A Method for Teaching the Binomial Option Pricing Model in Investments Courses

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One of the most important concepts in modern finance in general, and in teaching an investments course in particular, is option pricing. While the Black-Scholes model is the most well-known option pricing method, we have found that the binomial option pricing model is sometimes better for illustrating how different inputs affect the value of the option. The binomial option pricing model provides students with an intuitive understanding of the mechanics of option pricing. The binomial option pricing model is also widely used in practice. This paper presents an implementation of the one-period binomial model in Excel. By using spin buttons, students can directly observe the effects of changes in volatility, the underlying asset price, and the risk free rate, on the price of a call option. The model also allows students to compare prices of options with different strike prices. In addition to calculating binomial option prices and providing students with improved intuition, this spreadsheet provides a graphical representation of option value relative to the value of the underlying asset.

The spreadsheet is formed by first listing, on the left hand side, the input variables:

- S_0, current asset price
- S_mid, the midpoint (simple avg) of the expected stock prices
- S_spread, the spread (difference) between the expected stock prices
- Strike, the strike price
- Rf, the risk-free rate

The five input variables, S_0, S_mid, S_spread, Strike, and Rf, will all be derived from spin button outputs. We will demonstrate how to add a spin button in Excel 2007, using the first input variable, S_0, as an example. First, enter the items in figure 1 into a blank spreadsheet. In order to add spin buttons to the spreadsheet, it is convenient to first add a spin button icon to the Quick Access Toolbar. Click on the "Customize Quick Access Toolbar" icon and scroll down to "More Commands." Select "More Commands," then choose "All Commands."

Figure 1: First Entries.

	A	B	C	D
1	Binomial Call Option Pricing			
2	Inputs			
3	s_0			
4				
5				
6				

After choosing "All Commands," then scroll down to "Spin Button (Form Control)." Click "Add," then "OK." This will add the spin button to the toolbar.

Next, click on the spin button icon. This will give you a "+"-shape that you can use to draw your spin button. Go to the upper left-hand corner of cell C3, hold down the left mouse button, and drag to the lower right-hand corner of cell C4. Then, release the left mouse button. This creates the spin button.

Figure 2. Spin Button Inserted.

	A	B	C	D
1	Binomial Call Option Pricing			
2	Inputs			
3	s_0	0.9	▲	9
4			▼	
5				
6				

Right-click on the spin button. This will give you a box with a list of choices. Select Format Control. If the Format Control box is covering your spin button key, left-click on the blue banner at the top of the box, then drag the box off to the side. The raw output from the spin button is a whole number. In order to put the raw output in cell D4, click on the "Cell link" box, then click on cell D4 in the spreadsheet. Now we have linked the raw output from the spin button to cell D3. Click on the spin button a few times to see how the output changes

The stock price will vary in steps of $0.10. It is not possible to do this directly, because the smallest step size available is 1. In cell B3, reference the cell D3, and multiply the raw spin button output by the step size, in this case $0.10. It will be important to refer to this cell. It is convenient to give this cell, B3, a name. This allows absolute referencing without the need to remember the exact location of the cell. Click on B3, then click on the address of the cell in the name box. Create a variable name by typing "S_0" into the box. Hit enter. The first spin button is finished. Again, note that the raw output from the spinner, in column D, is a whole number. In order to create different step sizes, we multiply that raw output by the step size that is needed, in this case, $0.10. Create the next four spin buttons similarly. The step size for S_mid and Strike is $0.10. The step size for S_spread is $0.01. The step size for Rf is 0.001.These values are illustrated in figure 12. Each of the variables above, in the inputs list, is given the name to the left. For example, the stock price at time 0, 9.1, is given the name "S_0." Naming variables simplifies later references.

Figure 3. Completed Spin Buttons.

Binomial Call Option Pricing		
Inputs		
S_0	9.1	91
S_mid	9	90
S_spread	1.5	150
Strike	8.8	88
Rf	0.065	65
S_up	9.75	
S_down	8.25	

The one-period binomial option model is a two-state model. The stock price can take on one of two possible prices. The possible stock prices at time 1 are calculated from this data, the midpoint (S_mid) and the spread (S_spread). The resulting output is reported as S_up and S_down. S_up and S_down are calculated from the midpoint and spread as follows:

S_up = S_0 + S_spread/2
S_down = S_0 – S_spread/2

Figure 4. Spreadsheet inputs, including the possible values of the underlying asset at time 1.

The inputs can all be varied dynamically by adjusting the Excel

Binomial Call Option Pricing		
Inputs		
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Rf	0.065	65
S_up	9.75	
S_down	8.25	

spin buttons. The intermediate steps and final solution (call option value) are all reported under the heading "Outputs." The outputs include:

* C_up, the payoff of the call option in the high stock price state
* C_down, the low stock price state
* Delta, the value of delta
* Risk-free payoff, the payoff of the risk-free Portfolio at maturity
* PV(Risk-free payoff), the value of the risk-free portfolio at time 0.

Figure 5. Inputs and Outputs.

Binomial Call Option Pricing				
Inputs			Outputs	
S_0	9.1	91	C_up	$0.95
S_mid	9	90	C_down	$0.00
S_spread	1.5	150	Delta	0.633333
Strike	8.8	88	Risk-free payoff	$5.2250
Rf	0.065	65	PV(Risk-free payoff)	$4.9061
			Call option value	$0.8572
S_up	9.75			
S_down	8.25			

These variables are also named in Excel. The variable definitions are as follows and discussed in more detail in the following paragraphs:

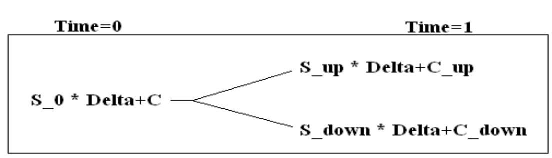
C_up = max(S_up - Strike, 0)
C_down = max(S_down - Strike, 0)
Delta = (C_up - C_down)/(S_up - S_down)
Payoff = Delta*S_up - C_up
PV_Payoff =Payoff*(1+Rf)^-1

The binomial option pricing model allows the modeler to price a risky call option on an underlying asset without knowing either the probabilities of the up and down states, the required return on the risky underlying asset, or the required return on the call option. The key to solving for the option price is the creation of the risk-free portfolio.

This case specifically investigates the pricing of a call. Because a call option provides the option holder with the right to buy, the option is in the money when the value of the underlying asset is greater than the strike price. The option is out of the money, or worth $0, when the value of the underlying asset is less than the strike price. This can be implemented with the Excel =max(.) function:

C_up = max(S_up - Strike, 0)
C_down = max(S_down - Strike, 0)

The modeler can create a risk-free portfolio by combining a short position in the call option with some amount of shares. This quantity of shares is called "Delta." The resulting portfolio value at time 0 and possible payoffs at time=1 look like this:



If the portfolio is a risk-free portfolio, then the two possible payoffs at time 1 should be equal. By setting the two payoffs equal to each other and solving for Delta, we obtain the value of Delta:

Delta = (C_up - C_down)/(S_up - S_down)

Because the two payoffs at time 1 are equal, we can use either one to find the dollar value of the payoff at time 1. To be specific, we will choose the first one, the payoff in the "up" state:

Payoff = S_up * Delta + C_up

Next, the value of the portfolio at time 0 should be equal to the present value of the payoff at time 1:

PV_Payoff =Payoff*(1+Rf)^-1

Set this equal to the value of the portfolio at time 0 and solve for C, the call option value:

S_0 * Delta – C = PV_Payoff
C = S_0 * Delta – PV_Payoff

This arrangement of inputs and outputs, and in particular, the inputting of the expected future stock prices by using the mid-point and spread, allows students to "see" the effects of volatility on the option price. Volatility is often a difficult concept to visualize. With this example, students have one specific type of volatility that is relatively straightforward to conceptualize--the spread between the two possible outcomes of the stock price at time one. By changing the value of the spread, the midpoint of the expected stock price remains the same, but the value of the call option increases.

The third part of the spreadsheet consists of the graph of the call option value, call option payoff, and call option time value. The graph provides students with additional insight. For example, when the underlying stock price changes, the value of the call option changes, but the option contract has not changed --only the *value* of the contract changes. However, different values of the strike price illustrate different call option contracts. The graph allows students to distinguish between changes in the value of an option contract and different option contracts. All recalculations of the option value are virtually instantaneous. The graph responds almost instantaneously as well.

Next, fill column J with stock prices that vary from \$8.00 to \$11.00, inclusive, in steps of \$0.01. Column L contains the payoff for a call option when the underlying asset price is equal to the corresponding stock price from column L. The formula entered into cell L3 is:

=MAX(J3-Strike,0)

This formula can be copied to fill column L. The variable that contains the strike price is named "Strike." This is an absolute reference to cell B9, which contains the strike price, the output from one of the spin buttons. The absolute references all have variable names, making referencing easier. In this case, the relative cell reference refers to the stock price in column J.

Column K contains the values of the call for each value of the underlying stock price from column J. The formula for the call value for \$8.00, entered in cell K3, is:

=MAX(J3*Delta-(1/(1+Rf))*(S_up*Delta-C_up),L3)

This formula can also be copied to fill column K. Again, the absolute references all have variable names, while the relative cell references refer to the stock price in column J and the option payoff in column L. The absolute reference Rf refers to the risk-free rate, one of the inputs to the problem, determined by the spin button setting. The value of S_up is determined by the value of the stock in the up state. This is an absolute reference to cell B14. The value of C_up is determined by the value of the stock in the up state and the strike price of the option. This is an absolute reference to cell G3. Delta refers to the number of shares needed to create a risk-free portfolio when taking a short position in one call option. This is an absolute reference to cell G7.

The time value of the option is calculated in column M. The formula for the time value found in cell M3 is:

=K3-L3

This formula uses relative addressing and so can be copied to fill column M.

The graph consists of 4 separate diagrams. The Call Value (column K), Payoff (Column L), and Time Value (Column M), are all plotted versus the stock price from column J. These are represented by the blue, red, and green lines, respectively. This aids the student in visualizing how different contracts with different strike prices are priced differently and how volatility affects the payoff structure of a given option.

In addition to being able to visualize how payoff structures and the graphs of option values change for different strike prices, it is also possible for the student to see how movements in the underlying asset price affect the value of a given option contract. By adjusting S_0, the stock price at time 0, it is possible to see how the value of the option changes, even though the option contract itself remains the same.

Figure 6. Screen view of the spin buttons, outputs, graph, and data for the graph.

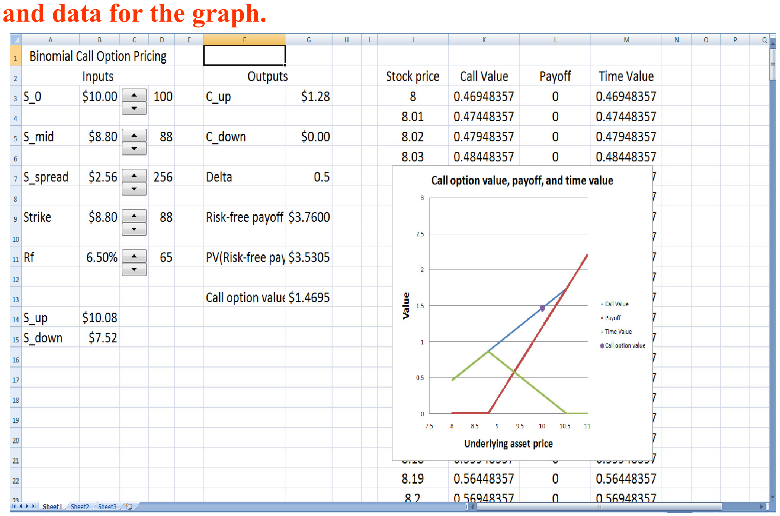


Figure 7. Inputs, outputs, and graph, underlying stock price = $9.10

Binomial Call Option Pricing				
Inputs			Outputs	
S_0	9.1	91	C_up	$1.43
S_mid	9	90	C_down	$0.00
S_spread	2.46	246	Delta	0.581301
Strike	8.8	88	Risk-free payoff	$4.5167
Rf	0.065	65	PV(Risk-free payoff)	$4.2410
			Call option value	$1.0488
S_up	10.23			
S_down	7.77			

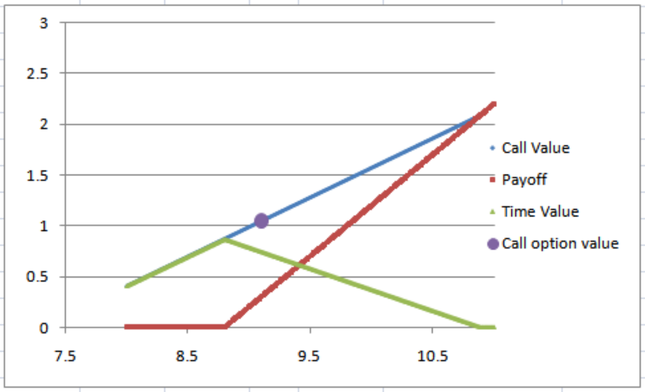


Figure 8. Inputs, outputs, and graph, underlying stock price = $10.00

Binomial Call Option Pricing				
Inputs			Outputs	
S_0	10	100	C_up	$1.43
S_mid	9	90	C_down	$0.00
S_spread	2.46	246	Delta	0.581301
Strike	8.8	88	Risk-free payoff	$4.5167
Rf	0.065	65	PV(Risk-free payoff)	$4.2410
			Call option value	$1.5720
S_up	10.23			
S_down	7.77			

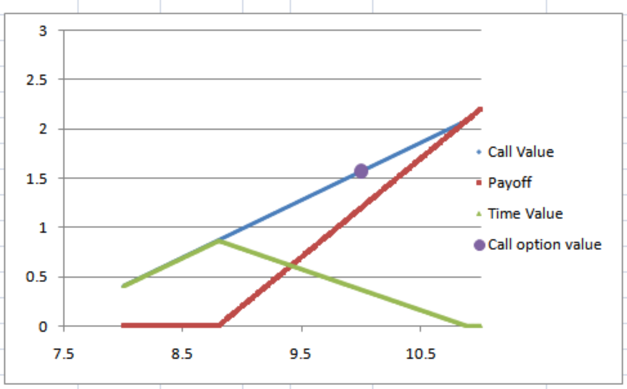


Figure 7 provides the following option diagrams:
* call option value
* option payoff at maturity
* time value

Figure 7 also illustrates the value of the call option for a particular underlying stock price, in this case, $9.10. Students can vary the values of the inputs. In particular, it is possible to see what happens to the value of the call option if the underlying stock increases to $10 in price.

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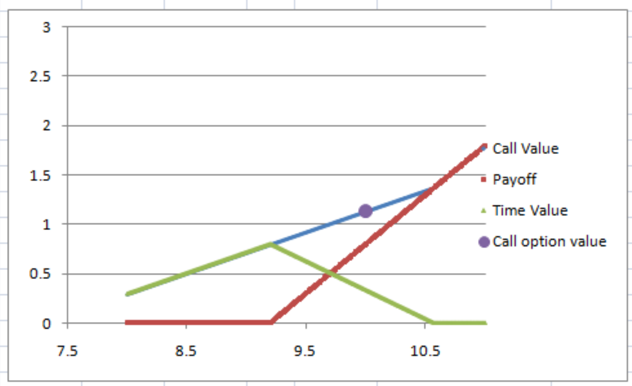
The graphs of the call value, payoff, and time value remain unchanged. Only the value of the call option itself has changed.

Students can see what would happen to the payoff, time value, and call value if the exercise price were different. Figure 9 shows the results for a strike price of $9.20, leaving the other inputs from Figure 16 unchanged:

Figure 9. Strike price of $9.20, other inputs unchanged from Figure 4.

Binomial Call Option Pricing				
Inputs			Outputs	
S_0	$10.00	100	C_up	$1.03
S_mid	$9.00	90	C_down	$0.00
S_spread	$2.46	246	Delta	0.418699
Strike	$9.20	92	Risk-free payoff	$3.2533
Rf	6.50%	65	PV(Risk-free payoff)	$3.0547
			Call option value	$1.1323
S_up	$10.23			
S_down	$7.77			



The “corner” in the payoff graph is further to the right, signifying the higher strike price, and the graphs of the call value and the time value are “lower” (have lower values). This allows the students to visualize the effects of different strike prices more easily.

It is also possible to study the effects of different expected future stock prices on the value of the call option today. In this spreadsheet, the future stock price information is input in two variables—the midpoint of the two expected future stock prices and the spread between the future stock prices. If the midpoint of the two expected future stock price increases, the call option should move deeper into the money. This is illustrated in Figure 10:

Figure 10. Midpoint of expected stock price = $9.50, otherwise unchanged from Figure 16.

Binomial Call Option Pricing					
	Inputs			Outputs	
S_0	$10.00		100	C_up	$1.99
S_mid	$9.50		95	C_down	$0.00
S_spread	$2.58		258	Delta	0.771318
Strike	$8.80		88	Risk-free payoff	$6.3325
Rf	6.50%		65	PV(Risk-free payoff)	$5.9460
				Call option value	$1.7672
S_up	$10.79				
S_down	$8.21				

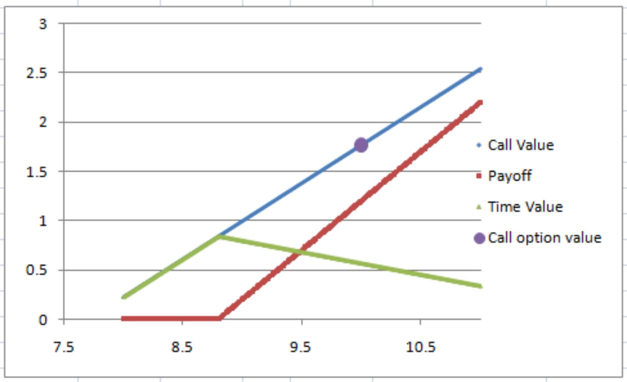
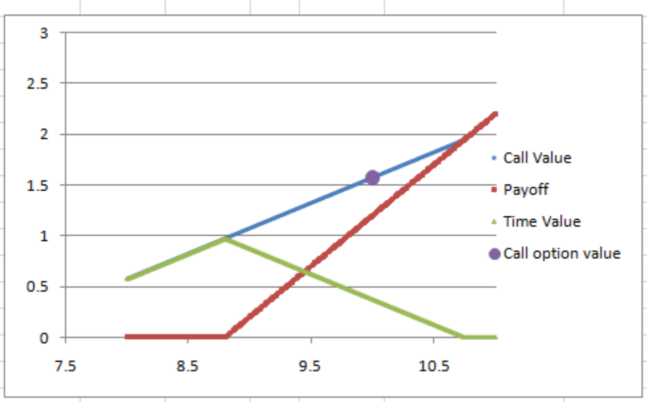


Figure 11. Spread of expected stock prices = $3.00, other inputs unchanged from Figure 16.

Binomial Call Option Pricing					
	Inputs			Outputs	
S_0	$10.00		100	C_up	$1.50
S_mid	$8.80		88	C_down	$0.00
S_spread	$3.00		300	Delta	0.5
Strike	$8.80		88	Risk-free payoff	$3.6500
Rf	6.50%		65	PV(Risk-free payoff)	$3.4272
				Call option value	$1.5728
S_up	$10.30				
S_down	$7.30				



By comparing this with figure 5 (or adjusting the input value in the spreadsheet), it is possible to see that the call value and time value have increased. The graph of the payoff is unchanged, because the graph of the payoff is determined uniquely by the strike price.

The effect of volatility of the underlying asset on option prices can be illustrated in a straightforward manner by the use of this spreadsheet. One of the input values is the spread between the two possible future stock prices. This spread is a rough measure of volatility. As the spread between the two possible future stock prices widens, the expected value of the option increases. This is illustrated in Figure 11.

If you try this method for enhancing options coverage, we would be grateful to receive feedback from you. Please contact us.The complete spreadsheet is available at www.jfcr.org in the downloads section for this issue of JITF.

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