Journal of Modern Applied Statistical Methods

Volume 18 | Issue 2 Article 29

10-12-2020

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Recommended Citation

Mishra, U., Siddiqui, S. & Singh, J. R. (2019). Robustness of the EWMA Sampling Plan to Non-Normality. Journal of Modern Applied Statistical Methods, 18(2), eP2626. https://doi.org/10.22237/jmasm/1604188860

Journal of Modern Applied Statistical Methods
November 2019, Vol. 18, No. 2, eP2626
https://doi.org/10.22237/jmasm/1604188860

ISSN 1538 − 9472

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The effect of non-normality on the OC function of the sampling plan under EWMA is studied by deriving the OC function for a non-normal population represented by the first four terms of an Edgeworth series.

Keywords: exponential weighted moving average (EWMA) sampling plan, OC function, non-normal population

Introduction

Acceptance sampling is a significant field of statistical quality control, familiarized by Dodge and Romig (1959), who explained that a sample should be chosen at random from the lot, and a conclusion can be drawn for the acceptance or rejection of the lot on the basis of the information that was received by the sample. Most acceptance sampling plans provide a conclusion on the basis of information from the current sample of a lot. The utilization of the prior information together with the current lot information can improve the inspection efficiency. According to Kalgonda et al. (2011), sampling plans based on the current information are called the memory less acceptance sampling plans. In the area of control charts, exponential weighted moving average (EWMA) statistic has been popularly used, which considers the present and past information by giving high weight to the present data and less weight to the previous data. This EWMA statistic is known to be efficient to detect a small shift in the process (see Montgomery, 2005; and Čisar & Čisar, 2011). Recently, Aslam et al. (2013) introduced the EWMA statistic in an acceptance sampling plan. Yen et al. (2014) developed a new variable sampling plan using the EWMA statistic for lot sentencing. They determined the sample size and the critical value of the proposed plan by considering the acceptable quality

doi: 10.22237/jmasm/1604188860 | Accepted: Sept. 15, 2017; Published: Oct. 12, 2020.
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level at the producer's risk and the lot tolerance percent defective at the consumer's risk. In many industrial situations, the distributional assumptions required for the sampling plans may be violated. Therefore, investigation of the robustness of EWMA sampling plan to the usual normality assumption is essential. Chou et al. (2006) obtained economic design of variable sampling intervals exponentially weighted moving average (VSI EWMA) control chart and also concluded in the same way. Chou et al. (2008) have also developed economic design of VSI EWMA chart. Epprecht et al. (2009) have compared performance of VSI EWMA and fixed sampling interval (FSI EWMA) control charts for attributes. The advantage of EWMA statistic is to consider the quality of the current lot and the preceding lots.

The aim of the current study is to study the effect of non-normality on the OC function of the sampling plan under EWMA. The OC function is derived for a non-normal population represented by the first four terms of an Edgeworth series.

OC Function for EWMA Model under Non-Normality:

Suppose that a process is on target μ initially and successive measurements $\bar{X}_t$ ($t = 1, 2, \ldots$) are taken (it could even be average of several measurements taken at time t) to check whether there is a shift from the target. Then to use a sampling plan based on the statistic W_t which satisfies the relation:

$$W_t = \lambda \bar{X}_t + (1-\lambda) W_{t-1},\ t \geq 0 \tag{1}$$

where $W_0 = \mu$.

This is a geometric mean of all the observations with $\bar{X}_t$; the most recent observation gets the greatest weight and all previous observations weight decreasing in geometric progression.

$E(W_t) = \mu$ and

$$\operatorname{var}(W_t) = \frac{\sigma^2}{n}\left(\frac{\lambda}{2-\lambda}\right) = \frac{\sigma^2}{n} T^2 \tag{2}$$

where $T^2 = \left(\frac{\lambda}{2-\lambda}\right)$.

Let the non-normal variable W_t have mean μ and variance σ^2 and λ_3 and λ_4 the measure of skewness and kurtosis. Assume the distribution of W_t is stationary and

EWMA model, and hence without loss of generality, denote the density function of W_t by $f(W)$ and is expressed it as

$$f\left(W\right)=\frac{1}{\sigma}\left[\begin{array}{l}\Phi\left(\frac{W-\mu}{\sigma}\right)-\frac{\lambda_3}{6}\Phi^{(3)}\left(\frac{W-\mu}{\sigma}\right)\\ \qquad\qquad+\frac{\lambda_4}{24}\Phi^{(4)}\left(\frac{W-\mu}{\sigma}\right)+\frac{\lambda_3^2}{72}\Phi^{(6)}\left(\frac{W-\mu}{\sigma}\right)\end{array}\right]. \tag{3}$$

where $\Phi\left(t\right)=\frac{1}{\sqrt{2\pi}}\exp\left(-t^2\!\big/_2\right)$ and $\Phi^r\left(t\right)=\frac{d^r}{dt^r}\Phi\left(t\right)$..

If U is the upper specification limit, then the acceptance criterion suggests to accept the lot if $\bar{x}+k\sigma\le U$ and reject it otherwise.

The formulae for the acceptance constant have been found by Bowker and Goode (1952) in the form

$$n=\left[\frac{\left(K_\alpha+K_\beta\right)}{\left(K_{p_1}-K_{p_2}\right)}\right]^2, \tag{4}$$

$$k=\left[\frac{K_\alpha K_{p_2}+K_\beta K_{p_1}}{\left(K_\alpha+K_\beta\right)}\right]. \tag{5}$$

The value of n and k are determined for a given set of values of $(p_1, 1-\alpha)$ and (p_2, β) by the above formulae.

If p is the proportion defective in the submitted lot, then we know that $(U-\mu)/\sigma = K_p$, which implies

$$\mu+K_p\sigma=U. \tag{6}$$

Under the assumption of normality, a lot having p percent defective items will be accepted, if $\bar{x}+k\sigma\le\mu+K_p\sigma$, where K_p is given by

$$\int_{K_p}^{\infty}\left[\Phi(t)-\frac{\lambda_3}{6}\Phi^{(3)}(t)+\frac{\lambda_4}{24}\Phi^{(4)}(t)+\frac{\lambda_3^2}{72}\Phi^{(6)}(t)\right]dt=p, \tag{7}$$

That is

$$\int_{K_p}^{\infty}\Phi(t)\,dt+\Phi(K_p)\left[\frac{\lambda_3}{6}H_2(K_p)+\frac{\lambda_4}{24}H_3(K_p)+\frac{\lambda_3^2}{72}H_5(K_p)\right]=p. \tag{8}$$

where $H_v(\mu)$ is well known Hermite Polynomial of degree 5 in μ given by $H_v(\mu) = (-1)^v\,\Phi(\mu)H_v(\mu)$.

The value of K_p corresponding to a given value of p can be approximately obtained by using the method of Cornish and Fisher (1938) as

$$K_p=x_p+\left[\frac{\lambda_3}{6}\left(x_p^2-1\right)+\frac{\lambda_4}{24}\left(x_p^3-3x_p\right)-\frac{\lambda_3^2}{72}\left(2x_p^3-5x_p\right)\right] \tag{9}$$

where x_p is given by

$$\int_{-\infty}^{x_p}\Phi(t)\,dt=1-p. \tag{10}$$

The probability of accepting a lot of incoming quality p, i.e. L_p, the OC function of the plan under EWMA model is

$$L_p=\int_{-\infty}^{\frac{\sqrt{n}}{T}(K_p-k)}\Phi(t)\left[1+\frac{\lambda_3 T}{6\sqrt{n}}H_3(K_p)+\frac{\lambda_4}{24}\frac{T^2}{n}H_4(t)+\frac{\lambda_3^2}{72}\frac{T^2}{n}H_6(t)\,dt\right]$$

$$L_p=\int_{-\infty}^{\xi_p}\Phi(t)\,dt-\Phi(\xi_p)\left[\begin{aligned}&\frac{\lambda_3 T}{6\sqrt{n}}H_2(\xi_p)+\frac{\lambda_4}{24}\frac{T^2}{n}H_3(\xi_p)\\&\qquad+\frac{\lambda_3^2}{72}\frac{T^2}{n}H_5(\xi_p)\,dt\end{aligned}\right] \tag{11}$$

where $\xi_p=\dfrac{\sqrt{n}}{T}\left(K_p-k\right)$.

Tabulation and Discussion of Results

Consider an example for known standard deviation under EWMA model sampling scheme, $p_1 = 0.05$, $\alpha = 0.05$, $p_2 = 0.30$ and $\beta = 0.10$. Further, consider a few non-normal population specified by the constants $(\lambda_3, \lambda_4) = (0, 0)$, $(-0.6, 0)$, $(0.6, 0)$, $(0, -1.0)$ and $(0, 2.0)$ for studying the robustness of EWMA model. The values of the OC function computed by using equation (11) are presented in Table 1. The nature of the effect for other non-normal populations being more or less, the same though with some variation in magnitude. Setting up an EWMA sampling schemes requires the specification of λ, which determines the relative amount of weight given to current and past observations. The overall discussion about these sampling schemes is that for the EWMA model considered here, the EWMA plan of the residuals is not necessarily better in terms of statistical properties than the EWMA sampling scheme of the original observations. This assumes the sampling scheme of the EWMA model of the observations is adjusted to account for the weight λ. In many applications, it might be preferable to prepare the EWMA sampling schemes of the observations because EWMA sampling plan gives an immediate estimate of the plan parameters and thus it will be easier for a product control practitioner to interpret the OC function. The EWMA forecast has been recommended as a simple alternative to the model-based forecast. The recovery rates for EWMA forecasts are different in many cases from those obtained for the model-based forecast.

In the AQL and LTPD plans considered above, the sample size n and the values of k determined on the assumption of normality are 7 and 1.0232 respectively. It may be observed that use of such values of k, though erroneous, is quite helpful in arriving at certain conclusions regarding the effect of non-normality on the OC function under EWMA model. From the table values it appears that when λ_4 is positive the OC function becomes steeper and reduces both producer's and consumer's risk; the reverse is the case when λ_4 is negative for $\lambda = 1$. When $\lambda = 1$ and λ_3 is positive, the OC function also tends to improve, making it cheaper; however, lot quality compares disfavorably to AQL, and the L_p is greater than that in the normal theory case.

The reverse holds well when λ_3 is negative in the known σ cases. When λ_3 is -0.6, λ_4 is zero and $\lambda = 1$, the OC function seems to be quite close to the normal theory OC function. In Table 1, the values of λ_3 and λ_4 are within the Barton and Denis (1952) limits, which means that for such values the population given by equation (3) is positive, definite and unimodal. The effect of moderate non-normality on the OC function when $|\lambda_3| \leq 0.6$ and $|\lambda_4| \leq 1.0$ may not be regarded as serious for $\lambda = 1$. However, a leptokurtic of the order $\lambda_4 = 2.0$, which is within

Table 1. Normal Theory OC Function as Distorted by Non-Normality with EWMA model, n = 7, k = 1.0232

	$\lambda_3 = 0, \lambda_4 = 0$		$\lambda_3 = -0.6, \lambda_4 = 0$		$\lambda_3 = 0.6, \lambda_4 = 0$		$\lambda_3 = 0, \lambda_4 = -1.0$		$\lambda_3 = 0, \lambda_4 = 2.0$	
	p	L_p	p	L_p	p	L_p	p	L_p	p	L_p
λ = .4	0.050	0.949490	0.028	0.997910	0.064	0.916440	0.052	0.954670	0.046	0.972825
	0.100	0.751750	0.090	0.798290	0.113	0.707000	0.118	0.695310	0.075	0.881873
	0.150	0.511960	0.155	0.508400	0.158	0.519520	0.169	0.484780	0.111	0.700436
	0.200	0.315610	0.218	0.289050	0.202	0.310840	0.228	0.261420	0.155	0.511836
	0.250	0.178780	0.279	0.139520	0.241	0.202040	0.273	0.139370	0.205	0.314212
	0.300	0.095100	0.336	0.056010	0.286	0.124540	0.321	0.069260	0.259	0.177238
	0.400	0.021178	0.443	0.008290	0.371	0.031580	0.412	0.012660	0.376	0.034523

	$\lambda_3 = 0, \lambda_4 = 0$		$\lambda_3 = -0.6, \lambda_4 = 0$		$\lambda_3 = 0.6, \lambda_4 = 0$		$\lambda_3 = 0, \lambda_4 = -1.0$		$\lambda_3 = 0, \lambda_4 = 2.0$	
	p	L_p	p	L_p	p	L_p	p	L_p	p	L_p
λ = .6	0.050	0.949490	0.028	0.998260	0.064	0.915586	0.052	0.956060	0.046	0.973009
	0.100	0.751750	0.090	0.798080	0.113	0.708691	0.118	0.693790	0.075	0.881083
	0.150	0.511960	0.155	0.506130	0.158	0.521809	0.169	0.488680	0.111	0.699350
	0.200	0.315610	0.218	0.290630	0.202	0.309571	0.228	0.263930	0.155	0.511746
	0.250	0.178780	0.279	0.139430	0.241	0.201823	0.273	0.139390	0.205	0.313199
	0.300	0.095100	0.336	0.054950	0.286	0.125050	0.321	0.068350	0.259	0.176128
	0.400	0.021178	0.443	0.007520	0.371	0.030726	0.412	0.010920	0.376	0.034626

	$\lambda_3 = 0, \lambda_4 = 0$		$\lambda_3 = -0.6, \lambda_4 = 0$		$\lambda_3 = 0.6, \lambda_4 = 0$		$\lambda_3 = 0, \lambda_4 = -1.0$		$\lambda_3 = 0, \lambda_4 = 2.0$	
	p	L_p	p	L_p	p	L_p	p	L_p	p	L_p
λ = .8	0.050	0.949490	0.028	0.998645	0.064	0.914613	0.052	0.957570	0.046	0.973253
	0.100	0.751750	0.090	0.797898	0.113	0.710663	0.118	0.692240	0.075	0.880030
	0.150	0.511960	0.155	0.503550	0.158	0.524404	0.169	0.493140	0.111	0.697909
	0.200	0.315610	0.218	0.292490	0.202	0.308187	0.228	0.266950	0.155	0.511627
	0.250	0.178780	0.279	0.139369	0.241	0.201628	0.273	0.139490	0.205	0.311847
	0.300	0.095100	0.336	0.053760	0.286	0.125657	0.321	0.067300	0.259	0.174648
	0.400	0.021178	0.443	0.006623	0.371	0.029750	0.412	0.008870	0.376	0.034763

	$\lambda_3 = 0, \lambda_4 = 0$		$\lambda_3 = -0.6, \lambda_4 = 0$		$\lambda_3 = 0.6, \lambda_4 = 0$		$\lambda_3 = 0, \lambda_4 = -1.0$		$\lambda_3 = 0, \lambda_4 = 2.0$	
	p	L_p	p	L_p	p	L_p	p	L_p	p	L_p
λ = 1	0.050	0.949490	0.028	0.999050	0.064	0.913570	0.052	0.959050	0.046	0.973590
	0.100	0.751750	0.090	0.797800	0.113	0.712870	0.118	0.690930	0.075	0.878580
	0.150	0.511960	0.155	0.500820	0.158	0.527160	0.169	0.497920	0.111	0.695920
	0.200	0.315610	0.218	0.294580	0.202	0.306840	0.228	0.270480	0.155	0.511460
	0.250	0.178780	0.279	0.139360	0.241	0.201530	0.273	0.139760	0.205	0.309980
	0.300	0.095100	0.336	0.052410	0.286	0.126340	0.321	0.066150	0.259	0.172610
	0.400	0.021178	0.443	0.005630	0.371	0.028710	0.412	0.006570	0.376	0.034950

Barton and Denis limits, may definitely be considered as causing a notable change in the OC function. As can be seen from Table 1, as the values of λ increases, OC function increases.

The result of examples considered, though not quite adequate to be generalized, certainly lead to some conclusions of general character. To a Leptokurtic population ($\lambda_4 > 0$), then the normal theory variables inspection plan is applied. An overall improvement is likely to result and for the values of λ_4 of order 2 it would be really a marked improvement. On the other hand, in the case of Platykurtic population, the OC function deteriorates. Positive skewness tends to improve the OC function only in a limited range of lot quality.

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