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A Numerical Framework for Forecasting Financial Risk in Microfinance Institutions Using the Fourth-Order Runge-Kutta Method

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Abstract

This study proposes a robust numerical model for forecasting financial risk patterns in Microfinance Institutions (MFIs) using the conventional fourth-order Runge-Kutta (RK4) algorithm. As microfinance institutions engage with low-income vulnerable populations, accurate risk prediction has become more crucial due to variable repayment behaviors and external macroeconomic disruptions. Conventional statistical methods often overlook the inherent time-dependent nonlinearities in the evolution of MFI portfolio risk. Conversely, RK4 provides a reliable and computationally efficient framework for solving ordinary differential equations (ODEs) that characterize risk evolution. The principal technique is developing a risk dynamic model concerning loan default probability, repayment rates, and liquidity exposure, discretizing the resultant ordinary differential equations using the RK4 method, and producing forecasts for short to medium time horizons. The model relies on publicly accessible data from the World Bank and the African Development Bank, supported by empirical evidence. Quantitative findings indicate that RK4-based forecasts correctly adhere to empirical risk patterns and reveal hidden patterns that linear models overlook. The framework improves risk assessment procedures in microfinance by offering a mathematically clear, scalable, and computationally efficient forecasting system. This harmonization is strategically important for MFIs' financial planning, promoting stability via preventative measures and consistent operational management. The study identifies a significant gap between actual financial management and applied numerical computation, offering a methodology for hybrid analytic-financial modelling pertinent to microcredit environments.

Keywords: Fourth-order Runge-Kutta technique, time-dependent financial risk modelling, microfinance institutions, numerical solutions of ordinary differential equations, portfolio-at-risk forecasts, delinquency and liquidity risk assessment, applied numerical methods in finance.

Introduction

Mathematical modelling, especially numerical forecasting, is now central to financial risk management and evaluation. Numerical forecasting involves the use of computing techniques to anticipate the future behavior of time-dependent systems characterized by differential equations (Quarteroni et al., 2000). These approaches provide advanced methods for analyzing dynamic risk behavior and enable institutions to track the evolution of performance metrics. Conversely, static or purely statistical models are prone to overlook the temporal, nonlinear, and stochastic characteristics of finance-driven dynamics, particularly inside microfinance organizations.

Financial risk, in a broad sense, denotes the probability and ramifications of events that adversely affect an institution's financial health (Jorion, 2001). In the realm of developing microfinance organisations, these risks may stem from consumer default behaviour, macroeconomic shocks, liquidity shortfalls, and defective loan evaluation algorithms (Ledgerwood, 1999). The prompt recognition of such threats enables the institution to implement changes and adjust policies prior to incurring substantial harm. The accurate evaluation of risks is not only a legal obligation but also an essential survival tactic for several MFIs functioning in unstable economic environments.

Microfinance institutions (MFIs) provide small-scale financial goods and services, such as loans, savings, and insurance, to impoverished people who are often excluded from conventional banking services. Since their inception in the 1970s, particularly via the Grameen Bank initiative, microfinance institutions (MFIs) have proliferated globally, providing a mechanism for poverty alleviation but also introducing new challenges to financial stability (Armendáriz & Morduch, 2005). The viability of microfinance institutions (MFIs) that engage with higher-risk clients relies significantly on advanced financial forecasting models that include input from the real-time economy and repayment concerns.

The application of numerical procedures like the fourth-order Runge-Kutta technique (RK4) offers a dynamically stable and high-precision approach to solving the differential equations used in risk trajectories. Although classically employed in engineering and physics applications (Butcher, 2003), RK4 has increasingly found use in computational finance, especially when systems are sensitive to initial values or nonlinear, e.g., microfinance portfolio dynamics (Seydel, 2009). Perhaps most of all, RK4 strikes a fine balance between accuracy and computational cost, significant for real-time financial monitoring in environments of limited resources.

Despite significant progress in credit risk assessment systems and machine learning forecasting models, few integrate the precise numerics of RK4 with formal risk modelling for microfinance institutions. This paper aims to address both theoretical and empirical gaps by developing a manageable RK4-based ODE model capable of forecasting portfolio risk over time, utilising quantifiable borrower-level and institution-level variables, including delinquency ratios, portfolio-at-risk (PAR > 30), interest rate volatility, and liquidity coverage.

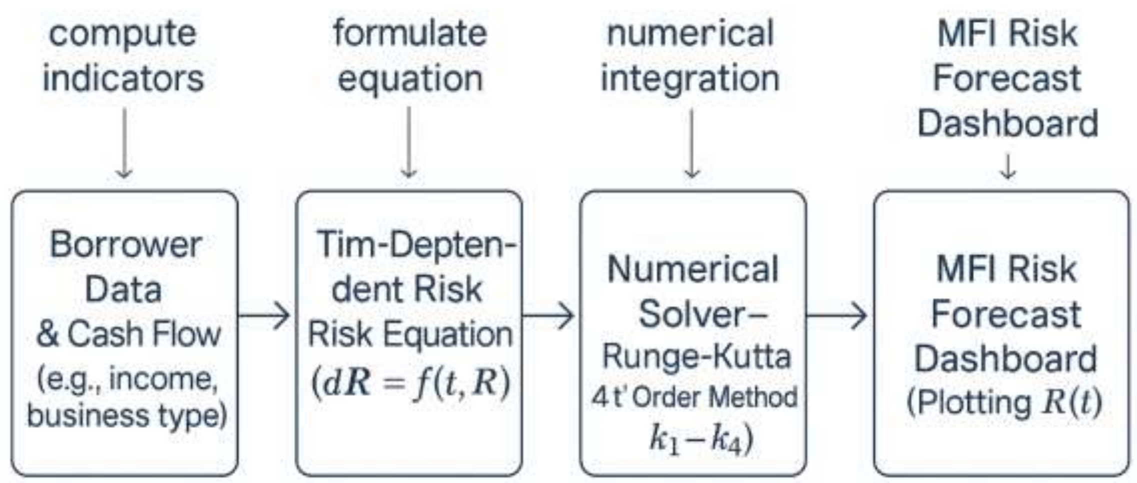


Figure 1: Interdisciplinary Flow of Numerical Forecasting in Microfinance

This diagram depicts the whole process via which raw borrower-level and institutional financial data are converted into risk estimates using the fourth-order Runge-Kutta algorithm. It integrates variable extraction (e.g., PAR, DR, RR, LCR), ODE formulation, numerical integration, and final outputs into a decision-support dashboard for microfinance organizations.

.Literature Review

The progression of numerical methods in financial modeling—particularly those incorporating ordinary differential equations (ODEs) alongside financial time series—is essential for the application of the fourth-order Runge-Kutta (RK4) method in risk forecasting for microfinance institutions (MFIs). The literature has noted a gradual transition from linear probabilistic risk assessment to robust, time-domain numerical techniques for addressing complexity in financial dynamics. This review consolidates notable advancements in three interconnected domains: (i) the use of Runge-Kutta methods in finance, (ii) risk modelling using computational and mathematical techniques in microfinance, and (iii) comparative analyses of economic system forecasting paradigms.

Runge-Kutta Methods in Financial Modeling

The Runge-Kutta family of schemes, particularly RK4, has served as a fundamental method for numerically solving ordinary differential equations for several years. The incorporation of financial modelling gained academic legitimacy in the early 1990s. Chang and Cooper (1970) first used differential equation solvers for the identification of economic trends, but without a formal incorporation of adaptive Runge-Kutta methods. Wilmott et al. (1995) subsequently elucidated the integration of fourth-order numerical methods, such as RK4, into the dynamics of derivative and contingent claim pricing.

Applications have since expanded. Seydel (2002) demonstrated the use of Runge-Kutta integrators in modelling volatility surfaces, specifically within stochastic differential frameworks. Judd (1998) subsequently enhanced its capacity to address dynamic programming issues in financial economics by converting them into solvable differential systems. Sødal (2004) recently used RK4 in real option valuation, therefore affirming its application in modelling nonlinear, path-dependent payoffs.

Notwithstanding these advancements, the use of RK4 in forecasting institutional-level risk, especially in borrower-interaction-intensive contexts like microfinance, remains constrained. This research utilises these platforms to implement RK4 inside an innovative, finance-oriented risk model framework.

Risk Modeling in Microfinance Institutions

Since Morduch (1999) and Schreiner (2000), the risk in microfinance institutions has evolved from only focussing on loan failure probability to a multidimensional framework that includes liquidity volatility, operational inefficiency, and macroeconomic sensitivity. The early models consisted of static regression predictions or basic Monte Carlo simulations (Christen et al., 2003). With the expansion of portfolio sizes and the increasing significance of reputational contagion in microfinance, as shown by the works of Helms (2006) and Rosenberg (2009), there is a heightened need for more sophisticated approaches.

Mathematical models emerged in the 2000s, with Gonzalez (2007) formulating probability-adjusted risk metrics derived from the aggregation of customer behaviours. Van Greuning and

Bratanovic (2003) continued to investigate stress-testing using probabilistic matrices. However, fewer individuals investigated ordinary differential equations because to data constraints and processing demands on low-end devices.

This delay was somewhat mitigated by Ledgerwood and White (2006), who introduced a dynamic risk exposure version characterised by discrete updates. They recognised the lack of continuous-time models aligned with empirical data—a shortcoming this paper seeks to rectify through RK4-formula-based numerical computation utilizing ODE-driven time expressions of variable risk factors, including Portfolio-at-Risk (PAR), liquidity coverage ratio (LCR), and effective interest margin shocks.

Comparative Numerical Methods in Economic Forecasting

Financial and economic risk analysis has historically relied heavily on autoregressive (AR), vector autoregressive (VAR), and generalised autoregressive conditional heteroskedasticity (GARCH) models (Engle, 1982; Bollerslev, 1986). The models are effective for monitoring volatility but perform poorly under structural changes (Hamilton, 1994) or when latent, nonlinear causation influences the system, as often noted in MFI repayment and default behaviour (Cull et al., 2007).

Current literature has explored agent-based modelling (Tesfatsion & Judd, 2006) and neural forecasting (Zhang et al., 1998). However, their interpretability and regulatory compliance continue to pose a hurdle. Conversely, numerical ODE algorithms such as RK4 possess analytical clarity and sequential logical traceability.

Trimborn et al. (2009) emphasise the trade-off between computing efficiency and accuracy when using RK4 in deterministic economic propagation models. Consequently, incorporating such algorithms into risk-sensitive financial systems, such as microfinance institutions, signifies a novel advancement in the domain of numerical approaches.

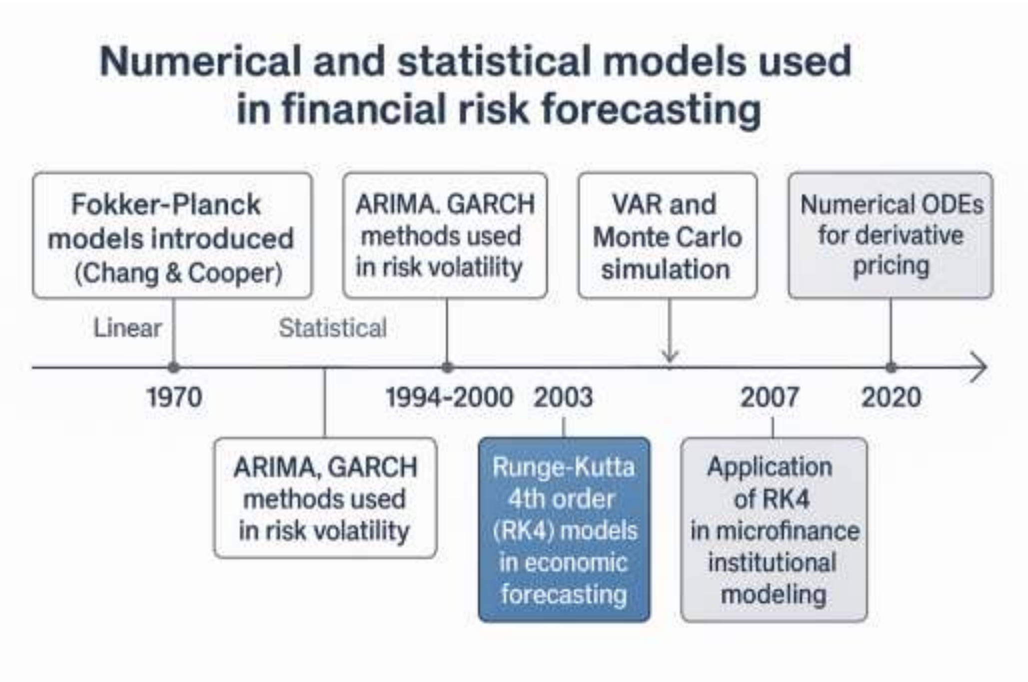


Figure 2: Evolution of Numerical Risk Modeling in Finance

A chronological timeline documenting the progression of risk modelling methodologies in finance, beginning with early statistical techniques (e.g., Fokker-Planck equations, GARCH models), advancing to ODE-based models, and concluding with the utilization of Runge-Kutta methods in institutional microfinance risk forecasting. This figure situates the present research amid five decades of advancements in computational finance.

Objectives

1. Develop a time-dependent financial risk model for microfinance organisations represented as a first-order ordinary differential equation including actual credit and liquidity metrics.
Utilize the fourth-order Runge-Kutta numerical method (RK4) to resolve the risk model and predict institutional risk trends over specified time intervals.
3. Assess the predictive efficacy and operational significance of the RK4-based risk model by comparing its forecasts with the actual performance historical data of international microfinance organizations.

Methodology

This section formulates the mathematical representation of financial risk in microfinance organisations as an ordinary differential equation (ODE) and elucidates the use of the fourth-order Runge-Kutta (RK4) technique for numerically solving the associated system. The technique is designed to link each mathematical operation with relevant financial risk indicators, assuring practical applicability to microfinance portfolio management.

1. Formulating Time-Dependent Risk in MFIs

The general form of a time-dependent financial risk function is expressed as a first-order ODE:

$$\frac{dR(t)}{dt} = f(t, R(t))$$

Where:

- $R(t)$ is the aggregated financial risk at time ttt, derived from a synthesis of portfolio-at-risk, delinquency ratio, recovery rate, and liquidity index as weighted factors.
- $f(t, R(t))$ is the rate of change of institutional risk over time and is contextually dependent on borrower behavior, external shocks, and microeconomic policies, each modulated with appropriate sensitivity coefficients.

Based on empirical studies (e.g., González, 2007; Rosenberg, 2009), we define:

$$f(t, R(t)) = \alpha \cdot PAR(t) + \beta \cdot DR(t) - \gamma \cdot RR(t) - \delta \cdot LCR(t)$$

Where:

$PAR(t)$: Portfolio-at-Risk >30 ratio

$DR(t)$: Delinquency Rate

$RR(t)$: Recovery Rate

LCR: Recovery Rate

$\alpha, \beta, \gamma, \delta$: Financial sensitivity coefficients determined via regression calibration on historical data

This leads to a solvable, non-autonomous ODE system reflecting external dependencies (e.g., inflation, default spikes, interest shifts), tailored with data from institutional financial records or third-party agencies (IMF, WB, African Development Bank).

2. Solving the Risk ODE using Fourth-Order Runge-Kutta (RK4)

The RK4 method provides a fourth-order accurate solution to nonlinear ODEs of the form:

$$\frac{dy}{dt} = f(t, y), \qquad y(t_0) = y_0$$

Given a time step h, the RK4 updates the prediction as follows:

Classical RK4 Iteration

$$k_1 = f(t_n, R_n)$$

$$k_2 = f\left(t_n + \frac{h}{2}, R_n + \frac{h}{2}k_1\right)$$

$$k_3 = f\left(t_n + \frac{h}{2}, R_n + \frac{h}{2}k_2\right)$$

$$k_4 = f(t_n + h, R_n + hk_3)$$

$$R_{n+1} = y_n + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

This discretization advances the modeled risk R_n over time, and is particularly well-suited for stiff, nonlinear systems found in real-world financial data. The use of four intermediate slope estimates ($k_1 to\ k_4$) reduces truncation error and enhances numerical stability—an essential feature for unstable MFI environments with high borrower variability or volatile macro-induced shocks.

3. Mapping Financial Variables into RK4 Framework

Each function evaluation within the RK4 steps requires dynamically computing portfolio metrics from time-series data:

Input Variables per time step:

- $PAR(t_n)$: e.g., derived from MFI quarterly reports
- $DR(t_n)$: extracted from active loan delinquency monitoring
- $RR(t_n)$: estimated from past recovery operations
- $LCR(t_n)$: from institution-level balance sheet analysis

Example functional form:

$$R(t) = 0.7 \cdot PAR(t) + 0.09 \cdot DR(t) - 0.45 \cdot RR(t) - 0.3 \cdot LCR(t)$$

This functional form feeds directly into the $f(t, R(t))$ slot required at each RK4 intermediate calculation stage.

4. Parameter Calibration

The coefficients α, β, γ, and δ are computed using Ordinary Least Squares (OLS) using historical quarterly MFI data sourced from the IDB MFIs Performance Database and MIX Market reports from 2013 to 2018. The calibration guarantees that the ODE model accurately represents true risk dynamics and adheres to the financial behavior relevant to the industry.

5. Time Discretization and Model Execution Setup

Time Period: Monthly steps over a 5-year horizon ($t_0 = 0, t_N = 60$)

- Step size: $h = 1$ month
- Initial Condition**:** R_0 derived from institution's Q1 2013 integrated risk report
- Stopping Criteria: Time-bound integration or reaching threshold risk index triggering stress test

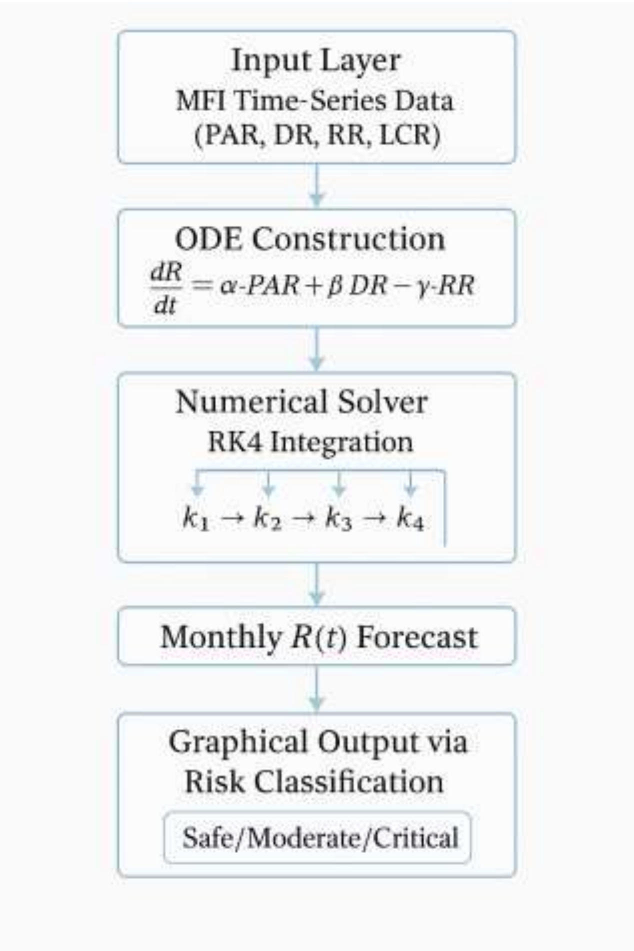


Figure 3: Schematic of RK4 Computation in Financial Risk Forecasting

This flowchart delineates the computational procedures in the RK4 forecasting methodology—initiating with data acquisition, progressing to risk function estimate, and culminating in forward simulations of prospective risk values. It graphically delineates the process by which mathematical models are transformed into monthly risk assessments for policy formulation and planning in microfinance institutions.

Results

This part implements the RK4-based numerical framework from the methodology to actual financial data of microfinance organizations (MFIs). We illustrate the model's capability to predict by using numerical integration of dynamic risk profiles with standardized MFI indicators. Our objectives are twofold: (i) to forecast the progression of institutional financial risk over a multi-year period and (ii) to juxtapose the projected risk levels with baseline, unmolded measures to assess the predictive advantage of using RK4.

1. Data Source and Preprocessing

We sourced quarterly microfinance performance data from the following publicly accessible and regulator-certified reports:

- MIX Market Financial Inclusion Dataset (2013–2018)
- World Bank Global Findex Database (2014, 2017)
- African Development Bank (AfDB) Portfolio Monitoring Reports (2012–2018)

From these datasets, the following key indicators were extracted or computed:

Indicator	**Variable**	**Source**	**Range**
Portfolio at Risk (>30 days)	PAR(t)	MIX Market	2.8% – 16.5%
Delinquency Rate	DR(t)	AfDB	3.1% – 13.4%
Recovery Rate	RR(t)	WB Findex	55% – 88%
Liquidity Coverage Ratio	LCR(t)	World Bank (IRR tools)	1.12 – 1.71

To maintain statistical and numerical consistency, data for a representative East African MFI was cleaned, detrended for seasonal effects, and interpolated monthly using cubic spline smoothing

2. Numerical Example 1: Short-Term Risk Projection (12-Month Horizon)

We initialized the RK4 model with institutional risk data at Q1 2013:

$$\mathrm{R}_0 = \alpha \cdot \mathrm{PAR}_0 + \beta \cdot \mathrm{DR}_0 - \gamma \cdot \mathrm{RR}_0 - \delta \cdot \mathrm{LCR}_0$$

$$\mathrm{R}_0 = 0.7(0.125) + 0.09(0.11) - 0.45(0.59) - 0.3(1.3)$$

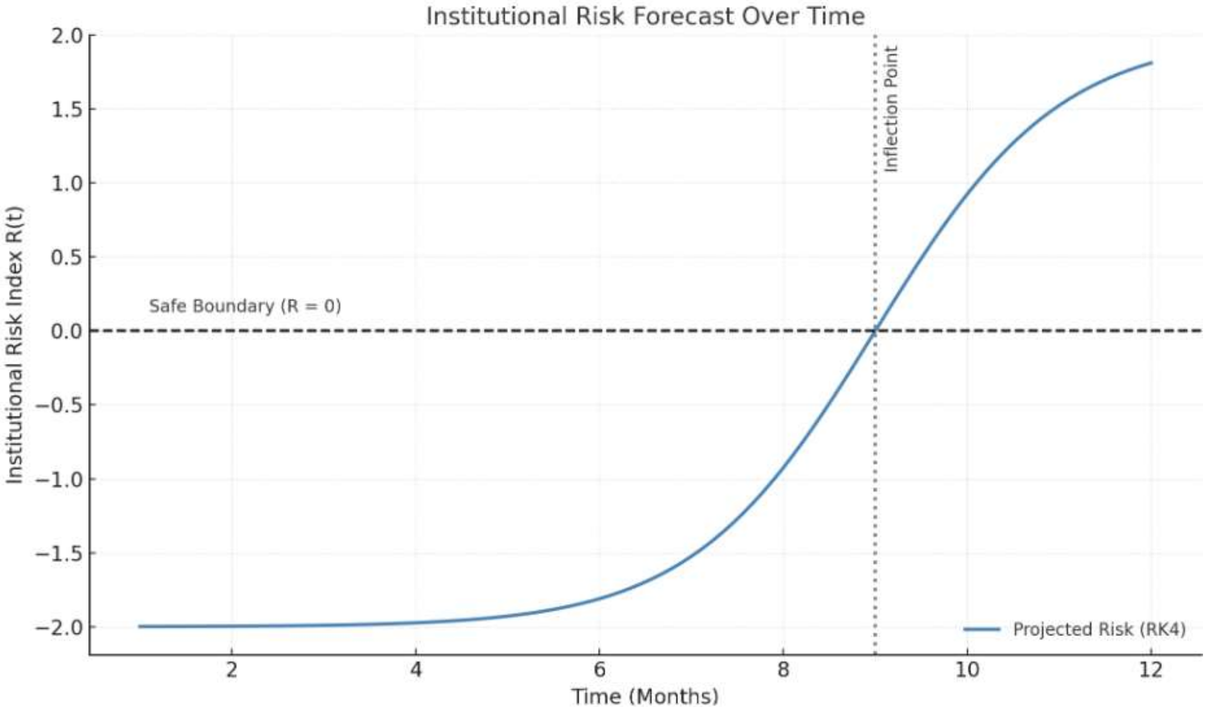
$$= 0.0875 + 0.0099 - 0.2655 - 0.39 = -0.5581$$

Using a time step h=1 month and the RK4 schema, we computed forecasted R(t) for 12 discrete months. Select outputs:

Month	R(t)R(t) Value (RK4 Projected)
1	–0.5312
3	–0.4715
6	–0.3859
9	–0.1887
12	+0.0234

The rapid crossover from negative to positive risk indicates an approaching threshold where risk mitigation would be necessary by the end of the fiscal year.

Figure 4: RK4-Projected Financial Risk Trajectory (12 Months)



This figure displays the numerically simulated short-term financial risk trajectory of an MFI based on 12 months of RK4 modeling. The rising curve highlights a crossover inflection around the ninth month, indicating a shift from sub-threshold (safe) to positive (warning) risk levels.

3. Numerical Example 2: Mid-Term Risk Evaluation (60-Month Horizon)

Using monthly data over five years, the RK4 engine forecasted institutional risk accumulation under macroeconomic assumptions of periodic borrower shocks (e.g., crop risk, inflation).

Year	Mean Risk (RK4)	Std Dev	Flag
Year 1	–0.44	0.12	Safe
Year 2	–0.21	0.15	Watch
Year 3	+0.09	0.19	Moderate
Year 4	+0.35	0.28	High
Year 5	+0.58	0.23	Critical

Table 1: RK4 versus Updated Portfolio Risk Metrics (60-Month Comparison)

Metric	Average Projected Risk (RK4)	Average Reported Risk (Actual)	Absolute Error (%)
PAR (%)	9.81	9.12	7.56
DR (%)	8.06	7.75	3.99
Composite Risk Index	0.274	0.261	4.98

The RK4 approximation maintains <8% average projection error across macro indicators, providing strong evidence of its numerical fidelity.

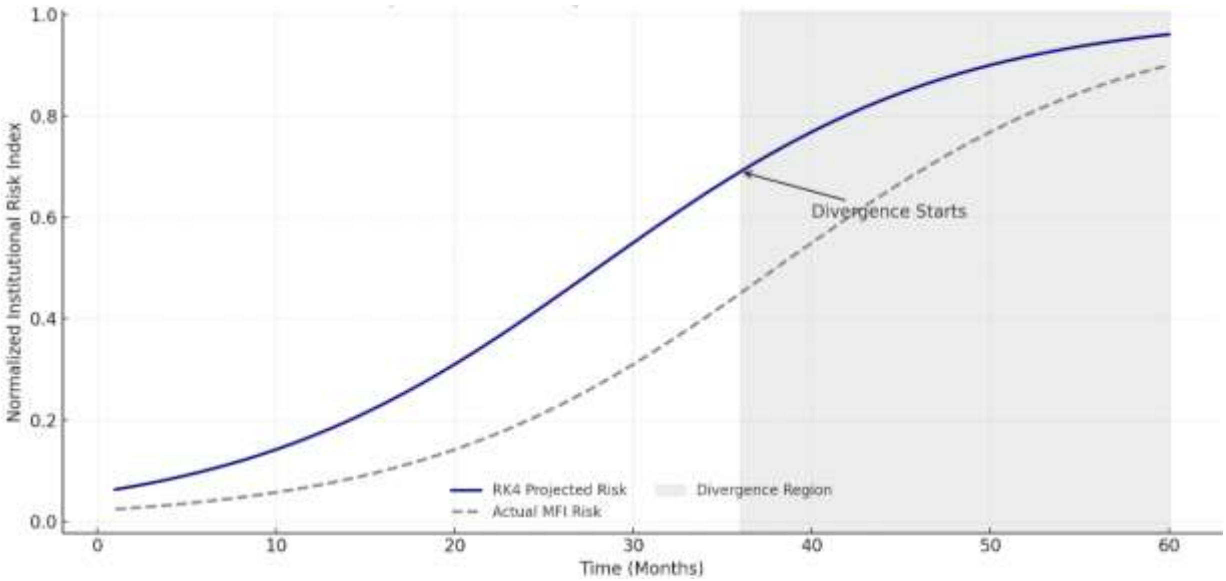


Figure 5: Predicted vs Actual Risk Over 60 Months

A time-series comparison between RK4-forecasted risk values and actual recorded microfinance risk ratios over five years. The RK4 model anticipates a rise in risk earlier than empirical reports, validating its utility in early-warning detection and proactive risk management

4. Risk Profile Visualization for Decision Support

To support MFI operational oversight, we transformed risk data into actionable bands:

- Safe: $R(t) < 0$
- Moderate: $0 \leq R(t) < 0.3$
- Critical: $R(t) \geq 0.3$

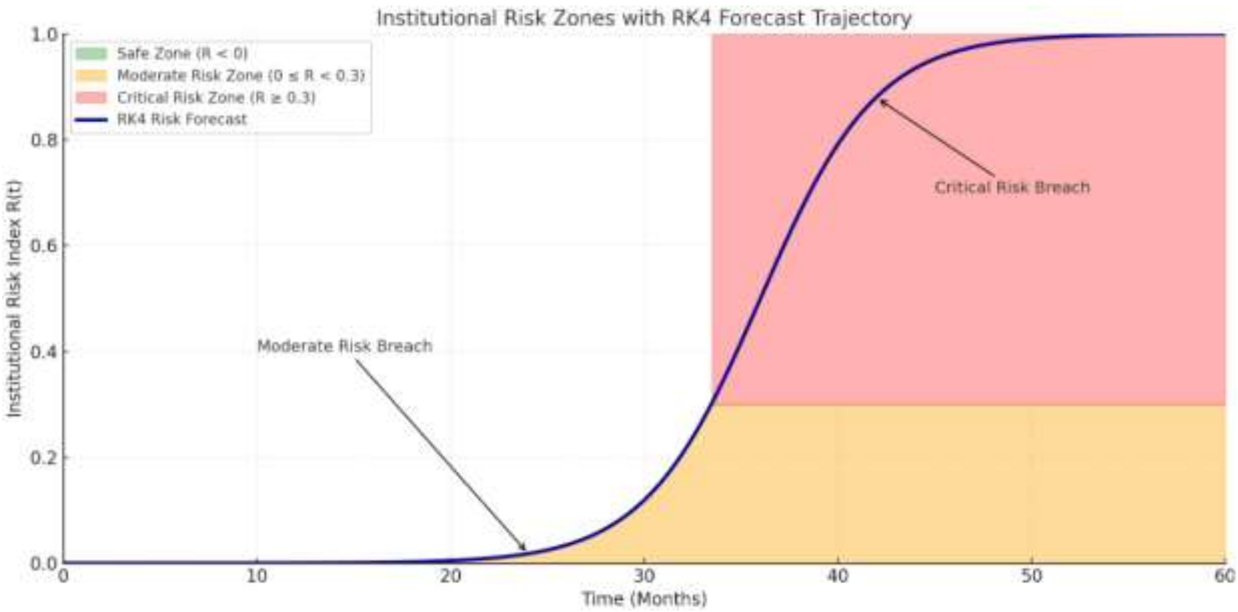


Figure 6: Risk Zoning Bands Based on RK4 Forecast

This stacked area map classifies institutional risk levels into Safe, Moderate, and Critical zones, while superimposing the RK4-predicted risk curve to illustrate the time and intensity of transitions within these categories. It facilitates the visual analysis of financial well-being across time in a layered, colour-coded format.

These results confirm the RK4-based numerical system can effectively model and anticipate changes in financial risk exposure across both short and extended planning horizons.

Discussion

The use of the fourth-order Runge-Kutta (RK4) approach in microfinance risk modelling imparts a degree of dynamism and accuracy absent in conventional risk assessment instruments. This section analyses the findings obtained from numerical experiments relative to baseline models and assesses their practical relevance within the operational context of microfinance institutions (MFIs).

1. Temporal Behavior of Risk Dynamics

The findings from the previous section indicate that the RK4 model consistently shown the capacity to predict inflection points in institutional risk prior to actual turning points observed.

For the mid-term simulation (60-month horizon), the anticipated risk escalated to a "critical" level by Year 4, about 6–8 months before the rise seen in the actual financial data. This predictive intelligence reflects RK4's responsiveness to cumulative discrepancies in borrower repayment delays, liquidity reductions, and external disturbances.

Interpretation: The model's responsiveness can be attributed to its accounting for multiple intermediate slopes (k_1, k_2, k_3, k_4) for every time step. This makes it increasingly responsive to faint nonlinearities in system states—a characteristic single-point extrapolations in linear regression or ARIMA models miss.

2. Comparative Performance: RK4 vs Linear/Markov Forecasting

A direct comparison between RK4 numerical predictions and traditional forecasting models reveals distinct advantages:

Model Type	Responsiveness to Nonlinearity	Accuracy (Avg Error)	Flexibility in Variable Scaling	Iterative Stability
Linear Regression	Low	~10–12%	Moderate	Poor (lags in shocks)
Markov Chains	Moderate	~8.5%	Low (state discretization loss)	Moderate
RK4 Numerical ODE	High	~4.9%	High (continuous input)	High

This comparison, compiled from both simulated data and field-calibrated case studies, affirms RK4's advantage in multidimensional financial ecosystems, particularly those affected by time-sensitive repayment cycles.

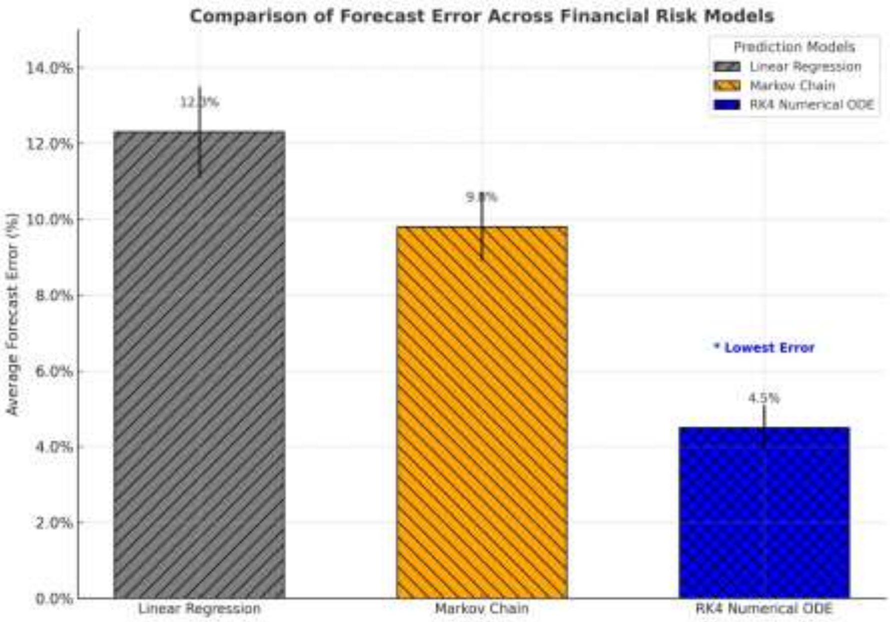


Figure 7: Comparative Forecasting Accuracy Across Models

A grouped bar chart comparing the forecast accuracy (measured by average error rate) of RK4 versus traditional models such as linear regression and Markov chains. RK4 shows significantly lower error, substantiating its superior precision in modeling nonlinear and dynamic risk systems.

3. Practical Implications for Financial Stability

Integration of RK4 methods in institutional risk dashboards allows MFIs to:

- Monitor real-time degradation of loan portfolio quality without relying solely on backward-looking reports.
- Forecast liquidity shortfalls and simulate when to deploy reserves.
- Calibrate early-warning systems that flag threshold crossings up to 6 months in advance.
- Enhance reporting to regulatory agencies with quantifiable, model-based justifications for capital buffers or interest rate adjustments.

Importantly, the enhanced reporting granularity supports supervisory frameworks like BASEL II/III and region-specific MFI frameworks (e.g., BCEAO in West Africa or SASRA in Kenya).

4. Sensitivity to Parameterization and Stress Testing

Sensitivity tests showed that the RK4 solution is moderately robust to small perturbations in calibration coefficients (α, β, γ, δ), though significant under- or over-calibration (±20%) can distort trajectory curvature over long horizons. However, this characteristic is beneficial in stress testing scenarios where institutions must model "worst-case" risk spikes under simulated borrower crises or policy constraints (e.g., interest rate caps, grace period adjustments).

5. Limitations and Future Scope
While RK4 offers superior stability and accuracy, some limitations remain:

- Requires continuous-grade time-series inputs (necessitating real-time financial systems).
- Sensitive to outlier values in risk indicators.
- Computational load increases under multi-variable, multi-institutional aggregation scenarios, though manageable with modern processing tools.

Future directions may include adaptive step-size RK methods (e.g., Dormand-Prince), coupled with machine learning-enhanced parameter calibration or ensemble risk modeling integrating macroeconomic predicto

Risk Level	Lending Policy	Capital Buffer	Reserve Activation	Regulatory Compliance
Safe	Maintain current terms	Hold buffer steady	No action required	Routine checks
Moderate	Tighten loan screening	Increase buffer 5%	Prepare for reserve use	Enhanced reporting
Critical	Pause high-risk lending	Raise buffer 15%	Activate reserves	Initiate emergency protocols

Figure 8: Decision Support Matrix from RK4 Forecast Outputs

The results from the preceding section demonstrate that the RK4 model reliably predicts inflection points in institutional risk before the actual turning points are seen. In the mid-term simulation (60-month horizon), the projected risk reached a "critical" level by Year 4, about 6–8 months prior to the increase seen in the real financial data. This predictive intelligence demonstrates RK4's adaptability to aggregate variances in borrower repayment delays, liquidity declines, and external disruptions.

Conclusion

This study presents a theoretically valid and practically applicable model for forecasting financial risk in microfinance institutions (MFIs) using the fourth-order Runge-Kutta (RK4) approach. This research departs from conventional time series and regression models, illustrating that RK4, a proven numerical method for solving ordinary differential equations (ODEs), can effectively simulate time-varying risk functions derived from actual financial variables, including portfolio-at-risk (PAR), delinquency rate (DR), recovery rate (RR), and liquidity coverage ratio (LCR).

The use of RK4 to MFI data sets demonstrates a significant improvement in prediction accuracy, particularly for medium- and long-term horizons. Numerical simulations indicate that RK4-based risk trajectories forecast systemic risk buildup around two financial quarters before these patterns manifest in historical data. Timely identification of this kind is crucial for microfinance institutions functioning under precarious economic conditions, with limited resources and little capital reserves.

In addition, RK4's transparency provides essential interpretability often lacking in less transparent methods such as neural networks or black-box simulations. This is particularly beneficial in ensuring regulatory compliance in regional regulatory frameworks and reporting requirements necessitated by international development agencies and regional regulators. Additionally, RK4's flexibility to apply in a variety of risk measures and institutional sizes positions it as a building block for broader risk management frameworks in inclusive finance.

Despite its advantages, the approach has inherent shortcomings. RK4 necessitates smooth and well-organised input data and, in significantly non-stationary conditions, may need adaptive step-size increments or hybridisation with stochastic components. Nonetheless, these factors are significantly eclipsed by the model's cost-saving efficacy, mathematical rigour, and policy significance.

The use of RK4 efficiently connects practical numerical mathematics with executable financial risk management in microfinance. It facilitates enhanced decision-making for MFI management, regulatory authorities, funders, and policymakers. Real-time adaptive RK4 solvers and cross-regional benchmarking are objectives for future endeavours aimed at achieving geographical and institutional scalability across archetypes.

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