

Using Excel Spreadsheets to Teach the Time Value of Money

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Possibly the most important precursor of academic success for students of finance is an understanding of the mathematics of interest and the time-value of money. Although a considerable portion of their first finance course is usually devoted to this topic, students often proceed to their advanced courses with inadequate knowledge of this vital subject. In this article, we present and make available Excel spreadsheet templates that provide students with a nearly unlimited collection of problems and solutions to help them efficiently perfect their time-value skills.

INTRODUCTION

In an earlier issue of this journal (Followill, 2010) we described our efforts to instill in our Finance and Real Estate majors, through a series of extracurricular seminars, the time-value of money problem-solving skills they will assuredly need in their future careers. This effort has been quite successful, resulting in enhanced learning in our upper-level finance courses, making them far more enjoyable for our students and their professors alike.

In order to entice our students to voluntarily engage in a series of late afternoon seminars that promises to be hard work for zero academic credit, we decided to provide all materials at no cost, and to recognize our majors who passed a difficult exam with only one or no errors. But we knew it was even more important to present to our students a learning experience that *they* found to be valuable. We realized that if the seminars and provided materials were not designed and delivered in an efficient and effective manner, word of mouth among our majors would surely end our project at its very beginnings. Fortunately, our students delivered a positive verdict, and the Mathematics of Interest Seminars have been held each semester at the University of Northern Iowa for the past seven years.

Our methods, as we recount them in the 2010 article, rely on an equations-oriented, rule-based approach that requires our majors to understand the underlying mathematics of time-value. We guide our students through the development of the time-value formulas, solve numerous problems during the lectures, and provide written versions of unsolved and solved problems. We also rely heavily on the use of Excel templates that generate a nearly unlimited set of problems and solutions in the different areas of time-value.

The purpose of this article is to present these time-value templates and, more importantly, make them available for others to use via the *Journal of Instructional Techniques in Finance* website, <http://www.jfcr.org/jitfvols.html>.

THE TEMPLATES

We divide the Excel time-value templates into four categories:

- 1) The present and future value of a lump sum amount,
- 2) Annuities and perpetuities,
- 3) Interest rate manipulation, and
- 4) Complex problem solving.

In each of the categories we provide an Excel template that presents students with a nearly infinite collection of problems to solve. Hints, formulas, cash flow diagrams, time-value rules, and solution procedures are also provided within the templates. Students are encouraged to drill themselves and check their solutions and thought processes against the solutions provided for each problem.

The present and future value of a lump sum amount; introducing the “building-block” equation

We refer to Equation 1 as the *building-block equation* because everything in the mathematics of time-value derives from it: the equations for annuities and perpetuities, and the procedure for manipulating interest rates.

$$FV = PV(1 + i)^n \quad (1)$$

where FV signifies future value, PV is present value, i is a simple interest rate across a period defined by n , and n represents the number of periods. The template generates questions that require students to solve PV , FV , i , or n . For some problems, i is an annual rate compounded annually, but for other problems, i will be an annual rate compounded other than annually. Figures 1 and 2 present the template problems and solutions for solving for PV and FV .

Figure 1. Sheet 1 of Template 1; PV and FV Problems

The Time Value of Money Templates **Template 1: PV and FV of a Lump Sum**

Rule 1: The interest rate, i , and the period defined by n must correspond.

Find the Present Value of a Lump Sum

PROBLEM: What is the value today of \$8,142.00 to be received at the end of 16 years if the annual interest rate is 8.10% compounded weekly?

HINT: Use the BUILDING BLOCK EQUATION

$$PV = \frac{FV}{(1 + i)^n}$$

Find the Future Value of a Lump Sum

PROBLEM: If you deposit \$4,809.00 in an account today that pays 4.80% compounded quarterly, how much will you have in 10 years?

HINT: Use the BUILDING BLOCK EQUATION

$$FV = PV(1 + i)^n$$

The solutions are provided in the 'PV-FV Solutions' tab.

For new problems, press 'F9'.

Figure 2. Sheet 2 of Template 1; PV and FV Solutions

The Time Value of Money Templates

Find the Present Value of a Lump Sum

PROBLEM: What is the value today of \$8,142.00 to be received at the end of 16 years if the annual interest rate is 8.10% compounded weekly?

Solution:
The value of the lump sum today is \$2,230.10.

Since the annual interest rate is 8.10% compounded weekly, be sure to express i in weekly periods.

Find the Future Value of a Lump Sum

PROBLEM: If you deposit \$4,809.00 in an account today that pays 4.80% compounded quarterly, how much will you have in 10 years?

Solution:
The value of the lump sum in 10 years is \$7,749.53.

The same principle applies to this problem as well. REMEMBER, i and n MUST correspond.

When we teach the material presented in these templates we stress the first of our time-value rules:

Rule 1: The interest rate, i , and the period defined by n must correspond.

If, for example, i is a nominal (or stated) annual rate compounded monthly, the number of periods, n , must be expressed in months. Understanding this rule is vital to correctly solving for i and n as shown in figures 3 and 4.

Each template is designed so that a new set of questions is presented each time the student presses F9. The number of individual questions that can be generated is not infinite, but the probability that a student will see the same question twice is quite small.

Once students have attained a complete understanding of the building-block equation, they are ready to tackle annuities and perpetuities.

Figure 3. Sheet 3 of Template 1; Solving for i and n .

The Time Value of Money Templates		Template 1: Solving for i and n
Find the Interest Rate	Find the Number of Periods	
<p>PROBLEM: Suppose you want to have \$5,610.00 in 12 years. You currently have \$2,805.00 to invest. What annual interest rate must you earn on your investment to reach your goal? Assume quarterly compounding.</p>	<p>PROBLEM: You have set a goal to have \$3,426.00. You currently have \$1,645.00 to invest. An account promises 3.00% interest, compounded semi-annually. How long, in years, will it take for you to reach your goal?</p>	
<p>HINT: Use the BUILDING BLOCK EQUATION \implies $FV = PV(1 + i)^n$</p>		
<p>Remember, i and n must correspond; if i is compounded monthly, n must be expressed in months, etc. Find the rate or number of periods using this rule, then convert to an annual rate, or annual periods.</p>		
<p>The solutions are provided in the "i and n solutions" tab.</p>		
<p>For new problems, press 'F9'.</p>		

Figure 4. Sheet 4 of Template 1; i and n Solutions.

The Time Value of Money Templates	
Find the Interest Rate	Find the Number of Periods
<p>PROBLEM: Suppose you want to have \$5,610.00 in 12 years. You currently have \$2,805.00 to invest. What annual interest rate must you earn on your investment to reach your goal? Assume quarterly compounding.</p>	<p>PROBLEM: You have set a goal to have \$3,426.00. You currently have \$1,645.00 to invest. An account promises 3.00% interest, compounded semi-annually. How long, in years, will it take for you to reach your goal?</p>
<p>Solution: The annual interest rate is 5.82 % compounded quarterly.</p>	<p>Solution: The number of periods is 24.64 years.</p>
<p>Since interest is compounded quarterly, then n must be expressed in quarterly periods. Now, convert to a stated annual rate compounded quarterly.</p>	<p>Since interest is compounded semi-annually, your solution for n will be in semi-annual periods. Now convert to years.</p>

Annuities and perpetuities

Our second template introduces our students to annuities and perpetuities. We present only the equations for ordinary annuities at this point.

The equation for the present value of an annuity is:

$$PVA = pmt \left(\frac{1 - \frac{1}{(1 + i)^n}}{i} \right) \tag{2}$$

where pmt is the annuity payment. While n is still the number of periods for an ordinary annuity, we stress that n is *always* the number of annuity payments.

The equation for the future value of an annuity is

$$FVA = pmt \left(\frac{(1 + i)^n - 1}{i} \right) \tag{3}$$

The equation for the present value of a perpetuity is:

$$PV_{\infty} = \frac{pmt}{i} \tag{4}$$

The rules we stress for ordinary annuities represented by the above equations are:

Rule 2: Pay attention to the time-line. The PVA of an annuity equation (or perpetuity equation) consolidates the stream of equal payments into a lump sum amount *one*

period before the first payment is made. The FVA of an annuity equation consolidates all payments at *the point in time the last payment is made.*

Rule 3: For annuities, *n* is the number of payments—**ALWAYS.**

Figure 5. The Interest Rate Manipulation On/Off Switch.

Interest Rate Manipulation Option

For problems that WILL NOT require Interest Rate Manipulation, place an X in the red box.

For problems that MAY require Interest Rate Manipulation, leave the red box blank.

Figure 6. Sheet 1 of Template 2; PVA and FVA Problems.

The Time Value of Money Templates *Template 2: PV and FV of Annuities; PV of a Perpetuity*

Rule 2: The PV of an annuity equation consolidates the stream of equal payments into a lump sum amount one period before the first payment is made. The FV of an annuity equation consolidates all payments at the point in time the last payment is made.

Rule 3: For annuities, *n* is the number of payments—ALWAYS.

Find the Present Value of an Ordinary Annuity

PROBLEM: What is the value today of a stream of 25 equal, semi-annual payments of \$1,500 if the nominal interest rate is 4.30%, compounded quarterly. Assume the first cash flow occurs six months from today.

HINT: Use the PVA EQUATION ==>

$$PVA = PMT \left[\frac{1 - \frac{1}{(1+i)^n}}{i} \right]$$

The solutions are provided in the 'PVA-FVA Solutions' tab.

For new problems, press 'F9'.

Find the Future Value of an Ordinary Annuity

PROBLEM: What is the value in period 13 of a stream of 13 equal, quarterly payments of \$3,250 if the nominal interest rate is 11.20%, compounded monthly. Assume the first cash flow occurs one quarter from today.

HINT: Use the FVA EQUATION ==>

$$FVA = PMT \left[\frac{(1+i)^n - 1}{i} \right]$$

Interest Rate Manipulation Option

For problems that WILL NOT require Interest Rate Manipulation, place an X in the red box.

For problems that MAY require Interest Rate Manipulation, leave the red box blank.

Figure 7. Sheet 2 of Template 2; PVA and FVA Solutions

Find the Present Value of an Ordinary Annuity

PROBLEM: What is the value today of a stream of 25 equal, semi-annual payments of \$1,500 if the nominal interest rate is 4.30%, compounded quarterly. Assume the first cash flow occurs six months from today.

Solution:

The value today of the cash flow stream is \$28,737.27.

Find the Future Value of an Ordinary Annuity

PROBLEM: What is the value in period 13 of a stream of 13 equal, quarterly payments of \$3,250 if the nominal interest rate is 11.20%, compounded monthly. Assume the first cash flow occurs one quarter from today.

Solution:

The value of the cash flow stream in period 13 is \$50,212.12.

Remember, if the compounding frequency differs from the cash flow frequency, you must modify the interest rate to match.

When we first introduce annuities to our students, we use the version of our second template that is designed to produce problems where the given interest rate fits exactly the time between payments. After we introduce interest rate manipulation in our third template we encourage students to return to the annuities template and switch to the version that requires interest rate manipulation. Figure 5 shows the switch that appears in Template 2 and in Template 4, the complex problems template.

Figures 6 and 7 show the problem and solution sheets for the PVA and FVA of ordinary annuities. Notice that in figure 6, the interest rate manipulation option is in force and the given interest rate does not fit the time span between payments. Students will have to manipulate the given

interest rate to find an effective rate consistent with the time between payments.

Sheets 3 and 4 of Template 2 present problems and solutions for the present value of a perpetuity. Since these pages are similar to those shown in figures 6 and 7, we do not present them here.

Interest rate manipulation

Interest rate manipulation may be necessary in order to compare two rates with different compounding periods, or to calculate an effective interest rate for use in the annuity and perpetuity equations. The equation for calculating an effective interest rate across any time span is:

$$EffectiveRate = \left(1 + \frac{Stated\ Annual\ Rate}{Compounding\ Periods\ Per\ Year} \right)^{\frac{Compounding\ Periods\ Across\ the\ Time\ Span}{}} - 1 \tag{5}$$

Figure 8. Sheet 1 of Template 3; Interest Rate Manipulation Problems

The Time Value of Money Templates
Template 3: Finding equivalent interest rates

There are two reasons why you might want to manipulate interest rates.

The First Reason is to make a comparison between two rates with different compounding periods.

The Second Reason for interest rate manipulation occurs when you are confronted with a problem involving an annuity where the given interest rate does not fit the time between payments. To use your annuity equation, you must manipulate the given interest rate to find an effective rate that fits the time span between annuity payments.

Which of these two rates will produce the greater Future Value?

A) 4.80% annual interest compounded annually

OR

B) 4.75% annual interest compounded semi-annually

To answer this question, find the effective annual rates (EARs).

You wish to find the Present Value of an annuity of semi-annual payments. The given interest rate is 6.20% compounded monthly. What effective semi-annual interest rate will you use in the annuity equation?

The solutions are provided in the 'Effective Rate Solutions' tab.

For new problems, press 'F9'.

Figure 9. Sheet 2 of Template 3; Interest Rate Manipulation Solutions

The Time Value of Money Templates

Which of these two rates will produce the greater Future Value?

A) 4.80% annual interest compounded annually

OR

B) 4.75% annual interest compounded semi-annually

You wish to find the Present Value of an annuity of semi-annual payments. The given interest rate is 6.20% compounded monthly. What effective semi-annual interest rate will you use in the annuity equation?

Solution:

A) The effective annual rate (EAR) for 4.80% annual interest compounded annually is 4.800%.

B) The effective annual rate (EAR) for 4.75% annual interest compounded semi-annually is 4.806%.

Rate B will produce the larger Future Value.

Solution:

Given the interest rate is 6.20% compounded monthly, the effective semi-annual interest rate is 3.140%.

Effective Interest Rate across any time span =

$$\left(1 + \frac{Stated\ Annual\ Rate}{Number\ of\ Compounding\ Periods\ per\ Year} \right)^{\frac{Number\ of\ Compounding\ Periods\ Across\ the\ Time\ Span}{}} - 1$$

$$EAR = \left(1 + \frac{Stated\ Annual\ Rate}{Number\ of\ Compounding\ Periods\ per\ Year} \right)^{Number\ of\ Compounding\ Periods\ per\ Year} - 1$$

Manipulating interest rates to fit a desired time period often proves to be a difficult concept for students to master. A key to understanding interest rate manipulation is to note the similarity between Equation 5 and the building block equation, $FV = PV(1+i)^n$, and the importance of Rule 1. In both Equation 1 and Equation 5, the interest rate and the period defined by n must correspond. Since students have now been introduced to annuities, we expand Rule 1 as follows:

Rule 1 (addendum): i and n must correspond. For the building block equation, the period for i is given, and n defines the number of periods between PV and FV . For annuities, the appropriate periodic rate should be used. Since the time between payments is given, i may have to be adjusted to fit the time period.

After some practice using the interest manipulation template, students are directed to return to the annuities and perpetuities template and engage the switch that requires interest rate manipulation in order to use the annuity equations.

Figures 8 and 9 present the problem and solution pages for interest rate manipulation.

Complex problem solving

Once students have mastered the basics of time-value, it is time to apply them to solving the complex problems presented in our final template. We follow the conventions of our previous templates, giving the student the option, when possible, of selecting problems that do or do not require interest rate manipulation. As with all of the templates, pressing F9 reconfigures each problem type with respect to the interest rate, the number and size of annuity payments, and timing of the cash flows.

A solution to each problem is presented on the adjacent sheet, along with a cash flow diagram, and a possible solution procedure. Problem-solving hints and the inviolate rules of time value that apply to each problem are also given as well.

We stress to students that there are usually several straightforward methods to arrive at the correct solution to any time-value problem, and that *the key to solving complex time-value problems* is to move all the money to a singular point in time. Once that is accomplished, the solution for the unknown, whether it is a lump sum amount, the payment size, or the interest rate, should be readily apparent.

For success in solving complex time-value problems, we stress our final time-value rule.

Rule 4: NEVER compare amounts of money unless they are at the same point in time.

The collection of complex problems is extensive and students who can work all of them should have little difficulty solving any time-value problem they may later encounter.

Figures 10 and 11 present just the first of the twenty complex problems found in our fourth template. Each problem is first presented on a question sheet. We encourage students to try to work the problem presented to them before moving to the next sheet which presents the solution.

Each problem is followed by a solution sheet that presents one possible solution procedure along with hints and problem solving suggestions. Each of the twenty problem formats has numerous permutations that may be accessed by pressing the F9 recalculation key.

Figure 10. Sheet 1 of Template 4; The First of Twenty Complex Time-Value Problems.

<i>The Time Value of Money Templates</i>	<i>Template 4: Complex problems</i>
<p>Q1</p> <p>PROBLEM: You have decided to make equal, annual deposits of \$602.00 to an account that pays 9.00% compounded monthly. You plan to make a total of 12 deposits. The first deposit will be made today and the final deposit will be made at the end of year 11.</p> <p>At the end of year 11, you will make the first of 10 annual withdrawals of \$662.00. How much money will remain in the account at the end of year 20 when the final withdrawal is made?</p>	
<p>Interest Rate Manipulation Option</p> <p>For problems that WILL NOT require Interest Rate Manipulation, place an X in the red box.</p> <p><input type="checkbox"/></p> <p>For problems that MAY require Interest Rate Manipulation, leave the red box blank.</p>	
<p>The solutions are provided in the 'A1' tab.</p>	
<p>For new problems, press 'F9'.</p>	

Figure 11. Sheet 2 of Template 4; A Solution to the First Complex Time-Value Problem

The Time Value of Money Templates

A1

PROBLEM: You have decided to make equal, annual deposits of \$602.00 to an account that pays 9.00% compounded monthly. You plan to make a total of 12 deposits. The first deposit will be made today and the final deposit will be made at the end of year 11.

At the end of year 11, you will make the first of 10 annual withdrawals of \$662.00. How much money will remain in the account at the end of year 20 when the final withdrawal is made?

HINT: You need to determine a value in the FUTURE. Thus, transform all cash flows into Future Values at the end of year 20.

$$FVA = PMT \left[\frac{(1+i)^n - 1}{i} \right]$$

Note to Students: The computer does not round in its internal calculations. The rate of 9.38% shown at left is actually 9.38069%. If you use 9.38%, your answer will be slightly lower than the computers answer even though your procedure is correct.

Solution:

The effective annual interest rate is 9.381%.

The value of the deposits in year 11 is \$12,403.86.

The value of the deposits in year 20 is \$27,798.59.

The value of the withdrawals in year 20 is \$10,242.30.

The amount of money remaining in year 20 is \$17,556.29.

CASH FLOW DIAGRAM

The diagram shows a bar chart with 'Cash flow' on the y-axis (ranging from -\$1,000 to \$1,000) and 'Years' on the x-axis (ranging from 0 to 20). Green bars representing deposits of \$602 are shown for years 0 through 10. Red bars representing withdrawals of \$662 are shown for years 11 through 20. Ellipses indicate intermediate years.

CONCLUSION

As the reader has already ascertained, we take a minimalist approach to teaching time-value to our students. We do not introduce some fairly basic topics such as annuities due, or conventions such as sinking fund loans, and we eschew more advanced topics such as growing annuities and perpetuities, gradient or step function annuities, or continuous compounding. Our entire motivation is to provide students with a basic understanding of time-value so that when they encounter topics such as the dividend growth model, duration, and the option pricing models in their higher level finance classes, they are quickly able to understand their presentation and grasp their meaning. When students have achieved this level of competence, teaching becomes much easier and vastly more rewarding.

We strongly believe in taking an equations-oriented, rule-based approach to teaching the mathematics of interest and the time-value of money, and we look upon the financial calculator-based instruction we see in many textbooks with some dismay. Financial calculators are useful, time-saving devices, and nearly all of our students who become adept at time-value learn to use the TVM buttons on their own. Almost invariably, however, when we

put a time-value problem on an exam along with the usual admonishment, “show your work,” and receive drawn pictures of the buttons and entries used to solve the problem, the answer is wildly incorrect.

At the University of Northern Iowa we have had measurable success instilling the basics of the time-value of money in our students. We gladly make these Excel templates available for other finance professors to use and improve upon. The Excel spreadsheets are available at www.jfcr.org. Comments regarding the templates are appreciated and may be directed to richard.followill@uni.edu and brett.olsen@uni.edu.

REFERENCE

Followill, R. (2010) *Time Value of Money Seminars: Immersion Therapy for Finance Majors*, Journal of Instructional Techniques in Finance, 2:1 9:12.

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