

## **Analyzing The Value of Proprietary Claims Under Uncertainty: A Real Options Approach**

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### **Abstract**

This paper develops and implements a general framework for evaluating proprietary claims generated by patents based on a real options approach. Our model considers both demand and competitive uncertainty as endogenous variables in the valuation process, and recognizes the interaction between the proprietary claim and the deferral and abandonment options embedded in an investment project. We find that the acquisition of a patent enhances the value of the deferral option, but reduces the value of the abandonment option as a consequence of reduced competitive uncertainty. Although we illustrate our approach with a product launch of a new technology in the automotive industry, our model applies just as easily to any capital budgeting problem where proprietary rights play an important role.

### **Introduction**

Given that project cash flows fluctuate over time and are a function of demand and competitive factors, most investment decisions are dynamic, rather than static, “now or never” decisions implied by conventional discounted cash flow (DCF) analysis. In fact, traditional DCF analysis can lead to erroneous accept/reject decisions. It is now widely accepted that most investment decisions are analogous to buying options, giving the buyer the right but not the obligation to go ahead with the project. A real option is a right (with no obligation) to take specific future investment actions contingent on how uncertain conditions regarding project output price and demand evolve over time. A real options approach to capital budgeting analysis recognizes that a manager has the freedom to defer work on the project, expand or scale it back, shut down the project temporarily, or abandon it permanently. A large body of literature has developed over the past twenty years demonstrating the important role of real options in a firm’s optimal investment strategies (Brennan and Schwartz 1985, McDonald and Siegel 1986, Ingersoll and Ross 1992, Dixit and Pindyck 1994). It is logical, therefore, that the real options approach should also play a significant role in analyzing the value of a firm’s investment in a patent. Patents create entry barriers for competitors up to a pre-determined future time, and hence may be regarded as proprietary claims on the underlying product. The evaluation of such claims is a challenging problem because their effect on the real investment is not directly observable, and therefore hard to quantify. The goal of this paper is to present a real options framework for valuing proprietary claims under uncertain market and demand conditions, wherein the uncertainty is endogenous to the evaluation process. We also examine the value of the deferral and abandonment options resulting from patenting decisions on the net present value of investments under uncertain market and demand conditions. The following relevant considerations, often overlooked or considered in piecemeal fashion, are addressed cohesively in our paper.

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First, the value of the patent is contingent upon the degree of demand and competitive uncertainty. The possible action by rival firms and potential entrants has a significant bearing on the opportunity cost of not imposing patent induced entry barriers, and is thus reflected in the patent value. The impact of competitive interaction on real investments has already been studied by several authors (Kulatilaka and Perotti 1998, and Reiss 1998), who recommend that the market structure be incorporated into the valuation model. Similarly, the product life cycle dynamics can enhance or diminish the benefits of the entry barrier provided by the patent. Several authors attempt to capture demand uncertainty by allowing for complex asset return dynamics driven, for instance, by jump-diffusion type stochastic differential equations (Angelis 2000, Pennings and Lint 1997, Willner 1995) or by Markov processes with switching parameters (Bollen 1998, 1999).

Second, a patent creates a proprietary claim over the new technology, giving its owner flexibility over new investments associated with the project. Firms holding a patent may still need to undertake further research and development to make the product ready for market, as well as expensive marketing and advertising to promote a new product (Blundell, Griffith, and Van Reenen 1999). However, since patents give their holders the exclusive rights to the new technology, they have the option to wait before making these sunk cost investments (Bloom and Van Reenen 2002). Thus, by providing a firm with a right to defer investment in a project, a patent also has a deferral value in addition to the proprietary value it confers on its owner.

Third, investing in a patent may affect the value of other real options embedded in the investment. Assume, for instance, that the firm has the right to abandon the project for a specified salvage value. It is possible that the new product is not embraced by the market, and cash flows turn out to be lower than anticipated, or alternatively, costs may turn out to be higher than expected. In either case, these catastrophic events could trigger the exercise of the abandonment option, whose value is substantial when demand and/or competitive uncertainty is large (Schwartz and Moon 2003). It is clear that investing in the patent diminishes the worth of the abandonment option because it reduces the demand and/or competitive risk.

In the following sections, we develop a market model and illustrate a methodology for computing the present value of real investments within our theoretical setting. We employ the explicit finite difference method for valuing proprietary claims as real options. Given the difficulty of obtaining a closed form expression for the project net present value, we derive an algorithm to evaluate it recursively. Next, we illustrate the valuation of proprietary claims within our framework with the help of a hypothetical example of the launch of an automobile based on a new technology. Finally, we offer some concluding observations from our example.

### The Market Model

Since the net present value of investment in a project depends upon the demand for the underlying output, we begin by specifying the demand function. Let  $Y_t$  denote the demand for the output of the real investment. Also, let us assume that  $Y_t$  solves the following diffusion type stochastic differential equation with random coefficients:

$$dY_t = Y_t(\mu(S_t)dt + \sigma(S_t)dW_t) \quad S_t = 0, 1, \dots, N \quad (1)$$

$S_t$  is continuous time Markov chain taking values in the state space  $\{0, 1, 2, \dots, N\}$ . Our market model permits migration to different states or market regimes that describe the random evolution of the environmental variables affecting the value of the real investment. These inter-state

migrations are governed by the continuous time Markov chain,  $S_t$ . Each state is assigned the vector  $\{\mu_i, \sigma_i, \theta_i\}$ , where  $\mu_i$  and  $\sigma_i$  respectively denote state  $i$  drift in demand growth and volatility coefficients in (1) and  $\theta_i$  is a percentage profit mark-up parameter related to the market competitive structure prevailing in state  $i$ . States  $\{0, 1, 2, \dots, M\}$  are described by specified drift, volatility and mark-up levels associated with distinct phases of demand evolution, when the technology is protected by the patent. Similarly, states  $\{M+1, M+2, \dots, N\}$  are described by drift, volatility and mark-up levels associated with each stage of demand evolution when the market is deregulated. The absorbing state  $N$  is a “bankruptcy” state that determines the random time when the project’s useful life comes to an end. Once the chain visits state  $N$ , the cash flows generated by the real investment become zero. Let the price of the real asset output be defined as:

$$\rho_i(Y_t, S_t) = \psi(Y_t, \theta_i) \quad S_t = 0, 1, \dots, N \quad (2)$$

for some known function  $\psi(\cdot, \cdot)$ . Equation (2) is intuitively appealing, since pricing policies are contingent both upon production levels, which affect marginal costs and revenues, and market competitive structure, which affects the price elasticity of demand.

The probabilistic behavior of the Markov chain,  $S_t$ , is entirely characterized by its infinitesimal generator, which takes the form of an  $N \times N$  matrix:

$$A(u) = [\lambda_{ij}(u)] \quad (3)$$

whose values,  $\lambda_{ij}$ , are the transition rates from state  $i$  to state  $j$  for  $i \neq j$  and  $\lambda_{ii} = -\sum_{j \neq i} \lambda_{ij}$ .

Given an initial probability vector,  $P(S_0)$ , the time  $t$  probability distribution of the chain,  $P(S_t)$  is easily computed as the solution to the Chapman Kolmogorov equations:

$$P(S_t) = P(S_0) \exp(A(u)t) \quad (4)$$

We hypothesize that the generator’s structure is a function of a parameter,  $u$ , which denotes the patenting strategy adopted by the firm. For simplicity, we shall assume that the firm can adopt a finite number of patenting strategies, and that there is a one to one correspondence between the form of the generator and the action taken by the firm. Let us use the following example to make this construction clear. Assume that a firm has the possibility of acquiring a patent for a finite time period, say five years, at a specified cost. The firm has thus two strategic possibilities (that is,  $u$  takes one of two distinct values), namely:

- a. acquire the patent and impose an entry barrier for the next five years; or
- b. do not acquire the patent and allow competition to enter the market immediately

Under case (a), the transition rates to competitive regimes are zero for the first five years of the life of the investment, whereas under case (b), the transition rates assume specified positive values for the entire life of the investment.

In the next section, we illustrate a methodology for evaluating real investments in our framework. We regard the real investment as a contingent claim paying a stream of cash flows, which depend upon the evolution of the underlying market. In this framework, the project value is the expectation of the future discounted cash flows generated by the investment. A critical feature in real options theory is the choice of an appropriate discount rate. The value of a real asset is, in fact, contingent upon the evolution of non traded state variables, such as demand, prices, or costs, thus making it difficult to justify risk neutral valuation. In the practice of capital budgeting, it is common to carry out all discounting operations at a risk adjusted rate, where,

risk adjustment, or premium, is determined by the risk preferences of the project stakeholders. We assume that the risk adjusted rate is given by the Capital Asset Pricing Model (CAPM):

$$\kappa = r + \beta(r_m - r) \tag{5}$$

where  $r$  is the risk-free rate,  $r_m$  is the market return and  $\beta$  is the risk-adjusted beta for the project.

**Project Valuation Methodology**

Let  $f(Y_t, t | S_t)$  denote the time  $t$  instantaneous cash flow function, conditional upon the state visited by the Markov chain in the time interval  $[t, t + \delta t]$ , henceforth referred to as an instantaneous move. The present value of the project  $\pi(Y_t, t)$  is computed as:

$$\pi(Y_t, t) = E^{Y_t, t} \left[ \int_t^T f(Y_s, s | S_s) \exp(-\kappa(s - t)) ds \right] \tag{6}$$

where  $E^{Y_t, t}[\cdot]$  stands for actuarial expectation conditional on the time  $t$  value of the process  $Y_t$  and  $T$  is the random useful life of the investment. Since  $f(Y_t, t | S_t)$  is a diffusion function with random coefficients and the upper bound of integration is a random time, this expectation is hard to compute in closed form; hence we develop an algorithm to compute (6) recursively.

Before developing our methodology, however, we need some preliminary results. Let  $Y_t$  be the solution to the stochastic differential equation in (1) and define the following function:

$$C(Y_t, t | S_t = i) = E^{Y_t, t} [e^{-\kappa(\delta t)} (\varphi(Y_{t+\delta t}, t + \delta t)) | S_t = i] \tag{7}$$

as the expectation of  $\varphi(Y_{t+\delta t}, t + \delta t)$ , conditional on  $Y_t$ , and on the state visited by  $S_t$  in an instantaneous move for some known function  $\varphi(\cdot, \cdot)$ . Note that  $C(Y_t, t | S_t = i)$  is conditionally Markovian with respect to the information generated by both the macroeconomic factor  $Y_t$  and the latent Markov chain,  $S_t$ . This implies that its value depends on the evolution of  $Y_t$  and  $S_t$  based only on their current level. By the Feynman-Kac formula,  $C(Y_t, t | S_t = i)$  satisfies the following partial differential equation:

$$\frac{\partial C}{\partial t} + \frac{1}{2} \sigma_t^2 Y^2 \frac{\partial^2 C}{\partial Y^2} + \mu_t Y \frac{\partial C}{\partial Y} - \kappa C = 0 \tag{8}$$

with terminal condition given by

$$C(Y_{t+\delta t}, t + \delta t | S_t = i) = \varphi(Y_{t+\delta t}, t + \delta t)$$

Although the coefficients of the above partial differential equation are random, inasmuch as they depend on the state of the Markov chain in the interval  $[t, t + \delta t]$ , one can safely assume they are constant in the infinitesimal. The probability of the chain jumping more than once in such time interval is, in fact  $o(\delta t)$ , signifying an inferior order of magnitude, and thus negligible. We use the above results to derive the following backward recursive relation for computing the time  $t$  present value of the project.

**Proposition 1:** under dynamic market conditions described by (1), given the instantaneous cash flow function  $f(Y, t)$ , the expectation in (6) can be computed recursively as follows:

$$\pi(Y, t) = \sum_{i=0}^N (f(Y, t | S_t = i) + C(Y, t | S_t = i))P(S_t = i) \quad (9)$$

where

- a)  $C(Y, t | S_t = i)$  satisfies (8) with terminal conditions  $C(Y_{t+\delta}, t + \delta) = \pi(Y_{t+\delta}, t + \delta)$
- b)  $P(S_t = i)$  denotes the probability of the chain visiting state  $i$  in  $[t, t + \delta]$  as in (4)
- c) the starting point for the recursion is the time  $T$  terminal condition  $\pi(Y_T, T) = f(Y_T, T)$

The proof of Proposition 1 is provided in the Appendix.

### Implementation Outline

In this section we outline the steps for computing (6). Let the domain  $\{Y, t\}$  be a two-dimensional mesh, where  $Y_m^k$  denotes the gridpoints with time coordinate  $k$  ( $k \in [0, 1, \dots, T]$ ) and  $Y$  coordinate  $m$  ( $m \in [0, 1, \dots, M]$ ). Let  $\delta Y$  be the size of the step in the  $Y$  direction and  $\delta t$  be the time-step size. Discretizing and rearranging (4) as in the implicit finite difference scheme:

$$C_m^{k+1}[i] = \left[ -\frac{\delta}{\delta Y^2} \frac{1}{2} \sigma_i^2 Y_m^{k2} - \frac{1}{2} \frac{\delta}{\delta Y} \mu_i Y_m^k \right] C_{m+1}^k + \left[ 1 + 2 \left( \frac{\delta}{\delta S^2} \frac{1}{2} \sigma_i^2 Y_m^{k2} \right) + \delta(+\kappa) \right] C_m^k + \left[ -\frac{\delta}{\delta S^2} \frac{1}{2} \sigma_i^2 Y_m^{k2} + \frac{1}{2} \frac{\delta}{\delta S} \mu_i Y_m^k \right] C_{m-1}^k \quad (10)$$

Equation (10) is valid for  $0 < m < M$ . The notation  $C_m^{k+1}[i]$  indicates that the equation coefficients are  $\{\mu_i, \sigma_i\}$ , e.g.  $S_{(k-1)\delta t} = i$ .

If  $C_m^{k+1}$  are known for all  $m$ , then, the difference equation (10) yields a linear system of  $M-1$  equations with  $M-1$  unknowns. Given boundary conditions,  $C_0^k = g(k\delta t)$  and  $C_M^k = h(k\delta t)$  provide the values of  $C_0^k$  and  $C_M^k$ , reducing the problem to a tridiagonal system of  $M$  equations in  $M$  unknowns, which is easily solved using standard methods. With this in mind, the implementation of the algorithm becomes straightforward. For clarity of exposition we present it in successive steps: Let  $T$  be the first time when the probability of being in the “bankruptcy” state is one, where  $T$  can be found using (4). This choice of  $T$  ensures that the expectation in (7) is taken over all possible stopping times. The final vector of grid-points

$$C_m^T = \pi(m\delta Y, T\delta t) = \sum_{i=1}^N f(m\delta Y, T\delta t | S_T = i)P(S_T = i) \text{ is known for every } m.$$

- 1) Compute  $C_m^{T-1}[i]$  solving the linear system (9) for each  $i$
- 2) Compute  $\pi(m\delta Y, (T-1)\delta t) = \sum_{i=0}^N (f(m\delta Y, (T-1)\delta t | S_t = i) + C_m^{T-1}[i])P(S_{(T-1)\delta t} = i)$  using

*Proposition 1*

- 3) Set  $C_m^{T-1} = \pi(m\delta Y, (T-1)\delta t)$
- 4) Repeat steps (1) to (3) until the 0 time step is reached.

Our finite difference model therefore uses a set of discrete time difference equations and solves these difference equations using a “rollback” process. Given the choice of discretization scheme, the algorithm inherits the error order and stability properties of the relative finite difference scheme. We opted for the implicit finite difference approach as opposed to the binomial, trinomial or other lattice based discrete time approximations of the continuous time model

because the finite difference approach maintains good stability properties and is easily implemented (Balsara et al, 2005).

### Evaluation of Proprietary Claims using a Hypothetical Example

In this section, we illustrate how proprietary claims are evaluated within the context of our proposed framework. Since we endogenize the uncertainty driving the value of the proprietary claim in the evaluation of the project investment, we are able to quantify the effect of acquiring the patent on the project net present value (NPV) as the difference between the project NPV with and without patent protection. Next, we evaluate the effect of the patent on the manager's right to defer and/or abandon the project. To illustrate our model, we discuss our methodology in relation to a hypothetical example. In particular, we analyze the market launch of an automobile based on new technology, as for example, a hybrid car which uses both gasoline and electricity.

#### 1. Investment Timing

At time  $t = 0$ , car manufacturer XYZ is preparing a business plan for the introductory phase of a hybrid vehicle. This is a breakthrough technology, with a high degree of uncertainty surrounding its acceptance by the market. Also, other car manufacturers are developing similar technologies, and their entry in the hybrid car market poses a serious competitive threat to XYZ. Management believes that first entrant advantage will be a crucial factor, but is uncertain as to whether to immediately commit to the investment, or defer it to some later time  $\tau$ , when demand conditions are clearer. However, the firm can purchase a patent for an instantaneous fee  $\alpha(t)$  to protect its investment up to a future date  $T_k$ ,  $k = 1, 2, 3, K$ , where  $T_k$  are fixed deadlines specified by sector regulations. The patent provides two distinct economic benefits to company XYZ:

- a. It gives XYZ a proprietary claim to the technology for the hybrid automobile, by creating entry barriers for its competitors.
- b. It allows XYZ to enter the market when demand conditions are favorable, thus providing the firm with the right to defer commercialization of the new technology.

We make the following simplifying assumptions regarding the entry strategy of the firm XYZ:

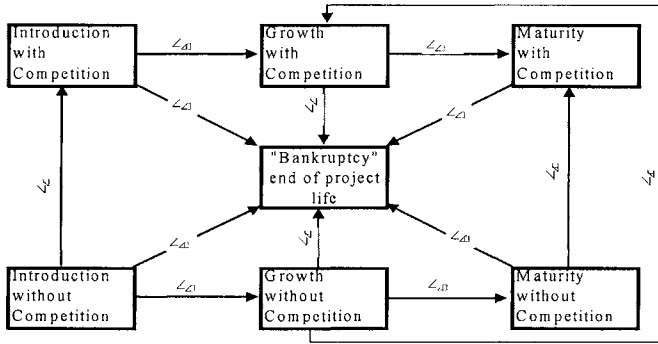
- 1) The entry time  $\tau$  takes values in the finite set  $\Lambda = \{0, t_0, t_1, t_2, \dots, t_n\} \cup \{\infty\}$  with  $t_n < T_k$ . This notation indicates that the entry time is either less than or equal to  $t_n$ , or the firm will defer the capital outlay indefinitely. If the firm decides not to buy the patent, it will enter the market immediately, e.g.  $T_k = 0$  implies  $\tau = 0$ .
- 2) At the time of entry, the firm incurs a lump-sum capital outlay,  $X(t)$ , that includes the cost of converting some of its manufacturing plants to hybrid production technologies, plus marketing and launch expenditures. Plant conversion time is assumed to be zero, so that production starts as soon as the entry decision is made. At any time after the launch, the firm can abandon the project and resell its manufacturing equipment for a salvage value  $K(t)$ .

#### 2. Demand Dynamics

Let the demand for XYZ hybrid vehicles,  $Y_t$ , evolve as in (1), with  $N=7$  states. States  $\{1\}$ ,  $\{2\}$ , and  $\{3\}$ , which we refer to as "monopoly" regimes, correspond to the introduction, growth, and maturity phases of the product life cycle when the market is protected by the patent

agreement. States {4}, {5} and {6}, denoted as “competitive” regimes, are the corresponding phases of the product life cycle in a deregulated market with no patent protection. Once the Markov chain visits the absorbing state {7}, the investment ceases to produce cashflows. Figure 1 presents a schematic representation of the interaction between the different states. To each state is associated the corresponding vector  $\{\mu_i, \sigma_i, \theta_i\}$ , where  $\mu_i$  and  $\sigma_i$  denote drift and diffusion coefficients of the demand process and  $\theta_i$  is the percentage mark-up on marginal costs in each market regime. We claim that in “monopoly” regimes, the parameters  $\mu_i$  and  $\theta_i$  are at least as high as their counterparts in “competitive” regimes, while the inverse relation holds for the volatility coefficients. For instance, state {1} corresponds to “introduction in monopoly regime” while state {4} models “introduction in competitive regime” and the market parameters are as follows:  $\mu_1 \geq \mu_4$ ,  $\theta_1 \geq \theta_4$ ,  $\sigma_1 \leq \sigma_4$ .

Figure 1. Structure of the Markov Chain



The infinitesimal generator of the Markov chain assumes different structures according to the patenting policy adopted by the firm. Recall from section 1 that the parameter  $u$  expresses the choice of the patent’s expiration date,  $T_k$ . It is easy to see that the chain’s infinitesimal generator  $A(u)$  can take different forms, each corresponding to a different patent expiration date alternative. Before the patent expires, the transition rates to the “competitive” states are equal to zero, because the market is legally protected from new entrants while, after  $T_k$ , the market is deregulated and competitive threats must be included in the model. Therefore,

$$\begin{aligned}
 \text{If } t \leq T_k \quad A^- &= \begin{bmatrix} -\sum \lambda_{1j} & \lambda_{12} & 0 & 0 & 0 & 0 & \lambda_{17} \\ 0 & -\sum \lambda_{2j} & \lambda_{23} & 0 & 0 & 0 & \lambda_{27} \\ 0 & 0 & -\sum \lambda_{3j} & 0 & 0 & 0 & \lambda_{37} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\
 \text{If } t > T_k \quad A^- &= \begin{bmatrix} -\sum \lambda_{1j} & \lambda_{12} & 0 & \lambda_{14} & 0 & 0 & \lambda_{17} \\ 0 & -\sum \lambda_{2j} & \lambda_{23} & 0 & \lambda_{25} & 0 & \lambda_{27} \\ 0 & 0 & -\sum \lambda_{3j} & 0 & 0 & \lambda_{36} & \lambda_{37} \\ 0 & 0 & 0 & -\sum \lambda_{4j} & \lambda_{45} & 0 & \lambda_{47} \\ 0 & 0 & 0 & 0 & -\sum \lambda_{5j} & \lambda_{56} & \lambda_{57} \\ 0 & 0 & 0 & 0 & 0 & -\sum \lambda_{6j} & \lambda_{67} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}
 \end{aligned}$$

This structure allows the chain to migrate within “monopoly” regimes only, while the market is guarded by the patent (e.g “introduction” to “growth”). Patent risks such as the issuing of similar patents or attacks for invalidity can be easily incorporated in the evaluation by letting the rates to competitive states take positive values before the expiration of the proprietary claim, or by increasing the rates to the “bankruptcy” regime. After the patent has expired, the chain is allowed to migrate to “competitive” states (e.g. “growth” to “growth with competitive threat”). Furthermore,  $S_t$  can only migrate to successive phases of the product life cycle ( $\{1\}$  to  $\{2\}$ , and  $\{2\}$  to  $\{3\}$ ), and once state  $\{4\}$  is reached, we are left with the possibility of migrating to states  $\{5\}$  and  $\{6\}$  only, because once the market is open to competition, it is not possible to regress to the “monopoly” regime. The bankruptcy state is accessible from each other state. The initial marginal probability vector is  $P(S_0) = [1,0,0,0,0,0,0]^T$ , because the market for product XYZ is currently at the introduction phase.

### 3. Valuation of the proprietary claim

Let the instantaneous cash flow function be defined as:

$$f(Y_t, t | S_t = i) = (Y_t \rho(\theta_t, Y_t) - OC(Y_t) - FC(t)) I_{\{t \geq \tau, S_t \neq 7\}} \tag{11}$$

where  $\rho$ ,  $OC$  and  $FC$  denote respectively unit price, operating and fixed cost functions and

$$I_{\{t \geq \tau, S_t \neq 7\}} = \begin{cases} 1 & \text{if } t \geq \tau \text{ and } S_t \neq 7 \\ 0 & \text{otherwise} \end{cases}$$

denotes the indicator function. Equation (11) states that the cash flows produced by the investment are zero if the firm has not yet entered the market or the project reaches the “bankruptcy” state.

By (5), the time  $t$  present value of the project is:

$$\pi(Y_t, t) = E^{Y_t, t} \left[ \int_t^T f(Y_s, s | S_s) \exp(-\kappa(s - t)) ds \right]$$

To compute the above, we apply *Proposition 1*, with the additional constraint,  $\pi(Y_t, t) \geq K(t)$ . If, at any time, the salvage value of the equipment exceeds the project value, the firm will exercise the option to abandon the project, hence the salvage value becomes the lower bound for the present value of the investment. Note that the distribution of the Markov chain  $P(S_t = i)$  is computed with the generator matrix  $A^-$  up to the patent expiry  $T_k$  and with the generator matrix  $A^+$  thereafter. Thus,

$$P(S_t = i) = P(S_0) \exp(A^- t) I_{\{t \leq T_k\}} + P(S_{T_k}) \exp(A^+ t) I_{\{t > T_k\}} \tag{12}$$

where the marginal probability vector of the Markov chain at the patent’s expiry date  $P(S_{T_k})$  is computed using (4):

$$P(S_{T_k}) = P(S_0) \exp(A^- T_k)$$

Terminal and boundary conditions are respectively:

$$C(Y_T, T) = f(Y_T, T) \quad (13)$$

and

$$\begin{cases} C(Y_{\max}, t) = \left( \int_t^T f(Y_{\max}, s) e^{-\kappa(T-s)} ds \right) \\ C(0, t) = K(t) I_{\{t \geq \tau\}} \end{cases} \quad (14)$$

where  $Y_{\max}$  denotes company XYZ maximum production capacity. We assume that capacity is fixed throughout the project duration. The current net present value of the project is computed as:

$$NPV(\tau, T_k^*) = \sup_{t_n \in \Lambda} E^{y,0} [e^{-\kappa t_n} (\pi(Y_{t_n}, t_n) - X(t_n))^+] - \alpha(T_k^*) \quad (15)$$

where  $\tau \in \Lambda = \{t_1, \dots, t_n\}$  and  $T_k^*$  are patent cost, optimal entry time and patent duration, respectively. Expression (15) is intuitively clear. Since the stream of cash flows will begin only after the firm introduces the hybrid vehicle in the market, the current NPV of the project is the expectation, conditional upon today's information, of the optimal entry time NPV discounted to the present time minus the patent cost. We make the following three remarks. First, the NPV computed in (15) comprises the value of the options to defer production and to abandon the venture. Secondly, the project NPV (net of patent fees) has a lower boundary of zero, because negative NPV projects are not undertaken. Finally, (15) is calculated by choosing the maximum value of  $NPV(t_n, T_k)$ , with  $t_n \in \Lambda = \{t_1, \dots, t_n\}$  and  $T_k \in \{T_1, T_2, \dots, T_k\}$ , at each demand level.

The formulae for computing the value of the proprietary claim, the deferral option and the abandonment option are given below:

$$\begin{aligned} V_{pc} &= NPV(\tau, T_k^*) - NPV(0,0) \\ V_{do} &= NPV(\tau, T_k^*) - NPV(0, T_k^*) \\ V_{ao} &= NPV(\tau, T_k^*) - NPV(\tau, T_k^*)_{wao} \end{aligned} \quad (16)$$

where  $V_{pc}$ ,  $V_{do}$  and  $V_{ao}$  denote the value of the proprietary claim, the values of the deferral and abandonment options respectively, and  $NPV(\tau, T_k^*)_{wao}$  is computed without imposing the constraint  $\pi(Y_t, t) \geq K(t)$ .

#### 4. Numerical Implementation

In this section we present a numerical implementation of the example previously formulated. The explicit forms for demand and cost functions and the parameters of the market model are summarized in the following Exhibits 1, 2, and 3:

**Exhibit 1. Functional Forms for Product Launch Model**

Unit Cost	Operating	$OC(Y_t) = c_1 + c_2 Y_t$
Unit Price		$\rho(Y_t, \theta_t) = OC(Y_t)(1 + \theta_t)$

Salvage Value of the plant	$K(t) = Ke^{-\kappa t}$
Profit function	$f(Y_t, t   S_t = i) = (Y_t \rho - OC(Y_t) - FC(t)) I_{\{t \geq \tau, S_t \neq 7\}}$

We make the following explanatory comments:

- a. The marginal cost function reflects the standard assumption that marginal costs increase with quantity produced.
- b. The instantaneous cash-flow is proportional to the time step used in the calculations. In our example, we use a 20 day time-step, within which the unit demand can vary between zero and the maximum production capacity per time-step. Since we assume that production capacity is fixed, firm XYZ is not able to meet demand in excess of its maximum capacity level.

**Exhibit 2. Cost and Demand Model Parameters**

Cost Parameters (x \$ 1000)			Demand Parameters			Market states					
Launch Outlay	$X$	100,000	Inst. Demand Range (units/dt)	$Y$	0-3000						
Unit Variable Cost	$c_1$	8.5									
Marginal Cost Increase	$c_2$	0.001				1	2	3	4	5	6
Yearly Fixed Cost	$F$	25,000	Risk Adj. Drift Regimes	$\mu$	0	0.4	0.01	-0.05	0.15	-0.065	
Yearly Patent Fee	$\alpha$	1,500	Volatility Regimes	$\sigma$	0.35	0.2	0.15	0.35	0.2	0.15	
			Mark-up (%)	$\theta$	0.4	0.6	0.25	0.2	0.3	0.125	
			Time Parameters								
			Time Step Size	$dt$	20 days						
Initial Salvage Value	$K$	100,000	Project Useful Life(yrs)	$T$	7						
Depreciation Rate	$k$	0.5	Patent Durations (yrs)	$T_k$	0,3,5						
Risk Adjusted Rate	$\kappa$	0.2	Set of entry times (yrs)	$A$	{0,5;Tk}						

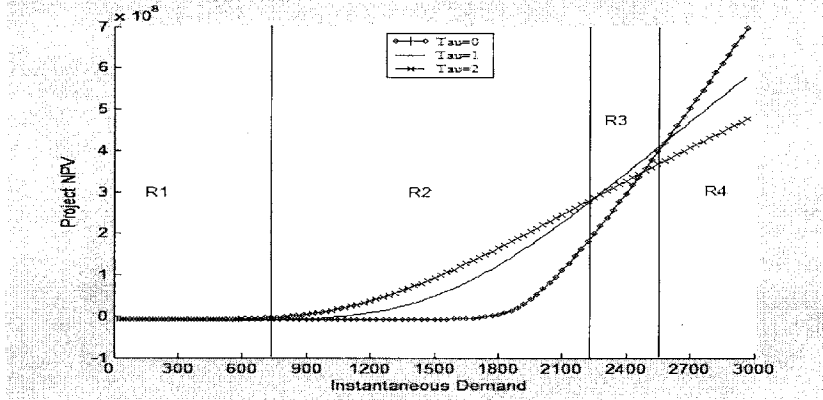
**Exhibit 3. Transition Rates between Market Regimes**

Before Patent Expiry								After Patent Expiry							
State	1	2	3	4	5	6	7	State	1	2	3	4	5	6	7
1	-0.31	0.3	0	0	0	0	0.01	1	-0.31	0.3	0.075	0	0	0	0.01
2	0	-0.085	0.075	0	0	0	0.01	2	0	-0.885	0.075	0	0.8	0	0.01
3	0	0	-0.15	0	0	0	0.15	3	0	0	-0.4	0	0	0.25	0.15
4	0	0	0	0	0	0	0	4	0	0	0	-0.31	0.3	0	0.01
5	0	0	0	0	0	0	0	5	0	0	0	0	-0.085	0.075	0.01
6	0	0	0	0	0	0	0	6	0	0	0	0	0	-0.15	0.15
7	0	0	0	0	0	0	0	7	0	0	0	0	0	0	0

In the following paragraphs, we analyze the interaction between current demand conditions and project NPV, paying special attention to the value of the options embedded in the patents on the project technology. Figure 2 illustrates the NPV of the project if investment were to occur today, 1 year from today, or 2 years from today, assuming that the life of the patent is five years. The figure shows four distinct exercise regions, *R1* through *R4*. If the current or instantaneous demand level is low enough to fall in region *R1*, it is not opportune to proceed with the investment now or at any time in the future. In regions *R2* and *R3*, the optimal entry times are years 2 and 1 respectively, whereas in region *R4*, it is best to proceed with the investment immediately. At moderately low demand levels, given by region *R2*, it makes sense to adopt a “wait and see” strategy for two years, whereas at moderately high demand levels, given by region *R3*, it makes sense to invest after a minimal delay of one year, since the cash flows foregone by deferring production greater than one year more than offset the benefits. At high

demand levels, given by region  $R4$ , it makes sense to go ahead with the project investment immediately.

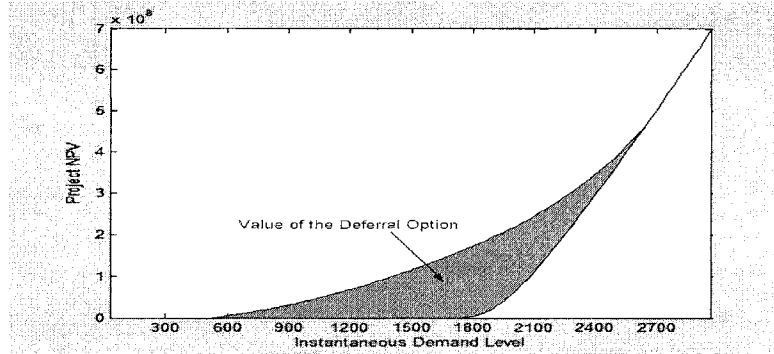
**Figure 2. NPV of a patent protected investment at different entry times**



### 5. The Value of the Deferral Option

Assume that after XYZ developed the hybrid car, it realized that it is still relatively expensive to manufacture the car with the new technology, and that, given these high manufacturing costs, the market for such a car is quite small right now. However, this does not mean that the rights to the new hybrid technology have no value. They have value since XYZ may be able to improve on this technology in the future, making the hybrid car manufacturing process more efficient and less expensive in years to come. Moreover, gasoline prices may increase dramatically in future, increasing the demand for fuel-efficient hybrid vehicles. In sum, the patent provides XYZ with the ability to delay the hybrid car project in the hope that technological and market conditions might change in the future, making the project more profitable. Hence the patent lends further value to the project. However, as is clear from Figure 3, the value of this deferral option is dynamic rather than fixed and is dependent on the current demand for the hybrid automobile. Figure 3 illustrates the value of the option to defer production, given that the optimal entry strategy is chosen in the set  $\Lambda$  (from Exhibit 2) and the patent duration is five years. The optimal exercise strategy is computed by choosing the entry time that maximizes NPV at each demand level. The deferral option value is represented by the width of the shaded region, representing the difference in NPVs for any given demand level, with and without the patent. Notice that the option to defer has a significant effect on the investment NPV at moderate current demand levels. This follows from the choice of parameters at the introduction phase of the product life cycle,  $\mu_1 = 0, \sigma_1 = .35$ , signifying zero growth and high demand uncertainty, in Exhibit 2. These market conditions obviously warrant a “wait and see” strategy at moderate demand levels. The negative NPV at very low demand levels makes it inappropriate to pursue the project at any time in the future, rendering moot the option to defer. Similarly, when demand is high enough to make an immediate profit, it is pointless to defer investment in the project, once again rendering ineffective the deferral option.

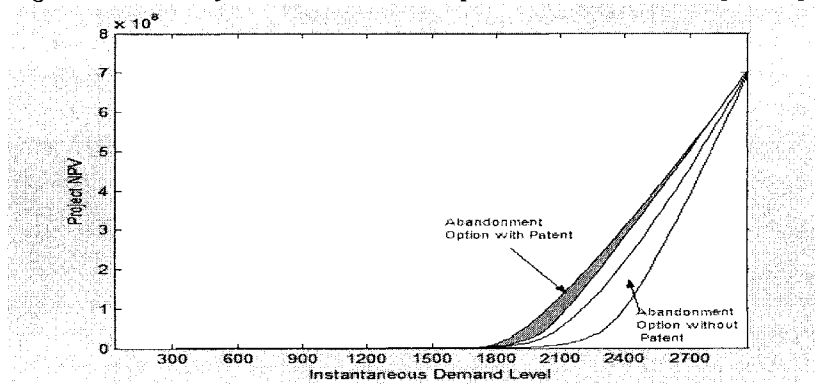
**Figure 3. Value of the Deferral Option**



**5. The Value of the Abandonment Option**

Let us now examine the option to abandon the project, given a set of adverse market conditions. Assume, for example, that the cost of gas has fallen dramatically a few years after the hybrid technology was introduced, while manufacturing costs remain high for the hybrid car. XYZ now wishes to abandon the hybrid car project and sell the technology, including any patent rights it might have acquired. In effect, the original hybrid car project might not be profitable, but the technology that has been developed could be valuable. This is categorized as the value of the abandonment option. Figure 4 illustrates the value of the option to abandon the project for salvage value in the presence of patent protection and without the benefit of patent protection. For sake of comparison, we assume that the investment is made today and the patent duration is five years. The value of the abandonment option is the difference in NPVs with and without the abandonment option, assuming patent protection exists and then assuming that we do not have the benefit of patent protection. This is represented by the width of the shaded areas at given demand levels.

**Figure 4. Value of the Abandonment Option with and without patent protection**



Observe that the option to abandon is worthless at low demand levels because under these conditions, the firm would either defer the investment or not invest at all. Similarly, the option to

abandon is negligible at high demand levels, given the high profitability of the project at high demand levels. The option to abandon is clearly more valuable in the absence of patent protection, when the absence of entry barriers increases the competitive uncertainty present in the market. Patent protection reduces competitive uncertainty and therefore reduces the value of the abandonment option. Although this result makes intuitive sense, it is not easily quantifiable using standard real options approaches because these do not take into account the effects of competitive interaction and product life cycle.

## **Conclusion**

Evaluating proprietary claims is a challenging problem, because their impact on the real investment is often unobservable and thus hard to quantify. In particular, the value of a patent is contingent on several factors concerning demand evolution and market structure. In this paper, we present an evaluation approach which addresses the fact that the value of the patent depends on the degree of demand and competitive uncertainty. Moreover, a sound approach must recognize that a patent creates a set of real options, and concurrently affects the value of other real options embedded in the investment. The main contribution of this paper is to formulate a general framework for pricing proprietary claims that allows us to model demand and competitive uncertainty as endogenous variables in the evaluation process and illustrates the interaction between the patent and managerial decision making. The impact of competitive and demand uncertainty is replicated by specifying a theoretical framework where the demand process evolves as a diffusion process modulated by a continuous time controlled Markov chain, and the market is allowed to migrate to regimes of different competitive intensity. A recursive algorithm, with stability and error order properties analogous to the implicit finite difference scheme, is developed for computing the net present value of the project. Finally, our model for evaluating patents is developed with the help of a hypothetical example which shows the interaction between a patent and managerial decision making. We show that the patent increases the value of the deferral option, especially at moderate demand levels. However, the existence of patent protection reduces the competitive uncertainty of the project, thereby reducing the value of the abandonment option inherent in the project.

## Appendix

**Proof of Proposition 1**

Let  $\pi(Y_T, T) = f(Y_T, T)$  be the instantaneous cash flow at the project's terminal date  $T$ . By (7),  $C(Y_{T-\delta t}, T - \delta t | S_t = i) = E^{Y_{T-\delta t}, T-\delta t} [e^{-\kappa(\delta t)} (f(Y_T, T)) | S_{T-\delta t} = i]$ , is the discounted, time  $T$  expected instantaneous cash flow. Thus

$$\begin{aligned} \pi(T - \delta t, Y_{T-\delta t} | S_{T-\delta t} = i) &= E^{Y_{T-\delta t}, T-\delta t} \left[ \int_{T-\delta t}^T f(Y_s, s | S_s) \exp(-\kappa(s - T + \delta t)) ds | S_{T-\delta t} = i \right] \\ &= E^{Y_{T-\delta t}, T-\delta t} [(f(Y_{T-\delta t}, T - \delta t | S_{T-\delta t}) + f(Y_T, T) \exp(-\kappa\delta t)) | S_{T-\delta t} = i] \\ &= f(Y_{T-\delta t}, T - \delta t | S_{T-\delta t} = i) + E^{Y_{T-\delta t}, T-\delta t} [f(Y_T, T) \exp(-\kappa\delta t) | S_{T-\delta t} = i] \\ &= f(Y_{T-\delta t}, T - \delta t | S_{T-\delta t} = i) + C(Y_{T-\delta t}, T - \delta t | S_{T-\delta t} = i) \end{aligned}$$

At this point  $\pi(T - \delta t, Y_{T-\delta t})$  is obtained by unconditioning over the state space  $\{0, 1, \dots, N\}$ .

$$\begin{aligned} \pi(T - \delta t, Y_{T-\delta t}) &= E[\pi(T - \delta t, Y_{T-\delta t} | S_{T-\delta t} = i)] \\ &= \sum_{i=0}^N (f(Y_{T-\delta t}, T - \delta t | S_{T-\delta t} = i) + C(Y_{T-\delta t}, T - \delta t | S_{T-\delta t} = i)) P(S_{T-\delta t} = i) \end{aligned}$$

where  $P(S_{T-\delta t} = i)$  denotes the probability that the Chain visits state  $i$  in the time interval  $[T - \delta t, T]$ .

Thus, (9) holds in the first step of the recursion. To show that it holds in general, it is enough to verify it at the next infinitesimal time-step.

Take  $\varphi(Y_{T-2\delta t}, T - 2\delta t) = \pi(Y_{T-2\delta t}, T - 2\delta t)$  as the terminal condition for equation (5), so that

$$\begin{aligned} C(Y_{T-2\delta t}, T - 2\delta t | S_{T-2\delta t} = i) &= E^{Y_{T-2\delta t}, T-2\delta t} [\pi(Y_{T-\delta t}, T - \delta t) | S_{T-2\delta t} = i] \\ \pi(T - 2\delta t, Y_{T-2\delta t} | S_{T-2\delta t} = i) &= E^{Y_{T-2\delta t}, T-2\delta t} \left[ \int_{T-2\delta t}^T f(Y_s, s | S_s) \exp(-\kappa(s - T + 2\delta t)) ds | S_{T-2\delta t} = i \right] \\ &= E^{Y_{T-2\delta t}, T-2\delta t} \left[ f(Y_{T-2\delta t}, T - 2\delta t | S_{T-2\delta t}) + \int_{T-\delta t}^T f(Y_s, s | S_s) \exp(-\kappa(s - T + \delta t)) ds | S_{T-2\delta t} = i \right] \\ &= f(Y_{T-2\delta t}, T - 2\delta t | S_{T-2\delta t} = i) + E^{Y_{T-2\delta t}, T-2\delta t} \left[ E^{Y_{T-\delta t}, T-\delta t} \left[ \int_{T-\delta t}^T f(Y_s, s | S_s) \exp(-\kappa(s - T + \delta t)) ds \right] | S_{T-2\delta t} = i \right] \\ &= f(Y_{T-2\delta t}, T - 2\delta t | S_{T-2\delta t} = i) + E^{Y_{T-2\delta t}, T-2\delta t} [\pi(Y_{T-\delta t}, T - \delta t) | S_{T-2\delta t} = i] \\ &= f(Y_{T-2\delta t}, T - 2\delta t | S_{T-2\delta t} = i) + C(Y_{T-2\delta t}, T - 2\delta t | S_{T-2\delta t} = i) \end{aligned}$$

Again, unconditioning over the state space of the Markov Chain

$$\begin{aligned} \pi(T - 2\delta t, Y_{T-2\delta t}) &= E[\pi(Y_{T-2\delta t}, T - 2\delta t | S_{T-2\delta t} = i)] \\ &= \sum_{i=1}^N (f(Y_{T-2\delta t}, T - 2\delta t) + C(Y_{T-2\delta t}, T - 2\delta t | S_{T-2\delta t} = i)) P(S_{T-2\delta t} = i) \end{aligned}$$

which is exactly equation (9).

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