows, etc. Qi and Xie (2015) explain these additional issues and demonstrate how to consistently restore the validity of the EAC methodology to arbitrary horizons.

**V. EAC Stream Can Start Anytime**

The actual costs have to be specified clearly on the time line. In the examples we have used in this study, the equivalent annual costs are matched exactly on the time line to those actual costs. However, no one has argued that this is necessary. Instead, we find one textbook suggesting that the EAC stream of cost cash flows may start anytime according to our like. Welch (2009) shows that the EAC does not have to match the corresponding actual costs. The example Welch (2009) uses is about determining a lease contract that substitute owning the machine.

We note that this practice is correct in theory and in that specific context; however, it can cause considerable misleading claims that may eventually lead to wrong decisions. Even to put it lightly, if we allow the EAC stream to start anytime or not match the horizon of the corresponding actual costs, the reliability of the EAC method is cast under great concern. Now we use the same example in the above section to illustrate this point. This is shown in Figure 2.



**Figure 2.** Machine E has three years of economic life. Here we show four equivalent annuities due starting at different times: two 3-year EAC’s (the first two cases,  $EAC_1$  and  $EAC_2$ ), and one 4-year and one 2-year EAC’s (the bottom two cases,  $EAC_3$  and  $EAC_4$ ).

The original 3-year cost cycle is now matched to four equivalent annuities due. At  $r = 10\%$ , PV of the costs of machine E from year-0 through year-2 is  $PV_E = \$20.34$ . Given the same present value of the costs, we find the equivalent annual costs as follows,

$$\left\{ \begin{array}{l} PV_E = \frac{EAC_1}{r} \left( 1 - \frac{1}{(1+r)^3} \right) \times (1+r) \rightarrow EAC_1 = \$7.44 \\ PV_E = \frac{EAC_2}{r} \left( 1 - \frac{1}{(1+r)^3} \right) \rightarrow EAC_2 = \$8.18 \\ PV_E = \frac{EAC_3}{r} \left( 1 - \frac{1}{(1+r)^4} \right) \times \frac{1}{1+r} \rightarrow EAC_3 = \$7.06 \\ PV_E = \frac{EAC_4}{r} \left( 1 - \frac{1}{(1+r)^2} \right) \times \frac{1}{(1+r)^4} \rightarrow EAC_4 = \$17.16 \end{array} \right.$$

Therefore, if we allow the EAC stream to start at any time and end at any point later on, then we would not have a unique equivalent annuity and the numbers can be “tuned” to pretty much any value, which makes comparing different machines a messy business because the numbers cannot be easily compare anymore. Nevertheless, we note that this calculation is handy for determining various possibilities of alternative lease financing. However, even though in theory one can do this, it would become a mess or misleading if we do so with even our simplest example of infinite horizon. Thus, we feel it is time to clarify that in a more generic environment, the stream of EAC cash flows should better match that of the corresponding actual costs.

## VI. Conclusions

The equivalent annual cost (EAC) methodology is a powerful tool that is taught in business schools and has wide applications in many areas outside the field of finance. However, there are misunderstandings about it. In this paper, we clarify two common ones. The first is that the EAC method only applies to infinite horizon, i.e., once a type of machine is chosen, it will be forever used except that a new set replaces the old one once it reaches the end of the machine's economic life. The second is the notion that the matching EAC cash flows may start at any point and end at any time. For the first, we show a few counter examples that finite horizons may still be handled by the EAC method. For the second, we make a caveat that from a practical viewpoint, it is better to match the EAC cash flow stream to the corresponding actual cost cash flow stream. Our work on the EAC methodology helps make it more accurate and applicable to situations that are more realistic.

There are two other important factors that are not considered in this study but deserve further investigation. One is transaction costs and another one is that, given the advancement of technology, a shorter-life machine may have a hidden advantage of being able to be upgraded sooner – a greater extent of flexibility that may be characterized as a call option on real assets. We leave these relevant issues for future investigations.

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