# Development of Optimal Stock Portfolio Selection Model in the Tehran Stock Exchange by Employing Markowitz Mean-Semivariance Model

Soheila Sadeghi, Taimoor Marjani, Ali Hassani, and Jose Moreno

**Abstract**

In an increasingly complex financial market, selecting the optimal stock portfolio has become a subject of intense debate. This study aims to develop a model for optimal stock portfolio selection. We apply Markowitz's mean-semivariance approach to determine the downside risk of portfolios, which reflects investors' intuitive perception of risk. In the first stage, the combination of the Analytic Hierarchy Process (AHP) and Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) with interval data is employed to identify and rank good quality stocks according to the recommended criteria by experts. After selecting qualified stocks, in the second phase, we create portfolios, and the weight invested in each stock is determined. Then, three portfolios are created for three groups of risk-averse, neutral to risk, and risk-taker investors. The mean-semivariance optimization model is used in this phase. The proposed approach in the paper is implemented in a real case study of the Tehran stock exchange (TSE). Three portfolios for three groups of investors were evaluated and compared to the market performance using sharp criteria. All three portfolios outperformed the market portfolio both in terms of risk and return. The proposed model of this study can be utilized as a decision support tool when forming an optimal stock portfolio by considering both experts' opinions on stock evaluation and investor risk preferences simultaneously.

*Keywords:* Stock Portfolio, Interval TOPSIS, Optimization, Markowitz mean-semivariance model

## I. Introduction

In an increasingly complex financial market, selecting the optimal stock portfolio has become a subject of intense debate. People around the world make investment decisions every day. It is often difficult to make such decisions due to the dynamic nature of capital markets. In developing countries, deciding about investing in financial markets, assessing portfolios, and selecting portfolios have become widespread and significant issues. Thus, portfolio selection needs to be studied in more detail (Li et al., 2017; Maykao & Yanpiranat, 2012; Wu et al., 2019). Iranian capital markets are more uncertain and riskier than other investment avenues due in part to the complex environment of global markets, rapid technological progress and globalization, and the convergence of industries. Due to the fact that investing can have a significant impact on a person's life, sophisticated mechanisms are usually required to guide the selection of a portfolio.

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## II. Literature review

An optimal investment portfolio selection is the cornerstone of any investment project. What investors hope to achieve through portfolio optimization is to maximize portfolio return and minimize portfolio risk. Harry Markowitz and his groundbreaking research in 1952 formulated a revolutionary portfolio risk-return model known as mean-variance optimization. Since that time, it has been extensively used to determine optimal portfolio allocation. In their study, Chen et al. (2017) proposed a multi-objective uncertain portfolio optimization model for maximizing expected return while minimizing the risk associated with removing skewness in portfolio returns. They developed the model in order to maximize expected return coupled with skewness and therefore minimize risk and increase returns. In the study by Sun et al. (2016), an optimal analytical solution was developed to overcome the problem of choosing multiperiod portfolios. Mehlawat (2016) developed two multi-objective multi-period fuzzy portfolio selection models based on credibility, in which a multi-choice goal programming approach was used to solve the problem. Zhang and Liu (2015) presented a novel genetic algorithm aiming to deal with a multi-period fuzzy portfolio selection problem based on ranking parameters as well as returns. Zhang and Li (2021) considered risky asset returns uncertain variables that experts estimated. By considering the return, risk, liquidity, and diversification degree of portfolios, they used a mean-variance-entropy model for uncertain portfolio optimization. Their model quantified the investment return as uncertain expected value, the investment risk as uncertain variance, and the diversification level of a portfolio by entropy.

Even though the Markowitz Mean-Variance framework is theoretically sound, there has always been criticism of how risk is measured in it (Michaud & Michaud, 2008; Zhang et al., 2018). The risk in Markowitz's mean-variance framework is measured in terms of the variance of expected portfolio returns. The underlying assumption of using variance as the appropriate measure for risk is that investors weigh the probability of negative returns equally against the probability of positive returns (Boasson et al., 2011; Grootveld & Hallerbach, 1999). Variance treats the favorable upside dispersion of investment returns over the mean as part of the risk and penalizes it as much as the unfavorable downside deviations from the mean; therefore, it is an inappropriate measure of risk (Leland, 1999). An incorrect risk measure makes it possible to construct an efficient frontier that yields a nonsensical result when optimizing a portfolio (Lam, 2016).

Markowitz advocated the use of semivariance rather than variance as a measure of risk in 1959 since semivariance was perceived to reflect downside losses rather than upside gains as a risk factor. In his view, since investors are more concerned with downside risks than overall volatility, assessing risk by semivariance produces more efficient portfolios. Eftekhari and Satchell (1996) defined semivariance as the weighted sum of square deviations from a certain threshold, considering only values below the expected value of returns (see section B for more detail). In this paper, the mean-semivariance model is employed in order to construct efficient portfolios.

We should also consider that Markowitz's portfolio theory only provides a solution to an optimization problem among predetermined stocks regardless of the quality of stocks. Hence, it is not recommended to use portfolio theory blindly to assess the value of low-quality stocks without first determining the quality of such stocks. In order to build a portfolio that is as valuable as possible, it is best to select a few high-quality stocks first and then use optimization techniques to develop the portfolio. Although it appears simple initially, the process of choosing stocks is very complex because there are so many different objectives to take into consideration, sometimes with conflicting objectives (Hilborn & Walters, 2013). The key question here is how to rank the stocks based on the factors that affect their performance. In order to make sound decisions about portfolio selections, it is important to consult experts and review financial reports (Zhang et al., 2018). It is, however, extremely difficult to choose the optimal alternative when there are a large number of possible solutions to a given problem. A problem may be exacerbated in instances where some criteria have an effect on certain problems, but in order to reach a satisfactory solution, the alternatives must all be based on the same criteria, which inevitably leads to a more informed and better decision. Multi-Criteria Decision Making (MCDM) pertains to structure and solving decision and planning problems involving multiple criteria (Aruldoss et al., 2013). The MCDM process evaluates alternatives by calculating weights for each criterion and then ranking them based on the weight of the criteria (Zeleny & Cochrane, 1973).

In recent decades, MCDM methods have been increasingly popular and employed across a wide spectrum of applications (Chakraborty & Chakraborty, 2021; Malik et al., 2021; Popović, 2021; Tan et al., 2021; Velasquez & Hester, 2013). In particular, MCDM has been employed in the ranking and selection of stocks. Jamei (2020) applied the MCDM model to pharmaceutical firms listed on the Tehran Stock Exchange (TSE). In order to rank the firms, he utilized the SAW and TOPSIS MCDM models. Ece and Oguzhan (2017) conducted Fuzzy TOPSIS to determine the optimal portfolio, indicating that the portfolio had positive returns, risks, as well as favorable performance. In Galankashi et al. 's (2020) study, the criteria were derived by using a Likert-type questionnaire, and the FANP was used to rank the TSE companies.

The TOPSIS technique employed in this study is considered to be one of the most popular MCDM approaches due to its simplicity, consistency, and reliability (Al-Aomar, 2010; Formisano et al., 2017; Ince et al., 2017; Roshandel et al., 2013; Sotoudeh-Anvari & Sadi-Nezhad, 2015). Furthermore, a rigorous evaluation of several MCDM methods showed that combining these methodologies eliminates some of the weaknesses of specific MCDM methods (Velasquez & Hester, 2013), in addition to providing a novel approach to decision making. This paper aims to address these issues by combining two MCDM methods, namely AHP and Interval TOPSIS, to produce a reliable ranking of companies.

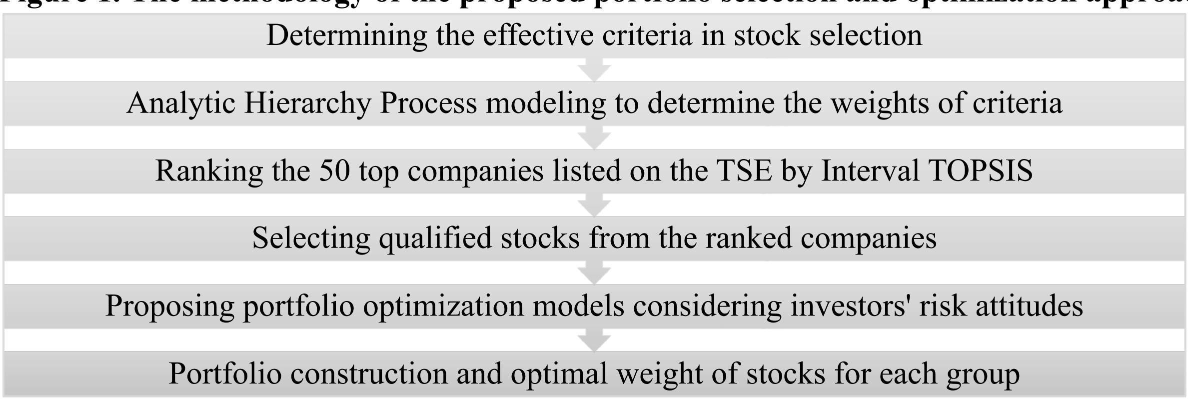
We also took into consideration in our proposed approach to portfolio construction the fact that different investors have different levels of risk tolerance. Different investors may see events very differently. For example, aggressive investors may be inclined to take risks, while conservative investors focus on the security of investment returns and the protection of their principal. A portfolio's optimal composition is determined by the investor's risk and return preferences. Taking into account the fact that investment returns vary with risk, investors need to find a balance between risk and return. In general, there is no single optimal portfolio for all investors. Thus, different decision-makers possessing different risk attitudes will require different portfolios (Deng et al., 2021; Makui & Mohammadi, 2019l Mansini et al., 2015). Investors in finance can be classified into three broad categories: risk-averse investors, risk seekers, and risk-neutral investors (Setiawan et al., 2022; Singh et al., 2020). Due to this, we construct three different portfolio models for each investment group.

In this paper, we present a robust and applicable optimal portfolio construction model using interval data in the TOPSIS matrix as opposed to definite numbers for ranking stocks, using semivariance instead of mean-variance as the risk factor, and presenting customized portfolios for different risk inclinations. In the first stage, the integration of AHP with Interval TOPSIS is used to identify and rank good quality stocks in accordance with the criteria recommended by experts. After selecting qualified stocks, we then proceed to create stock portfolios. Three portfolios are created for different investor types: risk-averse, risk-neutral, and risk-takers. The mean-semivariance is used in this step of the optimization process. Through these steps, the proposed model is applied to a real-life case study of the Tehran Stock Exchange (TSE). We will utilize sharp criteria to evaluate the performance of the portfolios in comparison with the performance of the stock market. The model developed in this study can be used in the selection of an optimal stock portfolio that takes both expert opinions in stock selection and investors' risk preferences into consideration simultaneously.

## III. Methodology

The developed model of this study consists of a few steps. First, we identify quantitative and qualitative criteria in stock selection and create a hierarchical structure of those criteria. After analyzing the questionnaire, we select the final criteria. Using AHP, we assign weights to the criteria. Then, a decision matrix is developed, and the top 50 companies will be ranked by interval TOPSIS. We then provide mathematical models of linear programming for optimizing the stock portfolio for each group of investors based on their level of risk-taking. To optimize the selected stock portfolio, the Colonial Competitive Algorithm (CCA) in Matrix Laboratory (MATLAB) will be used. The monthly returns and risks of companies are used as input data, and MATLAB determines the weight of each stock in a portfolio. The methodology is shown in Figure 1:

**Figure 1. The methodology of the proposed portfolio selection and optimization approach**




### A. Multi-Criterion Decision-Making

MCDM is a sub-discipline of operations research (OR) and a method of making complex decisions involving multiple criteria (quantitative and qualitative, conflicting criteria, etc.) (Xidonas et al., 2011). MCDM assists in evaluating and ranking top companies on TSE by considering important financial and non-financial criteria in uncertainty and determining the weight of criteria.

We analyzed and extracted factors affecting stock portfolio selection and developed a hierarchical chart of criteria and sub-criteria based on literature reviews and expert surveys. Afterward, AHP will be utilized to specify the importance of qualitative and quantitative criteria. In the following step, an interval TOPSIS will be employed to evaluate the stocks.

**A.1. Analytic Hierarchy Process modeling**

AHP is a methodology to determine the weight and importance of different criteria in a given situation, and it begins by providing a hierarchical tree (Mehregan, 2004). After collecting pairwise comparison matrices, weights were calculated by using the AHP technique and the Expert Choice software.

**A.2. TOPSIS technique**

In order to apply the TOPSIS method, information related to the decision-making process should be presented in the form of a decision matrix, as shown in Equation (1):

$$X = \left[X_{ij}\right]_{m*n} = \begin{array}{c} \\ A_1 \\ A_2 \\ \cdot \\ \cdot \\ \cdot \\ A_m \end{array} \begin{array}{c} \begin{matrix} C_1 & C_2 & \cdot & \cdot & \cdot & C_n \end{matrix} \\ \begin{bmatrix} X_{11} & X_{12} & \cdot & \cdot & \cdot & X_{1n} \\ X_{21} & X_{22} & \cdot & \cdot & \cdot & X_{2n} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ X_{m1} & X_{m2} & \cdot & \cdot & \cdot & X_{mn} \end{bmatrix} \end{array} \tag{1}$$

Where $A_i$ is the alternatives that experts should evaluate, $C_j$ is the decision criteria, and $X_{ij}$ is the values for each option.

**A.3. Interval TOPSIS technique**

The interval TOPSIS method is a new extension of the classical multicriteria TOPSIS method. In a world where the values of the criteria for evaluating each alternative can be characterized by uncertainty, interval analysis can prove to be a powerful tool for dealing with complex decision problems. Using interval analysis to introduce data uncertainty is a simple and intuitive way to deal with complex decision problems and can be used for a wide range of practical applications (Give, 2002; Jahanshahloo et al., 2009; Wentao & Han; 2010). In this model, unlike the classic TOPSIS model, interval data were used instead of definite and accurate numbers, which might be misleading in the real world and cannot provide plausible explanations. Additionally, under certain conditions, it is difficult to determine with certainty the values of matrix elements; therefore, values might be in the range (minimum-maximum) of the decision matrix.

An overview of the interval data is presented in Equation (2).

$$\left[a^{L}{}_{ij}, b_{ij}^{U}\right] \ , \forall i = 1, 2, 3, \ldots m, \ j = 1, 2, 3, \ldots n \tag{2}$$

In this regard, $a_{ij}^{L}$ represents the low limit or the minimum amount, and the $b_{ij}^{U}$ is for the high limit or the maximum amount available for the variable. The steps for using the interval TOPSIS method are as follows:

**Step 1:** Construct a decision matrix

$$X_{ij} = \begin{bmatrix} [a_{11} \quad ,b_{11}] & [a_{12} \quad ,b_{12}] \ldots & [a_{1m} \quad ,b_{1m}] \\ [a_{21} \quad ,b_{21}] & [a_{22} \quad ,b_{22}] \ldots & [a_{2m} \quad ,b_{2m}] \\ \vdots & \vdots & \vdots \end{bmatrix} \tag{3}$$

**Step 2:** Weighting the criteria: In this step weights assign to the criteria based on the weighting method or the decision maker's experience or discretion. It should be noted that the weights of the criteria are definite and accurate. The linear matrix of the weights of the criteria can be seen in Equation (4):

$$W = [W_1 \quad W_2 \; . \; . \; . \; W_n] \tag{4}$$

**Step 3:** Construct a normalized decision matrix in Equation (5):

$$r_{ij}^{l} = \frac{a_{ij}}{\sqrt{\sum a_{ij}^2 + \sum_{i=1}^{n} b_{ij}^2}} \;;$$

$$r_{ij}^{u} = \frac{b_{ij}}{\sqrt{\sum a_{ij}^2 + \sum_{i=1}^{n} b_{ij}^2}} \tag{5}$$

$$v_{ij}^{l} = w_i r_{ij}^{l} \;;$$

$$v_{ij}^{u} = w_i r_{ij}^{u};$$

Where $r_{ij}^{l}$ and $r_{ij}^{u}$ is the normalized values for the lower and upper limits of the interval data, and $w_i$ is the weight value of i-th criterion obtained from the weighting methods.

**Step 4:** Determining the ideal solutions of positive and negative intervals for each option. Equations (6) and (7) are used to find positive ideal solutions, and Equations (8) and (9) are used to find negative ideal solutions.

$$A_k^{+u} = \{(v_1^{+u}, v_2^{+u}, \dots, v_n^{+u})\} = \{(\max v_{ij}^u | i \in O), (\min v_{ij}^u | i \in I)\} \quad (6)$$

$$A_k^{+l} = \{(v_1^{+l}, v_2^{+l}, \dots, v_n^{+l})\} = \{(\max v_{ij}^l | i \in O), (\min v_{ij}^l | i \in I)\} \quad (7)$$

$$A_k^{-u} = \{(v_1^{-u}, v_2^{-u}, \dots, v_n^{-u})\} = \{(\min v_{ij}^u | i \in O), (\max v_{ij}^u | i \in I)\} \quad (8)$$

$$A_k^{-l} = \{(v_1^{-l}, v_2^{-l}, \dots, v_n^{-l})\} = \{(\min v_{ij}^l | i \in O), (\max v_{ij}^l | i \in I)\} \quad (9)$$

Where O and I are the profit and cost criteria, respectively.

**Step 5:** Calculate the positive and negative distances of the options using the Euclidean distance of each of the options from the ideal positive and negative solutions according to the Equations (10)-(13).

$$d_k^{+u} = \sqrt{\sum_{i\in I}(v_i^{+u} - v_{ij}^u)^2 + \sum_{i\in O}(v_i^{+u} - v_{ij}^l)^2} \quad (10)$$

$$d_k^{+l} = \sqrt{\sum_{i\in I}(v_i^{+l} - v_{ij}^l)^2 + \sum_{i\in O}(v_i^{+l} - v_{ij}^u)^2} \quad (11)$$

$$d_k^{-u} = \sqrt{\sum_{i\in I}(v_i^{-u} - v_{ij}^l)^2 + \sum_{i\in O}(v_i^{-u} - v_{ij}^u)^2} \quad (12)$$

$$d_k^{-l} = \sqrt{\sum_{i\in I}(v_i^{-l} - v_{ij}^u)^2 + \sum_{i\in O}(v_i^{+l} - v_{ij}^l)^2} \quad (13)$$

$$where\ d_k^{+1} \le d_k^{+u}\ and\ d_k^{-1} \le d_k^{-u}$$

**Step 6:** Calculating interval efficiency as follow:

$$\frac{d_k^{-l}}{d_k^{-u} + d_k^{+u}} \le C_i \le \frac{d_k^{-u}}{d_k^{-l} + d_k^{+l}} \quad (14)$$

**Step 7:** The next step is calculating the mid-point (m(E)) and half-width (HW(E)) of interval efficiency using equations (15) and (16):

$$m(E) = \frac{1}{2}(e^l + e^u) \quad (15)$$

$$HW(E) = \frac{1}{2}(e^u - e^l) \quad (16)$$

**Step 8:** We are now going to rank the alternatives in order to find the best alternative using the mid-point values from Equation (15). When the midpoints were equal, then one needed to calculate the half-width values (Equation (16)) and determine the rank accordingly. If the midpoints are not equal, the mid-point values will be used as the basis for comparison and ranking (Jahanshahloo et al., 2009). Interval data were used only for quantitative data in this study.

### B. Optimization and development of a model for optimal stock portfolio selection

In 1952, Markowitz showed that portfolios could be accurately defined through the use of optimization programs and by adjusting weights when sufficient information was available. Basically, optimization aims to improve performance in order to achieve the optimum point or points. There are two parts to this definition: 1- Search for ways to improve, and 2- Reaching the optimal point. Optimization can be defined as seeking an optimal solution to a problem.

Markowitz's mean-variance basic model does not take into account some important factors. In this study, we also applied behavioral biases (risk-taking level) to investment decision-making.

### B.1. Portfolio optimization and Markowitz's mean-semivariance optimization model

The modern theory of portfolio defines risk as the variability of total return around the average return, which is calculated based on variance, and is then classified as a symmetric risk criterion. Positive fluctuations, however, are more appealing to investors with short-term objectives, while negative fluctuations are deemed risky.

In 1959, Markowitz advocated the use of semivariance as a measure of risk rather than the variance in Post-modern portfolio theory. According to him, semi-variance-based analyses produce better stock portfolios than those that are based on variance. The assumption is that the target rate of return should be interpreted as the Minimum Required Return (MRR), which indicates the minimum rate of return that must be achieved in order to avoid losing certain important financial goals. According to Markowitz, there are two methods for calculating downside risk:

Method 1: The semivariance method, which is obtained from the average of the squared deviations (deviations less than the average rate of return) around the mean rate of return (below-mean semivariance)

Method 2: Semivariance is determined by the sum of undesirable deviations (deviations less than the target rate of return) relative to the target rate of return (below-target semivariance).

Equation (17) shows this point:

$$\sum\nolimits_{\rho B}^{2} = \frac{1}{T}\sum_{t=1}^{T} Min\ [(R_{\rho t} - B), 0]^{2} \qquad (17)$$

Where $T$ is the number of asset returns *R (return)* and a benchmark *B (expected return)* set by the investor, B is the average rate of return or target rate, $R_{\rho t}$ stands for the portfolio returns, and $\sum^2{}_{\rho B}$ is portfolio semivariance.

Portfolio risk is calculated through Equation (18) (Estrada, 2008).

$$\sum\nolimits_{\rho B}^{2} = \sum\nolimits_{i=1}^{n}\sum\nolimits_{j=1}^{n} X_i X_j S_{ijB}, \qquad (18)$$

$$S_{ijB} = (1/T).\sum\nolimits_{t=1}^{n}(R_{it} - B)(R_{jt} - B)$$

Where $X_i$ and $X_j$ denote the proportions of the portfolio invested in assets i and j, respectively, with returns Rit and Rjt for the expected assets, and n is the number of assets in the portfolio.

We used Markowitz's model to determine the optimal allocation of investors' capital to different stocks, which is based on two important parameters, risk and expected returns, with semivariance as the primary risk factor. A complex but realistic mathematical model in Equation (19) is formed in order to reflect the characteristics of the real stock market and consider amateurs' risk-taking levels:

$$Min\ ((1-\lambda)*R_p-\lambda*Var\ (R_p)) \tag{19}$$

In this pattern, $R_p$ is portfolio return, $Var\ (R_p)$ is portfolio risk, and λ is a parameter between 0 and 1, so that by $\lambda = 0$, the total value of the weighting factor is assigned to the return and regardless of the risk-taking factor, the stock portfolio will have the lowest risk value. In the interval between 0 and 1, portfolios were optimized considering risk and optimal return factors. In other words, by adding $\lambda$, risk-taking reduction became more important; in this case, since $(1-\lambda)$ was reduced, maximizing return became less important.

### B.2. Determining the weights of risk and return criteria for three investor groups

In step 1, the weights of each return and risk criteria used in Equation (19) were determined by asking individuals to give the criteria a weight between 1 and 9, with 1 showing equal importance and 9 showing the greatest importance.

### C. Performance evaluation of the formed portfolios

A number of criteria exist for evaluating the performance of stock portfolios, such as Sharpe, Treynor, Jensen, and Sortino. There are a number of traditional measures of stock portfolio performance that are widely used, including the Sharpe ratio and the Treynor index.

The Sharpe index is one of the most commonly used traditional metrics for evaluating performance relative to modified risk. This index measures the excess return that the portfolio has achieved over the return on a risk-free asset.

This index is calculated from Equation (20), which is also known as RVAR.

$$RVAR=\frac{\overline{TR_P}-\overline{R_F}}{SD_P}=\frac{Excess\ return}{Risk} \tag{20}$$

Where $\overline{TR_P}$ is the average return of the stock portfolio over a period of time, $\overline{R_f}$ is the average risk-free rate of return during the period under review, and $SD_P$ is the standard deviation of weekly returns of the portfolio throughout the study.

## IV. Model Analysis and Results

In this section, we perform the proposed model and provide optimized portfolios for three investment groups. Finally, we will evaluate the performance of three portfolios for three groups of investors using sharp criteria.

### A. Data

Every quarter, the Tehran Stock Exchange publishes a list of the top 50 companies, mainly based on financial criteria, that is, the liquidity of stocks, the extent to which the company has an impact on the market, as well as the company's financial performance. The primary criticism of this method is that it does not determine the top companies but rather the largest ones. As a result, the list provided by the stock exchange is not representative of the most important companies (Mirdamadian et al., 2016).

We selected the top 50 companies listed on the Tehran Stock Exchange in the first quarter of 2012, based on last year's data. The study period covered three years, from 2009 to 2012. In this study, our sample population and sample is divided into three levels:

**Level 1:** The top 50 companies listed on the TSE are used to select and evaluate stocks and rankings.

**Level 2**: Chief executive officers of brokerage firms, investment companies, and experts, including 190 individuals (faculty of financial studies at universities in Tehran that are familiar with stock markets).

**Level 3:** A random sample of 96 nonprofessional investors in the TSE comprising accounting and financial management graduate students and nonprofessional investors in the TSE.

### B. Determining the effective criteria in stock selection

Using the literature review, we identified quantitative and qualitative criteria in stock selection and created a hierarchical structure of those criteria. A questionnaire was used to refine and limit the criteria extracted from prior studies, with ten-point Likert scale questions. Experts determined the numerical value of each criterion, and the criteria with mean values higher or equal to seven were chosen as the final criteria. The specific details are presented in Table I.

According to the results shown in Table I, the criterion of *The impact of government decisions* had the highest importance, and *Dividend per Share (DPS)* and *Stock price fluctuations* had the lowest importance in stock selection and evaluation, respectively.

It should be noted that by considering any number below seven as the base average, the number of criteria examined would be at least 31, making the research process very complicated.

### C. Analytic Hierarchy Process modeling

To determine the importance weight of each criterion, pairwise comparisons and analyses based on the AHP model must be conducted. Modeling based on this method relies on the AHP hierarchical tree, which represents the problem under study. Stock rankings are the objective of this study. The hierarchy tree in Figure 2 represents this. As can be seen, the factors influencing the objective of stock ranking can be divided into four main categories, each of which includes sub-criteria.

**Figure 2. Criteria of the AHP model to rank shares**

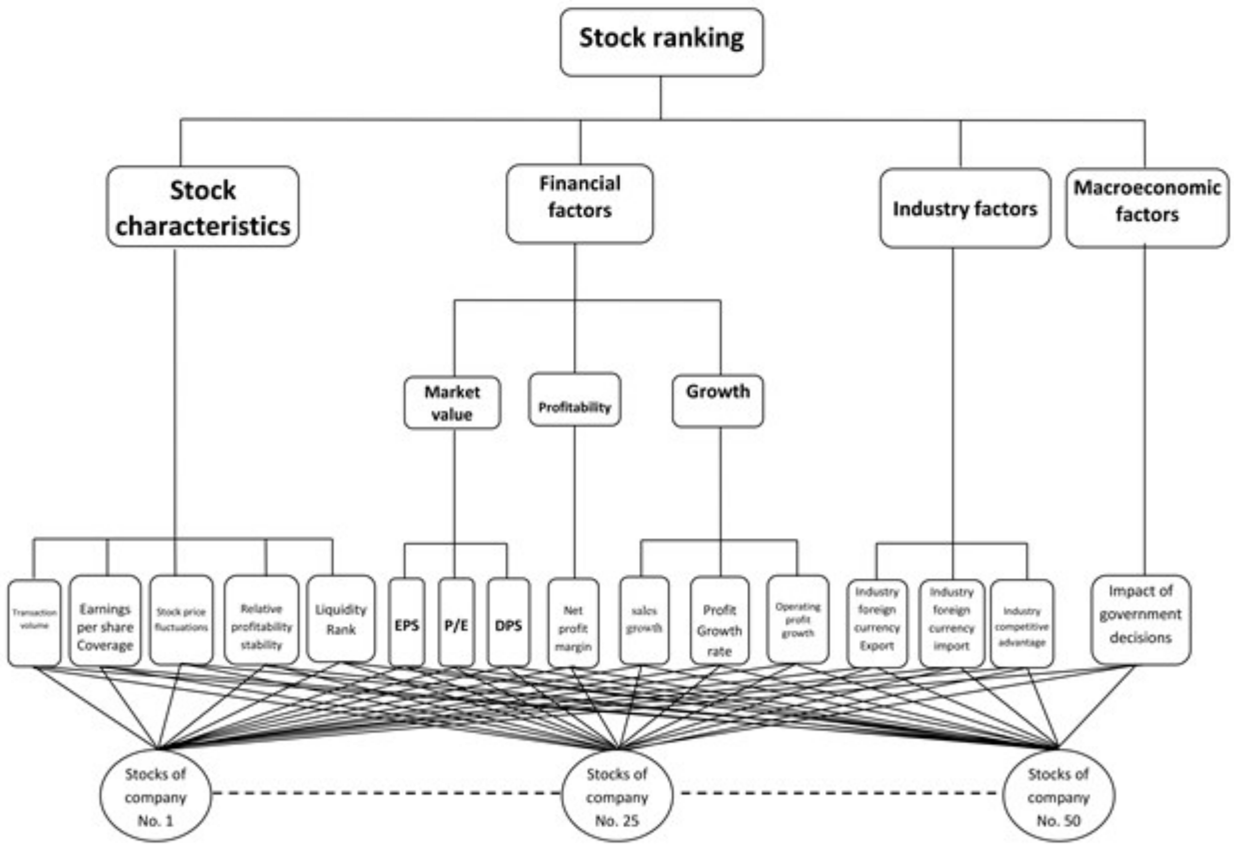



**Table I. Effective criteria in-stock selection**

| Row | Effective criteria in-stock selection | Description | Score |
|---|---|---|---|
| 1 | The impact of government policies | When the government's decisions impact a company, investors' behavior in stock selection and ranking is also affected. | 8.5 |
| 2 | The competitive advantage of the industry | An industry with a better perspective is preferred over other industries. | 8.1 |
| 3 | Foreign currency export of the company | By purchasing the required machinery and technology transfer, import companies need to source their products abroad. As a result, they need foreign currency, and any fluctuation in rates impacts their production rate. | 7.7 |
| 4 | Foreign currency import of the company | An increase in exchange rates increases the profitability of export companies. Increased foreign exchange rates have | 8 |

| | | | |
|---|---|---|---|
| | | positive consequences for export companies. | |
| 5 | Net profit margin | The ratio of net profit to net sales | 7.1 |
| 6 | Earnings per share (EPS) | The ratio of common share profit (loss) to the number of common shares | 8.7 |
| 7 | Price/earnings ratio (P/E) | The ratio of the current share price to its earnings per share (EPS) | 8.6 |
| 8 | Divided per share (DPS) | A part of the profit is the share of the company that is divided among shareholders:<br>The ratio of dividend per share (the sum of declared dividends issued by a company) to the number of shares | 7.3 |
| 9 | Sales growth | Growth in sales is defined as the proportion of net sales that grow from one fiscal period to another. | 7.7 |
| 10 | Net profit growth rate | Growth in net profit is defined as the proportion of net profit that grows from one fiscal period to another. | 7.2 |
| 11 | Operational profit growth | Operating profit growth is defined as the percentage of net operating profit that grows from one fiscal period to another. | 7.1 |
| 12 | Liquidity rate | A number that shows the liquidity of a share in the market | 7.3 |
| 13 | Transaction volume percentage | The ratio of transacted shares of a company in a day relative to its total number of shares | 7.3 |
| 14 | Stock price fluctuations | Stock trading price changes on the stock market. Stock price standard deviation is used for each company to determine stock price fluctuations. | 7.1 |
| 15 | Coverage of earnings per share | The ratio of real earnings per share in a period to predicted earnings per share based on the actual performance in the period | 7.4 |
| 16 | The relative stability of profitability | This ratio indicates the extent to which profitability deviates from the average profit or expected profit—the more stable the profitability, the lower the risk, and the higher the share value. | 7.8 |

## D. Determining the weights of criteria in stock assessment

Generally, to determine the importance or weight of criteria, the AHP technique is used.

Table II provides the descending order of the qualitative-quantitative scale, nature, and criteria weight.

**Table II. Criteria and their importance weights**

| Row | Criteria affecting stock evaluation | Scale | Nature | Criteria weight (In descending order) |
|---|---|---|---|---|
| 1 | The impact of government decisions | Qualitative [1] | Min | 0.4866 |
| 2 | The competitive advantage of the industry | Qualitative | Max | 0.1135 |
| 3 | Foreign currency export | Qualitative | Min | 0.0733 |
| 4 | Earnings Per Share (EPS) | Quantitative | Max | 0.0483 |
| 5 | Net profit margin | Quantitative | Max | 0.0401 |
| 6 | Foreign currency import | Qualitative | Max | 0.0355 |
| 7 | Profit coverage of each share | Quantitative | Min | 0.0331 |
| 8 | Sales growth | Quantitative | Max | 0.0288 |
| 9 | Price/earnings ratio (P/E) | Quantitative | Max | 0.0261 |
| 10 | The relative stability of profitability | Quantitative | Max | 0.0231 |
| 11 | The net profit growth rate | Quantitative | Max | 0.0203 |
| 12 | Liquidity rank | Quantitative | Min | 0.0194 |
| 13 | Transaction volume | Quantitative | Max | 0.0151 |
| 14 | Operational profit growth | Quantitative | Max | 0.0124 |
| 15 | Dividend per Share (DPS) | Quantitative | Max | 0.0122 |
| 16 | Stock price fluctuations | Quantitative | Min [2] | 0.0121 |

[1] To score qualitative criteria, a five-point Likert scale was used (from 1: very low to 5: very high).

[2] This criterion is equal to stock price changes and is of the cost (Min) type.

## E. Ranking the 50 top companies listed on the TSE

The mid-point and half-width values obtained from the interval TOPSIS technique were used to rank the 50 companies listed on the TSE during the first three months of 2012. The results are shown in Table III.

**Table III. The final ranking of the 50 top companies listed on TSE over the first three months of 2012**

| Company's name | Mid-point | Rank from Interval TOPSIS | Company's name | Mid-point | Rank from Interval TOPSIS |
|---|---|---|---|---|---|
| National Iranian Copper Industries | 0.506090 | 1 | Khazar Cement | 0.377461 | 26 |
| Sina Bank | 0.504921 | 2 | Leasing Ryan Saipa | 0.377134 | 27 |
| Iran Transfo | 0.472663 | 3 | Sahand Tire Industries | 0.375610 | 28 |
| Foolad Mobarake | 0.472347 | 4 | Sepah Investment | 0.375157 | 29 |
| Qadir Investment (Holding) | 0.443557 | 5 | Boali Investment | 0.374201 | 30 |
| Post Bank | 0.433875 | 6 | Pardis Investment | 0.373955 | 31 |
| Mellat Bank | 0.432990 | 7 | Razak Pharmaceutical | 0.373905 | 32 |
| Behshahr Industries Development | 0.432857 | 8 | Iranian Sanitary | 0.371906 | 33 |
| Petrochemical Industries Development (Holding) | 0.424957 | 9 | Mellat Investment | 0.367060 | 34 |
| Fars & Khuzestan Cement | 0.423784 | 10 | Techinco Inspection & Corrosion Control | 0.366143 | 35 |
| North Drilling | 0.423181 | 11 | Shahid Bahonar Copper Industries | 0.363306 | 36 |
| International Building Development | 0.422680 | 12 | Ardakan Industrial Ceramics | 0.362984 | 37 |
| Karafarin Bank | 0.422387 | 13 | Industry and Mining Industry | 0.353977 | 38 |
| Tehran Renovation and Building | 0.419879 | 14 | Jaber Ebne Hayyan Pharmaceutical | 0.352828 | 39 |
| Eghtesad Novin Bank | 0.418180 | 15 | National Investment of Iran | 0.346785 | 40 |
| Iranian Building Investment | 0.417487 | 16 | Ghadir Car Leasing | 0.332768 | 41 |
| Behceram Granite Production | 0.416685 | 17 | Bahman Investment | 0.337120 | 42 |

| | | | | | |
|---|---|---|---|---|---|
| Pars Tooshe Investment | 0.411592 | 18 | National Iranian Lead and Zinc | 0.316934 | 43 |
| Shahrood Cement | 0.404805 | 19 | Saipa | 0.301276 | 44 |
| Parsian Bank | 0.404086 | 20 | Insurance Industry Investment | 0.298237 | 45 |
| Bafgh Mines | 0.402689 | 21 | Iran Khodro Investment Expansion | 0.295394 | 46 |
| West Cement | 0.402458 | 22 | Pars Khodro | 0.293155 | 47 |
| Atiehe Damavand Investment | 0.389025 | 23 | Azarab Industry | 0.281667 | 48 |
| Informatics Services | 0.384706 | 24 | Piazer Agro-Industry | 0.279548 | 49 |
| Kowsar Pharmacology | 0.378872 | 25 | Gaz Looleh | 0.260488 | 50 |

## F. Stock portfolio selection from the ranked companies

Investors can determine the investment amount in each company's share based on the mean values obtained from the interval TOPSIS. However, mean-semivariance optimization with objective function and constraints is used to specify a more accurate weight for each share in the optimal stock portfolio.

Using the mean values obtained from interval TOPSIS, we created a portfolio of 24 companies whose mean values exceeded the total mean value (0.383515). The removal method is based on the method of interest described in previous studies. To determine the weight and number of the chosen stocks, an optimization method is conducted with certain constraints.

By analyzing existing databases, we collected monthly returns from companies over 36 months (March 2009 to February 2012), calculated semivariance and semi-covariance values from the returns, and then utilized these values as inputs to our model.

### F.1. Determining the weights of risk and return criteria for three investor groups

We determined the weights of return and risk criteria for each group of investors using the AHP method in steps 1 and 2.

**Step 1:** Calculating the geometric mean: The geometric mean of the matrices obtained from the completed questionnaires is shown in Table IV.

**Table IV. The geometric mean of criteria comparison by different investor groups**

| | **Risk-averse investors** | | **Neutral investors relative to risk** | | **Risk-taker investors** | |
|---|---|---|---|---|---|---|
| | **Return** | **Risk** | **Return** | **Risk** | **Return** | **Risk** |
| **Return** | 1.000 | 2.106 | 1.000 | 4.846 | 1.000 | 6.299 |
| **Risk** | 0.475 | 1.000 | 0.206 | 1.000 | 0.159 | 1.000 |

**Step 2:** Identifying the relative weight of criteria by different investor groups

Table V summarizes the number of investors in each group and the importance of each criterion to each group of investors.

**Table V. The importance of the return and risk criteria from the perspectives of different investors**

| The investor group | The number of investors (number) | The number of investors (%) | The importance of return (%) | The importance of risk (%) |
|---|---|---|---|---|
| **Risk-averse** | 38 | 36.5 | 0.322 | 0.678 |
| **Neutral towards risk** | 35 | 33.7 | 0.546 | 0.454 |
| **Risk-taker** | 31 | 29.8 | 0.863 | 0.137 |

**F.2. Multi-objective optimization model for portfolio selection for risk-averse investors:**

By using the return and risk coefficients from Table V, we present a model in Equation (21) for the optimal selection of stocks portfolios with an objective function and economic constraints for risk averse investors:

$$\begin{aligned}
&Min\ (0.678 * Var\ (R_p) - 0.322 * R_p) \\
&1)\ \sum W_i . R_i = R_p \\
&2)\ \sum W_i = 100\% \\
&3)\ W_i \leq \alpha_i\ (6.67\%) \\
&4)\ W_i \geq 0
\end{aligned} \tag{21}$$

Objective Function: $Min\ (0.678 * Var\ (R_p) - 0.322 * R_p)$
Constraint Functions: 1, 2, 3, 4
$R_i$= Expected return of stock i
$R_p$= Minimum expected portfolio returns by investor
$W_i$ = Weight of stock i
$\alpha_i$ = Maximum weight of each stock in portfolio
Objective function constraints for optimization were expressed as follows:

Constraint 1: The sum of the weight multiplication or the allocable stock ratio in the expected return must equal the return the investor considered to be satisfactory.

Constraint 2: Due to the fact that short transactions or exceeding the budget were not allowed, the total of the select stock weights had to be one (100 percent). This constraint is well suited to the real conditions of most shareholders.

Constraint 3: Stock weights had not to exceed 6.67 percent: $\frac{100}{15} = 6.67\%$

According to Evans and Archer (1968), unsystematic risk can be reduced by holding between 10 and 15 stocks. Accordingly, if there are 15 shares in a portfolio, we can accept the restriction that no single stock may exceed 6.67 percent in weight. Their study found that portfolio diversification with more than 15 or 16 stocks was not negative, but it was also not beneficial.

Constraint 4: Short selling is prohibited, so stock weights could not exceed 0. This assumption is logical and supported by market realities.

The results of performing the provided model for risk-averse investors are shown in Table VI. The portfolio's return level is 23.42 percent, and the risk level is 7.08 percent.

**Table VI. Optimal stock portfolio for risk-averse investors**

| Row | Company's name | Companies' stock weight | Row | Company's name | Companies' stock weight |
|---|---|---|---|---|---|
| **1** | National Iranian Copper Industries | 6.67 | 13 | Karafarin Bank | 6.67 |
| **2** | Sina Bank | 6.67 | 14 | Tehran Renovation and Building | 6.67 |
| **3** | Iran Transfo | 0 | 15 | Eghtesad Novin Bank | 6.67 |
| **4** | Foolad Mobarake | 0 | 16 | Iranian Building Investment | 5.58 |
| **5** | Ghadir Investment (Holding) | 6.67 | 17 | Behceram Granite Production | 0 |
| **6** | Post Bank | 0 | 18 | Pars Tooshe Investment | 1.1 |
| **7** | Mellat Bank | 6.67 | 19 | Shahrood Cement | 6.67 |
| **8** | Behshahr Industrial Development (Holding) | 6.67 | 20 | Parsian Bank | 6.67 |
| **9** | Petrochemical Industries Development | 0 | 21 | Bafgh Mines | 0 |
| **10** | Fars & Khuzestan Cement | 6.67 | 22 | West Cement | 0 |
| **11** | North Drilling | 0 | 23 | Atieh Damavand Investment | 6.67 |
| **12** | International Building Development | 6.67 | 24 | Informatics Services | 6.61 |

**F.3. Multi-objective optimization model for portfolio selection for investors neutral to risk:**

A model for selecting the optimal portfolio of stocks presented to investors that are neutral to risk using the return and risk coefficients from Table V and with the objective function and constraints defined by Equation (22):

$$
\begin{aligned}
&Min\ (0.454 * Var\ (R_p) - 0.546 * R_p) \\
&1)\ \sum W_i . R_i = R_p \\
&2)\ \sum W_i = 100\% \\
&3)\ W_i \leq \alpha_i\ (6.67\%) \\
&4)\ W_i \geq 0
\end{aligned}
\tag{22}
$$

Table VII shows the results of performing the model with a rate of return of 27.42 percent and a level of risk of 8.75 percent for investors neutral to risk.

**Table VII. Optimal stock portfolio for investors neutral to risk**

| Row | Company's name | Companies' stock weight | Row | Company's name | Companies' stock weight |
|---|---|---|---|---|---|
| **1** | National Iranian Copper Industries | 6.67 | 13 | Karafarin Bank | 6.67 |
| **2** | Sina Bank | 6.67 | 14 | Tehran Renovation and Building | 6.67 |
| **3** | Iran Transfo | 0 | 15 | Eghtesad Novin Bank | 6.67 |
| **4** | Foolad Mobarake | 0 | 16 | Iranian Building Investment | 0 |
| **5** | Ghadir Investment (Holding) | 6.67 | 17 | Behceram Granite Production | 0 |
| **6** | Post Bank | 0 | 18 | Pars Tooshe Investment | 6.67 |
| **7** | Mellat Bank | 6.67 | 19 | Shahrood Cement | 6.67 |
| **8** | Behshahr Industrial Development (Holding) | 6.67 | 20 | Parsian Bank | 6.67 |
| **9** | Petrochemical Industries Development | 0 | 21 | Bafgh Mines | 6.67 |
| **10** | Fars & Khuzestan Cement | 6.67 | 22 | West Cement | 0 |
| **11** | North Drilling | 0 | 23 | Atieh Damavand Investment | 6.67 |
| **12** | International Building Development | 0 | 24 | Informatics Services | 6.67 |

### F.4. Multi-objective optimization model for portfolio selection for risk-taking investors:

Equation (23) represents an optimal stock portfolio selection model with an objective function for risk-taking investors:

$$\begin{aligned}
&Min\ (0.137 * Var\ (R_p) - 0.863\ * R_p) \\
&1)\ \sum W_i . R_i = R_p \qquad (23) \\
&2) \sum W_i = 100\% \\
&3)\ W_i \leq \alpha_i\ (6.67\%) \\
&4) W_i \geq 0
\end{aligned}$$

After establishing mathematical models in this section, we implemented the models in MATLAB software and determined the weight of stocks in the portfolios for each group of investors.

The results of performing a risk-taker investor model with a return level of 29.32 percent and a risk level of 10.8 percent can be seen in Table VIII.

**Table VIII. Optimal stock portfolio for risk-taker investors**

| Row | Company's name | Companies' stock weight | Row | Company's name | Companies' stock weight |
|---|---|---|---|---|---|
| **1** | National Iranian Copper Industries | 6.67 | 13 | Karafarin Bank | 6.67 |
| **2** | Sina Bank | 6.67 | 14 | Tehran Renovation and Building | 6.67 |
| **3** | Iran Transfo | 6.67 | 15 | Eghtesad Novin Bank | 6.67 |
| **4** | Foolad Mobarake | 6.67 | 16 | Iranian Building Investment | 00 |
| **5** | Ghadir Investment (Holding) | 6.67 | 17 | Behceram Granite Production | 0 |
| **6** | Post Bank | 0 | 18 | Pars Tooshe Investment | 1.1 |
| **7** | Mellat Bank | 6.62 | 19 | Shahrood Cement | 6.67 |
| **8** | Behshahr Industrial Development (Holding) | 6.67 | 20 | Parsian Bank | 6.67 |
| **9** | Petrochemical Industries Development | 0 | 21 | Bafgh Mines | 6.67 |
| **10** | Fars & Khuzestan Cement | 0 | 22 | West Cement | 0 |
| **11** | North Drilling | 0 | 23 | Atieh Damavand Investment | 0 |
| **12** | International Building Development | 0 | 24 | Informatics Services | 6.67 |

### F.5. Optimal stock portfolio for investors in general

The risk shown in Table IX is in line with the expected returns for each group of investors. In other words, the stock portfolio risk increased in response to the expected returns. Investors had to tolerate more risk in order to achieve a higher return.

**Table IX. Expected return and risk in general**

| | Risk-averse | Neutral to risk | Risk-taker |
|---|---|---|---|
| **Return** | 23.42 | 27.42 | 29.32 |
| **Risk** | 7.08 | 8.75 | 10.8 |

### G. Performance evaluation of the formed portfolios

We utilized sharp criteria to evaluate and assess the performance of the portfolios in comparison with the performance of the stock market. As can be seen from Table X, this index for the formed stock portfolios was higher than the market portfolio; therefore, these portfolios outperformed the market in terms of risk and return.

**Table X. Stock portfolio performance evaluation criterion using Sharp index**

| Stock portfolio | Sharp index |
|---|---|
| A portfolio offered to risk-averse investors | 0.9 |
| A portfolio offered to neutral investors to risk | 1.2 |
| A portfolio offered to risk-taker investors | 1.1 |
| Market portfolio | -3.8 |

## V. Conclusion and Future Work

Taking into account uncertain conditions associated with investing in the TSE and the risk tolerance of investors, this study attempted to determine the optimal stock portfolio among fifty top companies. In selecting and evaluating stocks, we used both qualitative and quantitative criteria.

Portfolio selection was influenced by macroeconomic/political factors, such as government decisions affecting the industry, and industry-specific factors, such as competitive advantage and foreign exchange rate. Moreover, financial factors, such as growth factors (i.e., operational profit growth, net growth rate, and sales growth), profitability (net profit margin), and stock market value (i.e., DPS, EPS, P/E), were among the most important criteria. We also investigated the characteristics of stocks (e.g., liquidity rank, transaction volume percentage, stock price fluctuations, earnings coverage, and relative stability of profitability) and a variety of important factors influencing stock selection. The criterion of *The impact of government decisions* had the greatest importance, and *Dividend per Share (DPS)* and *Stock price fluctuations* had the lowest importance in stock selection and evaluation. Afterward, the companies were ranked using a technique called interval TOPSIS, which considers interval values for financial information. We should also note that the ranking of companies based on extracted criteria differed from the ranking of the 50 top-ranking companies on the TSE. Having selected qualified stocks, the next step consists of creating portfolios and determining the weight invested in each stock. Afterward, three portfolios were designed, one for an investor who was risk-averse, one who was neutral, and one for an investor who was risk-taking. At this stage of the optimization process, the mean-semivariance optimization model is used. Three portfolios for three groups of investors were evaluated and compared to the market performance using sharp criteria. On both a risk and return basis, each of the three portfolios has outperformed the market portfolio. This study proposes a model that can be utilized as a decision support tool for forming an optimal stock portfolio when considering both the expertise of experts as well as the preferences of investors when assessing stocks simultaneously.

Globalization, the growing economy, and the need to handle large amounts of data from various organizations around the globe necessitate the development of optimized algorithms to manage large amounts of data efficiently and timely (Cormen et al., 2022; Sepahyar et al., 2019). A variety of sorting algorithms can be employed in data processing, including quicksort, shell sort, and many others. Furthermore, machine learning and data mining algorithms (Qu et al., 2017; Pahwa & Agarwal, 2019) can be used to process large amounts of data in a very short period of time with a more efficient method. However, further research and study are required, and we will continue to study this in the future.

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