Insurance Contracting with Adverse Selection and Moral Hazard

Zhiqiang Yan and Hongbok Lee [1]

Abstract

The asymmetric information problem has been widely discussed in the context of insurance markets. Most of previous research treats adverse selection and moral hazard separately, though it is possible that they may coexist and interact with each other. In this paper, we build a principal-agent model to examine optimal contracts in a competitive insurance market facing adverse selection and moral hazard simultaneously. We apply the change-of-variable method and the Kuhn-Tucker conditions to solve the optimization programs. Our model brings richer separating Nash equilibria than pure moral hazard and pure adverse selection models, although separating Nash equilibria may not exist. It also retains some properties, for example, no full insurance and the positive correlation between insurance coverage and risk type, in those benchmark models. Our study on comparative statics indicates that, under some conditions and with some exceptions, the optimal indemnity and premium decrease with disutility from effort, increase with potential loss and decrease with the initial wealth of the insured.

Keywords: Insurance contracting; Adverse selection; Moral hazard

JEL Classification: G22

I Introduction

Insurance markets are well-known to be plagued with two types of asymmetric information problems: adverse selection and moral hazard. Under adverse selection, people are characterized by different levels of risk. High risk people, knowing that they are more likely to have an accident in the future, tend to purchase contracts with more complete coverage. However, in the context of moral hazard, people first choose different contracts due to exogenous reasons, and then they are faced with different incentive schemes: those who end up facing a contract with more complete coverage will have less incentive to adopt more cautious behaviors, which may result in higher probability of accident. Inspired by the seminal works of Arrow (1963) and Akerlof (1970), there is a flourishing literature that has theoretically examined the adverse selection problem (see, e.g., Rothschild and Stiglitz, 1976; Jean-Baptiste and Santomero, 2000) and the moral hazard problem (see, e.g., Pauly, 1974; Stiglitz, 1977; Shavell, 1979; Lambert, 1983; Smith and Stutzer, 1995). While we recognize their importance, most of these previous studies on asymmetric information take these two polar cases as mutually exclusive. In reality, however, it is possible that moral hazard and adverse selection can coexist in the same market and interact with each other. The approach to deal with each problem separately may provide us with limited or even biased information. In this paper, we develop a one-period principal-agent model with the simultaneous presence of moral hazard and adverse selection in a competitive insurance market. We analyze the characteristics of possible separating Nash equilibria and perform a comparative study to

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[1] Both Zhiqiang Yan and Hongbok Lee are Professor of Finance at the School of Accounting, Finance, Economics, & Decision Sciences, Western Illinois University. The authors can be reached via e-mail: Z-Yan@wiu.edu and H-Lee@wiu.edu, respectively.

investigate the effects of external shocks, such as initial wealth and potential loss, on contract design. Though we focus on insurance contracts in this paper, our model framework can be readily applied to other areas, such as financial contracts and labor contracts.

One important work on adverse selection is by Rothschild and Stiglitz (1976). They prove that in a competitive market with adverse selection only, a separating equilibrium (in a Nash sense) could exist, and that, in equilibrium, high-risk individuals self-select into a contract with full insurance coverage at a higher unit price, whereas low-risk individuals self-select into a contract with partial coverage at a lower unit price. However, a separating equilibrium may not exist under certain conditions. Stiglitz (1977) extends the Rothschild-Stiglitz model to the case of monopoly. In the monopolistic equilibrium, high-risk individuals purchase full insurance, while low-risk individuals purchase partial or no insurance. Cooper and Hayes (1987) investigate optimal multi-period insurance contracts with experience rating in both monopolistic and competitive environments. They demonstrate that, in both settings, the contract for high-risk individuals does not reflect their loss experience, whereas the contract for low-risk individuals does.

Under adverse selection, high-risk individuals are more likely to choose a contract with more complete coverage. The pure moral hazard model discovers a similar pattern between risk and coverage from an inverted causality, as people choose different contracts first. A person having better coverage and, therefore, weaker incentives will be less cautious, and hence becomes the riskier one (Arnott and Stiglitz, 1988).

Pauly (1974) investigates the moral hazard problem in a competitive setting. By assuming that insurance price is uniform over the quantity of insurance purchased, he shows that the first best solution is not feasible. Shavell (1979) argues that some imperfect information about the insured's level of care can still be useful and partial coverage is generally optimal in this context. Lambert (1983) examines the moral hazard problem in a two-period model. He demonstrates that an agent's compensation in the second period should depend on her performance in both periods, no matter whether she is pre-committed to the long-term contract or not. This implies that insurance companies should use both experience rating and retrospective rating to control moral hazard problems. Smith and Stutzer (1995) introduce aggregate uncertainty into the analysis of moral hazard problems. They show that insurance coverage varies across aggregate states, and insurance companies should offer less coverage to provide a greater incentive to mitigate the moral hazard problem in states where moral hazard is most costly.

To our best knowledge, there are only a few theoretical papers addressing insurance contracts when moral hazard and adverse selection coexist. Whinston (1983) considers a single-period social insurance model in a monopolistic setting with moral hazard and adverse selection, and demonstrates that the optimal equilibrium is a pooling one. Stewart (1994) assumes that agents only differ in their marginal costs of loss prevention effort and characterizes a separating reactive equilibrium (versus Nash equilibrium as in Rothschild and Stiglitz, 1976) in a competitive insurance market with both moral hazard and adverse selection. He shows that, in equilibrium, the adverse selection and moral hazard problems partially offset each other such that welfare losses are sub-additive. Jack (2002) examines the existence of pure strategy equilibria in health insurance markets that exhibit both *ex ante* adverse selection and *ex post* moral hazard. The *ex post* moral hazard in his paper is also labeled as hidden information moral hazard, in which insurers cannot observe the state of the world that occurs ex post. In contrast, it is hidden action moral hazard that we study in this paper, where individuals can take unobservable actions (efforts) that affect the probability distribution over states of nature. Huang, Liu and Tzeng (2010) graphically illustrate the separating Nash equilibria in the simultaneous presence of hidden confidence and hidden action on self-protection. They find that hidden overconfidence can result in advantageous selection in a perfectly competitive market. Gottlieb and Moreira (2014) introduce adverse selection in a standard moral hazard model, where agents are risk-averse and not subject to limited liability constraints. They characterize the solution and find that the optimal mechanisms are quite different from environments featuring only moral hazard or adverse selection. More recently, Chade and Swinkels (2021) analyze a principal-agent problem with moral hazard and adverse selection. They attempt to disentange moral hazard from adverse selection by utilizing a decoupling procedure that treats moral hazard and adverse selection sequentially. Gottlieb and Moreira (2022) study a principlal-agent model with moral hazard and adverse selection, where agents are risk-neutral and have limited liability. Under a multiplicative separability condition, they find that a single contract should be offered in the optimal mechanism.

Our paper is most allied with Laffont and Martimort (2002) and Chassagnon and Chiappori (1997). We follow the model setup in these two papers. Basically, we assume that there are two sorts of insureds differing *ex ante* according to their risk types. Each type could privately choose a discrete loss prevention effort level, and they both suffer from a disutility when exerting an effort. The insured's risk type or her effort level is private information and can not be observed by the insurer. Therefore, the insurer's task is to design optimal contracts to incentivize the insured to exert effort and truthfully report her type. In this setup, the probability of no loss depends on both the risk type and the effort level of the insured. Therefore, it is possible that by exerting an effort, a high-risk individual achieves a higher probability of no loss than a low-risk one who makes no effort.

Laffont and Martimort (2002) investigate insurance contracts in this context by assuming a monopolistic and risk-neutral insurer. Therefore, they maximize the insurer's expected profit by selecting the contracts offered to the high- and low-risk individuals, subject to participation constraints, adverse selection constraints, and moral hazard constraints. However, the insurance market is not monopolistic. Instead, it is often viewed as an archetypical example of a perfectly competitive market. The assumption of monopoly greatly simplifies the contract design problem as we only need to solve one optimization program. In a competitive market with two or more risk types, insurers' expected profits on each type of contracts are driven to zero. Hence, insurance contracts should maximize each type of insured's expected utility subject to the above mentioned constraints plus zero expected profit constraints. There are more than one constrained optimization programs to be solved in order to find the equilibrium, if it exists. In this paper, we extend the work of Laffont and Martimort (2002) to a competitive insurance market and characterize each possible equilibrium in detail.

Chassagnon and Chiappori (1997) examine insurance contracts with perfect competition in the presence of moral hazard and adverse selection as well. They find that, in the context of discrete effort levels, there are three types of separating Nash equilibria (in the sense of Rothschild and Stiglitz, 1976) and no pooling Nash equilibrium. Separating Nash equilibria may not exist under certain conditions. Furthermore, they extend the model to the case of continuous effort levels, and claim that pooling Nash equilibria are possible in this context. Our paper differs from Chassagnon and Chiappori (1997) in two major aspects. First, Chassagnon and Chiappori (1997) use the techniques of correspondence and sequences to characterize the equilibria. In this paper, we apply more straightforward mathematical techniques, that is, the change-of-variable method (Laffont and Martimort 2002) together with the Kuhn-Tucker conditions, to solve the optimization programs and find the equilibria. Second, Chassagnon and Chiappori (1997) allow zero effort from the insured under the optimal insurance contract in their model, whereas we require that the optimal insurance contracts always induce the insured to exert efforts, which results in different sorts of equilibria.

We find that there are five possible types of separating Nash equilibrium contracts based on our model setup, although separating Nash equilibria may not exist. The optimal contracts with both adverse selection and moral hazard retain some features in the pure adverse selection model and the pure moral hazard model. In equilibrium, the optimal contracts need to have the high-risk individual bear less risk than the low-risk one in order to reduce her incentive to lie on her type, as indicated in the pure adverse selection model. Consequently, the high-risk individual is offered more complete coverage but pays a higher unit price. Also, both types are offered partial insurance coverage as in the pure moral hazard model. In addition, the pure moral hazard model dominates in the sense that no agent in our model can obtain a coverage higher than that in the pure moral hazard model.

We further conduct a comparative analysis on each possible equilibrium. More specifically, we examine the effects of disutility, potential loss, and initial wealth on the optimal contracts offered to each risk type respectively. We find that, in general, the optimal premium and indemnity written on both types of contracts decrease with disutility, increase with potential loss, and decrease with initial wealth. However, some assumptions need to be made and some exceptions still exist. By comparing with the standard results from pure adverse selection and pure moral hazard models, we show that the mixed results in our model arise from the coexistence of moral hazard and adverse selection. Hence caution is needed in order to characterize the optimal contracts in our model.

The remainder of this paper is organized as follows. We describe the model framework in section 2 and brief the model with pure adverse selection and the model with pure moral hazard in section 3. We then develop a model with coexistence of adverse selection and moral hazard in the context of perfect competition, and graphically characterize possible separating Nash equilibria in section 4. We present the results of comparative analysis in section 5. Concluding remarks come in section 6.

II A Principal-Agent Model

The Model Setup

Following the literature, we assume that there are two parties in the insurance contract: a risk-neutral insurer group (or principals) and a risk-averse insured group (or agents). We assume that insurance markets are competitive and thus each insurer is constrained to earn zero expected profit. The insured has an initial wealth w and possesses von Neumann-Morgenstern utility function $u(w)$ with $u' > 0$ and $u'' < 0$ for all $w \in \mathbb{R}_+$.

In the simultaneous presence of moral hazard and adverse selection, defining an agent's risk type is a little tricky. In the standard adverse selection setting, the separation of high and low-risk types is clear-cut: a high-risk agent has a higher probability of accident, and vice versa. However, after introducing the moral hazard problem into the model of pure adverse selection, a high-risk agent may expend more effort to reduce her probability of accident, leading to a lower probability of accident than a low-risk agent who makes less or no effort. This possibility alone complicates traditional definitions of risk types.

A common approach taken in the literature (Stewart, 1994; Chassagnon and Chiappori, 1997) is to define an agent as a high-risk type if the agent's probability of accident is higher than that of the other type of agent, given the same level of effort. In Stewart (1994), the probability function of avoiding a loss is continuous and identical across agents, but one type of agent has a higher marginal cost of effort, which makes her a high-risk type. Chassagnon and Chiappori (1997) define the probability of accident in a similar fashion except that they use a discrete probability function.

In this paper, to make things simpler, we use a discrete probability function as in Chassagnon and Chiappori (1997). Assume that there are two types of agents who differ *ex ante* in their risk types $\theta \in \Theta = \{\underline{\theta}, \overline{\theta}\}$, where $\overline{\theta}$ represents a high-risk type and $\underline{\theta}$ corresponds to a low-risk type.[2] The two risk types are independently distributed, the distribution of which is common knowledge to both agents and principals. Both types of agents can choose to either make effort or not, i.e., the effort $e \in \{0,1\}$.

Therefore, the probability of no loss $\pi(\theta, e)$ is a function of an agent's risk type θ and the agent's loss prevention effort e. For the sake of simplicity, we suppress the notation to π_e with overline (or underline) on π to indicate that the agent is of a high- (or low-) risk type. Then, with probability $1 - \pi_e$ the agent incurs an accident loss with the amount equal to l. We assume that the loss l is so large that it is always optimal for the insurer to induce the insured to expend effort. It is natural to assume $\underline{\pi}_e > \overline{\pi}_e$ for every $e \in \{0,1\}$. Moreover, we rule out the non-generic case $\overline{\pi}_1 = \underline{\pi}_0$ to avoid peculiar equilibria. By exerting effort e, an agent suffers disutility $\psi(e)$, with $\psi(1) = \psi$ and $\psi(0) = 0$. To be more analytically tractable, we assume that the utility function is separable in wealth and effort, which essentially assumes away the non-convexity problem in the indifference curves and the zero expected profit curves.[3] In order to avoid the limited liability problem, we assume that the initial wealth w is greater than the potential accident loss l, i.e., $w > l$.

An insurer offers an insured a menu of insurance contracts, $\delta = \{P, I\}$, where P is the premium paid to the insurer and I is the indemnity less the premium if a loss claim is filed. The equilibrium in question will be a pure Nash equilibrium (i.e., a simultaneous game equilibrium) instead of a Stackelberg equilibrium (i.e., a sequential game equilibrium). Because a pooling Nash equilibrium does not exist in the adverse selection and moral hazard model in the context of discrete effort levels (Chassagnon and Chiappori, 1997), we only consider contracts supporting separating Nash equilibria in the following analysis.

A separating Nash equilibrium should be characterized by the following conditions: (1) for each contract, an insurer will earn zero expected profit. Otherwise, rival competitors can undercut the insurer and make a profit until the expected profit goes to zero; (2) since we assume that premium is actuarially fair, according to standard results of insurance economics, these risk-averse agents will always be better off by purchasing insurance (Mossin, 1968; Smith, 1968).

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[2] We use overline (or underline) on a variable or an incentive constraint associated with the high- (or low-) risk type, respectively.

[3] We refer interested readers to Arnott and Stiglitz (1983) for detailed discussion.

Constrained Utility Maximization

In a competitive insurance market, each contract maximizes an agent's expected utility

$$V = \max_{\{P,I\}} \pi_1 u(w-P) + (1-\pi_1)u(w-l+I) - \psi$$

subject to the agent's participation constraint, adverse selection constraint, moral hazard constraint, and the principal's zero expected profit constraint. We formulate these constraints in this section.

Participation Constraint (PC)

A type θ agent's participation constraint is:

$$V \geq U_0(\theta, e) \equiv \max_{e\in\{0,1\}} \pi_e u(w) + (1-\pi_e)u(w-l) - \psi(e),$$

where $U_0(\theta, e)$ is the reservation utility of the type θ agent without insurance.

It is natural to assume that

$$u(w) - u(w-l) \geq \frac{\psi}{\Delta\,\pi}$$

where $\Delta\,\pi = \pi_1 - \pi_0$.[4] This assumption means that the type θ agent will exert a positive effort if she is self-insured, which is consistent with the previous assumption that it is optimal for a principal to induce an agent to expend a positive effort due to the magnitude of claim l. With perfect competition and no transaction costs, risk-averse agents will always prefer insurance to self-insurance. Thus, the participation constraint is automatically satisfied.

Moral Hazard Constraint (MH)

Inducing the type θ agent to exert effort requires the following moral hazard incentive constraint to be satisfied:

$$\pi_1 u(w-P) + (1-\pi_1)u(w-l+I) - \psi \geq \pi_0 u(w-P) + (1-\pi_0)u(w-l+I),$$

which can be reduced to

$$u(w-P) - u(w-l+I) \geq \frac{\psi}{\Delta\,\pi}.$$

Adverse Selection Constraint (AS)

Here we need to analyze the adverse selection constraint for the low- and high-risk types, respectively.

To induce the high-risk agent to truthfully report her type, the following adverse selection incentive constraint must be met:

$$\overline{\pi}_1 u\left(w-\overline{P}\right) + (1-\overline{\pi}_1)u\left(w-l+\overline{I}\right) - \psi \geq \max_{e\in\{0,1\}} \overline{\pi}_e u\left(w-\underline{P}\right) + (1-\overline{\pi}_e)u\left(w-l+\underline{I}\right) - \psi(e).$$

[4] This assumption is automatically satisfied based on the moral hazard constraint.

To simplify the problem, we assume that, by exerting effort, the high-risk agent can increase her probability of no loss more effectively, i.e.,

$$\Delta \underline{\pi} < \Delta \overline{\pi}.$$

Together with the moral hazard constraint for the low-risk type, this condition implies that the moral hazard constraint for a high-risk type is easier to satisfy than that for a low-risk type. That is,

$$u(w-\underline{P}) - u(w-l+\underline{I}) > \frac{\psi}{\Delta \overline{\pi}},$$

or,

$$\overline{\pi}_1 u(w-\underline{P}) + (1-\overline{\pi}_1)u(w-l+\underline{I}) - \psi \geq \overline{\pi}_0 u(w-\underline{P}) + (1-\overline{\pi}_0)u(w-l+\underline{I}).$$

Therefore, the high-risk agent will always expend effort even if she lies and chooses the contract $\underline{\delta} = \{\underline{P}, \underline{I}\}$. The adverse selection constraint of the high-risk agent then becomes

$$\overline{\pi}_1 u(w-\overline{P}) + (1-\overline{\pi}_1)u(w-l+\overline{I}) - \psi \geq \overline{\pi}_1 u(w-\underline{P}) + (1-\overline{\pi}_1)u(w-l+\underline{I}) - \psi.$$

Similarly, the low-risk agent's adverse selection incentive constraint is:

$$\underline{\pi}_1 u(w-\underline{P}) + (1-\underline{\pi}_1)u(w-l+\underline{I}) - \psi \geq \max_{e\in\{0,1\}} \underline{\pi}_e u(w-\overline{P}) + (1-\underline{\pi}_e)u(w-l+\overline{I}) - \psi(e).$$

For the low-risk agent to exert effort while choosing the contract $\overline{\delta} = \{\overline{P}, \overline{I}\}$, we must have

$$\underline{\pi}_1 u(w-\overline{P}) + (1-\underline{\pi}_1)u(w-l+\overline{I}) - \psi \geq \underline{\pi}_0 u(w-\overline{P}) + (1-\underline{\pi}_0)u(w-l+\overline{I}),$$

which can be reduced to

$$u(w-\overline{P}) - u(w-l+\overline{I}) \geq \frac{\psi}{\Delta \underline{\pi}}.$$

We will prove later that it does not hold.

Zero Profit Constraint (ZPC)

The assumption of competitive insurance markets implies that a principal earns zero expected profit on every contract offered in equilibrium. Thus, given a contract $\delta = \{P, I\}$ offered to the type θ agent, we have

$$\pi_1 P - (1-\pi_1)I = 0$$ [5]

[5] By introducing moral hazard problems, the zero profit curve (and the indifference line) are partitioned into two portions. The zero profit curve above the moral hazard line is $\pi_1 P - (1-\pi_1)I = 0$ and it becomes $\pi_0 P - (1-\pi_0)I = 0$ when it is below the moral hazard line. We only present the upper portion of the zero profit curve here since we assume the optimal insurance contract will always induce the agent to exert an effort.

Change-of-Variable Method

When it comes to solve the optimization problem, a technical difficulty arises. That is, the maximization program may not be concave because the concave utility function appears on both sides of moral hazard constraints and adverse selection constraints, which renders the traditional Kuhn-Tucker method invalid.[6] To resolve this issue, we follow the change-of-variable method proposed by Laffont and Martimort (2002). Define $u_a = u(w - l + I)$ and $u_n = u(w - P)$, which represent the agents' utility levels whether a loss occurs or not. Denote the inverse function of $u(\cdot)$ by $h = u^{-1}$. $h(\cdot)$ is an increasing and strictly convex function ($h' > 0$ and $h'' > 0$), because $u' > 0$ and $u'' < 0$ by assumption. Using these new variables, we can obtain that $I = -w + l + h(u_a)$ and $P = w - h(u_n)$. Therefore, the utility maximization problem for the high-risk agent can be written as

$$\overline{V} = \max_{\{\overline{u}_n, \overline{u}_a\}} \overline{\pi}_1 \overline{u}_n + (1 - \overline{\pi}_1)\overline{u}_a - \psi$$

subject to the following constraints,

$$\begin{cases} \overline{MH}:\ \overline{u}_n - \overline{u}_a \geq \dfrac{\psi}{\Delta\, \overline{\pi}} \\ \overline{AS}:\ \overline{\pi}_1 \overline{u}_n + (1 - \overline{\pi}_1)\overline{u}_a - \psi \geq \overline{\pi}_1 \underline{u}_n + (1 - \overline{\pi}_1)\underline{u}_a - \psi \\ \underline{AS}:\ \underline{\pi}_1 \underline{u}_n + \left(1 - \underline{\pi}_1\right)\underline{u}_a - \psi \geq \max\limits_{e \in \{0,1\}} \underline{\pi}_e \overline{u}_n + \left(1 - \underline{\pi}_e\right)\overline{u}_a - \psi(e) \\ \overline{ZPC}:\ \overline{\pi}_1\left(w - h(\overline{u}_n)\right) - (1 - \overline{\pi}_1)\left(-w + l + h(\overline{u}_a)\right) = 0 \end{cases}$$

We can formulate the constrained optimization problem for the low-risk agent in a similar fashion. With the change of variables, the objective function, the adverse selection and moral hazard constraints are now linear, and the zero profit constraint becomes concave. This implies that the Lagrangian function is concave, and thus we can apply the Kuhn-Tucker procedure to solve the optimization problem.

After changing variables, the zero expected profit curve for a type θ agent is given by

$$\pi_1\left(w - h(u_n)\right) - (1 - \pi_1)\left(-w + l + h(u_a)\right) = 0.$$

According to the implicit function theorem, we can obtain,

$$\frac{\partial u_n}{\partial u_a} = -\frac{1 - \pi_1}{\pi_1} \frac{h'(u_a)}{h'(u_n)},$$

and

[6] Let f be a function of many variables with continuous partial derivatives of first and second order on the convex open set S and denote the Hessian of f at the point x by $H(x)$. Then f is concave if and only if $H(x)$ is negative semi-definite for all $x \in S$. $H(x)$ is negative semi-definite if and only if all the k-th order principal minors of A are non-positive if k is odd and non-negative if k is even. For the moral hazard constraints, the first order principal minors are $u''(w - P) < 0$ and $-u''(w - l + I) > 0$, the second order principal minor is $-u''(w - p)u''(w - l + I) < 0$. Therefore, the moral hazard constraints are not concave. We can check the adverse selection constraints similarly and neither of them is concave.

$$\frac{\partial^2 u_n}{\partial u_a^2} = -\frac{1-\pi_1}{\pi_1}\frac{\pi_1\big(h'(u_n)\big)^2 h''(u_a) + (1-\pi_1)\big(h'(u_a)\big)^2 h''(u_n)}{\pi_1\big(h'(u_n)\big)^3}.$$

Since $h' > 0$ and $h'' > 0$, we have $\partial u_n / \partial u_a < 0$ and $\partial^2 u_n / \partial u_a^2 < 0$. Therefore, each zero expected profit curve passes the uninsured point $E = \big(u(w-l), u(w)\big)$ and decreases at an increasing rate. Meanwhile, the zero expected profit curve gets flatter as the probability of no claim $\pi(\cdot)$ increases.

Note that the slope of an agent's indifference line is $-\frac{1-\pi_1}{\pi_1}$. We can obtain $\frac{1-\pi_1}{\pi_1}\frac{h\prime(u_a)}{h\prime(u_n)} \leq \frac{1-\pi_1}{\pi_1}$ because of $\frac{h\prime(u_a)}{h\prime(u_n)} \leq 1$, i.e, the slope of the zero profit curve is equal to or smaller than that of the indifference curve (in absolute terms). Therefore, for a type θ agent, the indifference line can cross the zero expected profit curve from above at any point (and they only cross once), except at the point $u_a = u_n$ where these two curves are tangent to each other.

III Pure Adverse Selection and Pure Moral Hazard

Before we go into details about our model of adverse selection and moral hazard, we first briefly present the standard models of pure adverse selection and pure moral hazard respectively, which serve as two benchmarks.

The Model with Pure Adverse Selection (PAS)

The competitive pure adverse selection model is proposed and characterized in great detail in Rothschild and Stiglitz (1976). They assume that risk type is an agent's private information but principals are able to observe the actions of agents, or effort that agents exert to prevent losses. Because of the assumption of perfect competition, contracts offered to each agent should maximize the agent's expected utility subject to the adverse selection constraint of each risk type and the zero expected profit constraint.

Therefore, for a type $\overline{\theta}$ agent, the optimal contract in equilibrium should maximize the agent's expected utility

$$\max_{\{\overline{u}_n, \overline{u}_a\}} \overline{\pi}_1 \overline{u}_n + (1-\overline{\pi}_1)\overline{u}_a - \psi$$

subject to the following constraints,

$$\begin{cases} \overline{AS}: & \overline{\pi}_1\overline{u}_n + (1-\overline{\pi}_1)\overline{u}_a - \psi \geq \overline{\pi}_1\underline{u}_n + (1-\overline{\pi}_1)\underline{u}_a - \psi \\ \underline{AS}: & \underline{\pi}_1\underline{u}_n + \big(1-\underline{\pi}_1\big)\underline{u}_a - \psi \geq \underline{\pi}_1\overline{u}_n + \big(1-\underline{\pi}_1\big)\overline{u}_a - \psi \\ \overline{ZPC}: & \overline{\pi}_1\big(w - h(\overline{u}_n)\big) - (1-\overline{\pi}_1)\big(-w + l + h(\overline{u}_a)\big) = 0 \end{cases}$$

It is well-known that, in the presence of pure adverse selection, high-risk agents self-select into a full insurance contract but pay a higher unit price for insurance coverage, while low-risk agents choose a partial insurance contract but pay a lower unit price, as illustrated in Figure 1.

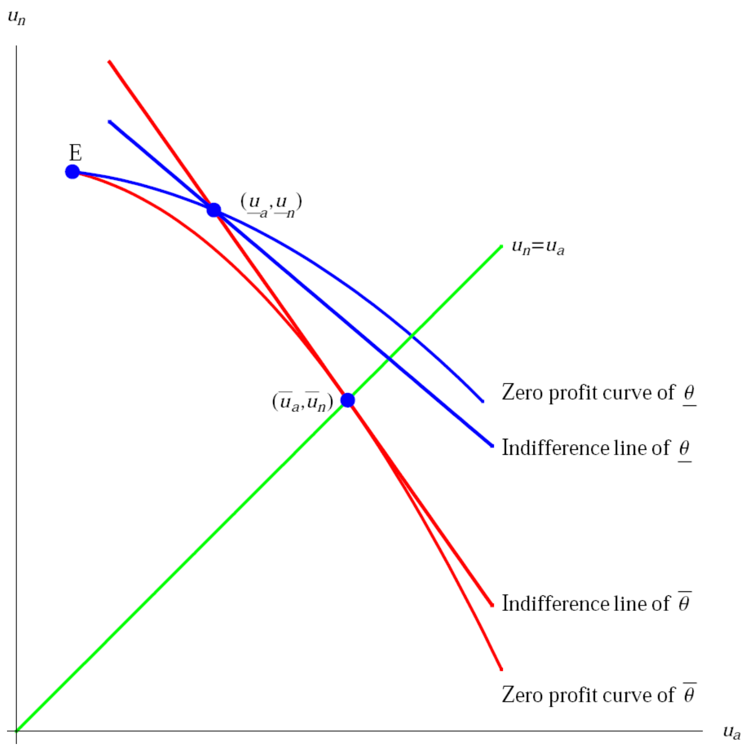

Figure 1: The Case of Pure Adverse Selection

Intuitively, the two adverse selection constraints indicate that $(\overline{u}_a, \overline{u}_n)$ must lie on or to the left of the indifference line of type $\underline{\theta}$ and $(\underline{u}_a, \underline{u}_n)$ must lie on or to the left of the indifference line of type $\overline{\theta}$. Since $\overline{\pi}_1 < \underline{\pi}_1$, the indifference line of type $\overline{\theta}$ is steeper than that of type $\underline{\theta}$. Therefore, we can shift the indifference line of type $\underline{\theta}$ rightward until it crosses the intersection of the indifference line of type $\overline{\theta}$ and the zero expected profit curve of type $\underline{\theta}$. The adverse selection constraint of type $\overline{\theta}$ must be binding and that of type $\underline{\theta}$ can never be binding. In order to maximize her expected utility, the high-risk agent's contract must be achieved at the point where type $\overline{\theta}$'s indifference line is tangent to her zero profit curve, which occurs at the point where $\overline{u}_n = \overline{u}_a$. The low-risk agent's contract is given by the intersection of type $\underline{\theta}$'s zero-profit curve and type $\overline{\theta}$'s indifference line.

The Model with Pure Moral Hazard (PMH)

In the case of pure moral hazard, an agent's risk type is publicly observable, but an agent's actions or loss prevention efforts are her private information. Since an agent's risk type is assumed to be observable by a principal, it is adequate to formally analyze one type of agent's equilibrium contract. Here we take a high-risk type as an example. Because of the perfect competition among insurers, the equilibrium contract offered to type $\overline{\theta}$ agent should maximize the agent's expected utility,

$$\max_{\{\overline{u}_n, \overline{u}_a\}} \overline{\pi}_1 \overline{u}_n + (1 - \overline{\pi}_1)\overline{u}_a - \psi$$

subject to the following constraints,

$$\begin{cases} \overline{MH}: \overline{u}_n - \overline{u}_a \geq \dfrac{\psi}{\Delta\,\overline{\pi}} \\ \overline{ZPC}: \overline{\pi}_1\big(w - h(\overline{u}_n)\big) - (1-\overline{\pi}_1)\big(-w + l + h(\overline{u}_a)\big) = 0. \end{cases}$$

According to the moral hazard constraint, the high-risk agent exerts effort when the contract is above the moral hazard line $\overline{u}_n = \overline{u}_a + \frac{\psi}{\Delta\overline{\pi}}$, whereas she makes no effort when the contract is below the line. Therefore, the indifference line and the zero profit curve are segmented into two parts by the moral hazard line. It is obvious that the agent achieves maximum utility at the intersection of the upper part of the zero profit curve and the moral hazard line. Hence, a principal offers a partial insurance contract to an agent in the context of pure moral hazard as illustrated in Figure 2.

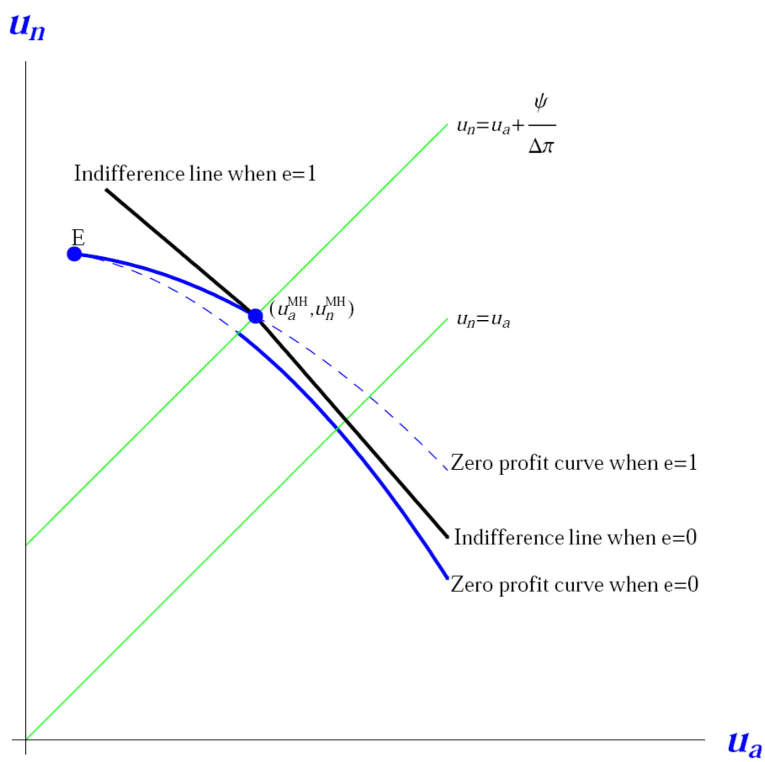

Figure 2: The Case of Pure Moral Hazard

IV Coexistence of Adverse Selection and Moral Hazard

When it comes to contract design, the majority of studies in the literature treat moral hazard and adverse selection separately. A few technical issues such as non-convex programming and random coverage (Winter, 2000) may be responsible for it. In this paper, by applying the change-of-variable technique and assuming a separable utility function in wealth and effort, we use the Kuhn-Tucker method to solve the maximization problem.

We use λ_{MH}, λ_{AS}, and λ_Z to denote the respective multipliers on the moral hazard constraint, the adverse selection constraint, and the zero profit constraint. The Lagrangian function of type $\overline{\theta}$ is:

$$\begin{aligned}\mathcal{L} = \quad & \max_{\{\overline{u}_n,\overline{u}_a\}} \overline{\pi}_1\overline{u}_n + (1-\overline{\pi}_1)\overline{u}_a - \psi \\ & +\overline{\lambda}_{MH}\left[\overline{u}_n - \overline{u}_a - \frac{\psi}{\Delta\,\overline{\pi}}\right] \\ & +\overline{\lambda}_{AS}\left[\overline{\pi}_1\overline{u}_n + (1-\overline{\pi}_1)\overline{u}_a - \psi - \overline{\pi}_1\underline{u}_n - (1-\overline{\pi}_1)\underline{u}_a + \psi\right] \\ & +\underline{\lambda}_{AS}\left[\underline{\pi}_1\underline{u}_n + \left(1-\underline{\pi}_1\right)\underline{u}_a - \psi - \underline{\pi}_{\underline{e}}\overline{u}_n - \left(1-\underline{\pi}_{\underline{e}}\right)\overline{u}_a + \psi\left(\underline{e}\right)\right] \\ & +\overline{\lambda}_Z\left[\overline{\pi}_1\left(w - h(\overline{u}_n)\right) - (1-\overline{\pi}_1)\left(-w + l + h(\overline{u}_a)\right)\right]\end{aligned}$$

Differentiating the Lagrangian function regarding $\overline{u}_n$ and $\overline{u}_a$ respectively, we can get the following first order conditions:

$$\frac{\partial\mathcal{L}}{\partial\overline{u}_n} = \quad \overline{\pi}_1\left(1+\overline{\lambda}_{AS}\right) + \overline{\lambda}_{MH} - \underline{\lambda}_{AS}\underline{\pi}_{\underline{e}} - \overline{\lambda}_Z\overline{\pi}_1 h'^{(\overline{u}_n)} = 0 \qquad (1)$$

$$\frac{\partial\mathcal{L}}{\partial\overline{u}_a} = \quad (1-\overline{\pi}_1)\left(1+\overline{\lambda}_{AS}\right) - \overline{\lambda}_{MH} - \underline{\lambda}_{AS}\left(1-\underline{\pi}_{\underline{e}}\right) - \overline{\lambda}_Z(1-\overline{\pi}_1)h'^{(\overline{u}_a)} = 0 \qquad (2)$$

By $(1)\times(1-\overline{\pi}_1) - (2)\times\overline{\pi}_1$, we can obtain

$$\overline{\lambda}_{MH} + \underline{\lambda}_{AS}\left(\overline{\pi}_1 - \underline{\pi}_{\underline{e}}\right) = \overline{\lambda}_Z\overline{\pi}_1(1-\overline{\pi}_1)[h'(\overline{u}_n) - h'(\overline{u}_a)] \qquad (3)$$

A close examination of Equation (3) leads to the results in Lemma 1. The proof can be found in the Appendix.

Lemma 1. *In equilibrium, if equilibrium exists, we must have* $\frac{\psi}{\Delta\overline{\pi}} \le \overline{u}_n - \overline{u}_a < \frac{\psi}{\Delta\underline{\pi}} \le \underline{u}_n - \underline{u}_a$. *In addition, the type* $\underline{\theta}$ *agent will not exert effort when she selects the type* $\overline{\theta}$ *agent's contract, that is,* $\underline{e} = 0$.

Intuitively, we can interpret $u_n - u_a$ as the risk level borne by the agent. If $u_n - u_a = 0$, i.e., (u_n, u_a) is on the full insurance line, the agent does not bear any risk. Lemma 1 tells us that, to prevent the high-risk agent from pretending that she is a low-risk type, the insurer would let the high-risk agent bear less risk than the low-risk agent, i.e., $\overline{u}_n - \overline{u}_a < \underline{u}_n - \underline{u}_a$.

According to Lemma 1, $\underline{e} = 0$, then Equation (3) becomes:

$$\overline{\lambda}_{MH} + \underline{\lambda}_{AS}\left(\overline{\pi}_1 - \underline{\pi}_0\right) = \overline{\lambda}_Z\overline{\pi}_1(1-\overline{\pi}_1)[h'(\overline{u}_n) - h'(\overline{u}_a)] \qquad (4)$$

Because $\overline{\lambda}_Z > 0$, $\overline{\lambda}_{MH} \ge 0$, $\underline{\lambda}_{AS} \ge 0$, and $\overline{u}_n > \overline{u}_a$, we can easily obtain Lemma 2 from Equation (4).

Lemma 2. $\overline{\lambda}_{MH}$ and $\underline{\lambda}_{AS}$ *can not equal zero simultaneously. In other words, at least one of the moral hazard constraints of the type* $\overline{\theta}$ *agent and the adverse selection constraint of the type* $\underline{\theta}$ *agent is binding.*

Therefore, there are three pairs of $\left(\overline{\lambda}_{MH}, \underline{\lambda}_{AS}\right)$, which are summarized below:

Case H1: $\overline{\lambda}_{MH} = 0$ and $\underline{\lambda}_{AS} > 0$, i.e., the moral hazard constraint of the high-risk type is not binding, while the adverse selection incentive constraint of the low-risk type is binding.

Case H2: $\overline{\lambda}_{MH} > 0$ and $\underline{\lambda}_{AS} = 0$, i.e., the moral hazard constraint of the high-risk type is binding, but the adverse selection incentive constraint of the low-risk type is not binding.

Case H3: $\overline{\lambda}_{MH} > 0$ and $\underline{\lambda}_{AS} > 0$, i.e., both the moral hazard constraint of the high-risk type and the adverse selection incentive constraint of the low-risk type are binding.

Similar to the derivation of Equation (3) for the high-risk type, we can obtain the following equation for the low-risk type:

$$\underline{\lambda}_{MH} + \overline{\lambda}_{AS}\left(\underline{\pi}_1 - \overline{\pi}_1\right) = \overline{\lambda}_Z \underline{\pi}_1\left(1 - \underline{\pi}_1\right)\left[h'\left(\underline{u}_n\right) - h'\left(\underline{u}_a\right)\right] \quad (5)$$

Lemma 3 follows directly from Equation (5).

Lemma 3. $\underline{\lambda}_{MH}$ and $\overline{\lambda}_{AS}$ *can not equal zero simultaneously. In other words, at least one of the moral hazard constraints of the type* $\underline{\theta}$ *agent and the adverse selection constraint of the type* $\overline{\theta}$ *agent is binding.*

There are also three possible pairs of $\left(\underline{\lambda}_M, \overline{\lambda}_{AS}\right)$:

Case L1: $\underline{\lambda}_{MH} = 0$ and $\overline{\lambda}_{AS} > 0$, i.e., the moral hazard constraint of the low-risk type is not binding, while the adverse selection constraint of the high-risk type is binding.

Case L2: $\underline{\lambda}_{MH} > 0$ and $\overline{\lambda}_{AS} = 0$, i.e., the moral hazard constraint of the low-risk type is binding, but the adverse selection constraint of the high-risk type is not binding.

Case L3: $\underline{\lambda}_{MH} > 0$ and $\overline{\lambda}_{AS} > 0$, i.e., both the moral hazard constraint of the low-risk type and the adverse selection incentive constraint of the high-risk type are binding.

Because the utility levels of the two types of agents, $(\overline{u}_n, \overline{u}_a)$ and $\left(\underline{u}_n, \underline{u}_a\right)$, are interdependent in equilibrium, we need to take the first order conditions of both agents into consideration in determining the optimal contracts. Based on Lemma 2 and 3, there are nine possible combinations of those Lagrangian multipliers. Lemma 4 below further rules out four of them. Therefore, there are five possible separating Nash equilibria, which are summarized in Proposition 1. The proofs are provided in the Appendix.

Lemma 4. *We can not have both* $\underline{\lambda}_{MH} > 0$ *and* $\underline{\lambda}_{AS} > 0$. *In other words, the adverse selection constraint and the moral hazard constraint of type* $\underline{\theta}$ *can not be binding at the same time.*

Proposition 1. *In a competitive insurance market with the simultaneous presence of adverse selection and moral hazard, there are five possible types of separating Nash equilibria in the sense of Rothschild-Stiglitz as follows:*

- Only Adverse Selection: *both of the adverse selection constraints are binding, but none of the moral hazard constraints is binding.*
- Only Moral Hazard: *both of the moral hazard constraints are binding, but none of the adverse selection constraint is binding.*
- Strong Adverse Selection: *both of the adverse selection constraints are binding, but only the moral hazard constraint of the high-risk type is binding.*
- Strong Moral Hazard: *both of the moral hazard constraints are binding, but only the adverse selection constraint of the high-risk type is binding.*

- Local Asymmetric Information: *the adverse selection constraint and the moral hazard constraint of the high-risk type are binding, but none of the asymmetric information constraints of the low-risk type is binding.*

Through the analysis of different equilibria, we can easily observe that $(\overline{u}_a, \overline{u}_n)$ is closer to the full insurance line than $(\underline{u}_a, \underline{u}_n)$, which means the type $\overline{\theta}$ agent is offered more coverage and bears less risk. Note that the unit price of insurance can be derived directly from the binding zero expected profit constraint, i.e., $\frac{P}{I} = \frac{1-\pi_1}{\pi_1}$. Therefore, the type $\overline{\theta}$ agent is offered a higher unit price than the type $\underline{\theta}$ agent, based on the assumption $\overline{\pi}_1 < \underline{\pi}_1$. The following proposition summarizes our findings comparing optimal contracts in the presence of both moral hazard and adverse selection with those in the context of pure adverse selection or pure moral hazard.

Proposition 2. *In the simultaneous presence of adverse selection and moral hazard, the moral hazard problem dominates in the sense that optimal contracts provide insurance coverage at most equal to the amount of coverage offered in the case of pure moral hazard, depending on model structures. Moreover, more complete insurance coverage is offered to type $\overline{\theta}$ at a higher unit price. Specifically,*

- *when $\overline{\pi}_1 > \underline{\pi}_0$: the optimal contract offered to type $\underline{\theta}$ provides an insurance coverage less than that in the case of pure moral hazard, while the optimal contract offered to type $\overline{\theta}$ provides an insurance coverage equal to or less than that in the case of pure moral hazard.*
- *when $\overline{\pi}_1 < \underline{\pi}_0$: the optimal contract offered to type $\underline{\theta}$ provides an insurance coverage equal to or less than that in the case of pure moral hazard, while the optimal contract offered to type $\overline{\theta}$ provides an insurance coverage equal to that in the case of pure moral hazard.*

Proposition 2 implies that optimal contracts with the coexistence of adverse selection and moral hazard retain some properties in the pure moral hazard model and the pure adverse selection model. As in the pure moral hazard case, no agent can obtain full insurance coverage. As in the adverse selection case, the positive correlation between insurance coverage and riskiness of agents still holds.

Furthermore, when $\overline{\pi}_1 > \underline{\pi}_0$, the type $\overline{\theta}$ agent is relatively riskier in the sense that the probability of loss of type $\overline{\theta}$ is higher if both types of agents expend the same level of effort. However, if the type $\overline{\theta}$ agent exerts effort while the type $\underline{\theta}$ agent does not, the latter becomes riskier. This additional layer of adverse selection complicates the principal's job of contract design even further and reduces the amount of coverage offered to the type $\underline{\theta}$ agent. When $\overline{\pi}_1 < \underline{\pi}_0$, the type $\overline{\theta}$ agent is absolutely riskier no matter whether the type $\underline{\theta}$ agent expends effort or not. In this case, the highest possible amount of coverage, which occurs at the intersection of type $\overline{\theta}$'s zero profit curve and moral hazard line, is offered to the type $\overline{\theta}$ agent.

V Comparative Statics

In this section, we conduct comparative analysis to investigate the effects of changing ψ (disutility), w (initial wealth), and l (loss) on I (optimal indemnity) and P (optimal premium) for both risk types. The separating Nash equilibria are defined by a system of nonlinear equations, which can not be solved analytically. We, therefore, linearize the system of nonlinear equations by differentiation and apply Cramer's rule to compute the partial derivatives of insurance premium and indemnity with respect to disutility, initial wealth, and potential loss. Proposition 1 demonstrates five possible types of separating Nash equilibria, and the comparative statics of the optimal contracts in those five Nash equilibria for the low-risk type and the high-risk type are reported in Table 1 and 2, respectively, together with the comparative statics of the two baseline models of pure adverse selection and pure moral hazard.[7]

Table 1: Comparative Statics of Optimal Contract for Low-Risk Type

Case	$\frac{\partial \underline{P}}{\partial \psi}$	$\frac{\partial \underline{P}}{\partial l}$	$\frac{\partial \underline{P}}{\partial w}$	$\frac{\partial \underline{I}}{\partial \psi}$	$\frac{\partial \underline{I}}{\partial l}$	$\frac{\partial \underline{I}}{\partial w}$
Case 1: Only adverse selection	-	+	-	-	+	-
Case 4: Local asymmetric information	-	+	?	-	+	?
Case 5: Only moral hazard	-	+	-	-	+	-
Case 6: Strong moral hazard	-	+	?	-	+	?
Case 7: Strong adverse selection	-	+	?	-	+	?
Pure adverse selection	0	+	?	0	+	?
Pure moral hazard	-	+	-	-	+	-

Note: "-"/"+" means that the partial derivative is negative/positive, "0" means that the partial derivative equals zero, and "?" means that the partial derivative is undetermined.

Table 2: Comparative Statics of Optimal Contract for High-Risk Type

Case	$\frac{\partial \overline{P}}{\partial \psi}$	$\frac{\partial \overline{P}}{\partial l}$	$\frac{\partial \overline{P}}{\partial w}$	$\frac{\partial \overline{I}}{\partial \psi}$	$\frac{\partial \overline{I}}{\partial l}$	$\frac{\partial \overline{I}}{\partial w}$
Case 1: Only adverse selection	-	?	?	-	?	?
Case 4: Local asymmetric information	-	+	-	-	+	-
Case 5: Only moral hazard	-	+	-	-	+	-
Case 6: Strong moral hazard	-	+	-	-	+	-
Case 7: Strong adverse selection	-	+	-	-	+	-
Pure adverse selection	0	+	0	0	+	0
Pure moral hazard	-	+	-	-	+	-

Note: "-"/"+" means that the partial derivative is negative/positive, "0" means that the partial derivative equals zero, and "?" means that the partial derivative is undetermined.

Comparative Statics for the PAS Model

In the pure adverse selection model, the contract offered to the high-risk type is at the intersection of the full insurance curve, $\overline{P} + \overline{I} = l$, and its zero profit curve, $\overline{\pi}_1 \overline{P} = (1 - \overline{\pi}_1)\overline{I}$. Obviously, a

[7] The derivation of the partial derivatives is not reported in the paper but available upon request.

change in disutility or initial wealth has no effect on the high-risk type's contract, while an increase in the loss results in an increase in both $\overline{P}$ and $\overline{I}$.

As to the low-risk type, its contract is determined by the binding adverse selection constraint of the high-risk type and the zero profit curve of the low-risk type. The binding adverse selection constraint means that type $\overline{\theta}$ is indifferent between the two types of contracts, thus the marginal utility resulting from an increase in ψ must be equal no matter whether she truthfully reports her type or not. Suppose an increase in ψ by one unit causes the indemnity to change by ΔI. If she chooses her own contract, one-unit increase in ψ will change her expected utility by $(1-\overline{\pi}_1)\left[u'\left(w-l+\overline{I}\right)-u'\left(w-\overline{P}\right)\right]\Delta\overline{I}-1$; if she lies, her expected utility will change by $\left[(1-\overline{\pi}_1)u'\left(w-l+\underline{I}\right)-\frac{\overline{\pi}_1(1-\underline{\pi}_1)}{\underline{\pi}_1}u'\left(w-\underline{P}\right)\right]\Delta\underline{I}-1$. To make the marginal changes equal, $\underline{I}$ and $\overline{I}$ must move in the same direction.[8] Recall that an increase in disutility has no impact on the high-risk type's contract, so it does not affect the low-risk type's contract either.

Now we investigate the impact of potential loss on the low-risk agent's contract. Suppose an increase in the loss by one unit causes the indemnity to change by ΔI. The expected utility changes by an amount of $(1-\overline{\pi}_1)\left[u'\left(w-l+\overline{I}\right)-u'\left(w-\overline{P}\right)\right]\Delta\overline{I}-(1-\overline{\pi}_1)u'\left(w-l+\overline{I}\right)$, if she truthfully reports her type, and by an amount of $\left[(1-\overline{\pi}_1)u'\left(w-l+\underline{I}\right)-\frac{\overline{\pi}_1(1-\underline{\pi}_1)}{\underline{\pi}_1}u'\left(w-\underline{P}\right)\right]\Delta\underline{I}-(1-\overline{\pi}_1)u'\left(w-l+\underline{I}\right)$, if she lies. We know that $\overline{I}$ increases with l. To make the marginal effects equal, we conclude that $\underline{I}$ increases with l as well.

For one-unit increase in initial wealth w, if the high-risk agent truthfully reports her type, it can increase the expected utility by $\overline{\pi}_1u'\left(w-\overline{P}\right)+(1-\overline{\pi}_1)u'\left(w-l+\overline{I}\right)$. If the high-risk agent pretends to be a low-risk type, the marginal benefit from the wealth change is $\overline{\pi}_1u'\left(w-\underline{P}\right)+(1-\overline{\pi}_1)u'\left(w-l+\underline{I}\right)$. The difference between them measures the effect of w on $\underline{I}$. Unless the marginal benefit from telling the truth is larger than that from lying on her type, an increase in initial wealth will decrease $\underline{I}$, and $\underline{P}$ accordingly.

Comparative Statics for the PMH Model

In the pure moral hazard model, the contract offered to the high- (low-) risk type is determined by its binding moral hazard constraint and zero profit constraint. We take the high-risk agent as an example in the following analysis. The binding moral hazard constraint, i.e., $u\left(w-\overline{P}\right)-u\left(w-l+\overline{I}\right)=\frac{\psi}{\Delta\overline{\pi}}$, implies that the risk borne by the high-risk agent, $\Delta\overline{u}=u\left(w-\overline{P}\right)-u\left(w-l+\overline{I}\right)$, is proportional to the disutility, ψ. An increase in the disutility ψ indicates that type $\overline{\theta}$ needs to bear more risk, leading to a decrease in both $\overline{P}$ and $\overline{I}$.

When the loss increases (but ψ remains unchanged), the risk borne by the high-risk agent increases if she keeps the same premium and indemnity as before. Therefore, the high-risk type have to increase $\overline{P}$ and $\overline{I}$ in order to keep the amount of risk unchanged.

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[8] Note that $u'\left(w-l+\overline{I}\right)-u'\left(w-\overline{P}\right)>0$ and $(1-\overline{\pi}_1)u'\left(w-l+\underline{I}\right)-\frac{\overline{\pi}_1(1-\underline{\pi}_1)}{\underline{\pi}_1}u'\left(w-\underline{P}\right)>0$, given $w-l+\overline{I}<w-\overline{P}$, $w-l+\underline{I}<w-\underline{P}$, $u''<0$ and $\overline{\pi}_1<\underline{\pi}_1$.

Now let us investigate the effect of initial wealth. Risk aversion implies that an increase in initial wealth results in a smaller increase in $u\left(w-\overline{P}\right)$ than that in $u\left(w-l+\overline{I}\right)$, given $w-\overline{P} > w-l+\overline{I}$. In other words, the risk borne by the high-risk agent goes down if she keeps the premium and indemnity unchanged. In order to offset this effect and keep the same amount of risk as before, the high-risk agent will demand less indemnity and thereby pay less premium.

Comparative Statics for the ASMH Model

In Cases 1, 4, and 5, there are exactly four equations and four unknown variables, i.e., $\underline{u}_n$, $\underline{u}_a$, $\overline{u}_n$, and $\overline{u}_a$ (see the proof of Proposition 1). All the other parameters ($\underline{\pi}_0$, $\underline{\pi}_1$, $\overline{\pi}_0$, $\overline{\pi}_1$, ψ, w, and l) are exogenous. We can compute the partial derivatives of $\underline{u}_n$, $\underline{u}_a$, $\overline{u}_n$, and $\overline{u}_a$ with respect to other parameters, based on the Cramer's rule. In Cases 6 and 7, there are five equations and we need to solve four unknowns ($\underline{u}_n$, $\underline{u}_a$, $\overline{u}_n$, and $\overline{u}_a$). Clearly, one of the other parameters must be endogenous and the choice of the endogenous variable surely affects the results of comparative statics in these two cases. The following analysis is based on the assumption that $\underline{\pi}_0$ is endogenous as it represents the simplest scenario.[9] We can compute the partial derivatives of $\underline{u}_n$, $\underline{u}_a$, $\overline{u}_n$, $\overline{u}_a$ and $\underline{\pi}_0$ with respect to other parameters in Cases 6 and 7. Recall the relationships: $P = w - h(u_n)$ and $I = -w + l + h(u_a)$. Using the chain rule, we can derive the partial derivatives of $\underline{P}$, $\underline{I}$, $\overline{P}$, and $\overline{I}$ with respect to other parameters.[10] Note that P and I change in the same direction as in the pure adverse selection or pure moral hazard model under the assumption that π_1 is exogenous.

Comparing the results in the pure adverse selection model and the pure moral hazard model with those in the model of both adverse selection and moral hazard, we can easily observe that the coexistence of adverse selection and moral hazard complicates the problem and generates some mixed results. First, Case 5 (only moral hazard) has the same comparative results as the pure moral hazard model. It is not surprising since in both models, if the equilibrium contracts exist, they must be jointly determined by the binding moral hazard and zero profit constraints, even though optimal contracts in Case 5 have to satisfy the adverse selection constraints as well. Second, in Cases 4, 6, and 7, the high-risk agent's contract is driven by the binding moral hazard constraint and therefore responds to external shocks in other parameters similarly as in the pure moral hazard model, while the contract offered to the low-risk agent is determined by the binding adverse selection constraint

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[9] If we choose $\underline{\pi}_0$ as the endogenous variable, the binding moral hazard constraint of type $\underline{\theta}$ in Case 6 and the binding adverse selection constraint of type $\underline{\theta}$ in Case 7 become redundant. $(\overline{u}_n, \overline{u}_a)$ is then determined by type $\overline{\theta}$'s binding moral hazard and zero profit constraints, and $(\underline{u}_n, \underline{u}_a)$ is determined by type $\underline{\theta}$'s binding zero profit constraint and type $\overline{\theta}$'s binding adverse selection constraint. Choosing any other variable as endogenous complicates our work of comparative analysis. If we choose $\underline{\pi}_1$ as the endogenous variable, for example, type $\overline{\theta}$'s contract remains unchanged as it is still determined by type $\overline{\theta}$'s binding moral hazard and zero profit constraints, but type $\underline{\theta}$'s contract will be jointly determined by type $\underline{\theta}$'s binding moral hazard and zero profit constraints and type $\overline{\theta}$'s binding adverse selection constraint.

[10] The partial derivatives computed in this way are expressed as functions of $h'(u_n)$ and $h'(u_a)$. We can express the partial derivatives in the forms of $u'(w-P)$ and $u'(w-l+I)$ once we notice the following relationships: $u'(w-P) = \frac{1}{h'(u_n)}$ and $u'(w-l+I) = \frac{1}{h'(u_n)}$.

of the high-risk type and thus behaves in the same manner as in the pure adverse selection model (except for the effect of disutility which affects the premium/indemnity of the high-risk agent and the low-risk agent in the same direction). Finally, Case 1 is the most complicated because the contracts are simultaneously determined by two binding adverse selection constraints. Hence, we cannot infer the results of comparative statistics of Case 1 from the pure adverse selection or the pure moral hazard model. The partial derivatives in this case can be derived following the procedure stated above. The results are rather mixed: the premium/indemnity profile of the low-risk type decreases with ψ, increases with l and decreases with w; for the high-risk type, the premium/indemnity decreases with ψ, but the effect of l or w is uncertain. It is in sharp contrast to the standard result that people buy more insurance when they face higher potential loss in the future. So, caution has to be used to characterize optimal contracts when both adverse selection and moral hazard exist.

VI Conclusion and Discussion

Since the early nineteen-seventies, the theoretical studies on contract theory have been explosive. Various optimal contracts are designed to deal with different asymmetric information problems, such as adverse selection and moral hazard. However, the majority of the asymmetric information literature treats adverse selection and moral hazard separately. In this paper, we consider a principal-agent model with the simultaneous presence of adverse selection and moral hazard in a competitive environment. To resolve the non-concavity issue in the optimization programming, we utilize the change-of-variable method proposed by Laffont and Martimort (2002), and then apply the Kuhn-Tucker method to solve the optimization programming. By analyzing the interaction between adverse selection and moral hazard, we find that there are several forms of separating Nash equilibria and graphically illustrate the characteristics of optimal contracts in each one of them. The equilibria in our paper are much richer than those in the pure adverse selection model and the pure moral hazard model, but retain some basic properties in these two benchmark models as well, such as no full insurance and the positive correlation between risk and coverage.[11] However, the equilibria may not exist. Even if an equilibrium exists, it may not be unique. A further examination of the conditions of existence and uniqueness of a separating Nash equilibrium may add new insight into our model. In addition, in the case of multiple equilibria, is there an equilibrium that Pareto-dominates all the others? These questions are beyond the scope of this paper but should be addressed in future research.

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[11] A common prediction of contract theory is the positive correlation property: everything being equal, people who face contracts with more comprehensive coverage should exhibit higher probability of accident. Therefore, if a data sample demonstrates a positive correlation between insurance coverage and accident occurrence, it implies the existence of asymmetric information. However, the positive correlation alone does not give us much insight into the nature of the underlying asymmetric information problems, as is well argued in the literature that the positive correlation can be the result of either adverse selection or moral hazard (Chiappori and Salanie, 2000, 2003), or even unobserved heterogeneous preferences (Meza and Webb, 2001). Proposition 1 of our paper implies that, in the simultaneous presence of adverse selection and moral hazard, the positive correlation property still holds.In addition, the comparative statics of the optimal contracts in our model predicts that the indemnity and premium decrease with disutility of effort, increase with potential loss, and decrease with the initial wealth of the insured, which may shed light on the regression analysis of insurance indemnity and/or premium.

One thing that makes the model with both adverse selection and moral hazard complicated is the insurer's inability to observe an agent's risk type and effort level. Therefore, the definition of "riskiness" alone is not an easy task. The low-risk agent may turn out to be even riskier if she is not incentivized to make effort. This simple possibility jeopardizes the Spence-Mirrless condition in the standard setup, and leads to more complex results (Chassagnon and Chiappori 1977, Laffont and Martimort 2002). In order to obtain more information on the "riskiness" of agents and provide corresponding incentive contracts, experience rating and retrospective rating are usually used. Further research on a more dynamic, two-period model based on these two rating approaches may be interesting.

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Appendix: Proofs in the Case of Moral Hazard and Adverse Selection

Proof of Lemma 1. Assume $\underline{e} = 1$, i.e., type $\underline{\theta}$ exerts effort when she selects type $\overline{\theta}$'s contract $\overline{\delta} = \{\overline{P}, \overline{I}\}$. Then we must have $\overline{u}_n - \overline{u}_a \geq \frac{\psi}{\Delta \underline{\pi}}$. Given $\Delta \underline{\pi} < \Delta \overline{\pi}$ by assumption, we can obtain $\overline{u}_n - \overline{u}_a \geq \frac{\psi}{\Delta \underline{\pi}} > \frac{\psi}{\Delta \overline{\pi}} > 0$.

Since $h''(\cdot) > 0$ and $\overline{u}_n > \overline{u}_a$, we have $h'(\overline{u}_n) - h'(\overline{u}_a) > 0$. In addition, given $\overline{\pi}_1 < \underline{\pi}_1$ by assumption and $\underline{\lambda}_{AS} \geq 0$ and $\overline{\lambda}_Z > 0$ by definition, we can obtain from equation (3) that $\overline{\lambda}_M > 0$. This implies that, if $\underline{e} = 1$, the moral hazard constraint of type $\overline{\theta}$ is binding, i.e., $\overline{u}_n - \overline{u}_a = \frac{\psi}{\Delta \overline{\pi}}$. This contradicts the inequalities $\overline{u}_n - \overline{u}_a \geq \frac{\psi}{\Delta \underline{\pi}} > \frac{\psi}{\Delta \overline{\pi}}$. Therefore, we must have $\underline{e} = 0$, and $\overline{u}_n - \overline{u}_a < \frac{\psi}{\Delta \underline{\pi}}$. Combining this inequality with the two moral hazard constraints, we can easily get that $\frac{\psi}{\Delta \overline{\pi}} \leq \overline{u}_n - \overline{u}_a < \frac{\psi}{\Delta \underline{\pi}} \leq \underline{u}_n - \underline{u}_a$. ■

Proof of Lemma 4. Suppose $\underline{\lambda}_{MH} > 0$ and $\underline{\lambda}_{AS} > 0$, which means that both the moral hazard constraint and the adverse selection constraint of type $\underline{\theta}$ are binding, i.e.,

$$\underline{u}_n - \underline{u}_a = \frac{\psi}{\Delta\, \underline{\pi}},$$

$$\underline{\pi}_1 \underline{u}_n + (1 - \underline{\pi}_1)\underline{u}_a - \psi = \underline{\pi}_0 \overline{u}_n + (1 - \underline{\pi}_0)\overline{u}_a$$

These equations essentially imply that, in equilibrium, if an equilibrium exists, the optimal contract offered to type $\underline{\theta}$ is at the intersection of her moral hazard line and indifference line $\underline{V}(\underline{e} = 1)$, while the optimal contract offered to type $\overline{\theta}$ is at the intersection of type $\overline{\theta}$'s indifference line $\overline{V}(\overline{e} = 1)$ and type $\underline{\theta}$'s indifference line $\underline{V}(\underline{e} = 0)$. Moreover, the indifference line $\overline{V}(\overline{e} = 1)$ should cross the indifference line $\underline{V}(\underline{e} = 0)$ from above, otherwise type $\overline{\theta}$ will select type $\underline{\theta}$'s contract since it yields a higher utility level to the type $\overline{\theta}$ agent (i.e., type $\underline{\theta}$'s contract lies above type $\overline{\theta}$'s indifference line).

A steeper indifference line $\overline{V}(\overline{e} = 1)$ implies that $\frac{1-\overline{\pi}_1}{\overline{\pi}_1} > \frac{1-\underline{\pi}_0}{\underline{\pi}_0}$, which can be simplified as $\overline{\pi}_1 < \underline{\pi}_0$. It means that type $\overline{\theta}$ is absolutely riskier than type $\underline{\theta}$, regardless of the effort level. In addition, when $\overline{\pi}_1 < \underline{\pi}_0$, for every u_a, the zero profit curve of type $\underline{\theta}$ when $\underline{e} = 0$ is flatter than the zero profit curve of type $\overline{\theta}$ at $\overline{e} = 1$. In equilibrium, the zero profit curve of type $\underline{\theta}$ when $\underline{e} = 0$ can not cross type $\underline{\theta}$'s indifference line $\underline{V}(\underline{e} = 0)$ (at most, to be tangent), otherwise an insurer can always offer another contract that shifts type $\underline{\theta}$ agent's indifference line rightwards and make a profit herself as well. Therefore type $\overline{\theta}$'s zero profit curve when $\overline{e} = 1$ can not cross the indifference line $\underline{V}(\underline{e} = 0)$, because the zero profit curve of type $\underline{\theta}$ when $\underline{e} = 0$ is flatter than the zero profit curve of type $\overline{\theta}$ at $\overline{e} = 1$ and the former is at most tangent to the indifference line $\underline{V}(\underline{e} = 0)$. Hence, there is no point on the indifference line $\underline{V}(\underline{e} = 0)$ that can be an optimal contract offered to the type $\overline{\theta}$ agent. This completes the proof that $\underline{\lambda}_M > 0$ and $\lambda_{AL} > 0$ can not hold simultaneously in equilibrium. ■

Proof of Proposition 1. In this proof, we investigate every case in turn. When there is a Rothschild-Stiglitz Nash equilibrium, we illustrate the equilibrium in a figure.

Case 1: (Case H1)+ (Case L1), that is, $\overline{\lambda}_{MH} = 0$, $\underline{\lambda}_{AS} > 0$, $\underline{\lambda}_{MH} = 0$, and $\overline{\lambda}_{AS} > 0$. This case represents the *Only Adverse Selection* equilibrium in Proposition 1.

In this case, the two moral hazard constraints are not binding, but the two adverse selection incentive constraints are binding. Coupled with the two binding zero-profit constraints, we now have four equations to solve for the four unknowns $\overline{u}_n$, $\overline{u}_a$, $\underline{u}_n$, and $\underline{u}_a$:

$$\begin{cases} \overline{AS}: \overline{\pi}_1 \overline{u}_n + (1 - \overline{\pi}_1)\overline{u}_a - \psi = \overline{\pi}_1 \underline{u}_n + (1 - \overline{\pi}_1)\underline{u}_a - \psi \\ \underline{AS}: \underline{\pi}_1 \underline{u}_n + (1 - \underline{\pi}_1)\underline{u}_a - \psi = \underline{\pi}_0 \overline{u}_n + (1 - \underline{\pi}_0)\overline{u}_a \\ \overline{ZPC}: \overline{\pi}_1\big(w - h(\overline{u}_n)\big) - (1 - \overline{\pi}_1)\big(-w + l + h(\overline{u}_a)\big) = 0 \\ \underline{ZPC}: \underline{\pi}_1\big(w - h(\underline{u}_n)\big) - (1 - \underline{\pi}_1)\big(-w + l + h(\underline{u}_a)\big) = 0 \end{cases}$$

It is difficult to analytically solve this system of equations without assuming the specific functional form of h. However, we can graphically demonstrate some features of the equilibrium contracts in Figure 3, if equilibria exist.

Since both of the adverse selection constraints are binding, $(\overline{u}_n, \overline{u}_a)$ should be at the intersection of type $\overline{\theta}$'s indifference line $\overline{V}(e=1)$ and type $\underline{\theta}$'s indifference line $\underline{V}(e=0)$, while $(\underline{u}_n, \underline{u}_a)$ should be at the intersection of type $\overline{\theta}$'s indifference line $\overline{V}(e=1)$ and type $\underline{\theta}$'s indifference line $\underline{V}(e=1)$. Furthermore, type $\overline{\theta}$'s zero profit curve should cross $(\overline{u}_n, \overline{u}_a)$, while type $\underline{\theta}$'s zero profit curve should cross $(\underline{u}_n, \underline{u}_a)$. From Figure 3, we can see that type $\underline{\theta}$'s indifference line $\underline{V}(e=0)$ crosses type $\overline{\theta}$'s indifference line $\overline{V}(e=1)$ from above, which leads to $\underline{\pi}_0 < \overline{\pi}_1$. $(\overline{u}_n, \overline{u}_a)$ and $(\underline{u}_n, \underline{u}_a)$ are above their respective moral hazard lines, which indicates that the amount of insurance offered to each type is even less than that offered in the case of pure moral hazard.

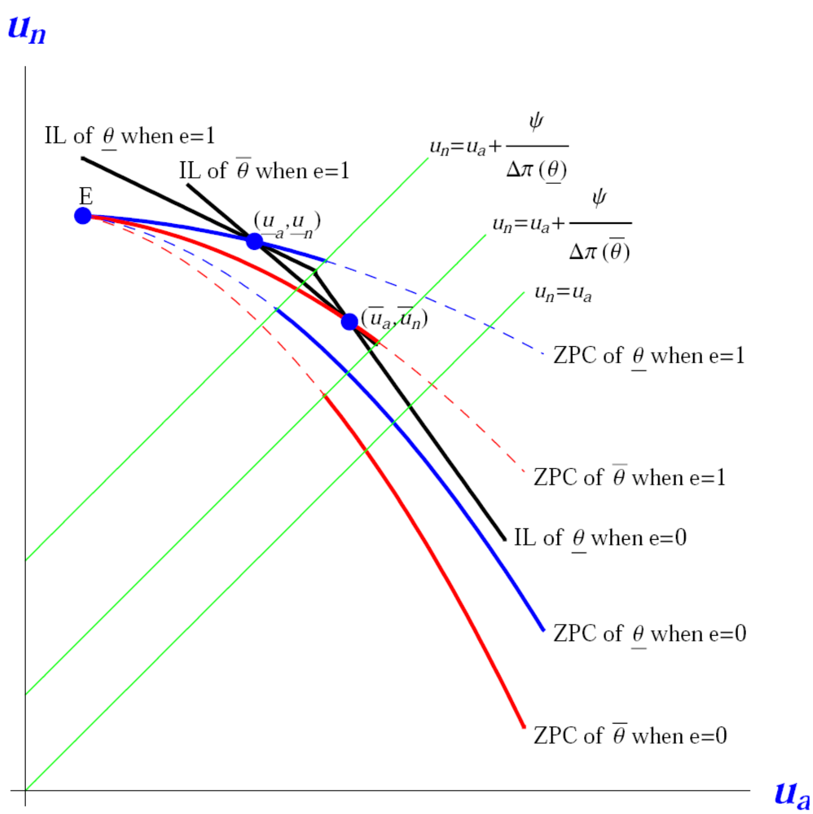

Figure 3: Case 1 of Adverse Selection and Moral Hazard

Case 2: (Case H1)+ (Case L2), that is, $\overline{\lambda}_{MH} = 0$, $\underline{\lambda}_{AS} > 0$, $\underline{\lambda}_{MH} > 0$, and $\overline{\lambda}_{AS} = 0$

In this case, the moral hazard constraint and the adverse selection constraint of type $\overline{\theta}$ are not binding, but those of type $\underline{\theta}$ are binding. According to *Lemma 2*, there is no equilibrium.

Case 3: (Case H1)+ (Case L3), that is, $\overline{\lambda}_{MH} = 0$, $\underline{\lambda}_{AS} > 0$, $\underline{\lambda}_{MH} > 0$, and $\overline{\lambda}_{AS} > 0$

In this case, the moral hazard constraint and the adverse selection constraint of type $\underline{\theta}$ are binding. Again, according to *Lemma 2*, there is no equilibrium.

Case 4: (Case H2)+ (Case L1), that is, $\overline{\lambda}_{MH} > 0$, $\underline{\lambda}_{AS} = 0$, $\underline{\lambda}_{MH} = 0$, and $\overline{\lambda}_{AS} > 0$. This case corresponds to the *Local Asymmetric Information* equilibrium in Proposition 1.

In this case, the moral hazard constraint and the adverse selection constraint of type $\overline{\theta}$ are binding, while none of the constraints of type $\underline{\theta}$ is binding. Combined with the two binding zero profit constraints, we have four equations with four unknowns:

$$\begin{cases} \overline{MH}: \overline{u}_n - \overline{u}_a = \dfrac{\psi}{\Delta\,\overline{\pi}} \\ \overline{AS}: \overline{\pi}_1\overline{u}_n + (1-\overline{\pi}_1)\overline{u}_a - \psi = \overline{\pi}_1\underline{u}_n + (1-\overline{\pi}_1)\underline{u}_a - \psi \\ \overline{ZPC}: \overline{\pi}_1\big(w - h(\overline{u}_n)\big) - (1-\overline{\pi}_1)\big(-w + l + h(\overline{u}_a)\big) = 0 \\ \underline{ZPC}: \underline{\pi}_1\left(w - h\left(\underline{u}_n\right)\right) - \left(1-\underline{\pi}_1\right)\left(-w + l + h\left(\underline{u}_a\right)\right) = 0 \end{cases}$$

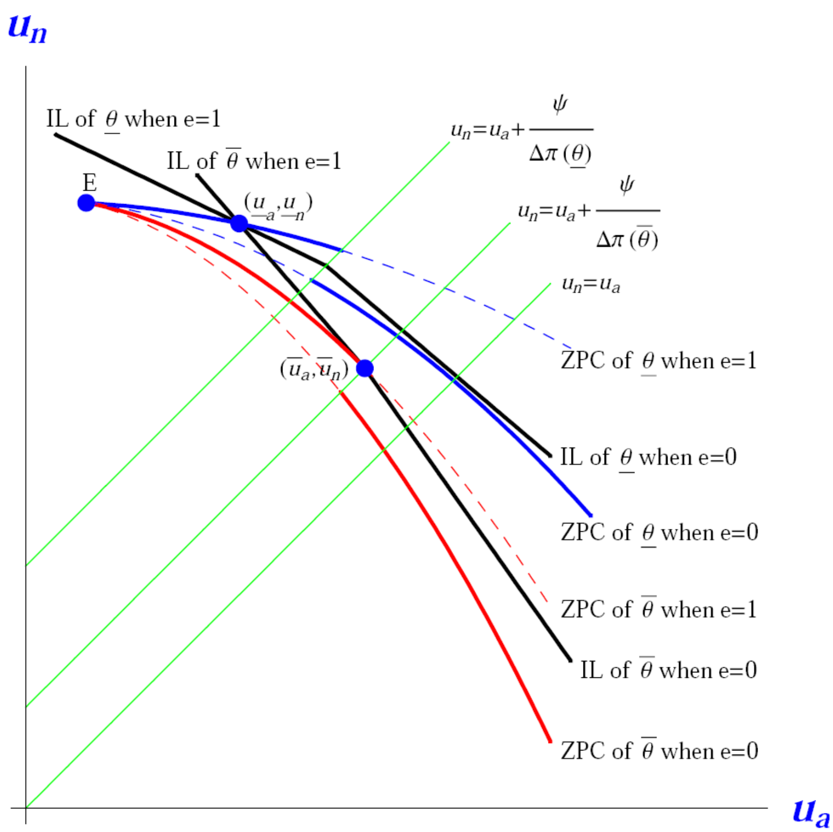

Figure 4: Case 4 of Adverse Selection and Moral Hazard

Since type $\underline{\theta}$'s moral hazard constraint is not binding but the adverse selection constraint of type $\overline{\theta}$ is binding, $(\underline{u}_a, \underline{u}_n)$ should locate above its moral hazard line and at the intersection of the indifference lines of the two types. As for $(\overline{u}_a, \overline{u}_n)$, since the moral hazard constraint of type $\overline{\theta}$ is binding, it is at the intersection of its indifference line and moral hazard line. Meanwhile, $(\overline{u}_a, \overline{u}_n)$ should be to the left of type $\underline{\theta}$'s indifference line, since the adverse selection constraint of type $\underline{\theta}$ is not binding. Figure 4 illustrates the possible equilibrium in this case. It is obvious that type $\overline{\theta}$

is offered the same contract as in the case of pure moral hazard, while type $\underline{\theta}$ is offered even smaller amount of insurance than that in the pure moral hazard case.

Case 5: (Case H2)+ (Case L2), that is, $\overline{\lambda}_{MH} > 0$, $\underline{\lambda}_{AS} = 0$, $\underline{\lambda}_{MH} > 0$, and $\overline{\lambda}_{AS} = 0$. This case represents the *Only Moral Hazard* equilibrium in Proposition 1.

In this case, both of the moral hazard constraints are binding, while none of the adverse selection constraints is binding. The two binding moral hazard constraints and the binding zero profit constraints give us a system of four equations with four unknowns:

$$\begin{cases} \overline{MH}: \overline{u}_n - \overline{u}_a = \dfrac{\psi}{\Delta\,\overline{\pi}} \\ \underline{MH}: \underline{u}_n - \underline{u}_a = \dfrac{\psi}{\Delta\,\underline{\pi}} \\ \overline{ZPC}: \overline{\pi}_1\big(w - h(\overline{u}_n)\big) - (1 - \overline{\pi}_1)\big(-w + l + h(\overline{u}_a)\big) = 0 \\ \underline{ZPC}: \underline{\pi}_1\left(w - h(\underline{u}_n)\right) - (1 - \underline{\pi}_1)\left(-w + l + h(\underline{u}_a)\right) = 0 \end{cases}$$

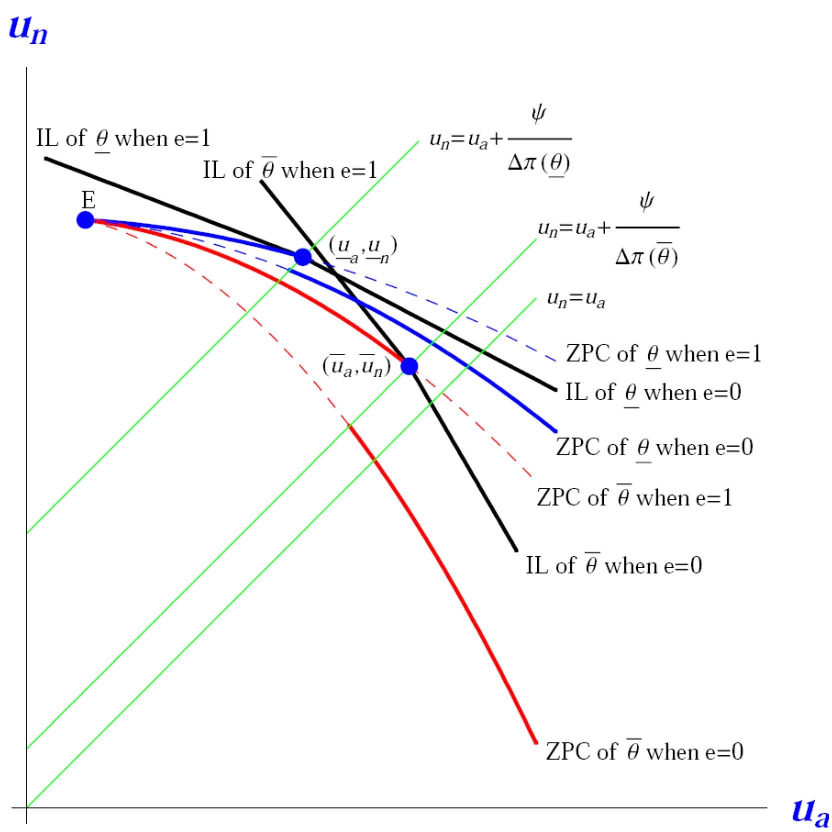

Figure 5: Case 5 of Adverse Selection and Moral Hazard

Since both of the moral hazard constraints are binding, $(\underline{u}_a, \underline{u}_n)$ and $(\overline{u}_a, \overline{u}_n)$ should be on their respective moral hazard lines, thus each agent is offered the same contract as in the case of pure moral hazard. Moreover, since both of the adverse selection constraints are not binding, the indifference line of the high-risk type $\overline{V}(e = 1)$ must cross the indifference line of the low-risk type $\underline{V}(e=0)$from above, as illustrated in Figure 5. This implies $\underline{\pi}_0 > \overline{\pi}_1$, i.e., type $\overline{\theta}$ is absolutely riskier than type $\underline{\theta}$ no matter whether the latter expends effort or not.

Case 6: (Case H2)+ (Case L3), that is, $\overline{\lambda}_{MH} > 0$, $\underline{\lambda}_{AS} = 0$, $\underline{\lambda}_{MH} > 0$, and $\overline{\lambda}_{AS} > 0$. This case represents the *Strong Moral Hazard* equilibrium in Proposition 1.

In this case, both of the moral hazard constraints and type $\overline{\theta}$'s adverse selection constraint are binding, while type $\underline{\theta}$'s adverse selection constraint is not binding. Therefore, similar to Case 5, each agent is offered the same contract as in the case of pure moral hazard and type $\overline{\theta}$ is absolutely riskier than type $\underline{\theta}$. The only difference is that type $\overline{\theta}$ is now indifferent between the two contracts offered, whereas in Case 5, she strictly prefers her own contract. The equilibrium is illustrated in Figure 6.

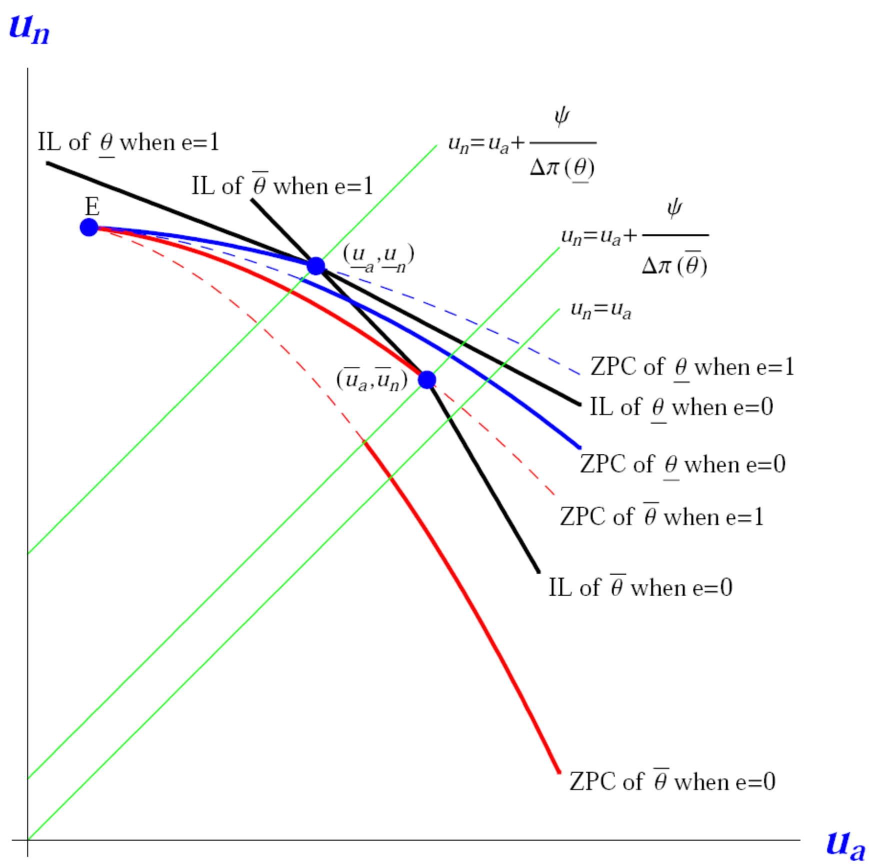

Figure 6: Case 6 of Adverse Selection and Moral Hazard

Case 7: (Case H3)+ (Case L1), that is, $\overline{\lambda}_{MH} > 0$, $\underline{\lambda}_{AS} > 0$, $\underline{\lambda}_{MH} = 0$, and $\overline{\lambda}_{AS} > 0$. This case corresponds to the *Strong Adverse Selection* equilibrium in Proposition 1.

In this case, both of the adverse selection constraints and type $\overline{\theta}$'s moral hazard constraint are binding, but type $\underline{\theta}$'s moral hazard constraint is not binding. Similar to Case 4, type $\overline{\theta}$ is offered the same contract as in the case of pure moral hazard, while type $\underline{\theta}$ is offered even smaller amount of insurance than that in the pure moral hazard case. One major difference between this one and Case 4 is that type $\underline{\theta}$ is now indifferent between the two contracts offered, whereas in Case 4, she strictly prefers her own contract. Also, two binding adverse selection constraints imply that the indifference curves of the two different types cross each other twice and thus $\underline{\pi}_0 < \overline{\pi}_1$. Figure 7 demonstrates the equilibrium in this case.

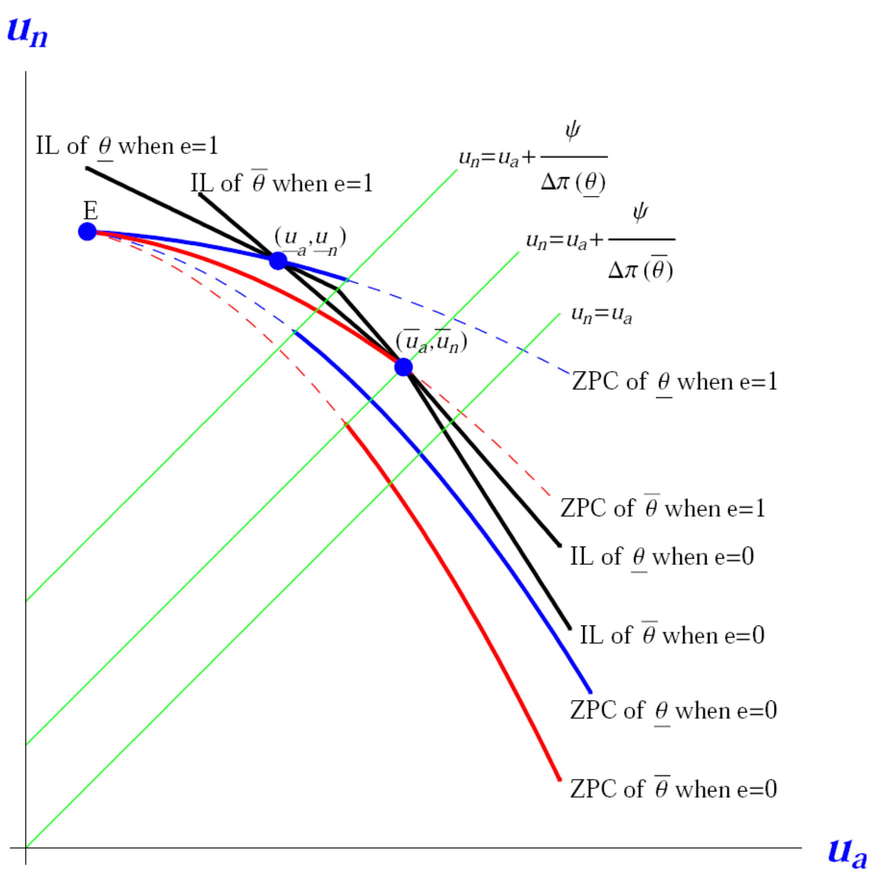

Figure 7: Case 7 of Adverse Selection and Moral Hazard

Case 8: (Case H3)+ (Case L2), that is, $\overline{\lambda}_{MH} > 0$, $\underline{\lambda}_{AS} > 0$, $\underline{\lambda}_{MH} > 0$, and $\overline{\lambda}_{AS} = 0$

In this case, both of the moral hazard constraints and type $\underline{\theta}$'s adverse selection constraint are binding, but type $\overline{\theta}$'s adverse selection constraint is not binding. According to *Lemma 2*, there is no equilibrium.

Case 9: (Case H3)+ (Case L3), that is, $\overline{\lambda}_{MH} > 0$, $\underline{\lambda}_{AS} > 0$, $\underline{\lambda}_{MH} > 0$, and $\overline{\lambda}_{AS} > 0$

In this case, all constraints are binding. According to *Lemma 2*, there is no equilibrium.

This completes the proof of Proposition 1. ■