Conditional Market Risk-Return Relationship Revisited

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Introduction:

The return on the market portfolio plays a central role in the capital asset pricing model (CAPM), which stipulates a positive relationship between stock market risk and return. Such a positive risk-return tradeoff has been an important research topic in asset pricing. Although there is general agreement that investors, within a given time period, require a higher expected return from a security that is riskier, there is no such agreement about the relationship between risk and return across time.[1] The intertemporal relationship between risk and return has been examined by several authors, but the empirical findings on this relationship are conflicting. For example, French, Schwert and Stambaugh (1987), Baillie and DeGennaro (1990), and Glosten, Jagannathan and Runkle (1993) show that this relationship is positive, but insignificant. Bollerslev, Engle and Wooldridge (1988), Harvey (1989), Turner, Startz and Nelson (1989), Scruggs (1998), and Veronesi (1999) find a significant positive relationship between the expected market risk premium and conditional volatility of the market; Whereas Campbell (1987), Turner, Startz and Nelson (1989), Nelson (1991), Glosten, Jagannathan and Runkle (1993), Whitelaw (1994), and Glabadanidis and Scruggs (2003) report a significantly negative relationship. As a result, this intertemporal properties of the stock market return are not yet fully understood and still remain an irresistible puzzle.

Motivated by Merton's (1973) intertemporal capital asset pricing model (ICAPM), Scruggs (1998) investigates these conflicting results and shows that the negative relationship between the expected market risk premium (hereafter, MRP) and conditional market variance reported in previous studies is due to the omission of a state variable (interest rate) in the conditional market risk premium equation. By estimating a conditional two-factor model, which incorporates the interest rate in both conditional mean and variance equations, Scruggs (1998) restores the positive and significant relationship between the MRP and conditional market variance. However, a more recent paper by Glabadanidis and Scruggs (2003), who examine the same issue using the same model, reports a totally different result - a significantly negative relationship. They argue that the intertemporal relationships are very sensitive to the specification of conditional first and second moments, which contributes the conflicting results.[2] Because of theses two conflicting results, one has to ask whether a conditional two-factor model is necessary to restore the positive and significant relationship between the MRP and conditional market variance, which is the main question that this paper attempts to address.

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[1] Glosten, Jagannathan, and Runkle (1993) argue that in general rational risk-averse investors would require a relatively larger risk premium during times when the payoff from a security is more risky, but this large risk premium may not be required because time periods which are relatively more risky could coincide with time periods when investors are better able to bear particular types of risk.

[2] Although the major difference between Scruggs (1998) and Glabadanidis and Scruggs (2003) is that the former uses EGARCH model, and the later uses Kroner and Ng's (1998) asymmetric dynamic covariance (ADC) model, there is another difference between these two papers, which may also contribute the conflicting results. That is, the second factor (interest-rate risk) is not included in the conditional bond risk premium equation in the first paper, but it is included in both conditional stock and bond risk premium equations in the second paper.

The theoretical positive relationship between the MRP and conditional market variance supported by asset pricing models requires the positivity of the market risk in all states. Since all previous studies use realized returns to test the relationship between the expected MRP and conditional market variance, the implicit assumption being made is that the realized MRP is an unbiased estimate of the expected MRP. However, since the realized MRP may be negative in some states, we believe that previous studies do not explicitly test the relationship between the MRP and conditional market variance. The purpose of this paper is to reexamine the puzzling relationship between the expected MRP and conditional market variance found in previous studies. Unlike these studies, this paper presents a new test which recognizes the impact of using realized MRP as a proxy for expected MRP. This test allows up and down-market volatility to have distinct impacts on the MRP. We show that the estimates of the single factor market risk-return relationship may be biased downwards due to the use of the realized MRP as a proxy for the expected MRP. Therefore, not accounting for states where the realized MRP is negative leads to an aggregation bias resulting from the compensation effects of the positive and the negative MRP. We find evidence of a positive (negative) and significant relationship between the MRP and conditional market variance in bull (bear) market context. This result is robust to whether the market volatility is proxied by the conditional variance generated from a GARCH(1,1) model or by the implied volatility index from Chicago Board Option's Exchange.

The paper is organized as follows. The next section presents the state-dependent CAPM. The econometric model – asymmetric GARCH(1,1)-M is described in Section III. Section IV discussed the data used. The empirical results are reported in Section V. Some concluding remarks are reserved for Section VI.

State-dependent CAPM:

Under the conditional version of the Sharpe-Lintner CAPM, the model predicts that expected MRP is related to the conditional volatility of the market according to the following equation:

$$E[r_{m,t} \mid \Omega_{t-1}] = \lambda_m \sigma^2_{m,t} \tag{1}$$

where $r_{m,t}$ is the excess return on a market-wide portfolio or MRP at time t and Ω_{t-1} is market information known at time $t-1$; $\sigma^2_{m,t}$ is the conditional market variance; and λ_m is the price of risk. Since λ_m is also the coefficient of relative risk aversion, and therefore should be positive, equation (1) implies a positive and linear relation between the expected MRP and conditional market variance. The theoretical association led researchers to directly test for a positive relationship. However, as these tests use realized MRP as a proxy for the expected MRP, we argue that they do not explicitly test the conditional single-factor market risk-return relationship. In fact, the positivity of the expected MRP in all states is a necessary condition for equation (1); however, there are states where the realized MRP is negative. Therefore, the traditional tests of the conditional single-factor market risk-return relationship should account for the negative portion of the MRP distribution. To characterize this, assume that the economy is represented by two states of the nature, Ω_1 and Ω_2, characterizing up and down-market periods, respectively. Theses two states of nature are assumed to be independent with the respective occurrence probabilities $p_1 = p_1(\Omega_1)$ and $p_2 = 1 - p_1 = p_2(\Omega_1)$. In this case, the expected MRP can be written as:

$$E[r_{m,t} \mid \Omega_{t-1}] = p_1 E_{t-1}[r_{m,t} \mid \Omega_1] + p_2 E_{t-1}[r_{m,t} \mid \Omega_2] \tag{2}$$

and its conditional variance can be obtained as:

$$\sigma^2_{m,t} = p_1^2\sigma^2_{m1,t} + p_2\sigma^2_{m2,t} \quad (3)$$

where $\sigma^2_{m1,t}$ and $\sigma^2_{m2,t}$ are conditional market variances associated respectively to up and down-market periods. Using equation (3), equation (1) can be re-written as:

$$E[r_{m,t} \mid \Omega_{t-1}] = (p_1^2\lambda_m)\sigma^2_{m1,t} + (p_2^2\lambda_m)\sigma^2_{m2,t} \quad (4)$$

A testable version of the relationship (4) is therefore:

$$r_{m,t} = \lambda_0 + \lambda_{m1}D\hat{\sigma}^2_{m,t} + \lambda_{m2}(1-D)\hat{\sigma}^2_{m,t} + \varepsilon_{m,t} \quad (5)$$

where $D = 1$ if the MRP is positive (up market) and $D = 0$ if the MRP is negative (down market). Since λ_{m1} (λ_{m2}) is estimated during up (down) market periods, the expected sign of this coefficient is positive (negative).

Econometric Methodology:

The estimation of the state-dependent CAPM in equation (5) requires modeling the conditional volatility of the MRP. The ARCH models pioneered by Engle (1982) allow researchers to generate time-varying conditional volatility. Following Glosten, Jagannathan, and Runkle (1993), we estimate a GARCH(1,1)-M model with asymmetric volatility effect. According to their model, the conditional variance of MRP can be written as:

$$\hat{\sigma}^2_{m,t} = c + a\hat{\sigma}^2_{m,t-1} + b\varepsilon^2_{m,t-1} + d\eta^2_{m,t-1};\ \varepsilon_{m,t} \mid \Omega_{t-1} \sim N(0, \hat{\sigma}^2_{m,t}) \quad (6)$$

where $\eta_{m,t-1} = \varepsilon_{m,t-1}$ if $\varepsilon_{m,t-1} < 0$, 0 otherwise. Under the assumption of conditional normality, the log-likelihood to be maximized can be written as:

$$\ln L = \sum_{t=1}^{N} \frac{1}{2}[-\ln(2\pi) - \ln(\hat{\sigma}^2_{m,t}) - \frac{\varepsilon_{m,t}}{\hat{\sigma}^2_{m,t}}] \quad (7)$$

Since the normality assumption is often violated in financial time series, a quasi-maximum likelihood estimation (QML) proposed by Bollerselv and Wooldridge (1992) which allows inference in the presence of departures from conditional normality is used. Under standard regularity conditions, the QML estimator is consistent and asymptotically normal and statistical inferences can be carried out by computing robust Wald statistics. The QML estimates can be obtained by maximizing (7), and calculating a robust estimate of the covariance of the parameter estimates using the matrix of second derivatives and the average of the period-by-period outer products of the gradient. Optimization is performed using the Broyden, Fletcher, Goldfarb and Shanno (BFGS) algorithm, and the robust variance-covariance matrix of the estimated parameters is computed from the last BFGS iteration.

Data and Summary Statistics:

Datastream U.S total market return index (dividend included) is used as a proxy for the market returns. The 7-day Eurodollar deposit rate is used as risk-free rate to compute the excess MRP.[3] All the data are extracted from Datastream and cover the period from July 17, 1987 through March 07, 2003, which is a 817-data-point series. However, this paper works with rates of return; that leaves 816 observations expanding from July 24, 1987 to March 07, 2003.

Table 1 presents descriptive statistics of the continuously compounded excess U.S. MRP. As can be seen, the average MRP is 0.177% with a standard deviation of 2.298%. Table 1 also

[3] Because weekly data is examined in this paper, I use the 7-day Eurodollar deposit rate, which is also available from Datastream, as a proxy for the risk-free rate.

reports Bera-Jarque and Ljung-Box statistics. The Bera-Jarque test statistic strongly rejects the hypothesis of normally distributed returns. The Ljung-Box test statistics, which is defined as: $LB(k) = T(T+2)\sum_{j=1}^{k} \frac{\rho_j^2}{T-j}$ where ρ_j is the j^{th} lag autocorrelation, k is the number of autocorrelations, and T is the sample size (See Ljung and Box (1978)), for raw returns, $LB(24)$, is not significant, implying that the U.S. stock market is weak-form efficient. For squared returns, $LB^2(24)$ is significant, indicating strong nonlinear dependencies in MRP. This is consistent with the volatility clustering observed in most financial asset returns: Large (small) changes in prices tend to be followed by large (small) changes of either sign. The GARCH models used in this study are well known to capture this property.

Empirical Evidence:

The first column of Panel A in Table 2 contains the basic estimation result for the state-independent CAPM (equation (1)) using GARCH(1,1)-in-mean (GARCH-M) parameterization where the market risk is parameterized by the conditional variance specified in equation (6).[4] To check the robustness of the result, the second column of Panel A (GARCH-VIX) presents the estimation result for the same model (equation (1)) except the market risk in the mean equation is replaced by an implied volatility index (VIX) from Chicago Board Options Exchange, which provides an objective, observable, and dynamic measure of stock uncertainty. Panel B presents the estimation results of the state-dependent CAPM (equation (5)). Similar to Panel A, the first (second) column reports the result using conditional volatility (VIX) as a proxy for the market risk. Finally, Panel C and Panel D present the estimation results similar to Panel A and Panel B except the error terms are assumed to follow a conditional student t-distribution. As can be seen from Panel A, the market price of risk, λ_m, is negative and insignificant when the market risk is proxied by the conditional volatility. Similarly, the λ_m is also negative when the market risk is proxied by the VIX. These results are consistent with previous studies that the positive relation between MRP and market risk fails to hold according to the theory. However, when allowing for market-risk-return to be state-dependent, the results are very encouraging. For example, the market price of risk is significantly positive during the bull (up) market periods and negative during bear (down) market periods. These results are independent of which variable is used as a proxy for the market risk. To provide further robustness of the results presented in Panel A and Panel B, we assume a student-t distribution for the error terms, and the results are virtually unchanged. That is, assuming state-independent CAPM, the market price of risk is not significant (the first column of Panel C) or negative (the second column of Panel C). However, when the market price of risk depends on the states of nature, it is significantly positive during the bull market periods and negative during the bear market periods regardless of whether conditional volatility or VIX is used as a proxy for the market risk.

Next, consider the parameter estimates for the conditional variance process. As can be seen in the table, almost all the parameters in the conditional variance equations are statistically significant, and none of the Ljung-Box portmanteau statistics to test the null hypothesis of zero autocorrelation up to 24 lags in both the standardized residuals ($LB(24)$) and the standardized

[4] In estimating equation (5), we include a constant term λ_0 since Scruggs (1998) shows that constraining the regression line to pass by the origin may result in an overestimation of the coefficient λ_m.

residuals squared ($LB^2(24)$) is significant except one case where $LB^2(24)$ is significant for GARCH-VIX with normally distributed error terms in Panel B. These results indicate that the asymmetric GARCH(1,1)-M specification used in this study performs quite well in capturing the dynamics of the conditional first and second moments of the U.S. market risk premia.

Summary and Concluding Remarks:

This paper re-investigates the relationship between the market risk premium and market risk in an attempt to resolve the conflicting results reported by previous studies. Results from the tests of traditional state-independent CAPM reveal a negative/insignificant relationship between the MRP and market risk. However, once the state-independent assumption is relaxed, significant positive relationship between MRP and market risk is restored. This result is robust with respect to the proxy used in modeling the market risk and the distributional assumption of the error terms, suggesting that a conditional two-factor ICAPM is not necessary to restore the positive relationship between MRP and market risk.

Table 1: Descriptive statistics [a]

	R_m
Mean (%)	0.177
Std. Dev. (%)	2.298
Minimum (%)	-14.391
Maximum (%)	8.952
$B-J$	818.936**
$LB(24)$	30.608
$LB^2(24)$	116.045**

[a] (i) The statistics are based on weekly data from 07/24/1986 to 03/07/2002 (816 observations). (ii) The Bera-Jarque ($B-J$) tests normality based on both skewness and excess kurtosis and is distributed χ^2 with two degrees of freedom. (iii) $LB(24)$ and $LB^2(24)$ denote the Ljung-Box test statistics for up to the 24th order autocorrelation of the raw and squared returns, respectively. (iv) * and ** denote statistical significance at the 5% and 1% level, respectively.

Table 2: Estimation of the Conditional Relation Between Market Risk Premium and Market Volatility

	Panel A: State independent				Panel B: State dependent			
	GARCH-M [a]		GARCH-VIX [c]		GARCH-M [a]		GARCH-VIX [c]	
	Coefficient	Std	Coefficient	Std	Coefficient	Std	Coefficient	Std
β_0	0.232	(0.061)**	1.373	(0.176)**	0.224	(0.015)**	0.313	(0.047)**
λ_m or (λ_{m1})	-0.001	(0.015)	-0.064	(0.010)**	0.621	(0.013)**	0.067	(0.003)**
λ_{m2}					-0.808	(0.007)**	-0.088	(0.005)**
c	0.121	(0.018)**	0.052	(0.009)**	0.023	(0.001)**	1.010	(0.169)**
a	0.851	(0.005)**	0.889	(0.003)**	0.973	(0.000)**	0.020	(0.082)
b	0.075	(0.006)**	0.071	(0.003)**	0.037	(0.000)**	0.394	(0.137)**
d	0.111	(0.012)**	0.080	(0.009)**	-0.037	(0.000)**	0.318	(0.326)
$LB(24)$	33.888		35.787		23.347		26.284	
$LB^2(24)$	19.522		17.677		36.299		99.439**	
log-LIK	-997.828		-974.423		-683.133		-602.398	

	Panel C: State independent				Panel D: State dependent			
	GARCH-M-t [b]		GARCH-VIX-t [c]		GARCH-M-t [b]		GARCH-VIX-t [c]	
	Coefficient	Std	Coefficient	Std	Coefficient	Std	Coefficient	Std
β_0	0.265	(0.106)*	1.337	(0.181)**	0.742	(0.158)**	0.425	(0.100)**
λ_m or (λ_{m1})	0.003	(0.026)	-0.060	(0.009)**	0.286	(0.084)**	0.052	(0.006)**
λ_{m2}					-1.002	(0.159)**	-0.086	(0.005)**
c	0.129	(0.053)*	0.057	(0.039)	1.367	(0.230)**	0.014	(0.008)
a	0.871	(0.030)**	0.899	(0.026)**	0.272	(0.068)**	0.957	(0.013)**
b	0.044	(0.033)	0.059	(0.025)*	0.062	(0.019)**	0.036	(0.014)**
d	0.111	(0.042)**	0.071	(0.034)*	0.081	(0.027)**	-0.006	(0.018)
v	9.469	(2.270)**	10.366	(2.981)**	3.404	(0.413)**	4.960	(0.734)**
$LB(24)$	33.962		34.624		34.176		19.249	
$LB^2(24)$	17.81		17.226		35.074		13.634	
log-LIK	-1268.235		-1248.466		-890.979		-816.259	

[a] State independent: $r_{m,t} = \lambda_0 + \lambda_m h_{m,t} + \varepsilon_{m,t}$; $\varepsilon_{m,t} \mid \Omega_{t-1} \sim N(0, h_t)$

$$h_t = c + ah_{t-1} + b\varepsilon^2_{m,t-1} + d\eta^2_{m,t-1}$$

State dependent: $r_{m,t} = \lambda_0 + \lambda_{m1} D\hat{\sigma}^2_{m,t} + \lambda_{m2}(1-D)\hat{\sigma}^2_{m,t} + \varepsilon_{m,t}$; $\varepsilon_{m,t} \mid \Omega_{t-1} \sim N(0, h_t)$

$$h_t = c + ah_{t-1} + b\varepsilon^2_{m,t-1} + d\eta^2_{m,t-1}$$

[b] State independent: $r_{m,t} = \lambda_0 + \lambda_m h_{m,t} + \varepsilon_{m,t}$; $\varepsilon_{m,t} \mid \Omega_{t-1} \sim Student\text{-}t(0, h_t, v)$

$$h_t = c + ah_{t-1} + b\varepsilon^2_{m,t-1} + d\eta^2_{m,t-1}$$

State dependent: $r_{m,t} = \lambda_0 + \lambda_{m1} D\hat{\sigma}^2_{m,t} + \lambda_{m2}(1-D)\hat{\sigma}^2_{m,t} + \varepsilon_{m,t}$; $\varepsilon_{m,t} \mid \Omega_{t-1} \sim Student\text{-}t(0, h_t, v)$

$$h_t = c + ah_{t-1} + b\varepsilon^2_{m,t-1} + d\eta^2_{m,t-1}$$

where $\eta_{m,t-1} = \varepsilon_{m,t-1}$ if $\varepsilon_{m,t-1} < 0$, 0 otherwise; $D = 1$, if $r_{m,t} > 0$, 0 otherwise

[c] In the mean equation, h_m is replaced by the implied volatility index (VIX).

Standard errors are given in parentheses. $LB(24)$ and $LB^2(24)$ are the Ljung-Box test statistics of order 24 For serial correlation in the standardized residuals and standardized residuals squared, respectively. * and ** denote statistical significance at the 5% and 1% level, respectively.

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