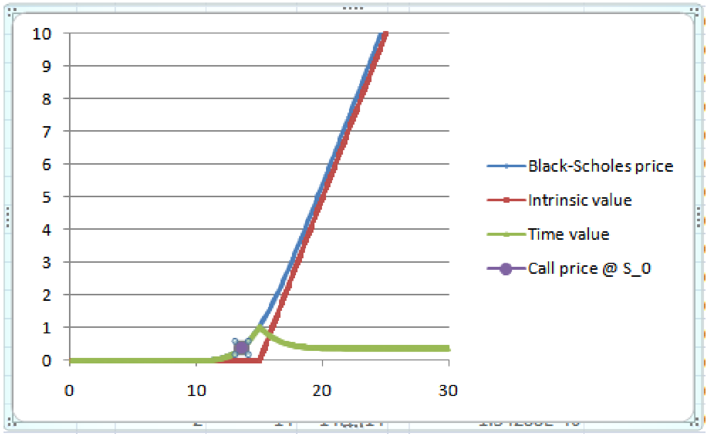
option using all five input values. Right-click on the graph, choose "Select Data," then select "Add." The title is found in cell A17, the x-value (just one x-value here) is found in cell B3, and the y-value (the value of the call option at the underlying asset price S_0 found in cell B3) is in B17.

Often the default shape for the fourth graph is an "X," which is hard to see. If this is the case, right-click on the X, choose "Format data series," then "Built in," then select a different shape. For illustration, in the graph, the ball shape has been chosen. See figure 24.

Figure 24: Graph complete except for header.



This completes the graph except for adding a title of one's choice. Move the ball on the graph by clicking on the spin button that controls S_0. Change the graph by clicking on any of the other input variables. In addition to gaining an understanding of how the different input variables drive option prices, the user can also understand how changes in the underlying asset, S_0, reflect changes in the value of the same option contract.

CONCLUSION

The method presented above is a very flexible way to develop a greater understanding of option pricing using the Black-Scholes model. One objective of the construction of this spreadsheet is to provide students an exercise in using the features of Excel to create desired analytical and graphical presentations. Virtually all business school graduates claim expertise in Excel, but few achieve fluency. This exercise can give them a degree of confidence in their abilities if they are required to create the spreadsheet from scratch. In addition, to complete the spreadsheet and graphs, the details of the Black-Scholes model must be translated into an analytical framework. This augments students' understanding of the model and why the option price moves as certain input variables (or distributions) change. It is a much more engaging teaching method than many others.

The preferences presented here, such as structure, input devices, graphical formats, and colors, are of course a matter of taste and can easily be adjusted by the user. In our teaching of the construction of the spreadsheet, students seem to desire an assigned structure rather than being given latitude, at least until after the initial spreadsheet is completed, at which point adjustments can easily be made.

This spreadsheet model can be extended to approximately value stocks that pay dividends. A standard textbook argument (ex Hull 283-5) reduces the problem to two cases: 1) finding the value of the option held until maturity or 2) exercising immediately before the dividends are paid. In case 1), the option holder does not receive the dividends. So, subtract the PV of the dividends from S0, the value of the stock at the beginning of the period. Everything else remains the same. In case 2), the option holder does receive the dividends. So, change the time to maturity to reflect the holder's decision to exercise the option early. Compare the value of the option under both scenarios. The value of the option is the higher of the two values.

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Black, F., and Scholes, M., 1973. The pricing of options and corporate liabilities. *Journal of Political Economy*, 81 637-59.

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Enhancing Student Understanding of the 1958 Modigliani-Miller Propositions

Dianne M. Lander and Glenn N. Pettengill

In their 1958 paper, Franco Modigliani and Merton Miller show that, in a theoretical "perfect world," a company's value and its cost of capital do not vary across levels of leverage. As a consequence, a linear relationship must exist between the debt-to-equity ratio and the cost of equity. Most textbooks first explore these fundamental theoretical relationships before transitioning on to "real world" issues such as corporate taxes and bankruptcy. In this teaching note, we suggest that textbook presentations of the 1958 MM propositions allow for confusion on the part of students because of ambiguity in discussions and graphical representations. To help avoid confusion on the students' part in this regard, we present a series of exercises that instructors may use to supplement and enhance textbook content.

In 1958, Franco Modigliani and Merton Miller published the first of a series of papers (Exhibit 1) that revolutionized capital structure theory and for which they were presented with the Nobel Prize. In this work, they show by using an innovative arbitrage proof (i.e., the "homemade leverage" argument) that, in a theoretical "perfect world[1]," the value of a company is independent of its leverage[2]. This proposition, known as the Irrelevance Proposition but generally referred to as MM

Proposition I (MMI), implies that a company's average cost of capital is constant across various levels of leverage and, in fact, is equal to its unlevered cost of equity. In this Modigliani-Miller ideal world, an optimal debt-to-equity ratio does not exist and managers ought to spend their time looking for value adding investment opportunities without consideration of financing alternatives.

MMI also leads to a second proposition, generally referred to as MM Proposition II. MMII states that a mathematically defined linear relationship exists between the debt-to-equity ratio and the levered cost of equity. As the company takes on more debt, it becomes more risky, but the effect of adding "cheaper" debt is exactly offset by having less, although now "more expensive," equity. If this were not the case, arbitrage would be imperfect and leverage would matter. These two MM propositions serve in most textbooks as the foundation for discussions of capital structure decisions faced by managers.

Although Modigliani-Miller show a linear relationship between the cost of equity and the debt-to-equity ratio, student understanding of leverage likely occurs in debt-to-value space. Further, many of the textbook discussions of follow-on capital structure topics theories are presented in debt-to-value space[3]. Finally, the concept of leverage in debt-to-value space is generally consistent with discussions of leverage in the popular press. So care must be taken in presentation of the MM concepts of the existence of the two measures of leverage. A related issue is that graphical representation of the MM concepts are often presented without fully identifying or scaling either or both axes[4], obscuring (i) the decidedly different relationships between the cost of equity and the two measures of leverage and also (ii) the extreme rise of the cost of equity as leverage increases.

This paper is a teaching note that seeks to aid instructors in enhancing student learning of the 1958 MMI and MMII propositions and, as a result, to better prepare students to transition their understanding from the Modigliani-Miller "perfect world" concepts to financial markets where frictions exist. We begin by discussing two different measures of leverage and present an exercise to underscore their differences. We next present several exercises utilizing the two measures of leverage that are designed to emphasize the understandings that the instructor may wish students to draw from the MMI and MMII propositions. In the last section, we provide concluding thoughts.

TEACHING THE MEASURES OF LEVERAGE

Textbooks do not always define the measure of leverage being used, or may have defined it in an earlier section or chapter. Sometimes the measure is switched within or between sections or chapters and with little or no notice. Graphing based on the debt-to-equity ratio and the debt ratio represent the same in that they are both accurate to the MM theory, but understanding the difference between the measures and being aware of which is used in a textbook at any particular point is critical to students understanding the discussions and graphs presented. Our experience suggests that students will not recognize these issues or think them through on their own. It then becomes critical for those of us teaching the theories and concepts of capital structure to start by ensuring that students are aware that there are different measures of leverage and that they understand what each one represents. In addition, students need to be able to recognize which of the measures their textbooks use for each discussion and graph presented. So we start by defining the measures of leverage most commonly used.

The introduction to this paper mentions two basic measures of leverage. The first is the debt-to-equity ratio (D/E), where D is defined as the market value of the company's debt and E is defined as the market value of the company's equity such that D+E is the market value of the company. The second measure of leverage is the debt ratio, also known as the percent of debt (financing, used), % leverage, debt-to-assets (D/A), and others too. The debt ratio is equal to D/V, where D is again defined as the market value of the company's debt and V is defined as the market value of the company (D+E), and the ratio is usually expressed as a percent (%).

Once students have developed an understanding of the commonly used measures of leverage, instructors could have students work an exercise, as developed below (Table 1), comparing changes in the D/E and in the debt ratio as leverage is increased. Such an exercise would help students to develop an understanding of the differences in the measures that is crucial for truly understanding the textbook presentations of the Modigliani-Miller model. Likewise, this understanding is crucial as students seek to understand capital structure in a world of frictions.

To illustrate the importance of the comparison, a worthwhile exercise is to ask students to identify the point on a simple graph, such as Figure 2A below, where the company is half financed by debt and half financed by equity. The range of the x-axis (D/E) could be from 0 to 9, 20, or even 40. Our experience is that, without working through an exercise such as the one below, the majority of students choose the midpoint–a D/E of 4.5, 10, or 20, respectively. Yet these three points actually represent about 82% debt, 91% debt, and 95% debt, respectively, with none being anywhere close to 50% debt. The students are thinking in terms of % debt, not D/E. Indeed, we find that our colleagues will often make the same choice as the students. Exercise 1 allows students to develop the correct answer that a D/E of 1.0 is equivalent to a 50% debt ratio and begin to see that the representation of capital structure concepts will look radically different depending on the measure of leverage used.

Table 1: Exercise 1 -- Comparing Leverage Changes on the D/E and the Debt Ratio

Market Value Debt	Market Value Equity	D/E	Debt Ratio
0	200	0	0%
20	180	0.11	10%
40	160	0.25	20%
60	140	0.43	30%
80	120	0.67	40%
100	100	1	50%
120	80	1.5	60%
140	60	2.33	70%
160	40	4	80%
180	20	9	90%
190	10	19	95%
195	5	39	98%
200	0	Undefined	100%

For students to complete Exercise 1, they will need reference market values for the firm and either the equity or debt and to be reminded that the total market value of the firm equals the total market value of the debt and the equity. Another key piece of information for the students is that (consistent with the Modigliani-Miller propositions presented below) the total value of the firm remains constant. Exercise 1 above assumes that the total market value of the company is 200.

The completed exercise shows that the two leverage measures have noticeably different values and that they are not scale comparable. (Also, note that 100% debt financing is not defined for D/E, or is a D/E of infinity.) To further illustrate these concepts, students could create graphs of the relationships depicted in Exercise 1 or simply be shown graphs of the D/E and debt ratios as a function of the amount of debt. These graphs are shown below[5]:

Figure 1A: D/E as a Function of Leverage

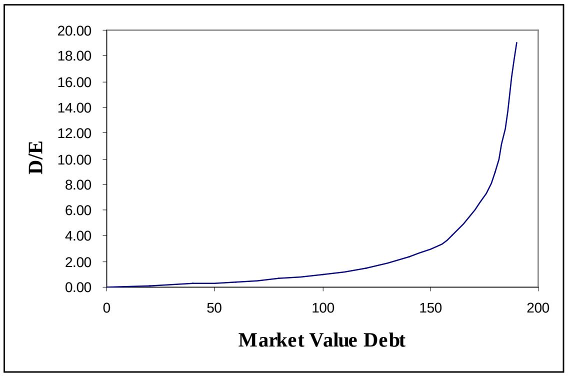
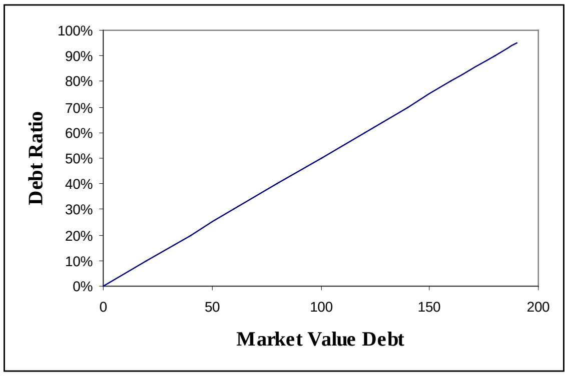


Figure 1B: Debt Ratio as a Function of Leverage



TEACHING THE 1958 MODIGLIANI-MILLER MODEL

Once the students have come to terms with the measures of leverage, they are ready to move on to the 1958 Modigliani-Miller propositions. MMI says that the value of a company is independent of its leverage and equal to its expected (constant) EBIT capitalized at a constant risk-adjusted discount rate. That is, the value of the company is a function of the earnings power of its assets and its risk-class, not of its level of debt financing. MMI also implies that a company's cost of capital is constant across various levels of leverage and is equal to its unlevered cost of equity. This relationship is shown in Equation 1:

$$VL = VU = EBIT/WACC = EBIT/keU \qquad (1)$$

VL is the value of the levered company
VU is the value of the unlevered company
EBIT is the company's (expected, constant, and into perpetuity) earnings before interest and taxes
WACC is the company's average cost of capital
keU is the company's unlevered cost of equity

Having the students graph MMI first using D/E on the x-axis and then using the debt ratio on the x-axis effectively results in the same graph. Assuming expected EBIT is a constant 20 into perpetuity and the WACC, which equals the company's unlevered cost of equity, is 10%, VL = VU = 20/0.10 = 200. Both graphs below clearly show that there is no optimal capital structure and that the value of the company is independent of its capital structure.

Figure 2A: MM Proposition I: D/E Space

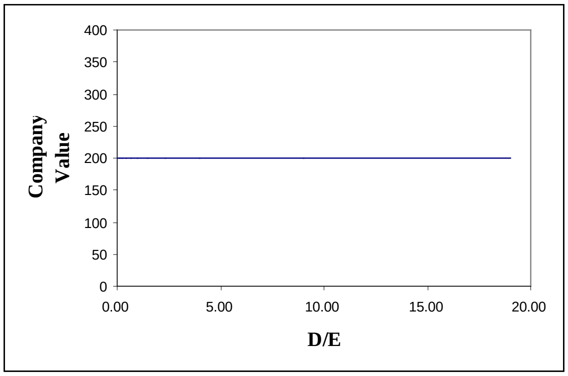
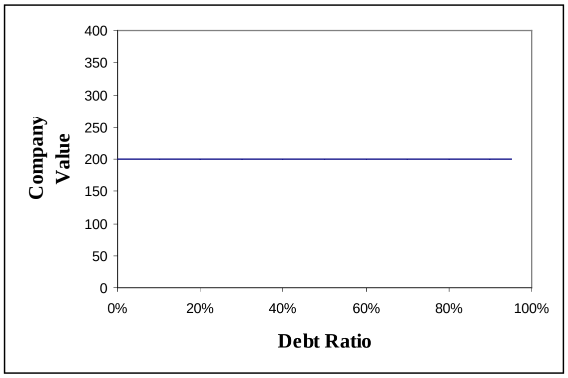


Figure 2B: MM Proposition I: Debt Ratio Space



The second MM proposition follows from the first and states that a linear relationship exists between D/E and the levered cost of equity. As the amount of debt financing increases, the cost of equity also increases, and does so in a specifically defined manner. This relationship is shown in Equation 2:

keL = keU + (keU – kd)*D/E (2)

keL is the company's levered cost of equity
keU is the company's unlevered cost of equity
kd is the constant cost of debt
D is the market value of the company's debt and E is the market value of the company's equity such that D+E is the market value of the company

Taken together, the two MM propositions imply that as the company issues more debt, risk increases for the equity stakeholder, the cost of equity increases, and the benefits of the "cheaper" debt exactly offset the increase in equity risk. Although the cost of equity has increased, the amount of the company financed by this "higher cost" equity is less. The net result is a constant company average cost of capital that is equal to the company's unlevered cost of equity. This trade-off or rebalancing may not be obvious to students and the instructor may find it desirable to use Equation 3 to show this relationship:

WACC = kd * (D/V) + keL * (E/V) (3)

WACC is the company's average cost of capital
kd is the constant cost of debt
keL is the company's levered cost of equity
D is the market value of the company's debt and E is the market value of the company's equity such that D+E is the market value of the company

When graphing the MMII relationships, it now matters which measure of leverage is used as a second set of graphs will show. First, however, students should be asked to work Exercise 2 below to understand (i) the levered cost of equity as a function of D/E (Equation 2) and (ii) the WACC (Equation 3). We again assume expected EBIT is a constant 20 into perpetuity and that the market value of the firm is 200. Columns 1 through 4 are as in Exercise 1. In order to work Exercise 2, students also need to be given a cost of debt, which is constant in the Modigliani-Miller world. For this exercise we assume a 6% cost of debt. The critical calculation for the students is the cost of equity in column 6, calculated using Equation 2. Because the value of the WACC is constant in this case, students may be tempted to simply fill out the value of 10%. Instructors, however, may want students to calculate each WACC using Equation 3 for learning purposes.

Table 2: Exercise 2 -- MM II: Cost of Equity and WACC

Market Value Debt	Market Value Equity	D/E	Debt Ratio	Cost of Debt	Cost of Equity	WACC
0	200	0	0%	6%	10.00%	10.00%
20	180	0.11	10%	6%	10.40%	10.00%
40	160	0.25	20%	6%	11.00%	10.00%
60	140	0.43	30%	6%	11.70%	10.00%
80	120	0.67	40%	6%	12.70%	10.00%
100	100	1	50%	6%	14.00%	10.00%
120	80	1.5	60%	6%	16.00%	10.00%
140	60	2.33	70%	6%	19.30%	10.00%
160	40	4	80%	6%	26.00%	10.00%
180	20	9	90%	6%	46.00%	10.00%
190	10	19	95%	6%	86.00%	10.00%
195	5	39	98%	6%	166.00%	10.00%
200	0	Undef.	100%	6%	Undef.	Undef.

Once Exercise 2 is completed, students can then graph the cost of debt, WACC, and cost of equity, first, using D/E on the x-axis and, second, using the debt ratio on the x-axis. This will allow the students to visualize the stark differences in the appearance of changes to the cost of equity in D/E versus debt ratio space. The results are shown in Graphs 2C and 2D.

Figure 2C: MM Proposition II: D/E Space

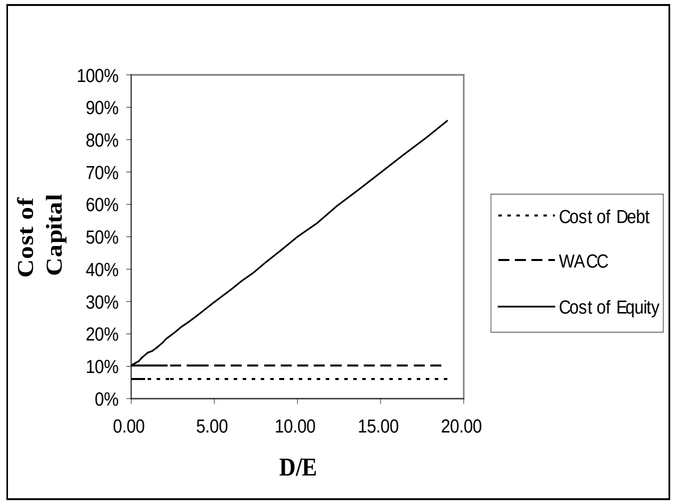
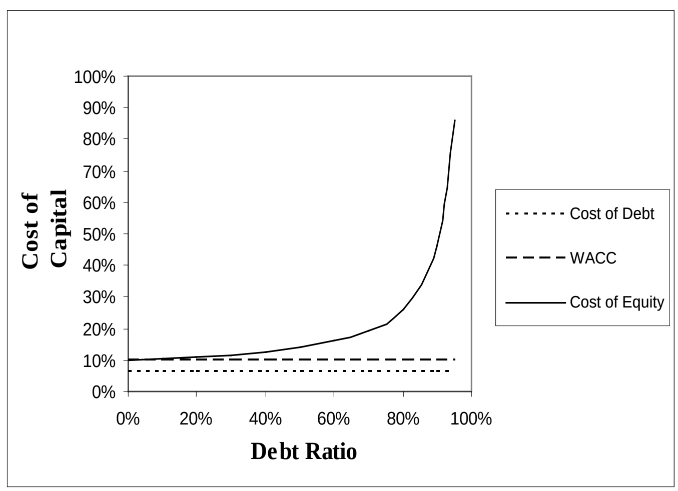


Figure 2D: MM Proposition II: Debt Ratio Space



Although the cost of debt and the WACC lines are the same in the new graphs, the cost of equity lines are dramatically different. When the x-axis is D/E, the cost of equity line is a straight line. This is expected and due to the second MM proposition: a linear relationship exists between D/E and the levered cost of equity (Equation 2). However, when the x-axis is the debt ratio, the cost of equity line is an upward sloping curve that, here, starts seriously increasing at around 50% debt. The change in the cost of equity due to increasing debt is exponential, not linear, when assuming a constant cost of debt and company average cost of capital.

As previously mentioned, students looking at the D/E graph but thinking of leverage in debt ratio terms are likely not to recognize the point where the company is half financed by debt and half financed by equity. Referring to the D/E graph above (Graph 2C), because the range of the x-axis is from 0 D/E to 20 D/E, students most likely will identify 50% debt with a D/E of 10, which is the midpoint of the x-axis. However, a D/E of 10 is equivalent to about 91% debt. Obviously, students will easily recognize the 50% debt point when the x-axis is the debt ratio (Graph 2D).

At this point, instructors may want to point out the severe degree of compression when the x-axis is D/E. A 50% debt ratio is a little shy of half way between 0% debt and 95% debt and is equivalent to a D/E of 1. Yet, using a range of 0 to 20, a D/E of 1 is close to the origin and only covers 5% of the total x-axis range. Similarly, using a range of 0 to 20, a D/E of 10 is the midpoint and covers 50% of the total x-axis range yet represents about a 91% debt ratio. Students will need to be guided through this phenomenon in order to truly understand the scales of the two leverage measures and how dramatically different they are. A table of equivalents surely helps, but "seeing" it on the graphs will make it real.

What can complicate student learning further is when a textbook graph does not provide scaling for the x-axis. That is, when the far right-end of the x-axis is not delineated. In that case, is the range 0 D/E to 1 D/E? 0 D/E to 20 D/E? 0 D/E to 40 D/E? Alternately, is the range 0% debt to 50% debt? 0% debt to 95% debt? 0% debt to 100% debt? The far right end point surely makes a difference when interpreting the graph. Without a far right end point or assuming a full range (0 D/E to say 40 D/E, 0% debt to 100% debt), the student cannot know what degree of leverage the graph represents. Especially when the line graphed is curved, there is no way for the student to know at what point the curve starts to noticeably increase.

Students looking at the D/E graph but thinking of leverage in debt ratio terms also are likely not to understand the effect of adding (excessive) debt on the cost of equity and, worst case, may even assume the effect is a linear, marginal, possibly even negligible, increase in the cost of equity. The debt ratio graph shows that this is plainly not the case and the instructor may want to emphasize this point. In addition, lack of scaling of the y-axis can obscure this result and hinder student understanding. The instructor may use Graph 2D to show the theoretical nature of the Modigliani-Miller results, including the dramatic increase in the cost of equity, and to begin the process of introducing "real world" considerations. Students could be asked to consider the likelihood that equity investors would actually expect a return of 166%, 86% or even 46%, and, if so, what type of market situation does this entail? Are securities likely to be priced to earn 166%? Under what conditions would a firm increase leverage to the point that equity investors would require a return of 166%? What type of job security would a CFO expect if (s)he undertook such a policy? Hopefully, students in upper division courses would find this behavior extremely unlikely!

TEACHING EXTENTIONS TO THE 1958 MM PROPOSITI0NS

Once the MM "no taxes" propositions have been presented, most textbooks start transitioning to a more "real world" of frictions by relaxing various assumptions. Often the first is to include corporate (only) taxes (i.e., MM 1963 "with taxes"). Without presenting details, we note that the exercises presented above can be expanded and adjusted accordingly and parallel sets of graphs created. All implications derived from the MM "no taxes" continue (e.g., measure of leverage used, axis scaling and compression) but now the graphs themselves will change shape as well, leading to changed or additional implications for the value of the firm or its costs of capital. For example, when allowing for corporate (only) taxes, the cost of debt is still constant but is lower, the cost of equity, at some debt ratio, still noticeably increases but does so at a slower rate, and the WACC is no longer constant but decreases in a linear fashion.

SUMMARY

Our goal in this teaching note was to provide instructors with a series of exercises that could be used to enhance student learning of the 1958 MM propositions and better prepare students for transitioning their understanding from the Modigliani-Miller "perfect world" concepts to financial markets where frictions exist. We have suggested that textbook presentations of the 1958 Modigliani-Miller model allow for confusion on the part of students because of ambiguity in discussions and graphical representations. Textbook discussions of capital structure theories may be presented using different measures of leverage. Students looking at a graph in D/E space but thinking of leverage in debt ratio space are likely not to recognize the point where the company is half financed by debt and half financed by equity. The severe degree of compression when the x-axis is D/E adds to their possible misunderstanding. Students also are likely not to understand the effect of adding (excessive) debt on the cost of equity and, may incorrectly assume the effect is a linear, marginal, possibly even negligible, increase in the cost of equity. Additionally, the full implications of the propositions cannot be seen when the supporting graphs do not provide complete scaling for either or both axes. Furthermore, these issues remain when presenting the effects of relaxing one or more of the "perfect world" assumptions.

Finally, a note of caution. Textbook discussions and graphs, as well as the exercises and graphs presented here, are general representations and no one graph will represent all firms. Referring to the MMII graph when the x-axis is the debt ratio (Graph 2D), the cost of equity line is an upward sloping curve that starts seriously increasing at around 50% debt. This is true for our example and may be true for some companies but most likely will not be true for many companies. Students would greatly benefit from more discussion about industries and companies having different acceptable ranges of leverage and why this is so. Not having this finishing discussion may leave students with a naive, black-white type perception and not yet ready to contribute credibly upon entering the workforce.

The authors wish to express their gratitude to the participants of the February 2010 Academy of Economics and Finance Annual Meeting and especially discussant David Lange.

END NOTES

1. A perfect world is one in which the typical perfect capital markets assumptions hold: homogeneous and perfect expectations, no taxes, no bankruptcy, no transaction costs, all participants have the same information and can borrow and lend at the same constant cost of debt.

2. In his 1938 book, *The Theory of Investment Value*, John Burr Williams also presented the idea that the value of a firm is independent of its capital structure. (North Holland Publishing, 1938; reprinted by Fraser Publishing, 1977)

3,4. The results of a brief textbook survey are available at www.jfcr.org in the downloads section for JITF.

5. In order to be able to more easily compare the graphs in this paper, and for purposes of consistency, graphs where an axis is in D/E space have a range from 0 D/E to about 20 D/E and graphs where an axis is in debt ratio space have a range from 0% debt to about 95% debt. Since a D/E of 20 is approximately equivalent to a 95% debt ratio, all graphs represent approximately the same range of leverage.

Exhibit 1: Selected Franco Modigliani and Merton H. Miller Papers

1958 The Cost of Capital, Corporation Finance and the Theory of Investment
American Economic Review, *48*, 261-297.

1959 The Cost of Capital, Corporation Finance and the Theory of Investment: Reply
American Economic Review, *49*, 655-669.

1963 Taxes and the Cost of Capital: A Correction
American Economic Review, *53*, 433-443.

1965 The Cost of Capital, Corporation Finance and the Theory of Investment: Reply
American Economic Review, *55*, 524-527.

A paper by Merton H. Miller.

1988 The Modigliani-Miller Propositions After Thirty Years
Journal of Economic Perspectives, *2* (4), 99-120.

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A Quick Note from the Editor

It has been a pleasure to serve as the editor of the Journal *of Instructional Techniques in Finance* for the past two years. During this past year, we have made arrangements for Steve Johnson from Sam Houston State University to take over as editor. Steve has been a past contributor to the *JITF*, and we are looking forward to his leadership in taking the *Journal* forward. Please be patient with us as we accomplish the transition.

Please consider sharing your techniques for effective teaching of finance with us and those who receive the *JITF*. Our goal is to augment the effectiveness with which finance educators impart knowledge to their students. Those interested in becoming a participant in our mission should contact Steve at sjj008@shsu.edu. I will remain as an Executive Editor to provide oversight. Any concerns in that regard may be addressed to me at rstretcher@shsu.edu. Thank you for your interest in finance education!

Robert Stretcher