

Illustrating Multiple IRRs Using Excel and Financial Calculator iPhone Applications

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In recent years, the capability of handheld computing devices has developed into a compelling argument for calculator applications. This article summarizes two calculator applications for the iPhone, the Texas Instruments BA II Plus and the Hewlett Packard 10B II. The obvious advantage to the user for both applications is cost. The BA II Plus app is \$5.99 and the Hewlett Packard 10B II app is 14.99, both substantially less than the equivalent calculators. The HP app discussed here, though, substantially extends the base calculator's capability. We illustrate this capability in an exposition of the multiple IRR problem in capital budgeting using Excel, and then using the two calculator applications. The extended capability of the iPhone applications is equally applicable to many other exercises in finance and other disciplines.

NONCONVENTIONAL CASH FLOWS AND THE MULTIPLE IRR PROBLEM

“Conventional” project cash flows are defined as cash flows that consist of one or more net outflows followed by net inflows in all subsequent periods. In this case below, the project under consideration will have a single IRR. On rare occasions, however, projects may have what are referred to as “nonconventional” cash flows. Nonconventional cash flows are defined as those where a net outflow occurs sometime after the net inflows have commenced. Stated differently, if the sign change (negative to positive or positive to negative) occurs more than once, then the cash flows are said to be nonconventional. In this atypical case, the project under consideration may have multiple IRRs. Examples:

Conventional					
-	+	+	+	+	+
-	-	-	+	+	+
Yr 0	Yr 1	Yr 2	Yr 3	Yr 4	Yr 5

Nonconventional					
-	+	+	+	+	-
-	+	+	-	+	+
+	-	+	+	+	-
Yr 0	Yr 1	Yr 2	Yr 3	Yr 4	Yr 5

An example of a project that may have nonconventional cash flows is the case of a paper manufacturer and the deforestation that occurs as a result of their business operations. If operating in the United States, the firm will need to repair the landscape and plant new trees in the place of the trees that were removed. The process of restoring the land is likely to be very expensive, thereby causing a net cash outflow in that year, usually the last year of the project. A hypothetical set of cash flows that captures the essence of this scenario could look like this

Project A	
Year	Cash Flow
0	\$ (1,000,000)
1	\$ 750,000
2	\$ 750,000
3	\$ 750,000
4	\$ (1,260,000)

Given these project cash flows, what is the IRR? Equation 1 states that the IRR is the discount rate that forces the NPV to equal zero. However, because equation 1 is a polynomial of degree n, it will have a maximum of n solutions. In fact, an nth degree polynomial can have as many solutions as the number of cash flow sign changes, however, having multiple sign changes does not mean that you will have multiple IRRs; it simply means that you might.

Equation 1.

$$NPV = 0 = CF_0 + \frac{CF_1}{(1+IRR)} + \frac{CF_2}{(1+IRR)^2} + \frac{CF_3}{(1+IRR)^3} + \dots + \frac{CF_n}{(1+IRR)^n}$$

For our example:

$$0 = 1,000,000 + \frac{750,000}{(1+IRR)} + \frac{750,000}{(1+IRR)^2} + \frac{750,000}{(1+IRR)^3} + \dots + \frac{-1,260,000}{(1+IRR)^n}$$

To assist in finding the IRR of this project, we calculate the NPV at various discount rates in order to create a NPV profile. We use rates beginning at 0% up to 34% in increments of 1%. Figure 1 shows the resulting NPVs as well as the NPV profile. From this we notice that the NPV is zero when the discount rate is 2.34% and again when it is 12.26%. In other words, there are two IRRs for this project. Given this scenario, which IRR should the decision maker use to decide

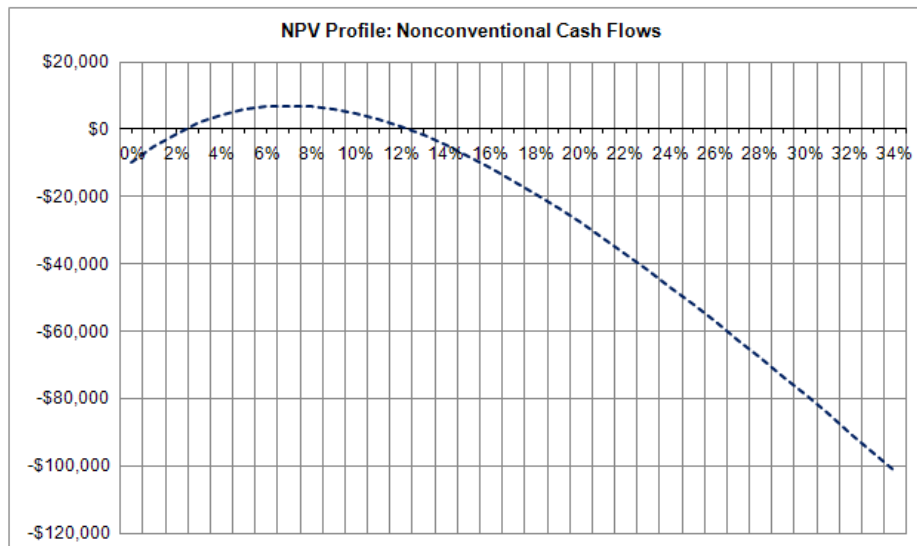
whether or not to invest? The answer to this question is ambiguous, which highlights one of the flaws when using the IRR to make capital budgeting decisions under the umbrella of nonconventional cash flows. In this instance, to answer the question, it is best to fall back on the NPV decision rule as its

assumption regarding reinvestment of cash flows is superior to that of IRR.

To complete the picture, we now explore a set of cash flows with multiple sign changes to see how many IRRs exist for this project.

Figure 1.

Project A	
Discount Rate	NPV
0%	\$ (10,000.00)
1%	\$ (5,096.33)
2%	\$ (1,132.78)
3%	\$ 1,964.84
4%	\$ 4,264.99
5%	\$ 5,830.90
6%	\$ 6,720.95
7%	\$ 6,989.07
8%	\$ 6,685.13
9%	\$ 5,855.23
10%	\$ 4,542.04
11%	\$ 2,785.01
12%	\$ 620.67
13%	\$ (1,917.15)
14%	\$ (4,797.13)
15%	\$ (7,990.25)
16%	\$ (11,469.63)
17%	\$ (15,210.34)
18%	\$ (19,189.29)
19%	\$ (23,385.05)
20%	\$ (27,777.78)
21%	\$ (32,349.05)
22%	\$ (37,081.79)
23%	\$ (41,960.14)
24%	\$ (46,969.42)
25%	\$ (52,096.00)
26%	\$ (57,327.22)
27%	\$ (62,651.36)
28%	\$ (68,057.54)
29%	\$ (73,535.66)
30%	\$ (79,076.36)
31%	\$ (84,670.98)
32%	\$ (90,311.45)
33%	\$ (95,990.34)
34%	\$ (101,700.72)



$$0 = 10,000,000 + \frac{-46,000,000}{(1+IRR)} + \frac{+79,100,000}{(1+IRR)^2} + \frac{-60,260,000}{(1+IRR)^3} + \frac{+17,160,000}{(1+IRR)^4}$$

In this pattern of cash flows, notice that there are four sign reversals. The sign reversals are from \$10,000,000 to \$(46,000,000), from \$(46,000,000) to \$79,100,000, from \$79,000,000 to \$(60,260,000), and from \$(60,260,000) to \$17,160,000. Because this cash flow pattern is a polynomial of degree four, it will have a maximum of four IRRs that will make the NPV equal zero. As we did previously, in order to close in on the IRRs, we calculate the NPV at various discount rates in order to create a NPV profile. The rates we

use for calculating the NPV begin at 0% and go to 34% in increments of 1%. Figure 6B.2 shows the resulting NPVs as well as the NPV profile. From this graph we see that the NPV is zero when the discount rate is 0%, 10%, 20%, and 30%. As the above function curves, it crosses the X axis four times with four IRR solutions. Other cash flow patterns and their resulting NPV profiles, while they may exhibit the same number of turns, may not cross the X axis as the turns occur. For these cases, fewer than n solutions will result, even for a polynomial of degree n.

For theoretical completeness, let us now consider a set of nonconventional cash flows which contain a total of four sign changes, but where the number of IRRs is less than four. More specifically, Project C is a simple modification of Project B, but includes one additional year of net cash flow totaling \$1,150 in Year 5.

Notice that the Year 5 cash flow is extremely small relative to the size of cash flows in prior years, but the impact is quite large in terms of the placement of the NPV profile. From the graph notice that the NPV profile sits above zero for most discount rates until it first falls below zero at 24.84%

and remains negative until 27.50%. Although an unlikely set of cash flows, this example is used to illustrate demonstrate

that a polynomial of degree n will have a maximum of n solutions

Project B	
Year	Cash Flow
0	\$ 10,000,000
1	\$ (46,000,000)
2	\$ 79,100,000
3	\$ (60,260,000)
4	\$ 17,160,000

Figure 2.

Project B	
Discount Rate	NPV
0%	\$ -
1%	\$ (476.55)
2%	\$ (744.99)
3%	\$ (856.41)
4%	\$ (853.44)
5%	\$ (771.28)
6%	\$ (638.74)
7%	\$ (479.02)
8%	\$ (310.48)
9%	\$ (147.28)
10%	\$ -
11%	\$ 123.91
12%	\$ 219.64
13%	\$ 284.64
14%	\$ 318.30
15%	\$ 321.61
16%	\$ 296.91
17%	\$ 247.67
18%	\$ 178.26
19%	\$ 93.80
20%	\$ -
21%	\$ (96.99)
22%	\$ (190.67)
23%	\$ (274.33)
24%	\$ (341.09)
25%	\$ (384.00)
26%	\$ (396.12)
27%	\$ (370.52)
28%	\$ (300.41)
29%	\$ (179.08)
30%	\$ -
31%	\$ 243.16
32%	\$ 556.53
33%	\$ 946.02
34%	\$ 1,417.29

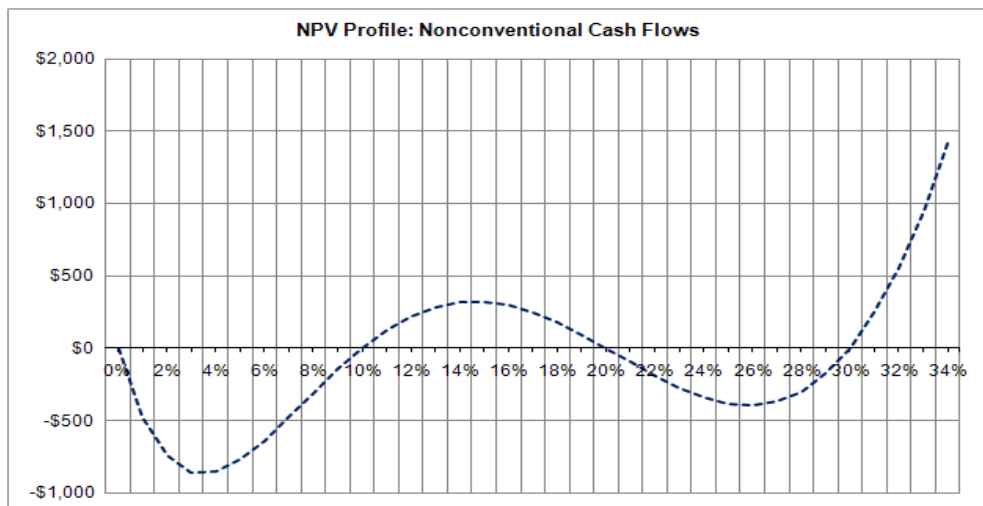
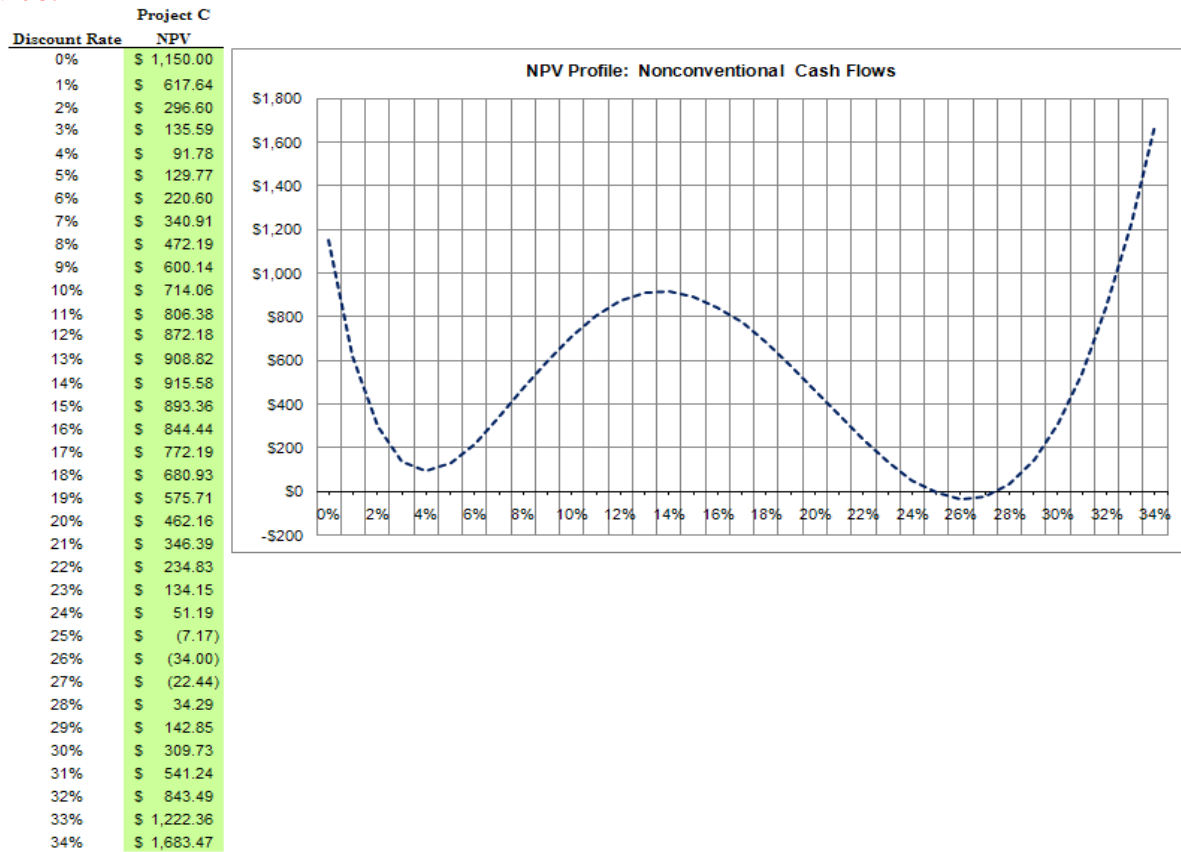


Figure 3.



Project C	
Year	Cash Flow
0	\$ 10,000,000
1	\$ (46,000,000)
2	\$ 79,100,000
3	\$ (60,260,000)
4	\$ 17,160,000
5	\$ 1,150

MULTIPLE IRRs USING iPhone APPS

We now present an exposition of the *TI BA II Plus* (TI) and the *HP-10B II* (HP) financial calculators to their respective *iPhone* applications¹, using the example of the multiple IRR problem in capital budgeting. Using either of the calculators (TI or HP), a problem that contains multiple IRRs becomes somewhat troublesome. In particular, if one were attempting to find the IRRs of Project A (below) using the TI calculator, difficulties arise because, by default, the TI calculator only displays the IRR closest to zero when multiple IRRs exist. Likewise, if using the HP calculator, an error message is displayed due to the fact that the HP calculator is unable to handle multiple IRR problems.

HP-10B II AND TI BA II Plus FINANCIAL CALCULATOR iPhone APPS

With regard to the TI *iPhone* application, there is very little improvement beyond what the calculator can do, at least thus far. The TI application, like the calculator, only displays the IRR closest to zero when multiple IRRs exist. The HP *iPhone* application, on the other hand, has computing power that greatly exceeds that of the corresponding calculator. More specifically, the HP *iPhone* application will display multiple IRRs, and will even graph the NPV profile, highlighting the points where the function crosses zero.

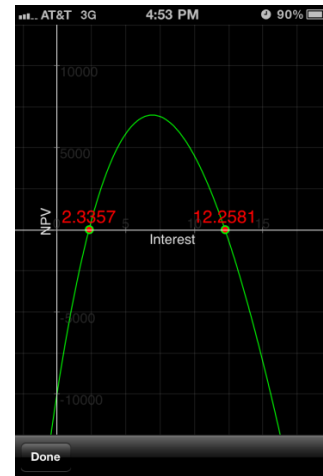
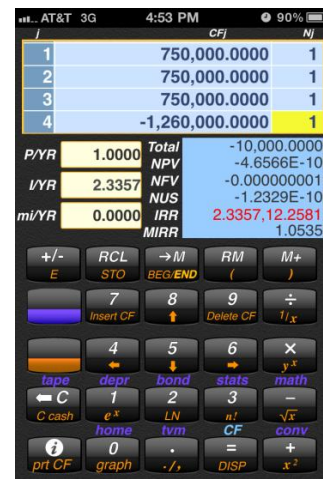
Project A	
Year	Cash Flow
0	\$ (1,000,000)
1	\$ 750,000
2	\$ 750,000
3	\$ 750,000
4	\$ (1,260,000)

The following set of screenshots show the results for computing IRR using both the TI and HP iPhone applications. Figure 4 is for the TI iPhone application, and Figure 5 for the HP iPhone application, making clear the difference in how comprehensively the HP iPhone application handles the nonconventional cash flows from Project A, relative to the TI iPhone application.

Figure 4. TI BA II Plus iPhone Application (Project A)



Figure 5. HP-10B II iPhone Application (Project A)



In figure 5, the first screen appears when prompting the calculator to calculate the IRR. By pressing the blue shift key and then "CF", the cash flows are displayed, showing the error and a more exact set of IRR solutions. Taking advantage of the extended capabilities of the HP emulator, by pressing the orange shift key and "graph", the NPV profile is displayed, showing the function and the two values where it crosses the axis (at a zero NPV).

Now refer to Project B, the set of cash flows with multiple sign changes. Recall from the prior example that the four IRRs for this set of cash flows are 0%, 10%, 20%, and 30%.

Project B	
Year	Cash Flow
0	\$ 10,000,000
1	\$ (46,000,000)
2	\$ 79,100,000
3	\$ (60,260,000)
4	\$ 17,160,000

As noted before, the TI iPhone application only displays the IRR closest to zero (Figure 6), thereby only providing the

first of four IRRs that we know to exist (0% in this case). The HP iPhone application also breaks down to some degree with this particular problem. As shown in Figure 7, the HP application provides two IRRs at or around 0% and two IRRs at 10% (first image). Pressing the blue shift key and "CF", the cash flows and more exact versions of the IRRs appear. Based on this example, it appears as though the HP iPhone application is unable to handle a problem with four solutions for IRR, providing only two of the known four solutions.

Despite the fact that the HP iPhone application fails to identify all possible IRR solutions, from a functional standpoint, the HP-10B II and TI BA II Plus iPhone applications do everything that their respective financial calculators do.

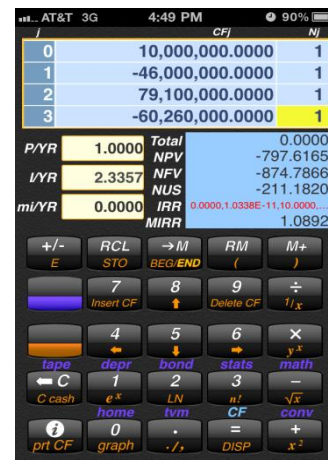


Figure 6. TI BA II Plus iPhone Application (Project B)

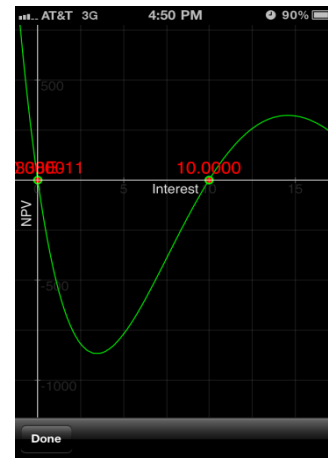
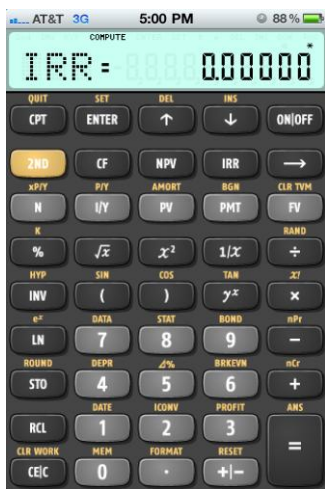
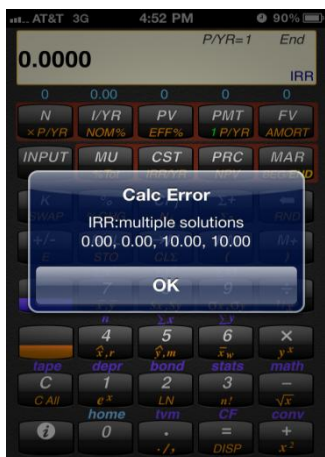


Figure 7. HP-10B II iPhone Application (Project B)



CONCLUSION

In the case of the HP-10B II, the iPhone application far exceeds the capability of the hand-held calculator, providing two IRR solutions, display capabilities concerning summaries of cash flows and solutions in a separate screen, and some fairly sophisticated graphing capabilities as well.

While the iPhone applications extend analytical capabilities, it is unlikely that professors would require iPhone and application purchases, and even less likely that communication devices would be allowable on exams. But this presentation does highlight the usefulness of the calculator applications beyond the handheld equivalents. As devices become more sophisticated, finance and business practitioners may find it convenient as well as quite robust to use handheld devices and appropriate applications to carry out useful analyses.