Price Dependencies and Investors' Choice

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Abstract

Under the efficient market hypothesis, where many investors resort to "buy and hold" strategies with maximum diversifications, "capitalization-weighted indexation" may offer investors the best risk-return combination. But the existence of non-linear dependences shows that prices of securities may be subject to temporary shocks that obscure their true value. In this case, "fundamental indexation" may achieve returns superior to the capitalization-weighted indexation.

I. Introduction

There are two distinctive groups of thought in the stock market analysis. One school of philosophy holds the assumption that the price of a stock is always in equilibrium. So the current prices of securities fully reflect all known information quickly and accurately. The other group, on the other hand, can be characterized as pursing active investment strategies that are based on the notion that there exists information available to at least one investor that has not been fully integrated into current market prices.

The literature of finance has traditionally assumed that the joint hypothesis including weak form efficiency is a necessary consequence of the general random walk model of security price changes. In this framework, tests of the weak form market efficiency are often presumed to require nothing more than a finding that security price changes over time are uncorrelated for any lag. However, this approach contains the possibility of a specification error. Zero autocorrelation coefficients for all lags of a security price series is a necessary, but not sufficient condition for concluding that the time series follows a random walk. In other words, a pair of variables can be uncorrelated, even though the variables contain a statistical dependence relationship. Independent random variables are uncorrelated, but not vice versa. Of course, the general random walk model requires that price changes for all lags be both uncorrelated and independent.

If stock price movements are more complicated than the random walk model suggests, it will take more sophisticated techniques to explain them. However, many of the traditional approaches (serial correlation, run test, etc) are too unsophisticated or too restrictive to pick up complicated patterns of the price behavior. The purpose of this paper is to fill the gap in the literature by applying rather sophisticated nonlinear models in the search for nonlinear dependence of the stock prices in the capital market. The simplest but the most useful class of nonlinear models is the bilinear model. This model is linear in the 'states' and also in the 'errors' separately but not in both. Thus bilinear models incorporate the class of linear models considered by Box and Jenkins [2], namely the integrated autoregressive moving average (ARIMA) models as special cases. In fact, bilinear series provides a natural generalization from the linear situation to the nonlinear one. In this paper, we find that the price generating mechanism of the security prices studied is nonlinear. In other words, using the Akaike's Information Criterion (AIC) [1], we

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find that the best subset of bilinear models perform better than any kind of autoregressive models.

The improved fit of the bilinear model analyzed, relative to the best linear model indicates that the bilinear model is capable of capturing a significant portion of the nonlinear dependencies which appear in the price generating process. When a nonlinear price generating process is modeled as a linear process, the specification error inherent in the test will be collapsed into the error terms of the model, thus inflating the value of the mean squared error. The significant reduction of the mean squared error achieved with a bilinear model is evidence which supports the hypothesis that price and return generating process contain a significant nonlinear dependency.

Under the efficient market hypothesis, where the current price represents the best, unbiased estimate of the true underlying value of the firm, "capitalization-weighted indexation" may offer investors the best risk-return combination. Because if investors think they can't "beat" the market, they may take passive strategies. But the existence of non-linear dependences shows that prices of securities may be subject to temporary shocks that obscure their true value. In this case, "fundamental indexation" may achieve returns superior to the capitalization-weighted indexation. The fundamental indexation means each stock in the portfolio is not weighted by its market capitalization, but by some fundamental metric, such as aggregate sales, or aggregate dividends. The "noisy market hypothesis" easily explains the size and value anomalies. If a stock falls for reasons unrelated to the changes in the fundamental values, then it's likely that overweighting such a stock will yield better than normal returns. Efficient market believers still dominate the finance research arenas, many practitioners and moonlighting academics including Siegal (2006) recommends that investors overweight value and small stocks in their portfolio.

In the next section, we will investigate relation between nonlinear dependence and market efficiency, followed by the best models for prices and returns. Then implications and conclusions will be discussed.

1. -2 max In (likelihood) + 2 (number of independent parameters). Hence the model with smallest AIC will usually be chosen as the right model.

II. Nonlinear Dependence and Market Efficiency

Random Walk and Price Dependence

The basic characteristic of the random walk hypothesis is that a specific price series follows a simple stochastic process. This implies that successive price changes are independent. In this kind of independent increment process, the past history of a series is of no use in predicting future changes in the series.

The assumptions of perfect capital markets are sufficient conditions for the market to be efficient. But they are not necessary conditions. The existence of the market imperfections, such as transaction costs, costly information, and heterogeneous beliefs among investors are not necessary sources of market inefficiency, they are only potential sources. On the other hand, even if stock prices deviate significantly from the random walk process, market participants may not have profitable trading opportunities by acting on the deviations because of market frictions. In this situation, it is possible to reject the random walk model without rejecting the notion of market efficiency. The random walk model can be viewed as an extension of the general expected return or the "fair game" model. Samuelson (1965) provided the efficient market notion that prices follow a martingale stochastic process. This is commonly regarded as the fair game model of the efficient market. However, the fair game model does not say much about the stochastic process generating returns, since it simply states that the conditions of market equilibrium can be expressed in terms of expected returns.

The notion that efficiency implies "current prices fully reflect all available information" is too general to be concretely tested. Hence we need a statistically tractable way of measuring this intuitive notion of an efficient market. Fama (1970) states if the current price of a security "fully reflects" available information then successive price changes are independent. He argues that market efficiency with the assumption of market equilibrium implies that the autocorrelations of the returns on any security are zero for all values of the lag t. Therefore, there is no way to use information available at t-1 as the basis of a correct assessment of the expected value of residuals (e_t) other than zero. Praetz (1979) elaborates this statement by saying that market inefficiency implies that from time to time the process of assessing information to stock prices is somewhat incomplete. These errors of assessment generally show up as correlated return sequences. Accordingly, in many cases, it has been presumed sufficient to test the autocorrelation of the time series and if it is zero for all lags then conclude that the series follows a random walk.

However, the above argument is true provided the time series follows a normal distribution. In other words, a pair of variables can be uncorrelated yet not be independent, but not vice versa. In this context, the correlation coefficient is just a measure of the degree to which two random variables are linearly related. Again, if two random variables come from bivariate normal distribution, uncorrelated variables are independent. If security prices are not normal, then even if they are uncorrelated they may not be independent. Therefore, if the linearity of the process is rejected we must also reject the normality assumption. But if the stock prices and returns are generated by the nonlinear processes, even though they might be uncorrelated, they might not be independent.

Accordingly, if the true processes generating daily stock prices and returns are nonlinear, the test of the zero correlation is not appropriate for the test of the random walk, or for the test of the market efficiency. The discovery of nonlinear process is inconsistent with the notion of an independent incremental process.

Independence and nonlinear process

For the general case of non-Gaussian processes, consider the problem of prediction the future value of a process, given observations up to time t. In the case of an independent process, 91' the past contains no information on the future. In the case of an uncorrelated process, el, the past contains no information on the future if the predictors are linear. However, for the nonlinear predictors, the past may well contain useful information on the future values. Consider the following simple example.

$$X_1 = e_t + \beta\ e_{t-1}\, e_{t-2} \qquad (1)$$

where e_t is an independent process with zero mean and constant variance.
Then, $E(X_t) = 0$, and $\{X_t\}$ is a white noise process, since Cov (X_t, Xs) = E (X_t X_s). However, the random variables X_t are not independent. In other words, there exists a relationship among X_{t-2}, X_{t-1}, and X_t for all t since

$$E\left(X_{t-2}X_{t-1}X_t\right) = E\left(e_{t-2}{}^2 e_{t-1}{}^2\right) = \beta\sigma_e{}^2. \tag{2}$$

Therefore, if daily stock prices and returns are generated by a nonlinear, non-Gaussian process, then the independence of price movements is questionable.

III. The Best Models for Stock Prices and Returns

For our investigation, we utilized 5 year daily closing prices of the S & P 500 drawn from the Wall Street Journal. The Index prices were for trading days during the period January 3, 2006 to December 31, 2010. In addition to the daily prices, daily index returns were obtained from the CRSP universe of stock and index returns for each trading day in the sample. All the models are determined based on daily prices (and returns) for one full year from 2006 to 2010, then we take the 5 year average values.

Fama (1970) showed that a time series of daily stock returns exhibit low levels of significant autocorrelation for lags varying between 1 and 10 days. Hence, for the S & P 500, the maximum lag is chose to be equal to 10. In this analysis, we employ the Akaike's Information Criterion (AIC) as the criteria for the model selection. It can be shown that AIC is a generalization of the maximum likelihood principle. Using AIC, we find that the best Autoregressive (AR) model for the stock price generating mechanism is the AR (1) model, where 5 year average mean sum of squares of errors ($\sigma_e{}^2$) is 2.2475 and AIC is 228.324. However, the best AR model for the stock price which has the lowest mean sum of squares of errors is the full AR (10) model with ($\sigma_e{}^2$) average of 2.2168 and AIC of 243.875. Following the procedures of the S. Rao et.al, (1981), the best bilinear model for the stock price generating process is Bilinear (BL) (1, 0:10, 10) with ($\sigma_e{}^2$) of 1.9828 and AIC of 218.158.

The best bilinear model has lower AIC value and lower mean sum of squares of errors than each of the ”best" AR models. The results are summarized in Table 1. The bilinear model not only has smaller mean sum of squares but also shows smaller AIC value when compared to other models. As a result, in the daily S & P 500 price generating process, the nonlinear model is a better fit than the linear models.

Table I

Value of $\sigma_e{}^2$, AIC, and No. of Parameters (Daily S & P 500 Price Data)

Model	Full AR (10)	Subset AR	Subset Bilinear
$\sigma_e{}^2$	2.13	2.25	1.98
AIC	243.88	228.32	218.16
No. of Parameters	11	2	7
The Best Choice			x

We then applied the same process above to the daily S & P 500 return data to derive the best subset of AR (10), and the best subset of BL (10, 0:10, 8). Table 2 presents the results of the scaled (x 10^4) Index returns with each model. In the return generating process as well, the bilinear model has less residual variance and less AIC than the autoregressive models.

Table II
Value of σ_e^2, AIC, and No. of Parameters (Daily S & P 500 Return Data: x 10^4)

Model	Full AR (10)	Subset AR	Subset Bilinear
σ_e^2	7391.29	7782.16	6499.04
AIC	3195.89	2769.93	2397.99
No. of Parameters	11	4	9
The Best Choice			x

IV. Implications

Why these dependences of the price and return date are observed? Three possible reasons can be suggested. Firstly, dependences may be because of the divergence of investors' expectations. That is each individual may have heterogeneous beliefs about future events. For example, each may assess different probability distributions on the asset returns. Secondly, even if individuals do have the same probability distributions on the future events, their interpretations about those events can be different. Lastly, such dependences may be because of the diverse risk aversions for investors. Depending on the characteristics of an individual's utility function, each may require different level of risk premium for a given project. Hence the actuarial value of that project will not be unique. Accordingly, some good opportunities to a specific risk averse individual will not be good enough to others and vice versa.

Under the efficient market hypothesis, where the current price represents the best, unbiased estimate of the true underlying value of the firm, "capitalization-weighted indexation" may offer investors the best risk-return combination. Because once investors believe that they can't "beat" the market, they will take passive strategies. But the existence of non-linear dependences shows that prices of securities may be subject to temporary shocks that obscure their true value. In this case, "fundamental indexation" may achieve returns superior to the capitalization-weighted indexation. The fundamental indexation means each stock in the portfolio is not weighted by its market capitalization, but by some fundamental metric, such as aggregate sales, or aggregate dividends. The "noisy market hypothesis" easily explains the size and value anomalies. If a stock falls for reasons unrelated to the changes in the fundamental values, then it's likely that overweighting such a stock will yield better than normal return.

V. Conclusions and Further Suggestions

Linear models have been used successfully for analyzing time series data. But there are cases when it has been recognized that linear time series may not be adequately explaining the underlying random movement. For these cases we may utilize nonlinear models in the time series analysis. Nonlinear models admit patterns of prices which are not serially independent and patterns of realized stock prices and returns which do not arise from an independent increment process. If the stock market has no "memory" or if the current stock prices are completely independent of the past, then is the study of historic data and fundamental factors utterly useless?

We find that the price and the return generating mechanisms are nonlinear. For the daily stock price and return generating process of the S & P 500 Index, we find that the best subset of bilinear models perform better than any kind of autoregressive models. The fact that the best bilinear model possesses superior explanatory abilities relative to the linear model suggests that the mechanism which generates security prices and returns is more complex than the literature suggests. An implication of this complexity is the possibility of earning economically significant arbitrage profits from a superior understanding of the nonlinear dependencies which exist in security price series.

For the practical purposes, implications for the portfolio management are rather significant. Under the efficient market hypothesis, where many investors resort to “buy and hold” strategies with maximum diversifications, “capitalization-weighted indexation” may offer investors the best risk-return combination. But the existence of non-linear dependences shows that prices of securities may be subject to temporary shocks that obscure their true value. In this case, “fundamental indexation” may achieve returns superior to the capitalization-weighted indexation. Further research is needed to determine what causes these nonlinear dependences of security prices and returns.

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