Multivariate Stable Distributions and Value at Risk: The Case of the Asian Currency Crisis

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Abstract

This paper explores practical applications of multivariate stable distributions to value at risk modeling during the Asian currency crisis. We fit multivariate stable distributions to daily foreign exchange rate data 1996 through 1998 for six Asian currencies using a rolling estimation procedure and backtest daily marginal and conditional probabilities under 95% and 99% value at risk nulls. We also examine gains in value at risk accuracy from using multivariate stable distributions relative to univariate benchmarks such as generalized autoregressive conditional heteroskedasticity or univariate stable models. We find multivariate stable distributions overstate the probability of extreme losses.

Introduction

It is widely accepted that empirical unconditional foreign exchange return distributions are characterized by leptokurtosis and volatility clustering, questioning traditional mean-variance portfolio theory and currency risk management applications based on Gaussian assumptions. For example, delta-normal value at risk (VaR) models perform poorly when applied to financial returns data relative to generalized autoregressive conditional heteroskedasticity (GARCH) models.

A heavy-tailed alternative related to the normal is the stable distribution. Known as the Stable Paretian Hypothesis, (Mandelbrot 1963) asserted that empirical asset price distributions have infinite variance and are appropriately modeled by non-Gaussian members of the stable family of distributions. Other advocates of the application of stable distributions to modeling financial returns include (Fama 1963), (Roll 1970), and (McCulloch 1978).

This paper conducts three experiments backtesting multivariate stable distributions during the Asian currency crisis. Specifically, we test VaR coverage rates using multivariate stable marginal and conditional probabilities. We find multivariate stable distributions overstate the probability of extreme losses evaluated at 95% and 99% VaR.

Stable Paretian Distributions

Stable Paretian distributions are limiting distributions of normalized sums of i.i.d. variates, where the normal is the only finite variance case. (Lévy 1924) showed tails of non-Gaussian stable distributions asymptotically follow the law of Pareto, giving rise to the term "stable Paretian." In addition, stable distributions possess the "stability" property, i.e., they are closed under addition. Stable distributions offer risk managers an alternative measure of risk to

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standard deviation when the 2nd moment is undefined. This is the stable index, which determines the general shape of distribution tails.

Stable distributions are analyzed in terms of characteristic functions, i.e., their spectral representation, since stable density functions typically have no closed-form solution (notable exception is the normal). Stable distributions have four parameters: characteristic exponent or index of stability $\alpha \in (0, 2]$, skewness parameter β, scale parameter γ, and location parameter δ. The key parameter governing the rate of decline of tail mass is the index of stability α, i.e., $P[|X| > x] \sim cx^{-\alpha}$. An important property of stable laws is that α determines the number of unbounded moments, i.e., moments of order $r > \alpha$ do not exist. Accordingly, if $\alpha < 1$, distribution tails are of such large mass that moment-generating integrals are unbounded. For $\alpha < 2$, variance and covariance do not exist.

(Nolan 1998) denotes a d-dimension α-stable random vector with spectral measure Γ and shift vector μ^0 as $\mathbf{X} \sim S_{\alpha,d}(\Gamma,\mu^0)$, with characteristic function:

$$(1) \quad \phi_{\mathbf{X}}(\mathbf{t}) = \mathbf{E}\exp\{i < \mathbf{X},\mathbf{t} >\} = \exp\left(-I_{\mathbf{X}}(\mathbf{t}) + i < \mu^0, t >\right)$$

$$\mathbf{I}_{\mathbf{X}}(\mathbf{t}) = \int_{S_d} \psi_\alpha(< \mathbf{t},\mathbf{s} >)\Gamma(ds)$$

Here: $< \mathbf{t}, \mathbf{s} > = t_1 s_1 + \ldots + t_d s_d$ is the inner product, and

$$\psi_\alpha(u) = \begin{cases} |u|^\alpha \left(1 - i\,sign(u)\tan\frac{\pi\alpha}{2}\right) & \alpha \neq 1 \\ |u|\left(1 + i\frac{2}{\pi}sign(u)\ln|u|\right) & \alpha = 1 \end{cases}$$

Accordingly, the complex valued function $\mathbf{I}_{\mathbf{X}}(\mathbf{t})$ determines the specific distribution of $\mathbf{X}$. The key parameter is Γ which contains information about the dependence structure in $\mathbf{X}$. Unfortunately, the general case of multivariate stable distributions is beyond current computational capabilities. (Nolan 2006) has developed computational capabilities under additional assumptions about the nature of the spectral measure. In this paper we employ the elliptically contoured/sub-Gaussian stable distribution which has characteristic function:

$$(2) \quad E\exp(i < \mathbf{u},\mathbf{X} >) = \exp\left(-\left(\mathbf{u}^T\mathbf{R}\mathbf{u}\right)^{(\alpha/2)} + i < \mathbf{u},\boldsymbol{\delta} >\right)$$

where $\mathbf{R}$ is a d x d positive definite covariation matrix and δ is a d x 1 location vector.

The Asian Currency Crisis

Over 1997-1998 regional Asian currencies plunged in response to speculative attacks reacting to unsustainable economic fundamentals such as large trade deficits, massive dollar-denominated debt, and overvalued currencies pegged to the U.S. Speculative attacks began in Thailand and quickly spread over the region. Indeed, a notable characteristic of the crisis was its contagious nature, with speculators attacking several emerging market currencies beginning summer 1997. Accordingly, the Asian currency crisis represents an important period in history that may render valuable information about the performance of risk management models in times of widespread currency speculation.

Daily data 1/1/1996 through 12/31/1998 were obtained for six currencies: Indonesian Rupiah [IDR], Malaysian Ringgit [MYR], Philippines Pesos [PHP], South Korean Won [KRW], Thai Baht [THB], and the Taiwan Dollar [TWD][1]. Loss distributions were created by taking log first-difference of the daily exchange rate measured in European terms. Accordingly, under home currency preference, investors look to the positive tail of the distribution of losses in risk analysis.

Our loss distributions showed significant deviations from normality. Specifically, we rejected Gaussian nulls of zero skewness and zero excess kurtosis for all currencies in the crisis period. In addition, we rejected the null of no serial correlation in squared losses for every currency. We also rejected the joint null of normality for all six currencies using the Shapiro-Wilk multivariate normality test at the .1% level of significance. Accordingly, our data are consistent with the stylized facts of leptokurtosis and higher-order dependence.

Univariate Stable Index Estimates and Goodness-of-Fit Analysis

Before we fit any multivariate stable distributions, we first examined evidence our data lie in the domain of attraction of stable distributions. Estimates of stable parameters were obtained using John Nolan's STABLE 4.0 program for Windows[2]. Null hypotheses of interest are (1) finite variance: H_0: $\alpha = 2$ and (2) finite expectation: H_0: $\alpha > 1$ against non-finite alternatives H_A: $\alpha < 2$ and H_A: $\alpha \leq 1$. As reported in Table I, the average estimated stable index using MLE is 1.49 for the pre-crisis period and 1.05 for the crisis period, showing an increase in tail masses of loss distributions during the crisis. The finding of average stable indexes near one during the crisis period is notable as the average stable index for stocks tends to be close to 2 (Fama 1965). Using 95% confidence intervals, the finite variance null is rejected for all currencies over both sample periods, suggesting our data are in the domain of attraction of non-Gaussian stable laws. The null of finite expectation is rejected for IDR, MYR, PHP, and KRW in the crisis period.

(Nolan 1999) suggested procedures to determine whether data are consistent with stabile distributions including empirical density plots, "variance stabilized" p-p plots, "thinned" q-q plots, and log(likelihood) values relative to the normal. Using these goodness-of-fit tools, we found the stable model was appropriate for our data and it outperformed the normal.

Backtesting Value at Risk

Value at Risk (VaR) is a quantile-based measure of market risk exposure, interpreted as the maximum holding-period loss measured with a specific level of confidence. Specifically, VaR_p is the p-percentage quantile of a loss distribution, exceeded with probability $(1 - p)$, conditional on information at time t-1, Ω_{t-1}:

[1] Data were obtained from the Pacific Exchange Rate Service at http://pacific.commerce.ubc.ca/xr/.
[2] Documentation on STABLE 4.0 is available at www.RobustAnalysis.com.

(3) $VaR_{p,t} = F^{-1}_{t|\Omega_{t-1}}(p)$

Our backtesting methodology consists of one-day performance comparisons recommended by the 1988 Basel Accord amended to VaR coverage rates. Specifically, we conducted three backtesting experiments designed to explore how multivariate stable distributions perform in VaR analysis. Each experiment compares daily estimated coverage rates based on multivariate stable marginal and conditional probabilities with target coverage rates under nulls of 95% and 99% VaR using failure rate analysis in (Christoffersen 1998). Accordingly, if we are to employ multivariate stable distributions in risk management, we expect estimated marginal probabilities to equal their target frequencies under the relevant VaR null.

We employed (Nolan 2006) STABLE 4.0 to estimate multivariate stable distributions and calculate multivariate stable probabilities. First, we estimated the spectral parameter from sample data then a particular multivariate stable distribution was defined. This multivariate stable distribution was then used to calculate marginal or conditional probabilities used in our three experiments. This procedure was done on a rolling daily basis based on the previous 240 days for the forecast period 1997-1998. Accordingly, our approach produces daily multivariate stable distributions which are time varying, allowing the stable index parameter to change as the Asian crisis evolved.

Once we have fit a multivariate stable distribution to the data we then calculate marginal and conditional probabilities by simulating 100,000 random stable vectors each day using the specific multivariate distribution for that day. Specifically, we estimate marginal probabilities for a given currency using relative frequencies:

(4) $\hat{p} = P[X \le x_t] = \dfrac{\#\{i^{th}\, element \le x_t\}}{100{,}000}$

where x_t is the loss to currency i on a given day. The set of daily probabilities produced can be used to judge whether an exception would have occurred at the 95% or 99% level. We define an exception, I_t, on a given day as the event an estimated marginal probability inferred from a multivariate stable distribution exceeds its desired coverage rate. The conditional coverage null is $E[I_t|\Omega_{t-1}] = p$, where p is the desired coverage rate. For instance, an estimated marginal probability of .98 is recorded as an exception under the 95% VaR null, but not the 99% null.

Experiment #1: MARGINAL STABLE DISTRIBUTIONS

We fit a multivariate stable distribution to all six currencies and estimated marginal probabilities of daily losses for each currency. The null is H_0: p = .95 for 95% VaR and H_0: p = .99 for 99% VaR.

As reported in Table II, all estimated marginal probabilities significantly exceeded their expected values under the nulls of 95% and 99% VaR. For instance, for IDR, under the 95% null, estimated stable marginal p-values exceed the 95% level approximately 24% of the time.

The fact multivariate stable distributions overstate expected coverage rates suggest they may also overstate VaR at extreme quantiles, although this is not tested directly.

Experiment #2: BIVARIATE STABLE DISTRIBUTIONS

We next estimated bivariate stable distributions for all currency pairs and applied our testing methodology. Consider two currencies X and Y. Two cases were considered:

1. Marginal distribution for a currency, $P[X < x_t \mid -\infty < Y < +\infty]$
2. Conditional distribution for a currency, $P[X < x_t \mid Y = y_{t-1}]$

In our first case, we fit daily bivariate stable distributions and calculated daily marginal probabilities of the day's loss on a given currency and tested 95% and 99% VaR nulls.

Our second case computes daily conditional probabilities of the loss of a target currency given knowledge of the value of a hedging currency on the previous day: $P[X < x_t \mid Y = y_{t-1}]$ evaluated at 95% and 99% nulls. We used the 'dmvstable' subroutine in STABLE 4.0 to generate density values over the range [-R, +R], where R was the maximum absolute daily loss from the full sample of six currencies (R = 0.302). Simpson's algorithm for numerical integration was then used to calculate the conditional probability:

$$(5)\qquad P\left[X < x_t \middle| Y = y_{t-1}\right] = \frac{\int_{-R}^{x} pdf_y(s)ds}{\int_{-R}^{R} pdf_y(s)ds},$$

where $pdf_y(s)$ denotes the conditional probability distribution function.

Tables III and IV report failure rates based on estimated marginal and conditional probabilities estimated from bivariate stable distributions for all currency pairs. As reported in Tables III and IV, bivariate stable estimated marginal and conditional probabilities consistently overstate conditional coverage rates rejecting their conditional coverage nulls in most cases. Notable exceptions are TWD/MYR, KRW/MYR, TWD/PHP, TWD/THB and IDR/PHP, since their marginal distributions had reasonable failure rates for 99% VaR. Thus our results show bivariate stable distributions consistently overstate desired coverage rates both at 95% and 99% VaR. In addition, failure fates tend to be higher for conditional probabilities, suggest prior knowledge of cross-currency values increase the incidence of overstating coverage rates.

Experiment #3: BACKTESTING A PORTFOLIO

This experiment examined an equally-weighted portfolio of currencies using our testing procedures. Using our rolling daily estimation procedures, we estimated the probability of a portfolio loss by taking the average of each vector using simulation:

$$(6)\qquad \hat{p} = P[X \le x_t] = \frac{\#\{\text{average of vector} \le x_t\}}{100{,}000},$$

where x_t is the portfolio loss on a given day. We then tested these estimated multivariate stable probabilities against their desired values under the VaR nulls. We also examined the following benchmarks:

1. Univariate Stable
2. Normal
3. GARCH (1, 1)
4. Historical

In the first case, Nolan's STABLE 4.0 univariate stable estimation and quantile routines were used. Normal VaR calculations follow (Jorion 2001) and the GARCH case fits a GARCH(1, 1) using a rolling estimation procedure similar to that used in this paper. Historical VaR estimates were based on actual (sample) quantiles.

Table V reports backtesting results for estimated multivariate stable marginal probabilities for the equally-weighted portfolio for 95% and 99% VaR.

As reported in Table V we fail to reject VaR nulls for the GARCH (1, 1) model. Specifically, only the GARCH values are not significantly different from their expected values, taking into account coverage statistics. Our results are consistent with (Basterfield and Bundt 2006) in that GARCH (1, 1) performed well over both 95% and 99% VaR, whereas the univariate stable model is useful only at 99% VaR. Finally, the multivariate stable model performed the worst systematically overpredicting VaR coverage rates.

Conclusion

Given the contagious nature of the Asian crisis, gains in VaR forecasting accuracy may result from applications of multivariate methods relative to univariate procedures such as GARCH models. However, the applications tested in this paper were largely unsuccessful. The multivariate stable model performed quite poorly systematically over-predicting VaR coverage rates. Of particular interest is how poorly multivariate stable distributions perform relative to univariate stable models for 99% VaR!

A plausible explanation may be the restriction that all currencies have the same stable index, our estimation procedures, and other issues. Accordingly, future research must consider multivariate stable estimation procedures and stable index parameter restrictions, as well as information flows between equity and currency markets. Finally, comparing daily with weekly VaR as well as examining non-crisis historical data for equities remains a priority for future research.

Table I: Stable Index ML Estimates

	IDR	MYR	PHP	KRW	THB	TWD
Pre-Crisis Period 1/1/1996-12/31/1996	1.56	1.37	1.63	1.36	1.47	1.30
Crisis Period 1/1/1997-12/31/19989	0.85	0.67	0.60	0.81	1.12	0.99

Table II: Multivariate Stable Marginal Probability Failure Rates

	IDR	MYR	PHP	KRW	THB	TWD
95% VaR	23.6%	26.1%	25.3%	18.2%	23.8%	11.4%
99% VaR	14.8%	17.4%	21.6%	11.0%	12.8%	3.6%

Table III: Bivariate Stable 95% VaR Failure Rates

	THB TWD	THB PHP	THB KRW	THB MYR	THB IDR	TWD PHP
Marginal	13.0%	7.6%	11.6%	10.2%	12.8%	9.6%
Conditional	21.0%	13.4%	19.4%	18.8%	22.4%	13.4%

	TWD KRW	TWD MYR	TWD IDR	PHP KRW	PHP MYR	PHP IDR
Marginal	6.8%	7.2%	6.4%	21.6%	16.4%	17.8%
Conditional	8.2%	16.8%	14.0%	22.2%	24.8%	22.6%

	KRW MYR	KRW IDR	MYR IDR	TWD THB	PHP THB	PHP TWD
Marginal	12.6%	11.6%	14.0%	9.0%	17.0%	24.2%
Conditional	16.8%	15.4%	18.0%	15.0%	23.8%	34.5%

	KRW THB	KRW TWD	KRW PHP	MYR THB	MYR TWD	MYR PHP
Marginal	15.0%	10.6%	14.6%	15.0%	14.2%	9.2%
Conditional	20.8%	16.0%	15.6%	23.6%	25.3%	14.4%

	MYR KRW	IDR THB	IDR TWD	IDR PHP	IDR KRW	IDR MYR
Marginal	14.0%	17.4%	14.0%	10.0%	13.2%	13.0%
Conditional	19.8%	26.3%	23.4%	20.0%	22.2%	23.6%

Table IV: Bivariate Stable 99% VaR Failure Rates

	THB TWD	THB PHP	THB KRW	THB MYR	THB IDR	TWD PHP
Marginal	13.0%	7.6%	11.6%	10.2%	12.8%	9.6%
Conditional	21.0%	13.4%	19.4%	18.8%	22.4%	13.4%

	TWD KRW	TWD MYR	TWD IDR	PHP KRW	PHP MYR	PHP IDR
Marginal	6.8%	7.2%	6.4%	21.6%	16.4%	17.8%
Conditional	8.2%	16.8%	14.0%	22.2%	24.8%	22.6%

	KRW MYR	KRW IDR	MYR IDR	TWD THB	PHP THB	PHP TWD
Marginal	12.6%	11.6%	14.0%	9.0%	17.0%	24.2%
Conditional	16.8%	15.4%	18.0%	15.0%	23.8%	34.5%

	KRW THB	KRW TWD	KRW PHP	MYR THB	MYR TWD	MYR PHP
Marginal	15.0%	10.6%	14.6%	15.0%	14.2%	9.2%
Conditional	20.8%	16.0%	15.6%	23.6%	25.3%	14.4%

	MYR KRW	IDR THB	IDR TWD	IDR PHP	IDR KRW	IDR MYR
Marginal	14.0%	17.4%	14.0%	10.0%	13.2%	13.0%
Conditional	19.8%	26.3%	23.4%	20.0%	22.2%	23.6%

Table V: Portfolio Marginal Probability Failure Rates

	95% VaR	99% VaR
Multivariate Stable	27.7%	20.0%
Univariate Stable	12.2%	2.2%
Normal	15.2%	8.8%
GARCH (1, 1)	5.2%	1.8%
Historical	14.0%	5.8%

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