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A New Test for Correlation on Bivariate Nonnormal Distributions

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A new method to conduct a right-tailed test for the correlation on bivariate non-normal distribution is proposed. The comparative simulation study shows that the new test controls the type I error rates well for all the distributions considered. An investigation of the power performance is also provided.

Keywords: Correlation, Edgeworth expansion, Cornish-Fisher inverse expansion, type I error rate, power performance

Introduction

Bivariate data is data that has two variables. In the bivariate case, the study of the relationship between the two variables is at least as important as analyzing each variable individually. The most popular measure of the strength of the linear relation between two variables is the correlation coefficient, denoted by ρ. The Pearson product-moment correlation, r, is the most frequently-used estimator for ρ. Another widely-used estimator is the Spearman's rank correlation, denoted by r_s.

Tests Based on Pearson Product-Moment Correlation

Pearson (1896) developed the initial mathematical formulas for the sample correlation coefficient. Let (X_i, Y_i), $i = 1,\dots, n$ be a random sample, the statistic r is given by:

$$r = \frac{s_{XY}}{s_X s_Y} = \frac{\sum\left(X_i - \bar{X}\right)\left(Y_i - \bar{Y}\right)}{\left[\sum\left(X_i - \bar{X}\right)^2 \sum\left(Y_i - \bar{Y}\right)^2\right]^{1/2}}$$

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where s_{XY} is the sample covariance of X and Y, n is the sample size, s_X and s_Y are the sample standard deviations, and $\bar{X}$ and $\bar{Y}$ are the sample means for the variables X and Y, respectively.

The Pearson product-moment correlation r is the maximum likelihood estimator of the parameter ρ when the population has a bivariate normal distribution. Although r is a biased estimator, the bias is negligible when the sample size is large. Researchers have done intensive work on the distribution of r when the population is bivariate normal (Fisher, 1915; Stuart & Ord, 1994).

r can be used to test $H_0: \rho = 0$ when the population is a bivariate normal distribution. The test statistic

$$t_r^* = r\sqrt{\frac{n-2}{1-r^2}}$$

follows the Student's t-distribution with $n - 2$ degrees of freedom under H_0.

r can also be used to test H_0: $\rho = \rho_0$, for $-1 \le \rho_0 \le 1$. The sampling distribution of r is complicated and unstable even when the population is bivariate normal. Fisher (1921) introduced a remarkable transformation of r, which tends to normality much faster. When the sample size n is moderately large, given

$$r^* = \frac{1}{2}\ln\frac{1+r}{1-r}, \quad \rho^* = \frac{1}{2}\ln\frac{1+\rho}{1-\rho}$$

the distribution of $r^* - \rho^*$ approaches to normal with an approximate mean $\rho \big/ 2(n-1)$ and variance $1 \big/ n-3$. Note that $n > 50$ is an adequate sample size for the above approximation (see David, 1938).

To test H_0: $\rho = \rho_0$, the test statistic is:

$$z_F = \frac{\frac{1}{2}\ln\frac{1+r}{1-r} - \frac{1}{2}\ln\frac{1+\rho_0}{1-\rho_0} - \frac{\rho_0}{2(n-1)}}{\sqrt{\frac{1}{n-3}}} \tag{1}$$

z_F has approximately a standard normal distribution under H_0.

Test Based on Spearman Rank Correlation

Spearman (1904) proposed a rank correlation which can be used to measure the relationship between two variables when the distribution is neither bivariate normal nor transformed to a bivariate normal. The Spearman rank correlation, r_s, is a non-parametric version of the Pearson product-moment correlation. Let (R_{1i}, R_{2i}), $i = 1,\ldots, n$ be the paired rank data of two variables, r_s is given by:

$$r_s = \frac{s_{R_1R_2}}{s_{R_1}s_{R_2}} = \frac{\sum\left(R_{1i}-\bar{R}_1\right)\left(R_{2i}-\bar{R}_2\right)}{\left[\sum\left(R_{1i}-\bar{R}_1\right)^2\sum\left(R_{2i}-\bar{R}_2\right)^2\right]^{1/2}}$$

where $s_{R_1R_2}$ is the sample covariance of the paired ranks and s_{R_1} and $\bar{R}_1$, s_{R_2} and $\bar{R}_2$ are the sample standard deviation and the sample mean of the ranks of the two variables, respectively.

The Spearman rank correlation r_s can be used to test:

$$\mathrm{H}_0 : \text{there is no association between the rank pairs}$$

The test statistic is

$$t_s^* = r_s\sqrt{\frac{n-2}{1-r_s^2}}$$

which follows the Student's *t*-distribution with $n - 2$ degrees of freedom under H_0.

Other Tests on Correlation

The test based on r can only be used when the population is bivariate normal or the sample size is relatively large. Although the test based on r_s is applicable to the distribution-free case, it is less powerful and limited to test for zero correlation. However, in real world situations, most distributions are not bivariate normal and the sample sizes may not be large. Furthermore, a test of non-zero correlation is often required. It is desired to develop methods to meet these needs.

Beasley et al. (2007) proposed two new approaches to test a non-zero by using the bootstrapping method. Their methods do not require any knowledge of the population. One is the hypothesis-imposed univarite sampling bootstrap (HI) and the other one is the observed-imposed univariate sampling bootstrap (OI). Two tests are conducted on populations with various combinations of normal and skewed variates with $\rho \geq 0.4$ and the sample size $n \geq 10$. Their study demonstrated that although OI is preferable to HI under the significance level of 0.05, the type I error rates are still slightly inflated. Also, the simulated populations are limited to the combinations between normal and skewed populations. The methods are not evaluated under the situations that both variables are non-normal. Another drawback of these two methods is that they are computer-intensive methods. Unfortunately, most practitioners do not have the computer programming skills to implement these methods.

Beversdorf and Sa (2011) proposed tests of correlation for bivariate non-normal data with small sample sizes. The tests investigated are Fisher's Z transformation z_F and the saddlepoint approximation r_L. They found that z_F and r_L have extremely similar performance which could control the type I error rates well when a left-tailed test was performed under all the bivariate non-normal distributions considered. Both methods essentially failed to control the type I error rates when a right-tailed test is desired.

The purpose of this study is to develop a new right-tailed test on bivariate non-normal distributions with non-zero correlation. The new test statistic is derived using the Edgeworth expansion and the Cornish-Fisher inverse expansion.

Methodology

Edgeworth Expansion

The Edgeworth expansion was derived by Edgeworth (1905), and uses a series to approximate a probability distribution in terms of its cumulants. Let $\hat{\theta}$ be an estimator of an unknown parameter θ, and $\sqrt{n}\left(\hat{\theta}-\theta\right)$ be asymptotically normally distributed with mean zero and variance σ^2. Hall (1983) developed the Edgeworth expansion of the distribution function of $\sqrt{n}\left(\hat{\theta}-\theta\right)$ as a power series in $\sqrt{n}$.

$$\mathrm{P}\left\{\frac{\sqrt{n}\left(\hat{\theta}-\theta\right)}{\sigma} \leq u\right\} = \Phi\left(u\right) + n^{-1/2}\,\mathrm{p}_1\left(u\right)\phi\left(u\right) + \ldots + n^{-j/2}\,\mathrm{p}_j\left(u\right)\phi\left(u\right) + \ldots \quad (2)$$

where $\Phi(u)$, $\phi(u)$, and $\mathrm{p}_j(u)$ denote the standard normal distribution function, its density function, and a polynomial function with coefficients depending on cumulants of $\hat{\theta}-\theta$, respectively.

The inverse of the Edgeworth expansion, obtained by inverting the formula (2), is known as the Cornish-Fisher expansion:

$$\mathrm{P}\left\{\frac{\sqrt{n}\left(\hat{\theta}-\theta\right)}{\sigma} \le z + n^{-1/2}\,\mathrm{p}_{11}\left(z\right) + n^{-1}\,\mathrm{p}_{21}\left(z\right) + \ldots + n^{-j/2}\,\mathrm{p}_{j1}\left(z\right) + \ldots\right\} = \Phi\left(z\right)$$

where z is the percentile of the standard normal distribution and the p_{j1} are polynomials defined in terms of p_js (Hall, 1992).

Proposed Test Procedure

Assume that a bivariate population has finite cumulants and a correlation coefficient ρ. Let κ_{01}, κ_{10}, κ_{02}, κ_{20}, κ_{11},… up to order six be the product cumulants for the bivariate population. Then $r_* = \sqrt{n}\left(r-\rho\right)$ has a limiting normal distribution with mean zero and constant variance σ^2, where σ^2 is of the form (Nakagawa & Niki, 1992):

$$\sigma^2 = \frac{1}{\kappa_{20}\kappa_{02}}\left(\frac{1}{4}\kappa_{40}\kappa_{20}^{-2}\kappa_{11}^{2} - \kappa_{31}\kappa_{20}^{-1}\kappa_{11} + \frac{1}{2}\kappa_{22}\kappa_{20}^{-1}\kappa_{11}^{2}\kappa_{02}^{-1} + \kappa_{22} + \kappa_{20}\kappa_{02} + \kappa_{20}^{-1}\kappa_{11}^{4}\kappa_{02}^{-1} \right.$$
$$\left. - \kappa_{13}\kappa_{11}\kappa_{02}^{-1} + \frac{1}{4}\kappa_{11}^{2}\kappa_{04}\kappa_{02}^{-2} - 2\kappa_{11}^{2}\right)$$

Bhattacharya and Ghosh (1978) provided the Edgeworth expansion of R, where

$$R = \frac{r_*}{\sigma} = \frac{\sqrt{n}\left(r-\rho\right)}{\sigma}$$

as

$$
\begin{aligned}
\mathrm{P}(R<u)=\Phi(u)-\phi(u)\Bigg[\frac{1}{\sqrt{n}}\left\{\frac{1}{6}\frac{\nu_3}{\sigma^3}\mathrm{H}_2(u)+\frac{\nu_1}{\sigma}\right\}+\frac{1}{n}\Bigg\{\frac{1}{72}\frac{\nu_3^2}{\sigma^6}\mathrm{H}_5(u)\\
+\left(\frac{1}{6}\frac{\nu_1\nu_3}{\sigma^4}+\frac{1}{24}\frac{\nu_4}{\sigma^4}\mathrm{H}_3(u)\right)\\
+\left(\frac{1}{2}\frac{\nu_2}{\sigma^2}+\frac{1}{2}\frac{\nu_2}{\sigma^2}\right)\mathrm{H}_1(u)\Bigg\}\Bigg]+\mathrm{O}\left(\frac{1}{n\sqrt{n}}\right)
\end{aligned}
\tag{3}
$$

where $\Phi(u)$ and $\phi(u)$ denote the standard normal distribution function and its density function, $\mathrm{O}\left(1\big/n\sqrt{n}\right)$ is the big-oh function of order $1\big/n\sqrt{n}$, $\mathrm{H}_1(u)$, $\mathrm{H}_2(u)$, $\mathrm{H}_3(u)$, and $\mathrm{H}_5(u)$ are Hermite polynomials with

$$
\mathrm{H}_1(u)=u;\quad \mathrm{H}_2(u)=u^2-1;\quad \mathrm{H}_3(u)=u^3-3u;\quad \mathrm{H}_5(u)=u^5-10u^3+15u,
$$

and v_1, v_2, v_3, and v_4 are parameters such that

$$
\frac{1}{\sqrt{n}}\frac{\nu_1}{\sigma},\quad 1+\frac{1}{n}\frac{\nu_2}{\sigma^2},\quad \frac{1}{\sqrt{n}}\frac{\nu_3}{\sigma^3},\quad \frac{1}{n}\frac{\nu_4}{\sigma^4}
$$

are the approximate cumulants of R. The explicit forms of v_1, v_2, v_3, and v_4 were provided by Nakagawa and Niki (1992). Formulas for calculating v_1 and v_3 are listed in Appendix A. Formulas for calculating v_2 and v_4 are not needed in this study.

Nakagawa and Niki (1992) applied the inverted Edgeworth expansion to the distribution of R of order $1/n$:

$$
\begin{aligned}
\mathrm{P}\Bigg(R\le z+\frac{1}{\sqrt{n}}\left\{\frac{1}{6}\frac{\nu_3}{\sigma^3}\left(z^2-1\right)+\frac{\nu_1}{\sigma}\right\}+\frac{1}{n}\Bigg\{z^3\left(-\frac{1}{18}\frac{\nu_3^2}{\sigma^6}+\frac{1}{24}\frac{\nu_4}{\sigma^4}\right)\\
+z\left(\frac{1}{2}\frac{\nu_2}{\sigma^2}+\frac{5}{36}\frac{\nu_3^2}{\sigma^6}-\frac{1}{8}\frac{\nu_4}{\sigma^4}\right)\Bigg\}\Bigg)\\
=\Phi(z)+\mathrm{O}\left(\frac{1}{n\sqrt{n}}\right)
\end{aligned}
\tag{4}
$$

If only order $1\big/\sqrt{n}$ is required, then (4) can be reduced to a simpler form:

$$\mathrm{P}\left(R \le z + B_1\right) = \Phi\left(z\right) + \mathrm{O}\left(\frac{1}{n}\right) \tag{5}$$

where $B_1 = \frac{1}{\sqrt{n}}\left\{\frac{1}{6}\frac{\nu_3}{\sigma^3}\left(z^2 - 1\right) + \frac{\nu_1}{\sigma}\right\}$.

To test H_0: $\rho = \rho_0$ versus H_a: $\rho > \rho_0$, the intuitive decision rule is:

reject H_0: $\rho = \rho_0$ when $R > z + B_1$, i.e.
reject H_0: $\rho = \rho_0$ under the significance level α, if

$$\frac{\sqrt{n}\left(r - \rho_0\right)}{\sigma} > z_\alpha + \frac{1}{\sqrt{n}}\left\{\frac{1}{6}\frac{\nu_3}{\sigma^3}\left(z_\alpha^2 - 1\right) + \frac{\nu_1}{\sigma}\right\} \tag{6}$$

Since negative values of B_1 might increase type I errors, the following adjustment is proposed: Define

$$B_2 = \max\left(z_\alpha, z_\alpha + \frac{1}{\sqrt{n}}\left\{\frac{1}{6}\frac{\nu_3}{\sigma^3}\left(z_\alpha^2 - 1\right) + \frac{\nu_1}{\sigma}\right\}\right) \tag{7}$$

The decision rule is adjusted to:

$$\text{reject } \mathrm{H}_0 \text{ if } \frac{\sqrt{n}\left(r - \rho_0\right)}{\sigma} > B_2 \tag{8}$$

All the parameters in (6) and (7) can be written in terms of the product cumulants. These product cumulants κ_{ij} are estimated by their corresponding unbiased estimators k_{ij}. Detailed formulas are provided in Appendix B.

For the special case of $\rho_0 = 0$, $\kappa_{01} = \kappa_{10} = 0$, $\kappa_{02} = \kappa_{20} = 1$, and $\kappa_{pq} = 0$ for $p + q \ge 5$, Nakagawa and Niki (1992) gave the simplified forms for parameters σ^2, ν_1, and ν_3 as follows:

$$\begin{aligned}
\sigma^2 &= 1 + \kappa_{22} \\
\nu_1 &= -\frac{1}{2}\left(\kappa_{13} + \kappa_{31}\right) \\
\nu_3 &= 3\kappa_{12}\kappa_{21} + \kappa_{03}\kappa_{30} - 3\kappa_{22}\left(\kappa_{13} + \kappa_{31}\right)
\end{aligned} \tag{9}$$

To test H_0: $\rho = 0$, (6) and (7) are evaluated with the parameters given in (9). Again, all the parameters are estimated by their corresponding unbiased estimators.

Simulation Study

The simulation study was implemented to evaluate type I error rates, to investigate the power performance, and to compare with the existing Fisher's Z transformation method on the type I error rates.

Simulation Description

Fleishman (1978) proposed a method to generate univariate non-normal random variables with desired coefficients of skewness β and kurtosis γ. Vale and Maurelli (1983) extended Fleishman's method to the bivariate non-normal case with a specified correlation coefficient. Five parameters, including two sets of skewness and kurtosis and one correlation coefficient, are required to generate the bivariate non-normal data using Vale and Maurelli method.

Seven levels of the skewness, -3.0, -1.2, -0.5, 0.0, 0.5, 1.2, and 3.0, and five levels of the kurtosis, 0.0, 4.0, 10.0, 14.0, and 25.0, were considered, and 24 combinations were selected. Moreover, five correlation coefficients, 0.0, 0.5, 0.6, 0.75, and 0.9, three significance levels, 0.10, 0.05, and 0.01, and two sample sizes, 15 and 30, were used in the simulation study.

Two new methods and the Fisher's Z transformation method were evaluated. The method using (6) was denoted by Zb, and the one using (8) was denoted by Zc. The Fisher's Z transformation method (1) was shortened as Zf. Both Zb and Zc methods were evaluated with two critical values, z_α and $t_{(\alpha,n-2)}$.

The Algorithm of the Test on Correlation:

1. Input the desired $\rho_{X,Y}$ and two sets of skewness and kurtosis, (β_1, γ_1) and (β_2, γ_2).
2. Generate n bivariate non-normal random variates (X, Y) based on the given parameters.
3. Calculate z_F in (1), Zb in (6), and Zc in (8).
4. Compare the tests with their critical values; count one if the test is rejected.
5. Repeat (2) – (4) 99,999 times.

6. Calculate the type I error rate, the proportion of the false rejection (out of 100,000) for each test.

In the power study, an extra parameter ρ_a is input in step (1) and used to generate the data as the true population correlation. However, all of the test statistics in step (3) are evaluated under ρ_0. All fo the simulations were run with Fortran 77 for Windows on an IBM T61 Laptop Computer.

Simulation Results

Type I Error Rate Comparisons

Tables 1-3 provide the comparative study of the type I error rates on various bivariate non-normal distributions with significance levels 0.10, 0.05, and 0.01 and sample size 30. Comparisons were made among the tests Zf, Zb, and Zc with two critical values, z_α and $t_{(\alpha,n-2)}$, while Zf only used the critical value, z_α. The correlation coefficients 0.00, 0.50, 0.60, 0.75, and 0.90 were targeted during the simulation study. A total of 24 bivariate non-normal distributions with various population conditions were examined.

Table 1 shows the results on testing a zero correlation. It can be observed that the Zc method controls the type I error rates well. On the contrary, the Zb method do not control type I error rates at all. Almost all of the type I error rates obtained by the Zb method are slightly inflated except for a few cases. The Zf method can control the type I error rates as long as the skewness and kurtosis are small. Once theses parameters increase, Zf becomes unstable and fails to control the type I error rates in many cases.

More specifically, in testing $\rho_0 = 0$ on a distribution which is bivariate normal or very close to bivariate normal, Zf controls type I error rates a bit better than Zc. However, for the non-normal distributions, Zc is better than Zf in controlling type I error rates.

Tables 2 and 3 give the results for right-tailed tests on the non-zero correlation. It is quite interesting to see that the hypothesized value ρ_0 actually affects the type I error rate performance. When $n = 30$ and $\rho_0 = 0.5$, both the Zf and Zb methods basically fail to control the type I error rates with very few exceptions. The type I error rates obtained by the Zc method have better performance. However, the cases with controlled type I error rates are restricted to the distributions with small to moderate skewness and kurtosis. When ρ_0 increases to 0.6, the Zc method successfully controls the type I error rates for nearly all the distributions considered with the *t* critical point. As ρ_0 increases, the type I error rates get more conservative. This tendency can be observed on both Zb and Zc methods.

The Zf method fails completely in the right-tailed test on non-zero correlation with only a few exceptions. This result confirms with the study by Beversdorf and Sa (2011). Their study shows that Zf can properly control the type I error rates on the left-tailed test but not on the right-tailed test. Therefore, it is fair to conclude that, for the right-tailed test, the only method that can properly control the type I error rates is the Zc method with the *t* critical point.

Due to the similar results in the study, only the moderate sample size 30 and significance levels of 0.05 and 0.01 are reported in the tables.

Power Results

The power performance of the proposed test is also evaluated. Tables 4 and 5 provide the power performance to test $\rho_0 = 0$ when $\rho_a = 0.0, 0.2, 0.4, 0.6$, and 0.8 with significance levels 0.05 and 0.01. Table 6 provides a small-scale investigation on the power performance to test $\rho_0 = 0.55$ and $\rho_a = 0.6$ and 0.7.

Both the Zf and Zc methods perform well in testing $\rho_0 = 0$. In testing on an exactly- or nearly-normal distribution, the power from Zf and Zc converges to 1 quickly. When $\rho_a = 0.6$, both achieve a power of 0.99; when $\rho_a = 0.8$, the power rates are essentially 1. For the distributions with large skewness and kurtosis, the Zc method, which is the only one with controlled type I error rates, converges to 1 more slowly but still reasonably well. A small-scale power study to test non-zero correlation is presented in Table 6. At significance level 0.10, sample size 30, $\rho_0 = 0.55$ versus $\rho_a = 0.6$ and 0.7, it is observed that the power of Zc steadily increases when ρ_a moves away from ρ_0.

Conclusions

This study proposed a new right-tailed test for the correlation of bivariate non-normal distributions. This new test adapts the inverse Edgeworth expansion for the standardized correlation $R = \frac{\sqrt{n}\left(r-\rho\right)}{\sigma}$ by Nakagawa and Niki (1992).

This newly proposed test can be conducted without any knowledge of the populations. The simulation study shows that this new right-tailed test has the best performance in controlling the type I error rates. The proposed method, along with the *t* critical point, can be used to test both $\rho_0 = 0$ and any value of ρ_0 when $\rho_0 > 0.5$.

The power performance of the new test was also evaluated. Zc is as powerful as Zf when testing $\rho_0 = 0$. To test non-zero correlations, it is meaningless to compare the two tests since Zf fails to control type I error rates. The power examination of the Zc method shows that the power steadily goes up when ρ_a moves away from ρ_0.

The new test does have its own limitations. It cannot control the type I error rates well when the population has a small correlation and it is a right-tailed test. In order to better control the type I error rates, a higher-order Edgeworth expansion may be considered. Unfortunately, this might lead to tedious computations when higher-order terms are introduced in the test.

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Appendix A: Formulae Used in Edgeworth Expansion of *R*

Let $\eta = \frac{1}{\sqrt{\kappa_{20}\kappa_{02}}}$. Then

$$\begin{aligned}
\nu_1 = \eta\Bigg(& \frac{3}{8}\kappa_{40}\kappa_{20}^{-2}\kappa_{11} - \frac{1}{2}\kappa_{31}\kappa_{20}^{-1} + \frac{1}{4}\kappa_{22}\kappa_{20}^{-1}\kappa_{11}\kappa_{02}^{-1} + \frac{1}{2}\kappa_{20}^{-1}\kappa_{11}^{3}\kappa_{02}^{-1} - \frac{1}{2}\kappa_{13}\kappa_{02}^{-1} \\
& + \frac{3}{8}\kappa_{11}\kappa_{04}\kappa_{02}^{-2} - \frac{1}{2}\kappa_{11}\Bigg)
\end{aligned}$$

$$\begin{aligned}
\nu_3 = \eta^3\Bigg(& -\frac{1}{8}\kappa_{60}\kappa_{20}^{-3}\kappa_{11}^{3} + \frac{3}{4}\kappa_{51}\kappa_{20}^{-2}\kappa_{11}^{2} - \frac{3}{2}\kappa_{42}\kappa_{20}^{-1}\kappa_{11} - \frac{3}{8}\kappa_{42}\kappa_{20}^{-2}\kappa_{11}^{3}\kappa_{02}^{-1} + \frac{9}{16}\kappa_{40}^{2}\kappa_{20}^{-4}\kappa_{11}^{3} \\
& - 3\kappa_{40}\kappa_{31}\kappa_{20}^{-3}\kappa_{11}^{2} + \frac{3}{2}\kappa_{40}\kappa_{22}\kappa_{20}^{-2}\kappa_{11} + \frac{3}{2}\kappa_{40}\kappa_{22}\kappa_{20}^{-3}\kappa_{11}^{3}\kappa_{02}^{-1} - \frac{3}{2}\kappa_{40}\kappa_{20}^{-2}\kappa_{13}\kappa_{11}^{2}\kappa_{02}^{-1} \\
& + \frac{3}{8}\kappa_{40}\kappa_{20}^{-2}\kappa_{11}^{3}\kappa_{04}\kappa_{02}^{-2} - 3\kappa_{40}\kappa_{20}^{-2}\kappa_{11}^{3} + 3\kappa_{40}\kappa_{20}^{-3}\kappa_{11}^{5}\kappa_{02}^{-1} + \frac{3}{2}\kappa_{33}\kappa_{20}^{-1}\kappa_{11}^{2}\kappa_{02}^{-1} + \kappa_{33} \\
& + \frac{15}{4}\kappa_{31}^{2}\kappa_{20}^{-2}\kappa_{11} - 3\kappa_{31}\kappa_{22}\kappa_{20}^{-1} - \frac{9}{2}\kappa_{31}\kappa_{22}\kappa_{20}^{-2}\kappa_{11}^{2}\kappa_{02}^{-1} + \frac{9}{2}\kappa_{31}\kappa_{20}^{-1}\kappa_{13}\kappa_{11}\kappa_{02}^{-1} \\
& - \frac{3}{2}\kappa_{31}\kappa_{20}^{-1}\kappa_{11}^{2}\kappa_{04}\kappa_{02}^{-2} + 12\kappa_{31}\kappa_{20}^{-1}\kappa_{11}^{2} - 12\kappa_{31}\kappa_{20}^{-2}\kappa_{11}^{4}\kappa_{02}^{-1} - \frac{1}{2}\kappa_{30}^{2}\kappa_{20}^{-3}\kappa_{11}^{3} \\
& + 3\kappa_{30}\kappa_{21}\kappa_{20}^{-2}\kappa_{11}^{2} - 3\kappa_{30}\kappa_{20}^{-1}\kappa_{12}\kappa_{11} + \kappa_{30}\kappa_{03} - \frac{3}{8}\kappa_{24}\kappa_{20}^{-1}\kappa_{11}^{3}\kappa_{02}^{-2} - \frac{3}{2}\kappa_{24}\kappa_{11}\kappa_{02}^{-1} \\
& + 3\kappa_{22}^{2}\kappa_{20}^{-1}\kappa_{11}\kappa_{02}^{-1} + \frac{3}{2}\kappa_{22}^{2}\kappa_{20}^{-2}\kappa_{11}^{3}\kappa_{02}^{-2} - \frac{9}{2}\kappa_{22}\kappa_{20}^{-1}\kappa_{13}\kappa_{11}^{2}\kappa_{02}^{-2} + \frac{3}{2}\kappa_{22}\kappa_{20}^{-1}\kappa_{11}^{3}\kappa_{04}\kappa_{02}^{-3} \\
& + 6\kappa_{22}\kappa_{20}^{-1}\kappa_{11}^{3}\kappa_{02}^{-1} + 6\kappa_{22}\kappa_{20}^{-2}\kappa_{11}^{5}\kappa_{02}^{-2} - 3\kappa_{22}\kappa_{13}\kappa_{02}^{-1} + \frac{3}{2}\kappa_{22}\kappa_{11}\kappa_{04}\kappa_{02}^{-2} - 12\kappa_{22}\kappa_{11} \\
& - 3\kappa_{21}^{2}\kappa_{20}^{-1}\kappa_{11} - \frac{3}{2}\kappa_{21}^{2}\kappa_{20}^{-2}\kappa_{11}^{3}\kappa_{02}^{-1} + 6\kappa_{21}\kappa_{20}^{-1}\kappa_{12}\kappa_{11}^{2}\kappa_{02}^{-1} + 3\kappa_{21}\kappa_{12} - 3\kappa_{21}\kappa_{11}\kappa_{03}\kappa_{02}^{-1} \\
& - 6\kappa_{20}\kappa_{11}\kappa_{02} - 12\kappa_{20}^{-1}\kappa_{13}\kappa_{11}^{4}\kappa_{02}^{-2} - \frac{3}{2}\kappa_{20}^{-1}\kappa_{12}^{2}\kappa_{11}^{3}\kappa_{02}^{-2} + 3\kappa_{20}^{-1}\kappa_{11}^{5}\kappa_{04}\kappa_{02}^{-3} - 18\kappa_{20}^{-1}\kappa_{11}^{5}\kappa_{02}^{-1} \\
& + 6\kappa_{20}^{-2}\kappa_{11}^{7}\kappa_{02}^{-2} + \frac{3}{4}\kappa_{15}\kappa_{11}^{2}\kappa_{02}^{-2} + \frac{15}{4}\kappa_{13}^{2}\kappa_{11}\kappa_{02}^{-2} - 3\kappa_{13}\kappa_{11}^{2}\kappa_{04}\kappa_{02}^{-3} + 12\kappa_{13}\kappa_{11}^{2}\kappa_{02}^{-1} \\
& - 3\kappa_{12}^{2}\kappa_{11}\kappa_{02}^{-1} + 3\kappa_{12}\kappa_{11}^{2}\kappa_{03}\kappa_{02}^{-2} - \frac{1}{8}\kappa_{11}^{3}\kappa_{06}\kappa_{02}^{-3} + \frac{9}{16}\kappa_{11}^{3}\kappa_{04}^{2}\kappa_{02}^{-4} - 3\kappa_{11}^{3}\kappa_{04}\kappa_{02}^{-2} \\
& - \frac{1}{2}\kappa_{11}^{3}\kappa_{03}^{2}\kappa_{02}^{-3} + 18\kappa_{11}^{3}\Bigg)
\end{aligned}$$

Appendix B: *k*-Statistics

Fisher (1930) introduced *k*-statistics as the unbiased estimator of the *m*th cumulant κ_m, i.e. $E(k_m) = \kappa_m$. Define the power sum of a univariate data as: $s_m = \sum_{i=1}^{n} x_i^m$, the first six *k*-statistics in terms of the corresponding κ_m are (See Stuart & Ord, 1994):

$$
\begin{aligned}
k_1 &= \frac{1}{n^{[1]}} s_1, \quad k_2 = \frac{1}{n^{[2]}}\left(n s_2 - s_1^2\right), \\
k_3 &= \frac{1}{n^{[3]}}\left(n^2 s_3 - 3n s_2 s_1 + 2 s_1^3\right), \\
k_4 &= \frac{1}{n^{[4]}}\left\{\left(n^3+n^2\right)s_4 - 4\left(n^2+n\right)s_3 s_1 - 3\left(n^2-n\right)s_2^2 + 12n s_2 s_1^2 - 6 s_1^4\right\}, \\
k_5 &= \frac{1}{n^{[5]}}\left\{\left(n^4+5n^3\right)s_5 - 5\left(n^3+5n^2\right)s_4 s_1 - 10\left(n^3-n^2\right)s_3 s_2 + 20\left(n^2+2n\right)s_3 s_1^2\right. \\
&\qquad \left. +30\left(n^2-n\right)s_2^2 s_1 - 60n s_2 s_1^3 + 24 s_1^5\right\}, \\
k_6 &= \frac{1}{n^{[6]}}\left\{\left(n^5+16n^4+11n^3-4n^2\right)s_6 - 6\left(n^4+16n^3+11n^2-4n\right)s_5 s_1\right. \\
&\qquad -15n\left(n-1\right)^2\left(n+4\right)s_4 s_2 - 10\left(n^4-2n^3+5n^2-4n\right)s_3^2 \\
&\qquad +30\left(n^3+9n^2+2n\right)s_4 s_1^2 + 120\left(n^3-n\right)s_3 s_2 s_1 + 30\left(n^3-3n^2+2n\right)s_2^3 \\
&\qquad \left. -120\left(n^2+3n\right)s_3 s_1^3 - 270\left(n^2-n\right)s_2^2 s_1^2 + 360n s_2 s_1^4 - 120 s_1^6\right\} \\
&= \frac{1}{n^{[6]}}\left\{\ell_1 s_6 - 6\ell_2 s_5 s_1 - 15\ell_3 s_4 s_2 - 10\ell_4 s_3^2 + 30\ell_5 s_4 s_1^2 + 120\ell_6 s_3 s_2 s_1\right. \\
&\qquad \left. +30\ell_7 s_2^3 - 120\ell_8 s_3 s_1^2 - 270\ell_9 s_2^2 s_1^2 + 360\ell_{10} s_2 s_1^4 - 120 s_1^6\right\}
\end{aligned}
$$

Stuart and Ord (1994) also provided an approach to derive the multivariate *k*-statistics. Define $s_{rt} = \sum_{i=1}^{n} x_i^r y_i^t$, where (x_i, y_i), $i = 1, 2, \ldots, n$ are the bivariate random observations. The following multivariate *k*-statistics can be derived:

$$
\begin{aligned}
k_{11} &= \frac{1}{n^{[2]}}\left(n s_{11} - s_{10} s_{01}\right) \\
k_{21} &= \frac{1}{n^{[3]}}\left(n^2 s_{21} - 2n s_{11} s_{10} - n s_{20} s_{01} + 2 s_{10}^2 s_{01}\right)
\end{aligned}
$$

$$k_{12} = \frac{1}{n^{[3]}}\left(n^2 s_{12} - 2ns_{11}s_{01} - ns_{02}s_{10} + 2s_{01}^2 s_{10}\right)$$

$$\begin{aligned}k_{31} = \Bigg\{ & \frac{1}{n^{[4]}}\left(n^3+n^2\right)s_{31} - 3\left(n^2+n\right)s_{21}s_{10} - \left(n^2+n\right)s_{30}s_{01} - 3\left(n^2-n\right)s_{20}s_{11} \\ & + 6ns_{11}s_{10}^2 + 6ns_{20}s_{10}s_{01} - 6s_{10}^3 s_{01}\Big\}\end{aligned}$$

$$\begin{aligned}k_{13} = \Bigg\{ & \frac{1}{n^{[4]}}\left(n^3+n^2\right)s_{13} - 3\left(n^2+n\right)s_{12}s_{01} - \left(n^2+n\right)s_{03}s_{10} - 3\left(n^2-n\right)s_{02}s_{11} \\ & + 6ns_{11}s_{01}^2 + 6ns_{02}s_{01}s_{10} - 6s_{01}^3 s_{10}\Big\}\end{aligned}$$

$$\begin{aligned}k_{22} = \Bigg\{ & \frac{1}{n^{[4]}}\left(n^3+n^2\right)s_{22} - 2\left(n^2+n\right)s_{21}s_{01} - 2\left(n^2+n\right)s_{12}s_{10} - 2\left(n^2-n\right)s_{11}^2 \\ & - \left(n^2-n\right)s_{20}s_{02} + 2ns_{02}s_{10}^2 + 8ns_{11}s_{10}s_{01} + 2ns_{20}s_{01}^2 - 6s_{10}^2 s_{01}^2\Big\}\end{aligned}$$

$$\begin{aligned}k_{51} = \frac{1}{n^{[6]}}\Big\{ & \ell_1 s_{51} - 5\ell_2 s_{41}s_{10} - \ell_2 s_{50}s_{01} - 10\ell_3 s_{31}s_{20} - 5\ell_3 s_{40}s_{11} - 10\ell_4 s_{30}s_{21} \\ & + 20\ell_5 s_{31}s_{10}^2 + 10\ell_5 s_{40}s_{10}s_{01} + 60\ell_6 s_{21}s_{20}s_{10} + 40\ell_6 s_{30}s_{11}s_{10} \\ & + 20\ell_6 s_{30}s_{20}s_{01} + 30\ell_7 s_{20}^2 s_{11} - 60\ell_8 s_{21}s_{10}^3 - 60\ell_8 s_{30}s_{10}^2 s_{01} \\ & - 180\ell_9 s_{20}s_{11}s_{10}^2 - 90\ell_9 s_{20}^2 s_{10}s_{01} + 120\ell_{10}s_{11}s_{10}^4 \\ & + 240\ell_{10}s_{20}s_{10}^3 s_{01} - 120s_{10}^5 s_{01}\Big\}\end{aligned}$$

$$\begin{aligned}k_{42} = \frac{1}{n^{[6]}}\Big\{ & \ell_1 s_{42} - 4\ell_2 s_{32}s_{10} - 2\ell_2 s_{41}s_{01} - 6\ell_3 s_{22}s_{20} - 8\ell_3 s_{31}s_{11} - \ell_3 s_{40}s_{02} \\ & - 6\ell_4 s_{21}^2 - 4\ell_4 s_{30}s_{12} + 12\ell_5 s_{22}s_{10}^2 + 16\ell_5 s_{31}s_{10}s_{01} + 2\ell_5 s_{40}s_{01}^2 \\ & + 24\ell_6 s_{12}s_{20}s_{10} + 12\ell_6 s_{21}s_{20}s_{01} + 48\ell_6 s_{21}s_{11}s_{10} + 8\ell_6 s_{30}s_{02}s_{10} \\ & + 12\ell_6 s_{21}s_{20}s_{01} + 16\ell_6 s_{30}s_{11}s_{01} + 12\ell_7 s_{20}s_{11}^2 + 6\ell_7 s_{20}^2 s_{02} \\ & - 24\ell_8 s_{12}s_{10}^3 - 72\ell_8 s_{21}s_{10}^2 s_{01} - 24\ell_8 s_{30}s_{10}s_{01}^2 - 72\ell_9 s_{11}^2 s_{10}^2 \\ & - 36\ell_9 s_{20}s_{02}s_{10}^2 - 144\ell_9 s_{20}s_{11}s_{10}s_{01} - 18\ell_9 s_{20}^2 s_{01}^2 + 24\ell_{10}s_{02}s_{10}^4 \\ & + 192\ell_{10}s_{11}s_{10}^3 s_{01} + 144\ell_{10}s_{20}s_{10}^2 s_{01}^2 - 120s_{10}^4 s_{01}^2\Big\}\end{aligned}$$

$$
\begin{aligned}
k_{24} = \frac{1}{n^{[6]}} \{ & \ell_1 s_{24} - 4\ell_2 s_{23} s_{01} - 2\ell_2 s_{14} s_{10} - 6\ell_3 s_{22} s_{02} - 8\ell_3 s_{13} s_{11} - \ell_3 s_{04} s_{20} \\
& - 6\ell_4 s_{12}^2 - 4\ell_4 s_{03} s_{21} + 12\ell_5 s_{22} s_{01}^2 + 16\ell_5 s_{13} s_{01} s_{10} + 2\ell_5 s_{04} s_{10}^2 \\
& + 24\ell_6 s_{21} s_{02} s_{01} + 12\ell_6 s_{12} s_{02} s_{10} + 48\ell_6 s_{12} s_{11} s_{01} + 8\ell_6 s_{03} s_{20} s_{01} \\
& + 12\ell_6 s_{12} s_{02} s_{10} + 16\ell_6 s_{03} s_{11} s_{10} + 12\ell_7 s_{02} s_{11}^2 + 6\ell_7 s_{02}^2 s_{20} \\
& - 24\ell_8 s_{21} s_{01}^3 - 72\ell_8 s_{12} s_{01}^2 s_{10} - 24\ell_8 s_{03} s_{01} s_{10}^2 - 72\ell_9 s_{11}^2 s_{01}^2 \\
& - 36\ell_9 s_{02} s_{20} s_{01}^2 - 144\ell_9 s_{02} s_{11} s_{01} s_{10} - 18\ell_9 s_{02}^2 s_{10}^2 + 24\ell_{10} s_{20} s_{01}^4 \\
& + 192\ell_{10} s_{11} s_{01}^3 s_{10} + 144\ell_{10} s_{02} s_{01}^2 s_{10}^2 - 120 s_{01}^4 s_{10}^2 \}
\end{aligned}
$$

$$
\begin{aligned}
k_{33} = \frac{1}{n^{[6]}} \{ & \ell_1 s_{33} - 3\ell_2 s_{23} s_{10} - 3\ell_2 s_{32} s_{01} - 3\ell_3 s_{13} s_{20} - 3\ell_3 s_{31} s_{02} - 9\ell_3 s_{22} s_{11} \\
& - 9\ell_4 s_{21} s_{12} - \ell_4 s_{30} s_{03} + 6\ell_5 s_{13} s_{10}^2 + 18\ell_5 s_{22} s_{10} s_{01} + 6\ell_5 s_{31} s_{01}^2 \\
& + 6\ell_6 s_{20} s_{10} s_{03} + 36\ell_6 s_{12} s_{11} s_{10} + 18\ell_6 s_{12} s_{20} s_{01} + 36\ell_6 s_{21} s_{11} s_{01} \\
& + 18\ell_6 s_{21} s_{10} s_{02} + 6\ell_6 s_{30} s_{01} s_{02} + 6\ell_7 s_{11}^3 + 12\ell_7 s_{20} s_{11} s_{02} \\
& - 6\ell_8 s_{10}^3 s_{03} - 54\ell_8 s_{12} s_{10}^2 s_{01} - 54\ell_8 s_{21} s_{10} s_{01}^2 - 6\ell_8 s_{30} s_{01}^3 \\
& - 54\ell_9 s_{11} s_{10}^2 s_{02} - 108\ell_9 s_{11}^2 s_{10} s_{01} - 54\ell_9 s_{20} s_{10} s_{01} s_{02} \\
& - 54\ell_9 s_{20} s_{11} s_{01}^2 + 72\ell_{10} s_{10}^3 s_{02} s_{01} + 72\ell_{10} s_{20} s_{10} s_{01}^3 \\
& + 216\ell_{10} s_{10}^2 s_{01}^2 s_{11} - 120 s_{10}^3 s_{01}^3 \}
\end{aligned}
$$

where

$$n^{[1]} = n \qquad n^{[2]} = n(n-1)$$

$$n^{[3]} = n(n-1)(n-2) \qquad n^{[4]} = n(n-1)(n-2)(n-3)$$

$$n^{[5]} = n(n-1)(n-2)(n-3)(n-4)$$

$$n^{[6]} = n(n-1)(n-2)(n-3)(n-4)(n-5)$$

and

$\ell_1 = n^5 + 16n^4 + 11n^3 - 4n^2$

$\ell_2 = n^4 + 16n^3 + 11n^2 - 4n$

$\ell_3 = n^4 + 2n^3 - 7n^2 + 4n$

$\ell_4 = n^4 - 2n^3 + 5n^2 - 4n$

$\ell_5 = n^3 + 9n^2 + 2n$

$\ell_6 = n^3 - n$

$\ell_7 = n^3 - 3n^2 + 2n$

$\ell_8 = n^2 + 3n$

$\ell_9 = n^2 - n$

$\ell_{10} = n$

Appendix C: Tables

Table 1. Comparison of type I error rates ($\rho_0 = 0$, $n = 30$)

			$\alpha = 0.10$			**$\alpha = 0.05$**			**$\alpha = 0.01$**		
β	**γ**	**CP**	**Zf**	**Zb**	**Zc**	**Zf**	**Zb**	**Zc**	**Zf**	**Zb**	**Zc**
0.0	0	z_α	0.0985	0.1171	0.1124	0.0491	0.0679	0.0632	0.0102	0.0262	0.0191
0.0	0	t_α		0.1124	0.1079		0.0628	0.0580		0.0227	0.0153
0.0	1	z_α	0.0987	0.1268	0.1153	0.0510	0.0755	0.0640	0.0101	0.0306	0.0173
0.0	1	t_α		0.1213	0.1095		0.0697	0.0579		0.0273	0.0137
0.0	10	z_α	0.1034	0.1512	0.1061	0.0549	0.0958	0.0476	0.0144	0.0588	0.0083
0.0	10	t_α		0.1453	0.0998		0.0900	0.0413		0.0570	0.0057
0.5	0	z_α	0.0988	0.1118	0.1051	0.0508	0.0637	0.0569	0.0110	0.0232	0.0163
0.5	0	t_α		0.1064	0.1021		0.0578	0.0541		0.0195	0.0140
1.0	0	z_α	0.1015	0.1014	0.0954	0.0559	0.0541	0.0499	0.0131	0.0164	0.0126
1.0	0	t_α		0.0959	0.0901		0.0488	0.0446		0.0138	0.0098
0.5	1	z_α	0.0999	0.1225	0.1117	0.0516	0.0709	0.0604	0.0111	0.0275	0.0162
0.5	1	t_α		0.1168	0.1063		0.0649	0.0548		0.0242	0.0128
1.2	4	z_α	0.1055	0.1265	0.1015	0.0562	0.0734	0.0495	0.0143	0.0330	0.0104
1.2	4	t_α		0.1209	0.0958		0.0674	0.0435		0.0304	0.0075
1.2	10	z_α	0.1048	0.1433	0.1022	0.0580	0.0927	0.0469	0.0156	0.0539	0.0078
1.2	10	t_α		0.1380	0.0963		0.0871	0.0412		0.0522	0.0056

Note: β: skewness; γ: kurtosis; CP: Critical Point

Table 1 (continued). Comparison of type I error rates ($\rho_0 = 0$, $n = 30$)

			$\alpha = 0.10$			**$\alpha = 0.05$**			**$\alpha = 0.01$**		
β	**γ**	**CP**	**Zf**	**Zb**	**Zc**	**Zf**	**Zb**	**Zc**	**Zf**	**Zb**	**Zc**
1.2	10	z_α	0.1048	0.1433	0.1022	0.0580	0.0927	0.0469	0.0156	0.0539	0.0078
1.2	10	t_α		0.1380	0.0963		0.0871	0.0412		0.0522	0.0056
1.2	25	z_α	0.1026	0.1508	0.0913	0.0593	0.1022	0.0361	0.0214	0.0745	0.0046
1.2	25	t_α		0.1456	0.0852		0.0975	0.0304		0.0734	0.0031
-1.2	4	z_α	0.1050	0.1262	0.1029	0.0561	0.0720	0.0482	0.0146	0.0322	0.0103
-1.2	4	t_α		0.1208	0.0972		0.0663	0.0429		0.0297	0.0076
-1.2	10	z_α	0.1051	0.1446	0.1021	0.0563	0.0915	0.0456	0.0159	0.0543	0.0075
-1.2	10	t_α		0.1386	0.0960		0.0853	0.0393		0.0525	0.0054
-1.2	25	z_α	0.1024	0.1507	0.0920	0.0589	0.1030	0.0358	0.0205	0.0739	0.0047
-1.2	25	t_α		0.1450	0.0855		0.0978	0.0303		0.0730	0.0033
1.2	4	z_α	0.1057	0.1375	0.1043	0.0579	0.0860	0.0490	0.0145	0.0436	0.0090
1.2	10	t_α		0.1315	0.0984		0.0797	0.0429		0.0416	0.0067
1.2	4	z_α	0.1064	0.1498	0.1012	0.0562	0.0947	0.0429	0.0155	0.0599	0.0065
1.2	25	t_α		0.1437	0.0950		0.0888	0.0373		0.0583	0.0045
-1.2	4	z_α	0.1047	0.1374	0.1032	0.0563	0.0841	0.0475	0.0143	0.0446	0.0087
-1.2	10	t_α		0.1313	0.0970		0.0781	0.0418		0.0425	0.0063
-1.2	4	z_α	0.1052	0.1477	0.1004	0.0575	0.0968	0.0437	0.0162	0.0590	0.0066
-1.2	25	t_α		0.1421	0.0940		0.0908	0.0377		0.0576	0.0050

Note: β: skewness; γ: kurtosis; CP: Critical Point

Table 1 (continued). Comparison of type I error rates ($\rho_0 = 0$, $n = 30$)

			$\alpha = 0.10$			$\alpha = 0.05$			$\alpha = 0.01$		
β	γ	CP	Zf	Zb	Zc	Zf	Zb	Zc	Zf	Zb	Zc
1.2	4	z_α	0.1109	0.1270	0.0877	0.0651	0.0779	0.0372	0.0206	0.0423	0.0055
3.0	14	t_α		0.1213	0.0817		0.0727	0.0323		0.0406	0.0036
1.2	4	z_α	0.1100	0.1375	0.0925	0.0623	0.0861	0.0393	0.0183	0.0488	0.0050
3.0	25	tα		0.1317	0.0862		0.0803	0.0336		0.0472	0.0034
1.2	14	z_α	0.1081	0.1466	0.0936	0.0611	0.0974	0.0374	0.0213	0.0656	0.0054
3.0	25	t_α		0.1414	0.0875		0.0922	0.0318		0.0643	0.0037
-1.2	4	z_α	0.1127	0.1298	0.0899	0.0630	0.0760	0.0375	0.0206	0.0415	0.0053
-3.0	14	t_α		0.1241	0.0837		0.0711	0.0328		0.0401	0.0034
-1.2	4	z_α	0.1108	0.1383	0.0926	0.0607	0.0844	0.0380	0.0188	0.0506	0.0054
-3.0	25	t_α		0.1328	0.0869		0.0789	0.0328		0.0490	0.0034
-1.2	14	z_α	0.1041	0.1447	0.0916	0.0631	0.0974	0.0381	0.0204	0.0635	0.0051
-3.0	25	tα		0.1391	0.0858		0.0924	0.0326		0.0625	0.0035
3.0	25	z_α	0.1138	0.1403	0.0843	0.0695	0.0936	0.0336	0.0276	0.0630	0.0040
3.0	25	t_α		0.1354	0.0781		0.0891	0.0287		0.0616	0.0024
-3.0	25	z_α	0.1113	0.1374	0.0817	0.0686	0.0923	0.0325	0.0276	0.0638	0.0042
-3.0	25	t_α		0.1328	0.0761		0.0875	0.0275		0.0628	0.0028

Note: β: skewness; γ: kurtosis; CP: Critical Point

Table 2. Comparison of type I error rates (n = 30, α = 0.05)

			ρ_0 = 0.50			ρ_0 = 0.60			ρ_0 = 0.75			ρ_0 = 0.90		
β	**γ**	**CP**	**Zf**	**Zb**	**Zc**	**Zf**	**Zb**	**Zc**	**Zf**	**Zb**	**Zc**	**Zf**	**Zb**	**Zc**
0.0	0	z_α	0.0488	0.0335	0.0155	0.0497	0.0183	0.0053	0.0499	0.0081	0.0007	0.0496	0.0009	0.0000
0.0	0	t_α		0.0320	0.0130		0.0189	0.0047		0.0085	0.0006		0.0009	0.0000
0.0	1	z_α	0.0517	0.0537	0.0221	0.0525	0.0362	0.0092	0.0529	0.0218	0.0025	0.0543	0.0044	0.0002
0.0	1	t_α		0.0522	0.0186		0.0373	0.0083		0.0226	0.0023		0.0044	0.0001
0.0	10	z_α	0.0917	0.1477	0.0502	0.1025	0.1265	0.0302	0.1180	0.1037	0.0153	0.1346	0.0474	0.0045
0.0	10	t_α		0.1476	0.0441		0.1293	0.0277		0.1063	0.0143		0.0478	0.0043
0.5	0	z_α	0.0555	0.0344	0.0158	0.0550	0.0202	0.0058	0.0567	0.0106	0.0011	0.0590	0.0014	0.0001
0.5	0	t_α		0.0330	0.0133		0.0205	0.0052		0.0111	0.0010		0.0014	0.0001
1.0	0	z_α	0.0489	0.0269	0.0121	0.0449	0.0156	0.0037	0.0308	0.0128	0.0010	0.0078	0.0072	0.0000
1.0	0	t_α		0.0256	0.0096		0.0162	0.0033		0.0135	0.0009		0.0073	0.0000
0.5	1	z_α	0.0558	0.0548	0.0225	0.0558	0.0366	0.0087	0.0571	0.0230	0.0028	0.0580	0.0049	0.0002
0.5	1	t_α		0.0534	0.0194		0.0375	0.0079		0.0239	0.0026		0.0050	0.0002
1.2	4	z_α	0.0760	0.0998	0.0386	0.0809	0.0776	0.0198	0.0875	0.0597	0.0083	0.0931	0.0223	0.0016
1.2	4	t_α		0.0982	0.0337		0.0790	0.0179		0.0612	0.0076		0.0224	0.0015
1.2	10	z_α	0.0973	0.1505	0.0529	0.1053	0.1264	0.0309	0.1218	0.1036	0.0162	0.1370	0.0490	0.0044
1.2	10	t_α		0.1501	0.0470		0.1288	0.0282		0.1057	0.0153		0.0494	0.0043
1.2	25	z_α	0.1551	0.2064	0.0770	0.1735	0.1803	0.0533	0.2002	0.1566	0.0337	0.2268	0.0936	0.0137
1.2	25	t_α		0.2065	0.0697		0.1829	0.0492		0.1595	0.0321		0.0943	0.0132

Note: β: skewness; γ: kurtosis; CP: Critical Point

Table 2 (continued). Comparison of type I error rates (n = 30, α = 0.05)

			ρ_0 = 0.50			ρ_0 = 0.60			ρ_0 = 0.75			ρ_0 = 0.90		
β	**γ**	**CP**	**Zf**	**Zb**	**Zc**	**Zf**	**Zb**	**Zc**	**Zf**	**Zb**	**Zc**	**Zf**	**Zb**	**Zc**
-1.2	4	z_α	0.0762	0.0984	0.0376	0.0803	0.0777	0.0199	0.0864	0.0609	0.0087	0.0926	0.0228	0.0018
-1.2	4	t_α		0.0968	0.0330		0.0790	0.0182		0.0623	0.0082		0.0229	0.0017
-1.2	10	z_α	0.0955	0.1500	0.0515	0.1060	0.1265	0.0320	0.1224	0.1073	0.0170	0.1363	0.0485	0.0050
-1.2	10	t_α		0.1501	0.0458		0.1286	0.0290		0.1097	0.0160		0.0488	0.0049
-1.2	25	z_α	0.1580	0.2076	0.0791	0.1731	0.1791	0.0529	0.1994	0.1570	0.0339	0.2240	0.0939	0.0144
-1.2	25	t_α		0.2068	0.0714		0.1816	0.0490		0.1601	0.0323		0.0947	0.0138
1.2	4	z_α	0.0837	0.1467	0.0511	0.0891	0.1318	0.0304	0.0864	0.0609	0.0087	0.1141	0.0664	0.0044
1.2	10	t_α		0.1470	0.0453		0.1348	0.0277		0.0623	0.0082		0.0667	0.0043
1.2	4	z_α	0.0996	0.2251	0.0668	0.1133	0.2153	0.0450	0.1224	0.1073	0.0170	0.2428	0.1272	0.0119
1.2	25	t_α		0.2269	0.0600		0.2204	0.0418		0.1097	0.0160		0.1274	0.0113
-1.2	4	z_α	0.0836	0.1474	0.0510	0.0884	0.1302	0.0305	0.1994	0.1570	0.0339	0.1120	0.0682	0.0047
-1.2	10	t_α		0.1472	0.0450		0.1329	0.0276		0.1601	0.0323		0.0685	0.0044
-1.2	4	z_α	0.0993	0.2243	0.0685	0.1123	0.2181	0.0480	0.0998	0.1185	0.0150	0.2419	0.1270	0.0117
-1.2	25	t_α		0.2263	0.0618		0.2228	0.0443		0.1213	0.0140		0.1273	0.0111
1.2	4	z_α	0.1037	0.1602	0.0628	0.1097	0.1332	0.0419	0.1360	0.2117	0.0283	0.1731	0.0189	0.0084
3.0	14	t_α		0.1585	0.0560		0.1342	0.0384		0.2149	0.0267		0.0191	0.0082
1.2	4	z_α	0.1134	0.2081	0.0737	0.1220	0.1919	0.0512	0.0993	0.1181	0.0154	0.1694	0.0871	0.0163
3.0	25	t_α		0.2085	0.0669		0.1946	0.0475		0.1210	0.0144		0.0872	0.0155

Note: β: skewness; γ: kurtosis; CP: Critical Point

Table 2 (continued). Comparison of type I error rates (n = 30, α = 0.05)

			ρ_0 = 0.50			ρ_0 = 0.60			ρ_0 = 0.75			ρ_0 = 0.90		
β	**γ**	**CP**	**Zf**	**Zb**	**Zc**	**Zf**	**Zb**	**Zc**	**Zf**	**Zb**	**Zc**	**Zf**	**Zb**	**Zc**
1.2	14	z_α	0.1364	0.1994	0.0757	0.1533	0.1719	0.0518	0.1363	0.2096	0.0293	0.1960	0.0680	0.0142
3.0	25	t_α		0.1983	0.0683		0.1739	0.0480		0.2133	0.0276		0.0683	0.0137
-1.2	4	z_α	0.1016	0.1565	0.0623	0.1089	0.1298	0.0405	0.1151	0.0949	0.0212	0.1734	0.0194	0.0083
-3.0	14	t_α		0.1550	0.0558		0.1304	0.0369		0.0964	0.0199		0.0196	0.0082
-1.2	4	z_α	0.1115	0.2082	0.0723	0.1239	0.1912	0.0521	0.1419	0.1719	0.0344	0.1682	0.0845	0.0155
-3.0	25	t_α		0.2082	0.0652		0.1942	0.0481		0.1742	0.0325		0.0847	0.0148
-1.2	14	z_α	0.1365	0.1981	0.0733	0.1534	0.1733	0.0526	0.1759	0.1429	0.0344	0.1931	0.0673	0.0141
-3.0	25	t_α		0.1976	0.0659		0.1747	0.0484		0.1453	0.0327		0.0676	0.0136
3.0	25	z_α	0.1648	0.2193	0.0833	0.1852	0.1898	0.0588	0.1147	0.0943	0.0216	0.2290	0.1044	0.0156
3.0	25	t_α		0.2183	0.0755		0.1913	0.0547		0.0958	0.0205		0.1049	0.0150
-3.0	25	z_α	0.1654	0.2186	0.0833	0.1820	0.1922	0.0587	0.1402	0.1696	0.0326	0.2257	0.1068	0.0159
-3.0	25	t_α		0.2172	0.0754		0.1938	0.0543		0.1723	0.0308		0.1074	0.0152

Note: β: skewness; γ: kurtosis; CP: Critical Point

Table 3. Comparison of type I error rates (n = 30, α = 0.01)

			ρ_0 = 0.50			ρ_0 = 0.60			ρ_0 = 0.75			ρ_0 = 0.90		
β	**γ**	**CP**	**Zf**	**Zb**	**Zc**	**Zf**	**Zb**	**Zc**	**Zf**	**Zb**	**Zc**	**Zf**	**Zb**	**Zc**
0.0	0	z_α	0.0107	0.0352	0.0034	0.0107	0.0272	0.0017	0.0103	0.0137	0.0003	0.0103	0.0013	0.0000
0.0	0	t_α		0.0375	0.0027		0.0297	0.0014		0.0145	0.0002		0.0013	0.0000
0.0	1	z_α	0.0115	0.0627	0.0056	0.0118	0.0526	0.0034	0.0118	0.0315	0.0008	0.0123	0.0049	0.0001
0.0	1	t_α		0.0671	0.0045		0.0562	0.0027		0.0330	0.0007		0.0050	0.0001
0.0	10	z_α	0.0350	0.1768	0.0160	0.0417	0.1615	0.0125	0.0512	0.1263	0.0072	0.0611	0.0501	0.0023
0.0	10	t_α		0.1858	0.0136		0.1694	0.0108		0.1295	0.0062		0.0502	0.0019
0.5	0	z_α	0.0125	0.0359	0.0036	0.0126	0.0285	0.0017	0.0135	0.0158	0.0004	0.0138	0.0017	0.0000
0.5	0	t_α		0.0386	0.0030		0.0308	0.0014		0.0167	0.0004		0.0017	0.0000
1.0	0	z_α	0.0138	0.0274	0.0025	0.0124	0.0245	0.0012	0.0083	0.0201	0.0003	0.0013	0.0093	0.0000
1.0	0	t_α		0.0293	0.0020		0.0265	0.0010		0.0215	0.0002		0.0097	0.0000
0.5	1	z_α	0.0124	0.0612	0.0054	0.0132	0.0513	0.0032	0.0138	0.0310	0.0009	0.0145	0.0057	0.0001
0.5	1	t_α		0.0651	0.0044		0.0543	0.0026		0.0325	0.0008		0.0057	0.0001
1.2	4	z_α	0.0246	0.1114	0.0114	0.0258	0.1011	0.0081	0.0296	0.0753	0.0042	0.0338	0.0236	0.0009
1.2	4	t_α		0.1175	0.0095		0.1060	0.0067		0.0778	0.0037		0.0236	0.0008
1.2	10	z_α	0.0378	0.1754	0.0173	0.0432	0.1597	0.0126	0.0538	0.1259	0.0083	0.0643	0.0528	0.0028
1.2	10	t_α		0.1834	0.0144		0.1662	0.0108		0.1292	0.0073		0.0530	0.0025
1.2	25	z_α	0.0833	0.2346	0.0292	0.0989	0.2168	0.0260	0.1169	0.1826	0.0187	0.1383	0.1010	0.0099
1.2	25	t_α		0.2430	0.0251		0.2238	0.0228		0.1866	0.0169		0.1014	0.0091

Note: β: skewness; γ: kurtosis; CP: Critical Point

Table 3 (continued). Comparison of type I error rates ($n = 30$, $\alpha = 0.01$)

			$\rho_0 = 0.50$			$\rho_0 = 0.60$			$\rho_0 = 0.75$			$\rho_0 = 0.90$		
β	**γ**	**CP**	**Zf**	**Zb**	**Zc**	**Zf**	**Zb**	**Zc**	**Zf**	**Zb**	**Zc**	**Zf**	**Zb**	**Zc**
-1.2	4	z_α	0.0237	0.1116	0.0110	0.0264	0.1010	0.0075	0.0300	0.0749	0.0039	0.0336	0.0243	0.0008
-1.2	4	t_α		0.1173	0.0092		0.1064	0.0064		0.0774	0.0034		0.0244	0.0006
-1.2	10	z_α	0.0381	0.1780	0.0175	0.0439	0.1605	0.0131	0.0531	0.1241	0.0080	0.0628	0.0519	0.0028
-1.2	10	t_α		0.1860	0.0146		0.1676	0.0113		0.1270	0.0071		0.0521	0.0025
-1.2	25	z_α	0.0833	0.2349	0.0295	0.0977	0.2169	0.0258	0.1206	0.1837	0.0196	0.1363	0.0997	0.0091
-1.2	25	t_α		0.2437	0.0254		0.2240	0.0226		0.1878	0.0178		0.1002	0.0085
1.2	4	z_α	0.0286	0.1740	0.0165	0.0318	0.1676	0.0120	0.0364	0.1403	0.0069	0.0446	0.0689	0.0023
1.2	10	t_α		0.1819	0.0136		0.1752	0.0101		0.1434	0.0061		0.0692	0.0020
1.2	4	z_α	0.0400	0.2715	0.0247	0.0457	0.2683	0.0201	0.0567	0.2376	0.0145	0.0995	0.1277	0.0064
1.2	25	t_α		0.2829	0.0206		0.2774	0.0174		0.2407	0.0126		0.1278	0.0057
-1.2	4	z_α	0.0281	0.1730	0.0161	0.0319	0.1672	0.0125	0.0371	0.1417	0.0072	0.0446	0.0698	0.0023
-1.2	10	t_α		0.1813	0.0133		0.1753	0.0105		0.1448	0.0060		0.0699	0.0020
-1.2	4	z_α	0.0390	0.2699	0.0244	0.0452	0.2660	0.0199	0.0575	0.2371	0.0140	0.1003	0.1284	0.0067
-1.2	25	t_α		0.2813	0.0203		0.2752	0.0172		0.2400	0.0123		0.1285	0.0059
1.2	4	z_α	0.0414	0.1707	0.0213	0.0436	0.1560	0.0166	0.0481	0.1080	0.0108	0.0818	0.0208	0.0059
3.0	14	t_α		0.1777	0.0177		0.1623	0.0142		0.1101	0.0094		0.0211	0.0054
1.2	4	z_α	0.0468	0.2369	0.0275	0.0512	0.2279	0.0234	0.0605	0.1915	0.0182	0.0621	0.0865	0.0089
3.0	25	t_α		0.2463	0.0231		0.2353	0.0204		0.1946	0.0161		0.0866	0.0078

Note: β: skewness; γ: kurtosis; CP: Critical Point

Table 3 (continued). Comparison of type I error rates (n = 30, α = 0.01)

			ρ_0 = 0.50			ρ_0 = 0.60			ρ_0 = 0.75			ρ_0 = 0.90		
β	**γ**	**CP**	**Zf**	**Zb**	**Zc**	**Zf**	**Zb**	**Zc**	**Zf**	**Zb**	**Zc**	**Zf**	**Zb**	**Zc**
1.2	14	z_α	0.0682	0.2232	0.0290	0.0783	0.2079	0.0248	0.0940	0.1625	0.0187	0.1061	0.0709	0.0090
3.0	25	t_α		0.2311	0.0252		0.2145	0.0219		0.1657	0.0165		0.0712	0.0081
-1.2	4	z_α	0.0425	0.1756	0.0223	0.0449	0.1566	0.0171	0.0479	0.1088	0.0107	0.0809	0.0205	0.0062
-3.0	14	t_α		0.1826	0.0192		0.1626	0.0150		0.1108	0.0092		0.0207	0.0059
-1.2	4	z_α	0.0466	0.2365	0.0269	0.0531	0.2306	0.0238	0.0593	0.1913	0.0171	0.0607	0.0880	0.0084
-3.0	25	t_α		0.2454	0.0225		0.2375	0.0209		0.1943	0.0150		0.0882	0.0075
-1.2	14	z_α	0.0671	0.2213	0.0283	0.0779	0.2024	0.0248	0.0939	0.1612	0.0179	0.1071	0.0708	0.0093
-3.0	25	t_α		0.2291	0.0242		0.2098	0.0218		0.1642	0.0163		0.0711	0.0085
3.0	25	z_α	0.0934	0.2429	0.0345	0.1058	0.2221	0.0295	0.1249	0.1873	0.0217	0.1403	0.1080	0.0098
3.0	25	t_α		0.2501	0.0300		0.2285	0.0262		0.1912	0.0197		0.1085	0.0088
-3.0	25	z_α	0.0936	0.2375	0.0345	0.1060	0.2196	0.0290	0.1241	0.1872	0.0206	0.1406	0.1094	0.0098
-3.0	25	t_α		0.2446	0.0301		0.2262	0.0253		0.1907	0.0188		0.1100	0.0090

Note: β: skewness; γ: kurtosis; CP: Critical Point

Table 4. Power performance for test $\rho_0 = 0$ ($n = 30$, $\alpha = 0.05$)

		ra = 0.0			ra = 0.2			ra = 0.4			ra = 0.6			ra = 0.8		
b	**g**	**Zf**	**Zb**	**Zc**	**Zf**	**Zb**	**Zc**	**Zf**	**Zb**	**Zc**	**Zf**	**Zb**	**Zc**	**Zf**	**Zb**	**Zc**
0.0	0	0.0498	0.0683	0.0635	0.2778	0.3148	0.3002	0.7231	0.7376	0.7156	0.9775	0.9785	0.9727	1.0000	1.0000	0.9993
0.0	0		0.0624	0.0576		0.2976	0.2827		0.7192	0.6950		0.9749	0.9674		1.0000	0.9991
0.0	1	0.0494	0.0739	0.0626	0.2814	0.3321	0.2964	0.7227	0.7489	0.6980	0.9776	0.9798	0.9546	1.0000	1.0000	0.9840
0.0	1		0.0681	0.0565		0.3133	0.2778		0.7314	0.6758		0.9768	0.9473		0.9999	0.9815
0.0	10	0.0553	0.0991	0.0485	0.3030	0.3849	0.2641	0.7436	0.7942	0.6416	0.9809	0.9837	0.8472	1.0000	0.9998	0.8527
0.0	10		0.0933	0.0427		0.3653	0.2419		0.7772	0.6128		0.9811	0.8330		0.9997	0.8428
0.5	0	0.0502	0.0625	0.0585	0.2821	0.3074	0.2927	0.7201	0.7334	0.7089	0.9766	0.9771	0.9680	1.0000	0.9999	0.9977
0.5	0		0.0564	0.0528		0.2888	0.2738		0.7147	0.6882		0.9738	0.9622		0.9998	0.9969
1.0	0	0.0553	0.0550	0.0511	0.2655	0.2531	0.2377	0.6519	0.6278	0.5973	0.9409	0.9341	0.9147	0.9988	0.9981	0.9957
1.0	0		0.0493	0.0457		0.2350	0.2193		0.6049	0.5726		0.9254	0.9022		0.9978	0.9948
0.5	1	0.0514	0.0704	0.0602	0.2832	0.3254	0.2907	0.7209	0.7451	0.6952	0.9763	0.9783	0.9497	1.0000	0.9999	0.9808
0.5	1		0.0650	0.0548		0.3063	0.2719		0.7263	0.6721		0.9753	0.9419		0.9999	0.9783
1.2	4	0.0570	0.0744	0.0510	0.2927	0.3284	0.2571	0.7218	0.7473	0.6451	0.9755	0.9765	0.8935	1.0000	0.9994	0.9213
1.2	4		0.0686	0.0452		0.3090	0.2368		0.7286	0.6192		0.9728	0.8809		0.9994	0.9146
1.2	10	0.0565	0.0919	0.0467	0.3046	0.3716	0.2568	0.7415	0.7841	0.6343	0.9798	0.9808	0.8440	1.0000	0.9997	0.8543
1.2	10		0.0863	0.0408		0.3518	0.2342		0.7672	0.6059		0.9780	0.8295		0.9996	0.8442
1.2	25	0.0597	0.1033	0.0359	0.3213	0.4029	0.2392	0.7433	0.7969	0.5898	0.9736	0.9780	0.7636	0.9999	0.9997	0.7636
1.2	25		0.0988	0.0311		0.3831	0.2147		0.7802	0.5592		0.9743	0.7470		0.9996	0.7519
-1.2	4	0.0556	0.0725	0.0493	0.2916	0.3273	0.2562	0.7208	0.7472	0.6445	0.9755	0.9764	0.8931	1.0000	0.9995	0.9211
-1.2	4		0.0666	0.0436		0.3078	0.2363		0.7282	0.6183		0.9729	0.8809		0.9994	0.9141

Note: β: skewness; γ: kurtosis; the “Zf”, “Zb”, and “Zc” results are calculated using the critical points z_α and t_α as the first and the second number

Table 4 (continued). Power performance for test $\rho_0 = 0$ ($n = 30$, $\alpha = 0.05$)

		ra = 0.0			ra = 0.2			ra = 0.4			ra = 0.6			ra = 0.8		
b	**g**	**Zf**	**Zb**	**Zc**	**Zf**	**Zb**	**Zc**	**Zf**	**Zb**	**Zc**	**Zf**	**Zb**	**Zc**	**Zf**	**Zb**	**Zc**
-1.2	10	0.0578	0.0925	0.0457	0.3050	0.3731	0.2574	0.7384	0.7819	0.6315	0.9790	0.9803	0.8443	1.0000	0.9995	0.8541
-1.2	10		0.0868	0.0399		0.3534	0.2344		0.7640	0.6029		0.9772	0.8305		0.9994	0.8444
-1.2	25	0.0597	0.1034	0.0362	0.3217	0.4027	0.2388	0.7440	0.7974	0.5898	0.9731	0.9775	0.7606	0.9999	0.9997	0.7667
-1.2	25		0.0986	0.0307		0.3836	0.2139		0.7801	0.5588		0.9739	0.7430		0.9996	0.7542
1.2	4	0.0565	0.0827	0.0472	0.3013	0.3557	0.2615	0.7359	0.7704	0.6452	0.9799	0.9802	0.8731	1.0000	0.9996	0.8910
1.2	10		0.0768	0.0413		0.3353	0.2396		0.7525	0.6178		0.9774	0.8600		0.9995	0.8821
1.2	4	0.0577	0.0956	0.0438	0.3152	0.3844	0.2569	0.7635	0.8037	0.6410	0.9864	0.9867	0.8343	1.0000	0.9999	0.8378
1.2	25		0.0897	0.0378		0.3647	0.2332		0.7874	0.6125		0.9847	0.8206		0.9999	0.8267
-1.2	4	0.0564	0.0843	0.0489	0.2957	0.3508	0.2590	0.7336	0.7677	0.6426	0.9802	0.9812	0.8737	1.0000	0.9997	0.8895
-1.2	10		0.0783	0.0433		0.3313	0.2374		0.7494	0.6149		0.9783	0.8608		0.9996	0.8811
-1.2	4	0.0571	0.0945	0.0431	0.3139	0.3849	0.2559	0.7627	0.8034	0.6414	0.9858	0.9868	0.8364	1.0000	0.9999	0.8388
-1.2	25		0.0889	0.0370		0.3656	0.2336		0.7867	0.6125		0.9846	0.8227		0.9999	0.8289
1.2	4	0.0633	0.0766	0.0372	0.3067	0.3263	0.2117	0.7135	0.7313	0.5620	0.9708	0.9731	0.8130	0.9999	0.9996	0.8384
3.0	14		0.0708	0.0323		0.3084	0.1918		0.7113	0.5319		0.9688	0.7957		0.9995	0.8275
1.2	4	0.0618	0.0849	0.0384	0.3182	0.3593	0.2356	0.7479	0.7744	0.6102	0.9804	0.9804	0.8228	1.0000	0.9997	0.8327
3.0	25		0.0792	0.0331		0.3397	0.2134		0.7557	0.5806		0.9775	0.8087		0.9996	0.8220
1.2	14	0.0629	0.0976	0.0383	0.3200	0.3859	0.2394	0.7464	0.7894	0.6004	0.9769	0.9785	0.7878	1.0000	0.9996	0.7894
3.0	25		0.0928	0.0330		0.3665	0.2153		0.7716	0.5698		0.9749	0.7715		0.9995	0.7781
-1.2	4	0.0638	0.0766	0.0369	0.3093	0.3297	0.2151	0.7127	0.7317	0.5633	0.9714	0.9738	0.8133	0.9999	0.9995	0.8377
-3.0	14		0.0715	0.0317		0.3107	0.1946		0.7123	0.5345		0.9698	0.7967		0.9993	0.8266

Note: β: skewness; γ: kurtosis; the “Zf”, “Zb”, and “Zc” results are calculated using the critical points z_α and t_α as the first and the second number

Table 4 (continued). Power performance for test $\rho_0 = 0$ ($n = 30$, $\alpha = 0.05$)

		ra = 0.0			ra = 0.2			ra = 0.4			ra = 0.6			ra = 0.8		
b	**g**	**Zf**	**Zb**	**Zc**	**Zf**	**Zb**	**Zc**	**Zf**	**Zb**	**Zc**	**Zf**	**Zb**	**Zc**	**Zf**	**Zb**	**Zc**
-1.2	4	0.0622	0.0855	0.0388	0.3186	0.3602	0.2388	0.7484	0.7755	0.6131	0.9806	0.9803	0.8225	1.0000	0.9997	0.8325
-3.0	25		0.0802	0.0334		0.3403	0.2159		0.7568	0.5834		0.9772	0.8074		0.9996	0.8216
-1.2	14	0.0618	0.0967	0.0378	0.3200	0.3855	0.2395	0.7459	0.7878	0.5980	0.9776	0.9785	0.7889	1.0000	0.9996	0.7923
-3.0	25		0.0918	0.0323		0.3662	0.2161		0.7702	0.5681		0.9752	0.7738		0.9996	0.7813
3.0	25	0.0689	0.0935	0.0331	0.3289	0.3661	0.2129	0.7241	0.7498	0.5456	0.9625	0.9629	0.7458	0.9999	0.9991	0.7631
3.0	25		0.0889	0.0279		0.3489	0.1918		0.7311	0.5149		0.9574	0.7274		0.9990	0.7509
-3.0	25	0.0682	0.0919	0.0334	0.3293	0.3686	0.2146	0.7257	0.7508	0.5431	0.9625	0.9629	0.7451	0.9998	0.9992	0.7676
-3.0	25		0.0875	0.0286		0.3502	0.1926		0.7317	0.5121		0.9573	0.7276		0.9991	0.7556

Note: β: skewness; γ: kurtosis; the “Zf”, “Zb”, and “Zc” results are calculated using the critical points z_α and t_α as the first and the second number

Table 5. Power performance for test $\rho_0 = 0.00$ ($n = 30$, $\alpha = 0.01$)

		ra = 0.0			**ra = 0.2**			**ra = 0.4**			**ra = 0.6**			**ra = 0.8**		
b	**g**	**Zf**	**Zb**	**Zc**	**Zf**	**Zb**	**Zc**	**Zf**	**Zb**	**Zc**	**Zf**	**Zb**	**Zc**	**Zf**	**Zb**	**Zc**
0.0	0	0.0103	0.0271	0.0202	0.1037	0.1508	0.1316	0.4585	0.4838	0.4451	0.9084	0.8966	0.8489	0.9996	0.9995	0.9871
0.0	0		0.0235	0.0161		0.1299	0.1095		0.4305	0.3908		0.8640	0.8044		0.9991	0.9797
0.0	1	0.0102	0.0319	0.0180	0.1046	0.1572	0.1185	0.4646	0.5000	0.4061	0.9090	0.9038	0.7928	0.9996	0.9995	0.9313
0.0	1		0.0283	0.0137		0.1348	0.0962		0.4451	0.3492		0.8728	0.7404		0.9990	0.9136
0.0	10	0.0147	0.0605	0.0087	0.1247	0.2016	0.0717	0.4893	0.5508	0.2901	0.9128	0.9203	0.6149	0.9997	0.9988	0.7141
0.0	10		0.0586	0.0061		0.1801	0.0525		0.4962	0.2281		0.8931	0.5508		0.9983	0.6823
0.5	0	0.0114	0.0226	0.0180	0.1074	0.1410	0.1245	0.4594	0.4744	0.4353	0.9026	0.8925	0.8371	0.9996	0.9992	0.9753
0.5	0		0.0192	0.0140		0.1208	0.1030		0.4210	0.3814		0.8597	0.7936		0.9984	0.9647
1.0	0	0.0129	0.0169	0.0130	0.1051	0.0961	0.0850	0.4051	0.3352	0.3022	0.8197	0.7457	0.6739	0.9929	0.9834	0.9398
1.0	0		0.0144	0.0101		0.0794	0.0682		0.2824	0.2524		0.6781	0.6026		0.9732	0.9098
0.5	1	0.0115	0.0286	0.0172	0.1047	0.1482	0.1134	0.4621	0.4886	0.4020	0.9055	0.8997	0.7877	0.9995	0.9990	0.9279
0.5	1		0.0249	0.0130		0.1256	0.0913		0.4331	0.3453		0.8665	0.7364		0.9986	0.9107
1.2	4	0.0148	0.0338	0.0110	0.1203	0.1512	0.0831	0.4721	0.4806	0.3217	0.9012	0.8896	0.6826	0.9996	0.9974	0.8191
1.2	4		0.0315	0.0082		0.1301	0.0637		0.4247	0.2662		0.8544	0.6223		0.9961	0.7932
1.2	10	0.0163	0.0544	0.0076	0.1310	0.1918	0.0676	0.4912	0.5330	0.2835	0.9086	0.9087	0.6116	0.9997	0.9980	0.7199
1.2	10		0.0527	0.0053		0.1706	0.0496		0.4801	0.2235		0.8782	0.5471		0.9972	0.6891
1.2	25	0.0215	0.0744	0.0047	0.1591	0.2325	0.0478	0.5077	0.5639	0.2222	0.8869	0.8971	0.5045	0.9989	0.9977	0.6111
1.2	25		0.0733	0.0032		0.2128	0.0316		0.5155	0.1640		0.8670	0.4369		0.9965	0.5767
-1.2	4	0.0145	0.0328	0.0104	0.1208	0.1503	0.0818	0.4723	0.4783	0.3207	0.9022	0.8890	0.6833	0.9996	0.9973	0.8200
-1.2	4		0.0303	0.0076		0.1300	0.0630		0.4228	0.2649		0.8541	0.6231		0.9960	0.7939

Note: β: skewness; γ: kurtosis; the “Zf”, “Zb”, and “Zc” results are calculated using the critical points z_α and t_α as the first and the second number

Table 5 (continued). Power performance for test $\rho_0 = 0.00$ ($n = 30$, $\alpha = 0.01$)

		ra = 0.0			ra = 0.2			ra = 0.4			ra = 0.6			ra = 0.8		
b	**g**	**Zf**	**Zb**	**Zc**	**Zf**	**Zb**	**Zc**	**Zf**	**Zb**	**Zc**	**Zf**	**Zb**	**Zc**	**Zf**	**Zb**	**Zc**
-1.2	10	0.0162	0.0538	0.0077	0.1289	0.1910	0.0665	0.4912	0.5327	0.2842	0.9081	0.9079	0.6091	0.9997	0.9980	0.7179
-1.2	10		0.0521	0.0053		0.1695	0.0482		0.4787	0.2262		0.8785	0.5449		0.9971	0.6858
-1.2	25	0.0211	0.0747	0.0048	0.1578	0.2318	0.0479	0.5102	0.5645	0.2215	0.8888	0.8984	0.5067	0.9988	0.9977	0.6081
-1.2	25		0.0740	0.0033		0.2124	0.0316		0.5174	0.1647		0.8695	0.4397		0.9964	0.5752
1.2	4	0.0156	0.0447	0.0094	0.1249	0.1740	0.0759	0.4840	0.5143	0.3082	0.9113	0.9057	0.6532	0.9998	0.9980	0.7733
1.2	10		0.0424	0.0065		0.1522	0.0568		0.4598	0.2487		0.8761	0.5924		0.9972	0.7447
1.2	4	0.0160	0.0586	0.0065	0.1348	0.2032	0.0643	0.5128	0.5580	0.2840	0.9298	0.9304	0.6169	1.0000	0.9994	0.6999
1.2	25		0.0572	0.0044		0.1807	0.0452		0.5023	0.2219		0.9057	0.5564		0.9991	0.6684
-1.2	4	0.0142	0.0456	0.0089	0.1245	0.1738	0.0757	0.4854	0.5144	0.3067	0.9111	0.9063	0.6515	0.9998	0.9978	0.7704
-1.2	10		0.0433	0.0063		0.1513	0.0560		0.4588	0.2477		0.8759	0.5899		0.9969	0.7406
-1.2	4	0.0157	0.0583	0.0062	0.1342	0.2019	0.0627	0.5101	0.5586	0.2853	0.9308	0.9322	0.6170	1.0000	0.9993	0.7001
-1.2	25		0.0567	0.0043		0.1801	0.0443		0.5040	0.2234		0.9074	0.5554		0.9991	0.6682
1.2	4	0.0207	0.0411	0.0050	0.1453	0.1614	0.0501	0.4815	0.4620	0.2219	0.8882	0.8708	0.5494	0.9991	0.9963	0.6948
3.0	14		0.0392	0.0034		0.1429	0.0350		0.4091	0.1699		0.8296	0.4796		0.9946	0.6610
1.2	4	0.0186	0.0501	0.0056	0.1423	0.1796	0.0551	0.5095	0.5184	0.2633	0.9139	0.9031	0.5867	0.9999	0.9986	0.6977
3.0	25		0.0483	0.0036		0.1599	0.0384		0.4641	0.2049		0.8712	0.5225		0.9978	0.6663
1.2	14	0.0216	0.0661	0.0053	0.1507	0.2115	0.0514	0.5075	0.5454	0.2422	0.8979	0.8984	0.5391	0.9995	0.9978	0.6459
3.0	25		0.0646	0.0036		0.1920	0.0348		0.4955	0.1849		0.8684	0.4746		0.9969	0.6133
-1.2	4	0.0194	0.0409	0.0052	0.1433	0.1614	0.0486	0.4814	0.4626	0.2221	0.8886	0.8722	0.5495	0.9991	0.9965	0.6940
-3.0	14		0.0393	0.0035		0.1428	0.0340		0.4106	0.1699		0.8318	0.4814		0.9947	0.6609

Note: β: skewness; γ: kurtosis; the “Zf”, “Zb”, and “Zc” results are calculated using the critical points z_α and t_α as the first and the second number

Table 5 (continued). Power performance for test $\rho_0 = 0.00$ ($n = 30$, $\alpha = 0.01$)

		ra = 0.0			ra = 0.2			ra = 0.4			ra = 0.6			ra = 0.8		
b	**g**	**Zf**	**Zb**	**Zc**	**Zf**	**Zb**	**Zc**	**Zf**	**Zb**	**Zc**	**Zf**	**Zb**	**Zc**	**Zf**	**Zb**	**Zc**
-1.2	4	0.0192	0.0501	0.0055	0.1434	0.1806	0.0550	0.5090	0.5181	0.2614	0.9142	0.9036	0.5878	0.9999	0.9983	0.6962
-3.0	25		0.0483	0.0036		0.1600	0.0385		0.4644	0.2026		0.8726	0.5256		0.9976	0.6636
-1.2	14	0.0216	0.0659	0.0053	0.1499	0.2138	0.0515	0.5060	0.5448	0.2406	0.9001	0.9015	0.5416	0.9994	0.9978	0.6478
-3.0	25		0.0648	0.0035		0.1941	0.0358		0.4956	0.1838		0.8722	0.4775		0.9968	0.6163
3.0	25	0.0277	0.0645	0.0040	0.1702	0.2099	0.0431	0.5080	0.5167	0.2028	0.8724	0.8581	0.4780	0.9978	0.9945	0.6159
3.0	25		0.0632	0.0025		0.1929	0.0293		0.4710	0.1516		0.8239	0.4133		0.9922	0.5829
-3.0	25	0.0282	0.0646	0.0044	0.1719	0.2110	0.0424	0.5079	0.5176	0.2031	0.8715	0.8589	0.4801	0.9978	0.9946	0.6142
-3.0	25		0.0633	0.0028		0.1938	0.0280		0.4734	0.1533		0.8239	0.4151		0.9923	0.5815

Note: β: skewness; γ: kurtosis; the "Zf", "Zb", and "Zc" results are calculated using the critical points z_α and t_α as the first and the second number

Table 6 Power performance for test $\rho_0 = 0.55$ ($n = 30$, $\alpha = 0.10$)

			$\rho_\alpha = 0.55$			$\rho_\alpha = 0.60$			$\rho_\alpha = 0.70$		
β	**γ**	**CP**	**Zf**	**Zb**	**Zc**	**Zf**	**Zb**	**Zc**	**Zf**	**Zb**	**Zc**
0.0	0	z_α	0.0991	0.1275	0.1139	0.3005	0.3038	0.2861	0.6538	0.4908	0.4779
0.0	0	t_α		0.1115	0.0978		0.2632	0.2452		0.4201	0.4073
0.0	1	z_α	0.1017	0.1352	0.1142	0.2990	0.3064	0.2795	0.6526	0.4850	0.4678
0.0	1	t_α		0.1219	0.0999		0.2729	0.2452		0.4226	0.4043
0.0	10	z_α	0.1483	0.1923	0.1358	0.3445	0.3497	0.2934	0.6604	0.4896	0.4611
0.0	10	t_α		0.1855	0.1253		0.3296	0.2690		0.4502	0.4190
0.5	0	z_α	0.1075	0.1312	0.1164	0.3058	0.3037	0.2853	0.6496	0.4919	0.4789
0.5	0	t_α		0.1153	0.1005		0.2662	0.2475		0.4245	0.4114
1.0	0	z_α	0.0914	0.1044	0.0918	0.2330	0.2350	0.2156	0.4929	0.4230	0.4030
1.0	0	t_α		0.0938	0.0813		0.2095	0.1901		0.3756	0.3561
0.5	1	z_α	0.1046	0.1355	0.1142	0.3026	0.3062	0.2798	0.6517	0.4870	0.4698
0.5	1	t_α		0.1220	0.0997		0.2717	0.2443		0.4259	0.4082
1.2	4	z_α	0.1280	0.1656	0.1259	0.3246	0.3255	0.2843	0.6491	0.4795	0.4556
1.2	4	t_α		0.1556	0.1143		0.3011	0.2573		0.4313	0.4057
1.2	10	z_α	0.1508	0.1968	0.1387	0.3491	0.3548	0.2969	0.6560	0.4845	0.4546
1.2	10	t_α		0.1889	0.1274		0.3348	0.2727		0.4466	0.4144
1.2	25	z_α	0.2114	0.2388	0.1621	0.3977	0.3716	0.3024	0.6554	0.4797	0.4407
1.2	25	t_α		0.2341	0.1523		0.3571	0.2827		0.4537	0.4114

Note: β: skewness; γ: kurtosis; CP: Critical Point

Table 6 (continued). Power performance for test $\rho_0 = 0.55$ ($n = 30$, $\alpha = 0.10$)

			$\alpha = 0.10$			$\alpha = 0.05$			$\alpha = 0.01$		
β	**γ**	**CP**	**Zf**	**Zb**	**Zc**	**Zf**	**Zb**	**Zc**	**Zf**	**Zb**	**Zc**
-1.2	4	z_α	0.1295	0.1656	0.1266	0.3213	0.3262	0.2836	0.6487	0.4781	0.4545
-1.2	4	t_α		0.1558	0.1148		0.2998	0.2550		0.4299	0.4046
-1.2	10	z_α	0.1517	0.1972	0.1394	0.3468	0.3545	0.2967	0.6554	0.4886	0.4593
-1.2	10	t_α		0.1899	0.1284		0.3342	0.2726		0.4492	0.4176
-1.2	25	z_α	0.2116	0.2375	0.1610	0.3954	0.3696	0.2989	0.6558	0.4755	0.4379
-1.2	25	t_α		0.2328	0.1509		0.3566	0.2803		0.4509	0.4097
1.2	4	z_α	0.1359	0.1958	0.1373	0.3357	0.3711	0.3103	0.6653	0.5225	0.4913
1.2	10	t_α		0.1872	0.1255		0.3491	0.2842		0.4801	0.4465
1.2	4	z_α	0.1600	0.2497	0.1594	0.3745	0.4387	0.3492	0.7149	0.5731	0.5348
1.2	25	t_α		0.2448	0.1480		0.4216	0.3255		0.5386	0.4972
-1.2	4	z_α	0.1358	0.1953	0.1373	0.3366	0.3704	0.3095	0.6645	0.5211	0.4904
-1.2	10	t_α		0.1879	0.1266		0.3486	0.2838		0.4789	0.4458
-1.2	4	z_α	0.1570	0.2459	0.1557	0.3746	0.4401	0.3518	0.7164	0.5735	0.5343
-1.2	25	t_α		0.2412	0.1447		0.4239	0.3287		0.5383	0.4955
1.2	4	z_α	0.1553	0.1993	0.1414	0.3418	0.3569	0.3005	0.6380	0.4908	0.4662
3.0	14	t_α		0.1937	0.1320		0.3395	0.2798		0.4554	0.4288
1.2	4	z_α	0.1724	0.2435	0.1626	0.3790	0.4097	0.3344	0.6958	0.5346	0.5011
3.0	25	t_α		0.2378	0.1515		0.3927	0.3116		0.5009	0.4644

Note: β: skewness; γ: kurtosis; CP: Critical Point

Table 6 (continued). Power performance for test $\rho_0 = 0.55$ ($n = 30$, $\alpha = 0.10$)

			$\alpha = 0.10$			**$\alpha = 0.05$**			**$\alpha = 0.01$**		
β	**γ**	**CP**	**Zf**	**Zb**	**Zc**	**Zf**	**Zb**	**Zc**	**Zf**	**Zb**	**Zc**
1.2	14	z_α	0.1942	0.2334	0.1578	0.3843	0.3659	0.3004	0.6628	0.4698	0.4376
3.0	25	t_α		0.2278	0.1479		0.3510	0.2806		0.4415	0.4064
-1.2	4	z_α	0.1586	0.2051	0.1459	0.3404	0.3566	0.3006	0.6378	0.4939	0.4685
-3.0	14	tα		0.1990	0.1363		0.3395	0.2802		0.4584	0.4315
-1.2	4	z_α	0.1700	0.2413	0.1599	0.3790	0.4073	0.3333	0.6958	0.5321	0.4995
-3.0	25	t_α		0.2365	0.1498		0.3901	0.3109		0.4983	0.4631
-1.2	14	z_α	0.1955	0.2326	0.1588	0.3865	0.3638	0.2994	0.6620	0.4616	0.4303
-3.0	25	t_α		0.2270	0.1484		0.3495	0.2805		0.4329	0.3992
3.0	25	z_α	0.2214	0.2517	0.1699	0.3995	0.3760	0.3003	0.6430	0.4740	0.4292
3.0	25	t_α		0.2473	0.1604		0.3630	0.2821		0.4489	0.4009
-3.0	25	z_α	0.2210	0.2514	0.1685	0.3996	0.3776	0.3012	0.6411	0.4731	0.4284
-3.0	25	tα		0.2464	0.1584		0.3644	0.2822		0.4480	0.4001

Note: β: skewness; γ: kurtosis; CP: Critical Point