A Method for Teaching the Black-Scholes Option Pricing Model Using Excel

Steve Johnson and Robert Stretcher

One of the most important concepts in modern finance practice and education is option pricing. The Black-Scholes model of option pricing is possibly the most commonly-used model. This paper presents an implementation of the Black-Scholes model of option pricing that includes graphs of the option value, the intrinsic value, and the time value. These graphs are dynamic, allowing the user to change the value of the volatility of underlying asset returns, time to maturity, and the risk-free rate. The user can also see how price of an option changes with movements in the underlying asset price. By illustrating both option prices and the components of option prices (intrinsic and time value) over a range of underlying asset prices, students can more easily visualize the effects of the drivers of option prices.

INPUT VARIABLES

The first step in designing the spreadsheet is to list the input variables. These are recorded in table 1.

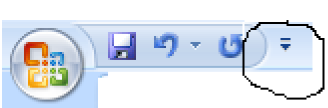
Table 1: Input variables.

S_0,	the current price of the underlying asset
Strike,	the strike price of the option
Rf,	the annual risk-free rate
Time,	the time until maturity, measured as a fraction of a year
sigma,	the standard deviation of the returns of the underlying asset.

The five input variables, S_0, Strike, Rf, Time, and sigma are all derived from spin button outputs. A spin button is a convenient input variable adjustment tool to use with the model in Excel 2007. The first input variable, S_0, is used here as an example.

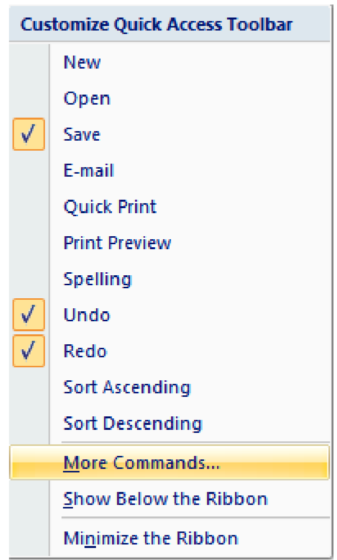
First, in order to be able to conveniently add spin buttons to the spreadsheet, add the spin button icon to the Quick Access Toolbar. Once the icon is added, it will be available every time the spreadsheet is opened. Click on the "Customize Quick Access Toolbar" icon. This icon is the right-most icon in the toolbar. See figure 1.

Figure 1: Customize Quick Access Toolbar icon.



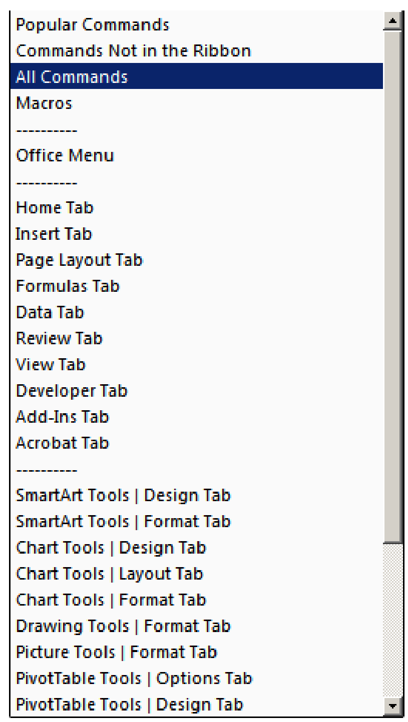
Scroll down to "More Commands." See figure 2.

Figure 2: Select "More Commands."



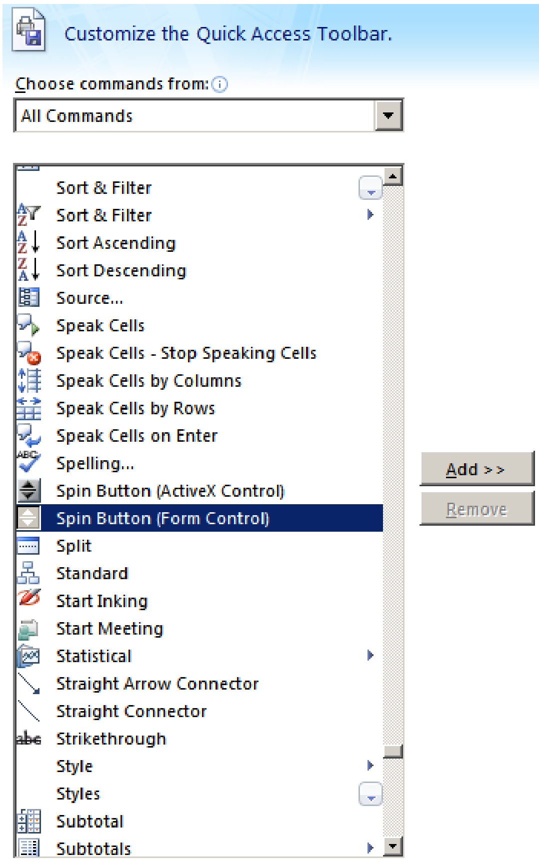
Select "More Commands," then choose "All Commands." See figure 3:

Figure 3: All Commands.



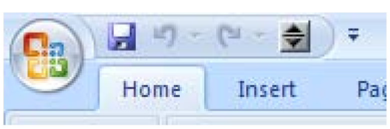
After choosing "All Commands," scroll down to and select "Spin Button (Form Control)," then click on Add. See figure 4:

Figure 4: Add Spin Button (Form Control).



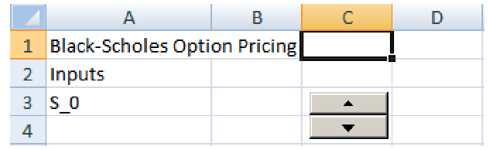
After clicking on Add, it should be possible to see the spin button icon in the toolbar. The updated toolbar is illustrated in figure 5.

Figure 5: Excel toolbar with spin button icon.



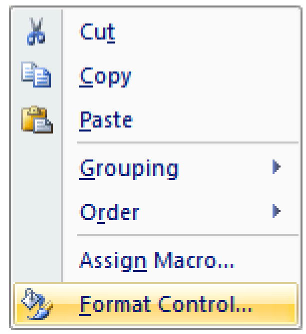
Enter the text, "Black-Scholes Option Pricing" in cell A1 and "Inputs" in cell A2. In cell A3, enter the text, "S_0." This is the name representing the value of the underlying asset. Next, click on the spin button icon. This will render a "+"-shape that can be used to draw the spin button. Go to the upper left-hand corner of cell C3. Hold down the left mouse button and move to the lower right-hand corner of cell C4. Then, release the left mouse button. The result should be similar to figure 6.

Figure 6: The S_0 (Underlying asset price) spin button.



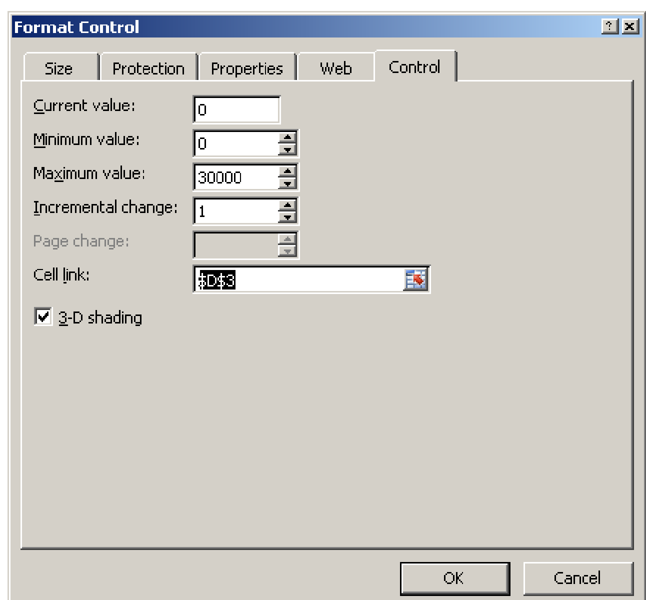
Next, the spin button must be linked to a cell, and then scaled to provide the appropriate range of values. For example, the underlying asset price could be set to vary between $0 and $30 in steps of $0.10. In order to link the spin button output to a cell, right-click on the cell and choose "Format Control." See figure 7.

Figure 7: Choose format control.



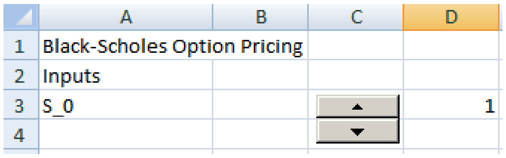
After choosing format control, put the cursor next to the box labeled "Cell Link," then click on cell D3. See figure 8.

Figure 8: Format Control with cell link.



Click on OK. This will close the Format Control box. Now the spin button will send its raw output to cell D3. Click on the spin button once. The result should be similar to figure 8.

Figure 9: Spin button that sends raw output to cell D3.



Finally, scale the output to the range and step size needed. Here, the minimum price is set to be $0 and the step size to be $0.10. Use the relation

desired output = starting value + (step size) * (# of steps)

The starting value = 0, the step size is 0.10, and the number of steps is the raw output from the spin button, D3. So, in cell B3, enter the formula

= 0 + 0.1 * D3.

Set the range for the input variable S_0 such that 0≤ S_0 ≤30, which implies: 0≤ 0+0.1*D3≤30 or 0≤ D3≤300. Open Format Control again and enter the maximum value of 300. This is illustrated in figure 10.

Figure 10: Format Control with maximum value of 300.

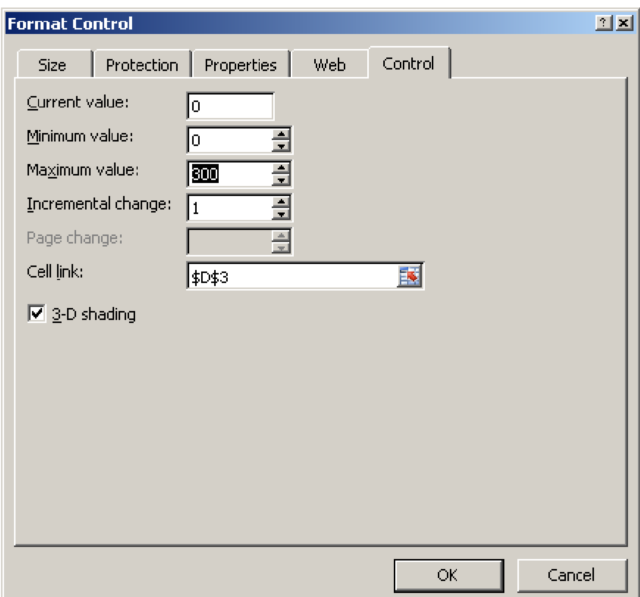
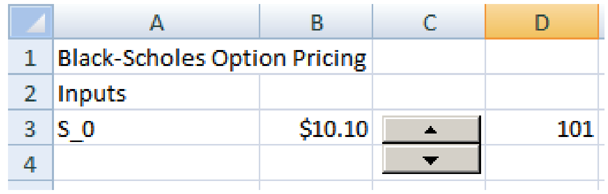


Figure 11 illustrates the final result with the raw spin button output set to 101 and cell B3 formatted to two-decimal US currency.

Figure 11: Final result for the S_0 (underlying asset price) spin button.

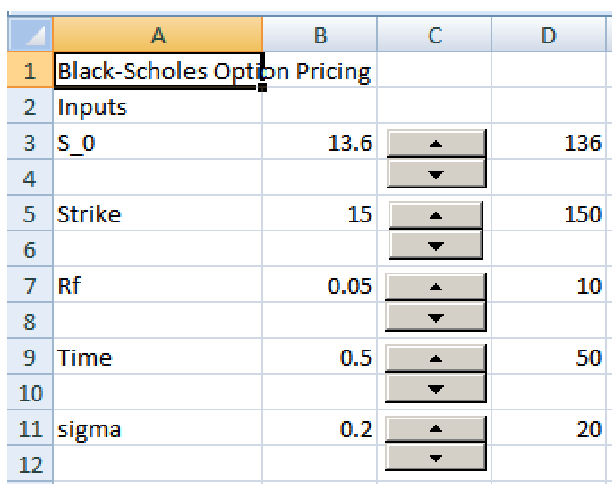


The next step is to add the other input variables:

Strike, the strike price of the option
Rf, the annual risk-free rate
Time, the time until maturity, measured as a fraction of a year
sigma, the standard deviation of the returns of the underlying asset.

Repeat the process described above for the other input variables. Add the remaining input variables every other row, so that Strike is in cell A5, Rf is in cell A7, Time is in cell A9, and sigma is in cell A11. Insert spin buttons in cells C5..C6, C7..C8, C9..C10, and C11.C12. Use Format Control to put the raw outputs of the spin buttons in cells D5, D7, D9, and D11, respectively. Use the same relation used before in cell B3 for the value of Strike in cell B5. Allow the interest rate (Rf) and the standard deviation (sigma) to vary from 1%, in steps of 0.2%, in B7 and B11, respectively. Allow time to vary from 0.01, in steps of 0.01, in cell B9. The final result should be similar to figure 12.

Figure 12: Input variables: Black-Scholes call option.



Note that, in order to cap the input values of any of the input variables that feed into the option pricing model, use the maximum option on the Format Control step. For example, it is possible to set upper and lower limits of 1% and 5% for the input variable Rf such that 1%≤ Rf ≤5% with increments of 0.2%.

This would imply that

$$0.01 \leq 0.01 + 0.002*D7 \leq 0.05 \text{ or } 0 \leq D7 \leq 20.$$

Use the name box to give each of the values in column B the corresponding name in column A. For example, for cell B3, click on B3, then enter "S_0" in the NameBox immediately above the column header for column A. See figure 13.

Figure 13: Variable name for cell B3 identified in the NameBox.

S_0 | fx | =D3*

	A	B	C
1	Black-Scholes Option Pricing		
2	Inputs		
3	S_0	13.6	
4			

INTERMEDIATE CALCULATIONS AND CALL PRICE CALCULATION

The next step is to calculate the values, denoted here as d_1 and d_2, of the intermediate step. According to the Black-Scholes theory of option pricing,

call option price = S_0 * normdist(d_1, 0, 1) - Strike * exp(-Rf*Time)*normdist(d_2, 0, 1)

where d_1 = (ln(S_0/Strike) + (Rf + (sigma^2)/2)*Time)/(sigma*sqrt(Time))
and d_2 = (ln(S_0/Strike) + (Rf - (sigma^2)/2)*Time)/(sigma*sqrt(Time))

Enter the text "d_1" and "d_2" in cells A14 and A15, respectively. Enter the above formulas in cells B14 and B15, respectively. Rename cell B14 as d_1. Rename cell B15 as d_2. The result should resemble figure 14:

Figure 14: Input values and intermediate values d_1, d_2.

	A	B	C
1	Black-Scholes Option Pricing		
2	Inputs		
3	S_0	13.6	
4			
5	Strike	15	
6			
7	Rf	0.05	
8			
9	Time	0.5	
10			
11	sigma	0.2	
12			
13			
14	d_1	-0.44534	
15	d_2	-0.58676	
16			

At this point, all the inputs for the calculation of the call price are complete. The Black-Scholes value of the call option with the given inputs is

=S_0*normdist(d_1,0,1)-Strike*exp(-Rf*Time)*normdist(d_2,0,1)

Enter the text "Call Price @ S_0" in cell A17 and the above formula in cell B17. This completes the Black-Scholes pricing of a call option with the above inputs. See figure 15.

Figure 15: Black-Scholes call price calculation for one set of inputs.

	A	B	C
1	Black-Scholes Option Pricing		
2	Inputs		
3	S_0	13.6	
4			
5	Strike	15	
6			
7	Rf	0.05	
8			
9	Time	0.5	
10			
11	sigma	0.2	
12			
13			
14	d_1	-0.44534	
15	d_2	-0.58676	
16			
17	Call price @ S_0	0.384284	

GRAPHS

One of the powerful features of Excel is the ability to create graphs that change when the input values change. Implementing this feature makes it is possible to see what happens to option prices when inputs are changed. In this model, a useful image is a plot of the option value vs. the underlying stock price. Changes in the risk-free rate (Rf), the volatility (sigma) of underlying asset returns, the time to maturity (Time), and the strike price (Strike) impact the entire graph. Changes in the underlying stock price are represented by different values on the same graph. An effective visualization is to sketch four graphs on the same chart. The first three are: the option price, the intrinsic value of the option, and the time value of the option. Also included is a graph of the option price at one point--the value of the underlying stock price found in cell B3.

Enter the following text into the following cells:

"S_0" in cell F1,
"d_1" in cell G1,
"d_2" in cell H1,
"Black-Scholes price" in cell J1,
"Intrinsic value" in cell K1, and
"Time value" in cell L1.

Figure 16: Column headers for graph input data.

F	G	H	I	J	K	L
S_0	d_1	d_2		Black-Scholes price	Intrinsic value	Time value

Enter the value 0.1 in cell F2, then 0.2 in F3, then copy and drag down to cell F301, which should have a value of 30. These will be the "x-values," the values on the horizontal axis, of the graph. These are the different underlying asset prices.

In column G, use the formula for d_1 that was written in B14. However, replace S_0 with the value in column F. In column H, repeat the same procedure for the value of d_2 that is written in cell B15. Copy and drag over G2:H301.

In column J, use the formula for the Black-Scholes call option price found in cell B17. Replace S_0, d_1, and d_2 with the corresponding values from columns F, G, and H, respectively. This way, the spreadsheet will calculate the Black-Scholes call option price for every value of the underlying asset from 0.10 to 30.00.

The intrinsic value of the option is the value if the option were to be exercised immediately. The call option gives the holder of the option the right to buy the underlying asset at the strike price. So, the intrinsic value would be the maximum of either 0, if the strike price is greater than or equal to the underlying asset price, and (underlying asset price - strike price), if the strike price is less than the underlying asset price. In cell K2, enter

= max(0, F2 - Strike)

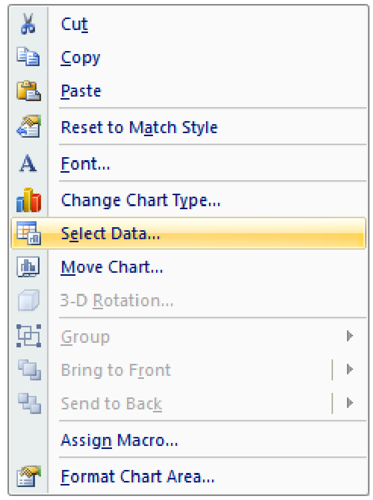
The value of the underlying asset is found in column F. The time value of the option is the part of the option value that comes from the possibility that the underlying asset price might change before maturity. Take the difference of the Black-Scholes call option price and the underlying asset price. In cell L2, enter

= J2 - K2

Copy and drag J1:L1 to J1:L301. This completes the construction of the input data for the graphs.

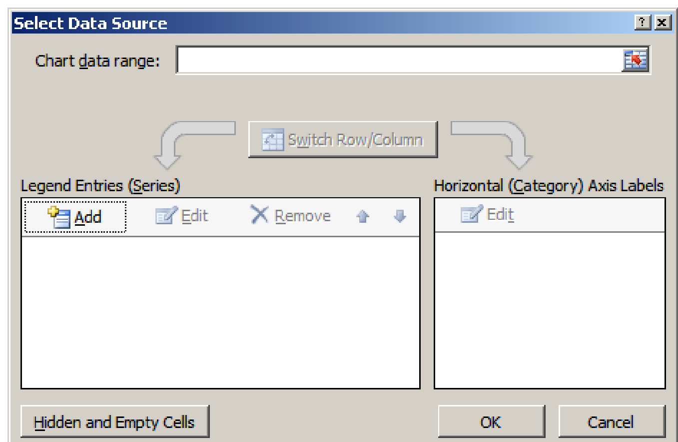
To begin construction of the graph, click on the "Insert" tab, then choose "Scatter," then choose the first option, "Scatter with only markers." A blank graph results. Right-click on the blank graph. Choose select data from the list of options. See figure 17.

Figure 17: Scatterplot options.



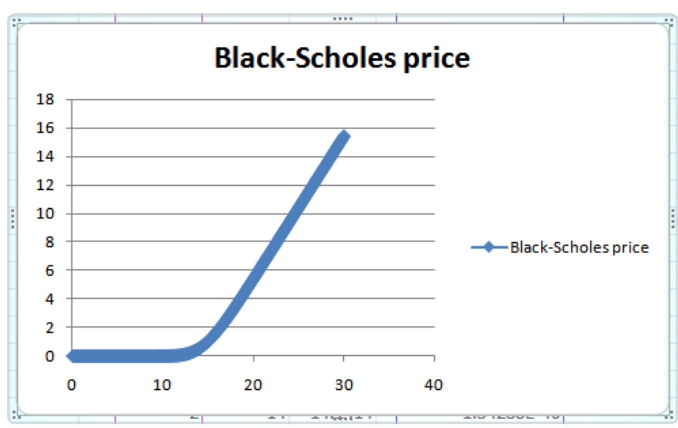
Next, click on "Add." See figure 18.

Figure 18: Selecting the data sources.



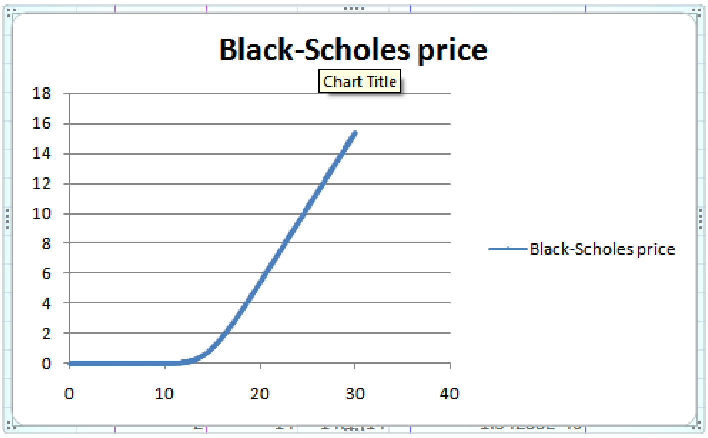
For the series name, click on cell J1. For the x-values, select or enter F2:F301. For the y-values, first delete the text in the box, then select or enter J2:J301. Then click OK. See Figure 19.

Figure 19: Partially completed graph.



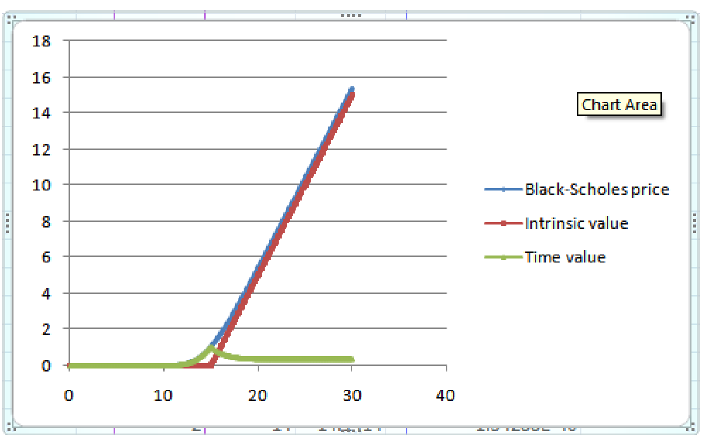
It is possible to make the markers smaller, facilitating reading the graph. Right-click on the curve and choose "Format data series." Click on "Marker options," then "Built-in," then type or enter "2" using the scroll button. Then, click on close. The graph should look like figure 20:

Figure 20: Partially complete graph, with line width adjustment.



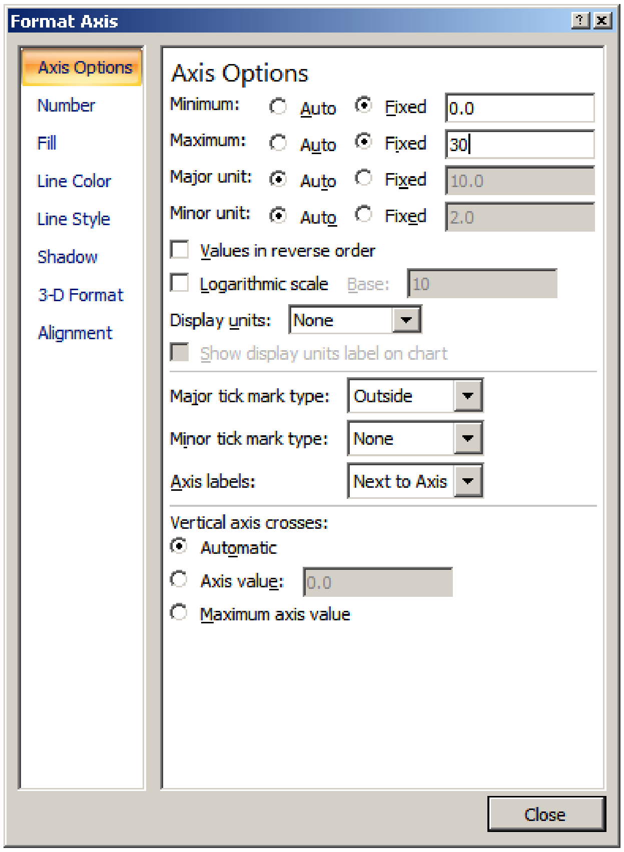
Use the same procedure to enter the graphs of the intrinsic value and the time value of the option. Upon completion, the graph should be similar to figure 21.

Figure 21: Partially complete graph, with option price, time value, and intrinsic value.



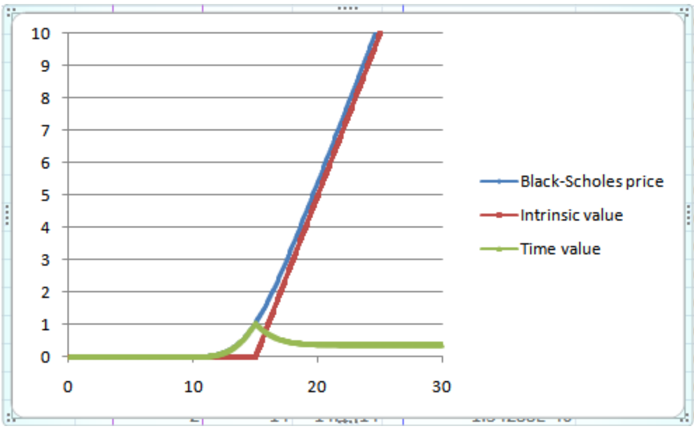
The scale of the graph will need to be adjusted. Since the highest value of the underlying asset is 30, adjust the horizontal axis to display results between 0 and 30 as follows: Right click on one of the values on the horizontal axis. Then, choose format axis. For the minimum value, under "Axis Options," choose "Fixed" and leave the value at 0.0. For the maximum value, choose "Fixed" and enter 30. Then click close. The menu of choices appears in figure 22.

Figure 22: Menu of choices for adjusting the horizontal axis.



For the vertical axis, choose values between 0 and 10. This scale will allow the investigator to see how the option price behaves near the strike price while also allowing observation of some of the behavior when the stock price is more distant from the strike price. Right-click on a value on the horizontal axis of the graph, then follow a procedure similar to that used for the horizontal axis. The graph is now near completion, as can be seen in figure 23.

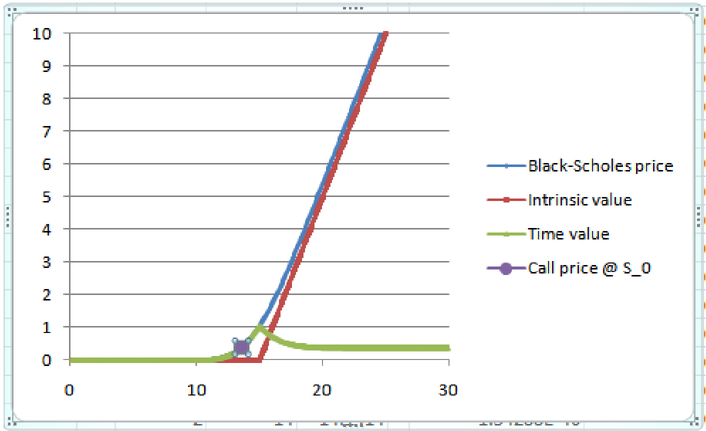
Figure 23: Partially completed graph, near completion.



In addition to the three graphs above, it is also of value to observe how the option price changes for different values of the underlying stock price. In this case, all the values lie on the same graph. A single point is added to represent the value of the option using all five input values. Right-click on the graph, choose "Select Data," then select "Add." The title is found in cell A17, the x-value (just one x-value here) is found in cell B3, and the y-value (the value of the call option at the underlying asset price S_0 found in cell B3) is in B17.

Often the default shape for the fourth graph is an "X," which is hard to see. If this is the case, right-click on the X, choose "Format data series," then "Built in," then select a different shape. For illustration, in the graph, the ball shape has been chosen. See figure 24.

Figure 24: Graph complete except for header.



This completes the graph except for adding a title of one's choice. Move the ball on the graph by clicking on the spin button that controls S_0. Change the graph by clicking on any of the other input variables. In addition to gaining an understanding of how the different input variables drive option prices, the user can also understand how changes in the underlying asset, S_0, reflect changes in the value of the same option contract.

CONCLUSION

The method presented above is a very flexible way to develop a greater understanding of option pricing using the Black-Scholes model. One objective of the construction of this spreadsheet is to provide students an exercise in using the features of Excel to create desired analytical and graphical presentations. Virtually all business school graduates claim expertise in Excel, but few achieve fluency. This exercise can give them a degree of confidence in their abilities if they are required to create the spreadsheet from scratch. In addition, to complete the spreadsheet and graphs, the details of the Black-Scholes model must be translated into an analytical framework. This augments students' understanding of the model and why the option price moves as certain input variables (or distributions) change. It is a much more engaging teaching method than many others.

The preferences presented here, such as structure, input devices, graphical formats, and colors, are of course a matter of taste and can easily be adjusted by the user. In our teaching of the construction of the spreadsheet, students seem to desire an assigned structure rather than being given latitude, at least until after the initial spreadsheet is completed, at which point adjustments can easily be made.

This spreadsheet model can be extended to approximately value stocks that pay dividends. A standard textbook argument (ex Hull 283-5) reduces the problem to two cases: 1) finding the value of the option held until maturity or 2) exercising immediately before the dividends are paid. In case 1), the option holder does not receive the dividends. So, subtract the PV of the dividends from S0, the value of the stock at the beginning of the period. Everything else remains the same. In case 2), the option holder does receive the dividends. So, change the time to maturity to reflect the holder's decision to exercise the option early. Compare the value of the option under both scenarios. The value of the option is the higher of the two values.

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Enhancing Student Understanding of the 1958 Modigliani-Miller Propositions

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In their 1958 paper, Franco Modigliani and Merton Miller show that, in a theoretical "perfect world," a company's value and its cost of capital do not vary across levels of leverage. As a consequence, a linear relationship must exist between the debt-to-equity ratio and the cost of equity. Most textbooks first explore these fundamental theoretical relationships before transitioning on to "real world" issues such as corporate taxes and bankruptcy. In this teaching note, we suggest that textbook presentations of the 1958 MM propositions allow for confusion on the part of students because of ambiguity in discussions and graphical representations. To help avoid confusion on the students' part in this regard, we present a series of exercises that instructors may use to supplement and enhance textbook content.

In 1958, Franco Modigliani and Merton Miller published the first of a series of papers (Exhibit 1) that revolutionized capital structure theory and for which they were presented with the Nobel Prize. In this work, they show by using an innovative arbitrage proof (i.e., the "homemade leverage" argument) that, in a theoretical "perfect world[1]," the value of a company is independent of its leverage[2]. This proposition, known as the Irrelevance Proposition but generally referred to as MM