

# Are Increasing Interest Rates From Leverage Consistent with Modigliani-Miller Arbitrage?

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*Among the most important contributions to the finance literature is Modigliani and Miller's seminal work on capital structure. As still taught in virtually all corporate finance textbooks, their famous Propositions I and II provide the foundation for the analysis of capital structure. Proposition I argues that under certain assumptions capital structure is irrelevant to the value of the firm. Proposition II describes the change in the required return to equity in response to a change in financial risk due to a change in leverage. In establishing Proposition II, Modigliani and Miller rely critically on an arbitrage process that forces the cost of equity to assume a value consistent with Proposition I. Although most of their analysis assumes constant debt rates, Modigliani and Miller allow that interest rates do vary with the level of leverage but that this variation does not affect the arbitrage process. In this paper, we argue that the arbitrage process is no longer dependable when debt rates change due to variation in leverage and discuss implications for teaching capital structure.*

## INTRODUCTION

Modigliani and Miller (1958<sup>1</sup>) establish two propositions concerning the capital structure decision, both of which are widely presented in textbooks as the foundation from which more advanced topics are examined. Proposition I argues that, under conditions of a perfect capital market (Table 1), capital structure has no impact on the value of the firm (p. 268<sup>2</sup>), implying that financial managers may rely on any mixture of debt and equity to finance the firm. Stated alternatively, the use of leverage has no effect on a firm's cost of capital (Figure 1). This relationship will be enforced by arbitrage opportunities between firms having the same level of business risk.

Leverage does, however, affect the firm's cost of equity (p. 268). Taking Proposition I as true, Proposition II states that the cost of equity linearly increases as leverage increases, reflecting the additional financial risk to the shareholder. Further, Proposition II states that the increase in required return for the equity stakeholder must be in direct relationship to the debt to equity ratio (p. 271). These relationships are summarized by Figure 1 and in equations (1) and (2).

$$WACC = kd * (D/V) + keL * (E/V) \quad (1)$$

$$\text{Where } keL = keU + (keU - kd)*D/E, \quad (2)$$

The variables used in the equations (1) and (2) include the following:

keL is the company's levered cost of equity.

keU is the company's unlevered cost of equity.

kd is the constant cost of debt.

D is the market value of the company's debt.

E is the market value of the company's equity such that

D+E is the market value of the company.

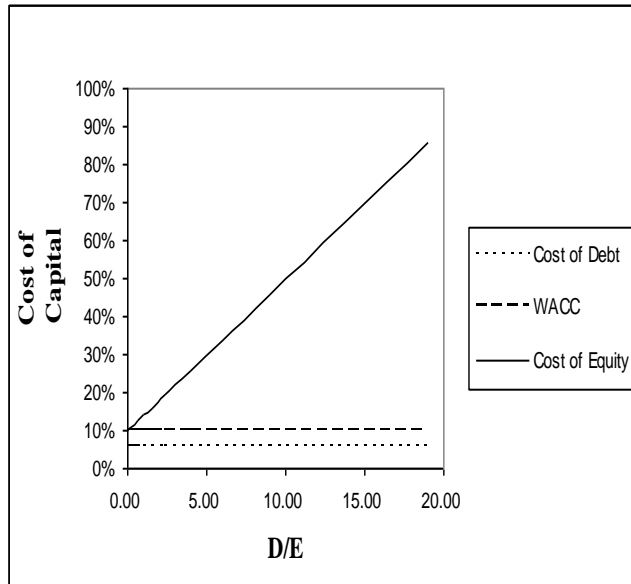
WACC is the company's average cost of capital.

Modigliani and Miller argue that a number of scholars had developed arguments similar to Proposition I but that their development of Proposition II represents a departure from previous works (p. 271). Previous economic theory, they argue, developed the cost of capital with a "cavalier" treatment of uncertainty (p. 263). Paradoxically, although they investigate the impact of risk from the viewpoint of the equity investor, they appear to be themselves a bit "cavalier" with regard to the impact of risk on the debt

## Table 1: Perfect Capital Market Assumptions

One possible set of assumptions related to Modigliani and Miller's paper is:

- Asset markets are frictionless (no transaction costs and information is costless and simultaneously available to all investors (i.e., no asymmetric information, no agency costs)).
- There are no market imperfections such as taxes (personal, corporate), regulations, and bankruptcy costs.
- The law of one price holds.
- Firms can be divided into risk classes based on business risk (volatility of operating income).
- There is one debt interest rate, investors and firms can borrow and lend at this rate, and all bonds are perpetuities.
- Operating earnings are not affected by leverage and are modeled as an expected no growth perpetuity (all profits are paid to the shareholders).

Figure 1 (Source: Lander and Pettengill (2010, p. 38)<sup>3</sup>)

investor. With regard to equity holders, Modigliani and Miller argue that the unlevered required return will vary with the risk level of the operating income (p. 267)<sup>4,5</sup> and that leverage will additionally affect the variability of the equity returns within classes of risk given operating income (p. 268). The exact response to an increase in risk from leverage on the required return for the shareholder is determined by Proposition II. Modigliani and Miller introduce an arbitrage process that is critical in supporting Proposition II and is described in more advanced corporate finance texts. This arbitrage process is the focus of our paper and is reviewed and examined below. We find that the process is not dependable when interest rates increase due to increased leverage.

Unlike their treatment of risk for the equity holder, as Modigliani and Miller develop Propositions I and II, they assume that all debt issues provide constant and certain income streams, as well as yield the same rate of return<sup>6</sup>, and the certainty of the debt income stream is not affected by the issuer (e.g., whether debt issued by a firm to purchase physical assets or debt issued by a household to buy shares) (p. 268). By implication, they assume the required return of the debt holder does not vary with the creditworthiness or the amount of leverage of the debt issuer.

After establishing Propositions I and II, Modigliani and Miller consider the impact of three qualifications. First, the impact of corporate taxes when interest payments are tax deductible. Second, the impact of the possibility that bond interest rates change with the issuer's credit worthiness and degree of leverage. Third, the impact of market imperfections that might interfere with the arbitrage process. Modigliani and Miller argue that, given certain additional assumptions, these qualifications do not affect their conclusions (pp. 272-281). In their 1958 paper, they conclude that taxes have no effect on Proposition I. They correct this position in their famous 1963 paper. As for the impact of multiple bond rates, they argue that, as long as the

interest rate function is the same for all borrowers, which they suggest is a reasonable assumption, Proposition I continues to hold as is and Proposition II changes only in that the relationship between the cost of equity and leverage is no longer strictly linear (p. 273). The impact of market imperfections is dismissed with the argument that these imperfections will be neither large nor systematic (p. 281).<sup>7</sup>

Our focus in this paper is on the second qualification, the impact of the possibility that bond interest rates change with the issuer's credit worthiness<sup>8</sup> and degree of leverage. We examine the impact of relaxing this assumption on the arbitrage process supporting the Modigliani-Miller propositions and discuss teaching implications of the result. The rest of the paper is organized as follows. The next section deals with the impact of relaxing the assumptions made about the impact of the cost of debt on Propositions I and II. We provide a numerical example that illustrates the results and may be suitable for classroom use. The subsequent section presents a way to teach capital structure that allows for a smoother transition from the more theoretical Modigliani-Miller propositions to the more "real world" tradeoffs. The last section offers a brief conclusion.

## RISK, LENDERS' REQUIRED RETURNS, AND THE MM PROPOSITIONS

The Modigliani-Miller (MM) propositions are developed with the presumption that the required return for equity investors varies with regard to both business risk and financial risk, but that the required return for lenders is not. Ignoring micro-structure effects of debt, three key issues challenge this assumption. First, just as equity investors will demand differing levels of return as business risk varies so should bond investors. Second, similarly, lenders should demand different levels of return depending on the credit standing of borrowers. This assertion directly challenges the assumption made in the Modigliani and Miller arbitrage argument that individual investors may borrow at the same rate as corporations. Third, a lender's required return will surely vary with the borrower's level of leverage. We investigate these in the order presented above and analyze each with respect to the assumption that the same debt interest rate applies across firms and individuals. When relaxing an assumption provides a critical challenge to the MM propositions, we developed arguments as to why and explore the teaching implications of our arguments.

The first challenge to a constant debt interest rate is that the required return for bond holders will certainly differ across firms with differing business risk. Corporations whose bond issues are provided with an AA rating borrow at a different rate than do corporate issuers whose bonds are given a BB rating. Recognition of this difference in lending rates does not, however, affect the development of the MM propositions. Modigliani and Miller recognize this difference with regard to equity investors and develop their arbitrage proof with the caveat that the arbitrage occurs within risk classes. Thus, relaxing this assumption only impacts their proof in terms of scale.

The other two challenges to the assumption of a constant debt interest rate provide more critical challenges to the arbitrage process that establishes the two MM propositions. If individual equity investors cannot borrow at the same rate as the corporation because lenders view them as inherently more risky or if different degrees of leverage create different required returns to debt, arbitrage is limited. As to the latter, Modigliani and Miller state that both theory and empirical observations indicate that borrowing rates tend to increase with the level of leverage (p. 273). They argue, however, given all borrowers face a common yield curve, Proposition II is impacted by this reality but Proposition I is not. In the remainder of this section, we first examine the validity of the presumption of equal borrowing rates and then examine the validity of the argument that varying borrowing rates due to leverage do not affect the validity of Proposition I.

The first question is: Can individual equity investors borrow at the same rate as can the corporation, as assumed when Modigliani and Miller develop their arbitrage argument? Modigliani and Miller defend this assumption on the basis that the rate on brokers loans, which is the rate at which arbitrageurs would borrow, had been similar to the rate on corporate loans (p. 274). Brealey, Myers and Allen (2008, p. 486) also defend this assumption, arguing that both home mortgage rates offered to individual home buyers and call rates on margin buying offered to individual investors compare favorably with rates paid by corporate bond issuers. Home mortgage rates, however, involve substantial collateral obligations on the part of the borrower. A more relevant question is whether the call rate at which equity investors would indeed borrow for purposes of equity investing is commensurate with the corporate lending rate. This comparison is also hampered with micro-market effects. The call loan requires margin. And the call loan is, well, callable. Thus, these rate comparisons are not sufficient to establish equality in borrowing rates. Individual equity investors cannot presume to borrow at the same rate as the corporation and, thus, the arbitrage process is not guaranteed.

The question then becomes: Are rates sufficiently different to make arbitrage implausible? When students are asked if they can borrow at the same rate as, say, IBM, they inevitably answer no, but this response misses the point. Some large equity investors, as Modigliani and Miller argue (p. 274), unlike most corporate finance students, may borrow at rates not substantially different than corporate rates. Indeed, some large equity investors may have higher credit standing than the corporations in which they are investing. One may argue, however, that equity investors should not be able to borrow at a lower rate to invest in the assets of a corporation than can the corporation itself. The argument that an equity investor can borrow at a rate sufficiently low as to provide arbitrage may not be certainly true, but it does not seem on the face to be a false assertion that the marginal equity investor can borrow at a sufficiently low rate to undertake the arbitrage activity necessary to validate MM I. Thus, we now turn to the final challenge to

the arbitrage proof and the central issue of our paper, the impact of the amount of leverage.

In the remainder of this section, we develop the argument that the impact of increased borrowing rates due to leverage could prevent the arbitrage process developed by Modigliani and Miller and illustrate our argument with a numerical example. We assume throughout that individual investors can indeed borrow at the same rate as the corporation and all profits are paid to the shareholders.

We believe this numerical example is appropriate for classroom presentation and present the example in three stages. In the first stage, we show returns to equity holders in a levered and unlevered firm where results are consistent with Propositions I and II. In the second stage, we disturb the equilibrium posited by Propositions I and II and show how the arbitrage process developed by Modigliani and Miller corrects the market mispricing and restores equilibrium. In the third stage, we again disturb the Proposition I and Proposition II equilibrium but, in this stage, we assume that the leverage of the levered firm is sufficiently high such that the cost of additional debt increases for the levered firm.<sup>9</sup> We show how the arbitrage process, in this situation, then cannot re-establish equilibrium.

In all three stages of the numerical example, an instructor could postulate two firms, Firm A and Firm B, both of which fall into the same class of business risk and so have identical expected return distributions. An additional assumption that could be used in all three stages is that the expected annual return for both of these firms, as held by all investors, is 8% on assets of \$1,000,000. The firms differ, however, in terms of their financing. Firm A is all equity financed. Firm B is financed only partially with equity. According to MM I, the firms must be priced to have equal value, and, if not, arbitrage activity would equate the value of the two firms. ROE, however, is different for the two firms, in agreement with MM II.

In stage 1 of the example, the instructor could illustrate the standard Modigliani-Miller equilibrium with the firms priced to have equal value. The instructor could postulate two consecutive years in which the firms get the same returns, as they should on average. Further, the example could assume that for both firms ROA is 10% (\$100,000 profit) in the first year and 6% (\$60,000 profit) in the second year, consistent with an expected return of 8%. The stage 1 example assumes market participants get it right: Firm A's equity is valued at \$1,000,000; Firm B's equity is valued at \$500,000. The total value of Firm B, including the \$500,000 debt, is \$1,000,000. Now the students could participate in calculating ROE.

For Firm A, ROE is easy. Given no interest payments, ROE is 10% in year 1 ( $100,000/1,000,000$ ) and 6% in year 2 ( $60,000/1,000,000$ ). For Firm B, additional work is required because of the leverage. And it may be well for the instructor to remind students by way of query that ROE is a percent determined by dividing the dollar returns available to the equity holders by total equity. For year 1, Firm B's ROE is 13% ( $(100,000 - 0.07 * 500,000) / 500,000$ ), which is

greater than Firm A's. The benefit of leverage! For year 2, however, Firm B's ROE is lower than Firm A's. Firm B's ROE is 5%  $((60,000 - 0.07 * 500,000) / 500,000)$ . The two-edged sword of leverage! The average return to equity for Firm A is 8% and for Firm B is 9%. Firm B's equity investors get a higher return, on average, to compensate for the greater risk inherent with the leverage. The instructor could complete stage 1 by showing that the results obtained are consistent with Proposition II.

**Table 2: Numerical Example Stage 1 Key Assumptions**

- All equity investors rationally price financial risk.
- Corporations borrow at a constant rate regardless of leverage.
- Firm A is all equity financed.
- Firm B is financed with 50% equity and 50% debt, and pays 7% for its debt.
- Individual investors can borrow at the same rate as the corporation.
- All profits are paid to the shareholders.

$$\begin{aligned} \text{ROE}_{\text{Firm A}} &= keU + (keU - kd) * D/E \\ &= 8\% + 0\% = 8\% \end{aligned} \quad (3)$$

$$\begin{aligned} \text{ROE}_{\text{Firm B}} &= keU + (keU - kd) * D/E \\ &= 8\% + (8\% - 7\%) * (0.50/0.50) = 9\% \end{aligned} \quad (4)$$

**Table 3: Numerical Example Stage 2 Key Assumptions**

- Some equity investors irrationally price financial risk, but other equity investors will benefit from this irrational pricing through arbitrage.
- Corporations borrow at a constant rate regardless of leverage.
- Firm A is all equity financed.
- Firm B is financed with 50% equity and 50% debt, and pays 7% for its debt.
- Individual investors can borrow at the same rate as the corporation.
- All profits are paid to the shareholders.

In stage 2 of the example, the instructor could illustrate the Modigliani-Miller arbitrage process by assuming a disturbance to equilibrium. The instructor could assume that, at the end of year 2, Firm A is still valued at \$1,000,000, commensurate with the expected return, but postulate that naïve equity investors behaved foolishly and overpriced the equity of Firm B because they observed the higher average return to equity and think there is a benefit to leverage. The example could assume that the value of Firm B's equity is irrationally priced at \$600,000 while the value of its debt remains at \$500,000.

Now the instructor could postulate an equity investor who owns 1% of Firm B and who has read and understands Modigliani and Miller. This investor knows that his stock is

overpriced relative to Firm A, so he conducts an arbitrage. He sells his shares in Firm B and receives \$6,000 from the sale  $(0.01 * 600,000)$ , has a really great party with \$1,000, and uses the remaining \$5,000, along with \$5,000 borrowed at 7%, to buy a 1% stake in Firm A. This action could be presumed to occur at the end of year 2, just before two more consecutive years of returns equal to those for year 1 and year 2. That is, both firms experience a 10% return on assets in year 3 and a 6% return on assets in year 4. With this 1% stake in Firm A, this astute arbitrageur will get \$1,000 in year 3 and pocket \$650 after paying off the hard-nosed lender who demands her \$350. Likewise he will pocket \$250  $(600 - 350)$  in year 4. Now other equity investors in Firm B catch on, and also sell their shares and buy shares in Firm A until equilibrium is restored with Firm B's equity again priced at \$500,000.

Next, ignoring the problem that the arbitrage process might overprice Firm A, the instructor could posit a second investor who is considering buying 1% of the equity of either Firm A or Firm B and then ask the students which investment would be preferred? The higher average return to equity of Firm B or the lower risk of Firm A? The example shows that, regardless of his preference, the investor is indifferent between the two. If he prefers the tradeoff in Firm B, he can borrow \$5,000 at 7%, use \$5,000 of his own money, invest the \$10,000 in Firm A, and receive exactly the same payoff as if he invested \$5,000 in Firm B. Likewise, if he prefers the safer return of Firm A, he can lend \$5,000 at 7%, invest \$5,000 of his own money in Firm B, and receive exactly the same payoff as if he had invested \$10,000 in Firm A. By borrowing or lending different amounts, this investor can achieve an infinite number of risk return tradeoffs using either firm. The end result is that investors are indifferent between owning 1% of Firm A or 1% of Firm B and the total value of Firm A and Firm B must be the same.

Thus far the example has supported the validity of the MM propositions. But now the example should continue by postulating a different equilibrium disturbing event. In the later part of their paper, Modigliani and Miller postulate that interest rates likely increase as leverage increases, but they argue that there is no basic change to Proposition I as the arbitrage is still alive. Investors just have to face a common yield curve with the firm (p. 273). But there is the problem. The common yield curve assumes that both the investor and the firm are at the exact same place with respect to their D/E ratios. But, if the required return to debt changes with the D/E ratio, the arbitrage falls apart, and, as the example continues, the students will see why.

In stage 3 of the example, the instructor could postulate that Firm B extends leverage to a total debt of \$600,000 (lowering total equity to \$400,000) with the last \$100,000 in debt issued at a higher rate of interest, say 7.5%. The instructor may want to point out that, although the firm's D/E has changed, the income stream from operations remains the same. Given this assumption, the students could again help calculate ROE for a 1% investment in the equity of Firm B given two more consecutive years with ROAs of

10% and 6%, respectively. For the next year, Firm B's ROE is  $14.375\% = ((100,000 - 0.07 * 500,000 - 0.075 * 100,000) / 400,000)$ .

**Table 4: Numerical Example Stage 3 Key Assumptions**

- Some equity investors irrationally price financial risk.
- Corporations with higher levels of leverage borrow at higher rates.
- Firm A is all equity financed.
- Firm B is financed with 40% equity and 60% debt, and pays 7% for the first \$500,000 borrowed and 7.5% for the last \$100,000 borrowed.
- Individual investors can borrow at the same rate as the corporation.
- All profits are paid to the shareholders.

For the following year, Firm B's ROE is  $4.375\% ((60,000 - 0.07 * 500,000 - 0.075 * 100,000) / 400,000)$ . The average return for Firm B is now 9.375%. More leverage, higher overall average return. No surprise here.

Continuing on, the example should now assume a third investor, Risky Ruby, who likes what has happened at Firm B. She thinks the increase in return to this average of 9.375% more than compensates for the added financial risk. She does not realize that 9.375% just adequately compensates for the increased level of financial risk. The question now is: Can she get that same return by investing in Firm A using 60% borrowed funds? The previous investors did using 50% borrowed funds when Firm B was 50% debt financed, and Modigliani and Miller assume she can, but now we have to make some serious assumptions about where she is on her yield curve. The following calculations will show the students that investors are no longer indifferent between investing in Firm A and Firm B. Personal leverage no longer assures the same return from the two firms.

There is no reason to assume that the previous investors and Ruby differ in terms of where they are in their levels of personal leverage. But, as long as the required return to debt changes as the level of personal leverage changes, we cannot be certain about the rate at which this risk-loving investor can borrow. The example could first ask what Ruby would earn from investing in Firm A if she did so by borrowing 60% of her invested funds at 7%, a rate reflecting her personal leverage. That is, if she borrows \$6,000 at 7% and also invests \$4,000 provided by her Uncle Harry, what is her return on the gift money from Uncle Harry?

To show her return, the instructor could use two more years of a 10% ROA followed by a 6% ROA. ROE calculations show Ruby's return in Firm A to be 14.5% in the first year  $((1,000 - 0.07 * 6,000) / 4,000)$  and 4.5% in the second year  $((600 - 0.07 * 6,000) / 4,000)$ . In both years, her return from investing in 1% of Firm A would be higher than the return from investing in 1% of Firm B. She is no longer indifferent between investing in Firm A and investing in Firm B. She may have the same leverage in her investment

in Firm A as Firm B, but she can still borrow at 7% because her personal leverage is different from the leverage of Firm B. She prefers investing in Firm A, and, if she is a typical investor, as is implicitly assumed in the arbitrage example, values for Firm A and Firm B will not be equalized. The market is saying that Firm B is too levered for a 7% cost of debt.

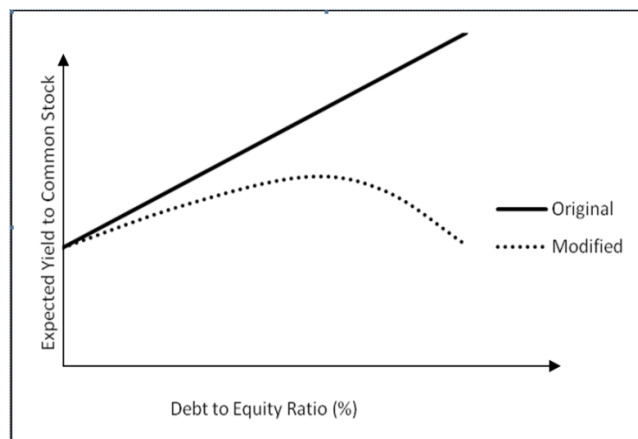
So far, the example has shown that if Ruby could borrow at 7% because her personal leverage allowed her to borrow at that rate, a rate less than paid by Firm B, she is no longer indifferent between Firm A and Firm B. Further, if other investors can duplicate Ruby's investing, these two firms will not have the same value. Firm B suffers from too much leverage. The stage 3 example could finish by considering the possibility that Ruby's lender is concerned with the percentage of the investment funds that she borrows (60%) and charges her accordingly. If, at the margin, this risk requires a 7.5% rate, as for Firm B, that is the rate she gets. Now what would Ruby's return look like if she was to borrow \$6,000 at 7.5% and also use the \$4,000 gift money to invest in Firm A? The instructor could walk the students through the calculations one last time. Her return would be 13.75% in the first year  $((1,000 - 0.075 * 6,000) / 4,000)$  and 3.75% in the second year  $((600 - 0.075 * 6,000) / 4,000)$ . This is definitely not a good plan. Ruby would earn a lower average return but with the same level of financial risk. If she wants more leverage than her investment in Firm A would provide, she best directly invest in Firm B. Again, the values of the firms are affected by the amount of leverage used because market borrowing rates change with the amount of leverage.

The only way Ruby is indifferent between investing in Firm A and investing in Firm B is if the lender charges her 7.083%, the weighted average of the rates for Firm B's two debt issues. But will the market lend to Ruby at 7.083%? The lender will charge Ruby either 7% based on her personal leverage or 7.5% based on the percentage of the investment funds that she borrows (60%) to invest in Firm A, which itself is 0% debt financed. If the market will not lend to her at 7.083%, and there seems to be no rational reason for it to do so, arbitrage may not re-establish equilibrium, MM I may no longer hold, and, thus, MM II may not adjust as Modigliani and Miller suggest.

We can look at the impact of higher borrowing rates from higher leverage by examining their impact on the cost of equity. Modigliani and Miller develop their propositions assuming that all debt issues "must yield the same rate of return" (p. 268). As noted early on, after developing their propositions, they consider three qualifications, one being that there are multiple debt interest rates. They state that both theory and empirical observations indicate that borrowing rates tend to increase with the level of leverage (p. 273). But then they argue that, as long as all borrowers face a common yield curve, Proposition I maintains and Proposition II changes in that the required return to equity for the levering firm that suffers an increase in the cost of debt tends to increase at a slower rate or even fall (p. 275) (Figure 2)12. Proposition II must change in this way

because of Modigliani and Miller's assertion that arbitrage can still work and, as a result, the overall cost of capital will be constant.

**Figure 2: MM Proposition II in D/E Space with Multiple Debt Interest Rates**



Source: Figure 2 above approximates Modigliani and Miller's Figure 2 (1958, p. 275).

According to Modigliani and Miller, the relationship between the Expected Yield to Equity and the Debt to Equity Ratio (%) could be represented by the modified curve, "although in practice the curvature would be much less pronounced" (p. 275).

But how can it be that the required return to equity falls with an increase in leverage? Returns to equity decreasing is even somewhat inconsistent with Modigliani and Miller's own statement that the required return to equity tends to increase with an increase in the use of leverage. Modigliani and Miller do provide an explanation, but it is contingent on arbitrage holding. They say, "Remember, however, that the yield curve of Proposition II is a consequence of the more fundamental Proposition I. Should the demand by the risk-lovers prove insufficient to keep the market to the peculiar yield-curve MD, this demand would be reinforced by the action of arbitrage operators." (p. 276)

Yet the numerical example presented above demonstrates that arbitrage will not always be able to restore equilibrium. Moreover, the required return to equity cannot decrease with increased leverage because financial risk increases with greater leverage. And, if financial risk increases with greater leverage, then, in response to an increased D/E, the required return to equity could decrease only if that higher D/E suddenly had less financial risk. Logically, just the opposite will be true. In the case of increasing required returns to debt at high levels of D/E, the risk of bankruptcy will adversely affect equity holders as well. Witness the recent extreme fall in the price of stocks, reflecting higher required returns, as firms in the automobile industry approached bankruptcy.

Our last example clearly shows that leverage matters if interest rates change with the D/E ratio, that Proposition I, as well as Proposition II, may be impacted by this reality,

and, if so, required returns to equity likely will not fall but increase, maybe even at an increasing rate. Now the question is how to relate this result back to Modigliani and Miller's original work? How would they have represented our last example? A good guess is that they would have said that Firm B has \$6,000 of debt at 7.083%. What is different in our example is that there are two issues of debt, each at a different rate, and so two perfect market assumptions are relaxed. First is the assumption of one constant and certain rate to debt. But second is that firms have only one issue of debt. This combination of multiple issues each at a different rate, which certainly is "real world," leads to the results above.

### TRANSITIONING FROM MM TO MORE "REAL WORLD" CONSIDERATIONS

Modigliani and Miller argue that increases in the required return to bond holders affects Proposition II but not Proposition I. We argue above that both propositions can be impacted when the cost of debt is allowed to vary. If one accepts our argument, then obviously there exist implications for teaching capital structure theory and practice. We suggest that recognizing the relationship allows for a smoother transition from the more theoretical MM propositions to the more "real world" tradeoffs.

In our experience, traditional finance textbooks that present the MM propositions typically transition to "real world" applications with somewhat of a disconnect. One example is Brealey, Myers and Allen (2008). Chapter 18, which asks the question "Does Debt Policy Matter,?" presents the Modigliani and Miller concepts, including showing a non-linear increase in the required return to equity as leverage becomes high, and concludes that "debt policy rarely matters in well-functioning capital markets with no frictions" (p. 496). It seems a bit incongruous, therefore, that the next chapter, Chapter 19, is titled: "How Much Should a Firm Borrow,?" It is not surprising that Chapter 19 does not include any of the cost of capital graphs used in Chapter 18 to present the Modigliani-Miller foundation, but students may wonder why they have studied Modigliani and Miller in Chapter 18 if there is no application when they study the question "How Much Should a Firm Borrow,?" in Chapter 19.

In an introductory corporate finance course, depending on the enrollment, the instructor may not want to introduce the Modigliani-Miller concepts when discussing capital structure. We suggest, however, that presentations of capital structure and cost of capital in upper division and graduate courses should indeed begin with MM I and MM II and that advanced students be provided an exercise, such as the numerical example above, showing the standard arbitrage opportunities when levered and unlevered firms are not valued equally, thus allowing the instructor the option to emphasize the concepts of financial risk for the equity investor and increased variability in returns to the equity investor as leverage increases.

After the arbitrage and financial risk concepts are discussed, the instructor may consider relaxing the key assumptions regarding debt and taxes. An exercise, such as the one in this paper, could be used to illustrate that arbitrage can fail when borrowing rates change with the degree of leverage. This may then be followed with an exercise to show the impact of taxes. Combining these two influences allows for the presentation of the tradeoff theory with graphical presentations showing the influence of both taxes and increasing interest rates as the required return to bond holders increases with increasing leverage and the chance for default.

The instructor may wish to take the opportunity to contrast the changes in the required return to equity and required return to bond holders as leverage increases. Does the required return to bond holders stay virtually the same over some range as leverage increases when the probability of default is quite low? If this is the case, can the same principle apply to equity holders? In the absence of arbitrage enforcing a particular relationship between the required return to equity and the debt to equity ratio, might not initial increases in the required return to equity holders be more moderate than under MM II because the investors do not notice the change in financial risk?

Such discussions could also facilitate micro-market examinations. To what extent can an equity investor borrow at the same rate as the corporation? Would arbitrage ever take place between levered and unlevered firms with similar levels of business risk? Or, are such differences insufficiently transparent for the operation of arbitrage in the "real world?" All of these discussions easily follow from a presentation of the MM propositions and allow for a guided transition to decisions that the students, when financial managers, will indeed make.

## CONCLUSION

For over fifty years, Modigliani and Miller's seminal work has stood as the starting point for the analysis of capital structure. Their famous Proposition I argues that, under certain assumptions, capital structure is irrelevant to the value of the firm and the use of leverage has no effect on a firm's cost of capital. Modigliani and Miller argue that this proposition stands even given the favored treatment of debt under the tax laws and increasing borrowing rates due to the possibility of financial distress as leverage increases. In their 1963 paper, they acknowledge that corporate tax laws do invalidate Proposition I. In this paper, we examined the impact of increasing borrowing rates due to leverage on the validity of Proposition I and argue that increases in leverage and the accompanying changes in the required returns to bond holders impact both Proposition I and Proposition II and influence the optimum capital structure. We suggest that recognition of this relationship should influence the teaching of capital structure because the presentation of capital structure theory in standard finance textbooks may inadequately bridge the Modigliani-Miller foundation and more "real world" considerations.

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## END NOTES

1. Unless otherwise noted, references to Modigliani and Miller's paper are to their 1958 paper.
2. Page references to selected statements are given to assist the reader in finding the original statements in the 1958 Modigliani and Miller paper.
3. Lander and Pettengill (2010) provide a review and numerical examples and show that, although the cost of equity is linear in D/E space, it is not linear in Debt Ratio space and that, in Debt Ratio space, the cost of equity increases rapidly at high levels of debt in accordance with MM I and MM II. They argue that most students and even faculty and practitioners think in terms of Debt Ratio space and may not notice that presentations such as shown in Figure 1 are in D/E space. This may allow the mistaken assumption that the cost of equity grows linearly with increasing debt ratios.
4. Modigliani and Miller (1958) define risk classes as being similar to industries. They say:  
We shall assume that firms can be divided into "equivalent return" classes such that the return on the shares issued by any firm in any given class is proportional to (and hence perfectly correlated with) the return on the shares issued by any other firm in the same class. This assumption implies that the various shares within the same class differ, at most, by a "scale factor." Accordingly, if we adjust for the difference in scale, by taking the ratio of the return to the expected return, the probability distribution of that ratio is identical for all shares in the class. (p. 266)
5. Recently Ghosh and Ghosh (2010) explore the impact of relaxing the assumption of constant operating income on Proposition I and II. We explore the Modigliani-Miller propositions leaving this assumption intact.
6. Modigliani and Miller's assumption about this debt interest rate is not clear. In some places, they assume this is the risk-free rate and, in others, they assume this is a rate commensurate with the business risk of the firm.
7. This argument is more extensively developed in Modigliani and Miller's 1959 reply to Durand (1959). Much of the discussion centers on margin and short-selling restrictions.
8. We do not purport to be the first to raise the issue concerning the equality of borrowing rates. Rose (1959) raises the issue but concentrates on the equality of share ownership in levered and unlevered firms, largely missing the argument developed as Proposition II.
9. In a classic article, Baxter (1967) explores the role of increased risk with leverage, but not from the viewpoint of conducting arbitrage. Rather, Baxter (1967) argues that the risk of ruin changes the assumption that the levered and

unlevered firms enjoy the same stream of operating income. The increased chance of bankruptcy decreases the expected cash flow to the levered firm due to expected bankruptcy costs. Thus, Baxter (1967) develops the tradeoff between the tax advantage and bankruptcy costs in determining the optimum level of leverage.

10. In their arbitrage example, Modigliani and Miller do not allow the investor in the overvalued firm to have a party. Instead the investor dutifully invests all of his proceeds from the shares of the levered firm into the purchase of the unlevered firm, borrowing enough to own a greater percentage stake in the unlevered firm. Durand (1959) in his examination of the arbitrage process allows the investor to keep excess funds. He simply does not specify the use of these funds. We agree with Durand that the investor in the levered firm disgorge his excess cash for two reasons. First, students rather appreciate the ability to have a really great party. Second, by investing only part of the proceeds, the investor from the unlevered firm has a debt to equity ratio equal to that which was originally established by the levered firm and which the levered firm maintains in equilibrium. The latter rationale is important to Durand's example.

11. In their example, Modigliani and Miller show equilibrium to be reestablished by selling pressure lowering the price of the overvalued levered firm. Elsewhere, they identify price changes in both stocks. In the former case, the question exists as to why buying pressure affects only one firm. In the other case, the question becomes, if the unlevered firm was originally priced to be in equilibrium, how would a new higher price also be in equilibrium? And what price shift would occur for all other firms of similar risk levels?

12. The authors wish to note that Figure 2 appears very similar to graphs depicting the tradeoff theory when corporate taxes and bankruptcy costs are jointly considered. Despite the similarity between the two graphs Figure 2 depicts a different concept altogether.

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