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Generally, in empirical financial studies, the determination of the true conditional variance in GARCH modelling is largely subjective. In this paper, we investigate the consequences of choosing a wrong conditional variance specification. The methodology involves specifying a true conditional variance and then simulating data to conform to the true specification. The estimation is then carried out using the true specification and other plausible specification that are appealing to the researcher, using model and forecast evaluation criteria for assessing performance. The results show that GARCH model could serve as better alternative to other asymmetric volatility models.

Keywords: Forecasts, GARCH, misspecification, specification

Introduction

Since the seminal articles of Engle (1982) and Bollerslev (1986), the class of Generalized Autoregressive Conditionally Heteroscedasticity (GARCH) models has been a key model in financial industries. Due to wide applications of this model in financial industries and related areas, Lee and Hansen (1994) referred to the model as the workhouse of the industry. Considered here is the misspecification of variants of GARCH models. The variants include the GARCH model of Bollerslev (1986), Exponential GARCH model of Nelson (1991), Glosten Jagannathan and Runkle-GARCH (GJR-GARCH) model of Glosten, Jagannathan and Runkle (1993) and Asymmetric Power ARCH (APARCH) model of Ding, Granger and Engle (1993). Using model and forecast evaluation

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criteria, both correctly specified and the misspecified model performances are judged.

Specification of the form of GARCH model depends on the behavior and properties of the series. For example, there are asymmetric GARCH specifications which are preferred for the asymmetric series. Unlike Smooth Transition Autoregressive (STAR) model of Teräsvirta (1994) which allows selection of model between the Exponential STAR (ESTAR) and Logistic STAR (LSTAR) model based on the model specification tests, GARCH model is yet to develop such tests which selects among many alternatives. A particular GARCH model is often considered on the asset returns/residuals based on the properties of the series. The GARCH specification is a parametric model in which a particular structure is imposed at a time, and therefore, it is important to perform misspecification tests to check for the consequence of choosing a wrong model structure. Engle and Ng (1993) and Li and Mak (1994) proposed an adequacy test using the squared standardized error process. Recently, Lundbergh and Teräsvirta (2002) proposed tests for remaining ARCH in standardized errors, linearity and parameter constancy. None of the specification tests were designed to select or reject a particular GARCH specification.

Misspecification of GARCH model may pose serious problem to forecast values hence it deserves to be investigated. Wang (2002) affirmed that spurious and inefficient inference is expected when pure GARCH models are misspecified, this as well may affect the Quasi Maximum Likelihood Estimates (QMLEs) of the misspecified model. The QMLE of a pure GARCH (1,1) model indicates that the ARCH parameter is small, GARCH parameter is close to unity and the sum of both parameters approaches unity as the sampling frequency increases (Engle & Bollerslev, 1986; Bollerslev & Engle, 1993; Baillie, Bollerslev, & Mikkelsen, 1996; Ding & Granger, 1996; Andersen & Bollerslev, 1997 and Engle & Patton, 2001). This fact is reflected in the Integrated GARCH (IGARCH) model of Engle and Bollerslev (1986). More recent paper by Jansen and Lange (2010) shows that in a GARCH (1,1) model, the estimates of $\hat{\alpha}_1$ and $\hat{\beta}_1$ tend to 0 and 1, respectively as the sampling frequency increases, which is an IGARCH effect.

In a situation whereby the GARCH series is fitted to any other variants of the model, particularly those ones with asymmetric effect, do we still expect this IGARCH convergence? This paper therefore considers the misspecification of GARCH models using simulation approach. The model and forecast evaluation criteria are used to judge the alternative models.

Variants of GARCH model

The (GARCH) model predicts the volatility in the residuals ε_t of the mean equation

$$y_t = \phi_0 + \phi_1 y_{t-1} + \varepsilon_t \tag{1}$$

where y_t is the time series or returns series under investigation, ϕ_0 and ϕ_i are the constant and Autoregressive (AR) parameters of the model. In volatility modelling, autoregressive order is usually less than 3 and in some cases autoregression as well as constant may not be significant, which is the case of a pure GARCH process. The residuals of this model often violate normality assumption and are serially correlated. In that case, the non-normal residuals ε_t are modelled using variance equation.

Engle (1982) proposed the first variance equation for predicting volatility in the asset returns/innovations ε_t, and this has been the origin of other volatility models in the literature. Bollerslev (1986) proposed using lags of the conditional volatility in the model specification. The GARCH (1,1) model, proposed in Bollerslev (1986) is,

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \tag{2}$$

where ε_t are the log-returns series of the financial asset. The residuals relates to the volatility as $\varepsilon_t = \sigma_t z_t$ with $z_t \approx N(0, 1)$. The σ_t is the unconditional standard deviation expressed by the variance equation (GARCH model). The parameter is conditioned as $w > 0$, $\alpha_1 \geq 0$ and $\beta_1 \geq 0$ in order to ensure positive definite variance. These α_1 and β_1 are the ARCH and GARCH parameters for the ARCH term ε_{t-1}^2 and GARCH term σ_{t-1}^2, respectively while the stationarity imposition on the GARCH (1,1) is that the sum of the ARCH and GARCH parameters should be less than unity, that is $\alpha_1 + \beta_1 < 1$. Then, combining the AR model in (1) with GARCH model in (2) gives AR (1)-GARCH (1,1) model.

The Exponential GARCH (EGARCH) model is given in Nelson (1991). This model was developed based on the fact that GARCH (1,1) model of Bollerslev (1986) uses the magnitude of the innovations to predict future volatility but do not consider the effect of the positivity or negativity of the innovations on the volatility. The positive constraint imposed on the intercept ω often poses serious estimation problems. In that case, Nelson (1991) considered the

GARCH (1,1) model as symmetric type while the EGARCH (1,1) is asymmetric in the sense that it assumes different conditional volatility responses for either positive or negative innovations. The simplest EGARCH (1,1) specification is

$$\log \sigma_t^2 = \omega + \alpha_1 \left| \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right| + \beta_1 \log \sigma_{t-1}^2 - \gamma_1 \left(\frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right) \tag{3}$$

This model can also be re-specified as,

$$\log \sigma_t^2 = \omega + \alpha_1 \left| z_{t-1} \right| + \beta_1 \log \sigma_{t-1}^2 - \gamma_1 \left(z_{t-1} \right) \tag{4}$$

because $\varepsilon_t = \sigma_t z_t$. Here, there is good news if $\varepsilon_{t-1} > 0$ and bad news if $\varepsilon_{t-1} < 0$ which have different effect on the conditional variance. The response of either good news or bad news on the conditional volatility is then measured by the asymmetric parameter, γ_1.

The Asymmetric Power ARCH (APARCH) model is proposed in Ding et al. (1993) with power specification δ. The proposition was based on modelling standard deviation instead of the variance as in the case of GARCH and EGARCH models. This ideas was earlier considered in Taylor (1986) and Schwert (1989). The power parameter is estimated simultaneously with other parameters in the model. The specification of the APARCH (1,1) model is,

$$\sigma_t^\delta = \omega + \alpha_1 \left(\left| \varepsilon_{t-i} \right| - \gamma_1 \varepsilon_{t-1} \right)^\delta + \beta_1 \log \sigma_{t-1}^\delta \tag{5}$$

where $\delta > 0$ and $|\gamma_1| \leq 1$. At $\delta = 2$ and $\gamma_1 = 0$, the APARCH (1,1) model reduces to GARCH (1,1) model.

Estimation and Forecasts Evaluation

Estimation of GARCH model is often carried out by numerical derivatives. Numerical derivatives are used in GARCH estimation since the model lacks closed form estimation (Xekalaki & Degiannakis, 2010). The derivatives simplifies and maximises the QML log likelihood function

$$L_t = -\frac{1}{2} \left[N \log(2\pi) + \sum_{t=1}^{N} \frac{\varepsilon_t^2}{\sigma_t^2} + \sum_{t=1}^{N} \log \sigma_t^2 \right] \tag{6}$$

where ε_t are the innovations from the initial AR model, σ_t^2 are the conditional volatility realized from the variance equation and N is the sample size. Berndt, Hall, Hall and Hausman (BHHH) algorithm of Berndt, Hall, Hall and Hausman (1974) is often preferred to other numerical derivatives such as Marquadt and Gauss Newton, since it uses only the first derivatives of the likelihood function to estimate the parameter values. The algorithm is

$$\psi^{(i+1)} = \psi^{(i)} - \left(\sum_{t=1}^{N} \frac{\partial L_t^{(i)}}{\partial \psi} \cdot \frac{\partial L_t^{(i)}}{\partial \psi'} \right)^{-1} \frac{\partial L_N^{(i)}}{\partial \psi} \tag{7}$$

with initial parameter set as $\psi^{(0)}$, the parameter set which maximizes the likelihood function is denoted as $\psi^{(i+1)}$ and the log-likelihood L_t as given in (6) above. The number of iteration is denoted by i, and the iteration stops once there is no further improvement in the likelihood function. Ideally, EViews software allows setting the number of iteration and the level of precision for the estimation.

Forecast evaluation criteria considered are the Root Mean Squares Forecast Error (RMSFE), Mean Absolute Forecast Error (MAFE), Mean Absolute Percentage Forecast Error (MAPFE) and Theil Inequality of Theil (1961; 1966). The MSFE is defined as,

$$MSFE = \frac{1}{m} \sum_{t=1}^{m} \left(\hat{\sigma}_t^2 - \sigma_t^2 \right)^2 \tag{8}$$

where $\hat{\sigma}_t^2$ is the predicted in-sample conditional variances, and this depends on the scale of the variance series, σ_t^2. The square root of MSFE is the RMSFE,

$$RMSFE = \sqrt{\frac{1}{m} \sum_{t=1}^{m} \left(\hat{\sigma}_t^2 - \sigma_t^2 \right)^2} \tag{9}$$

The MAFE and MAPFE are obtained by taking the absolute differences of the predicted conditional volatilities and the observed volatilities as,

$$MAFE = \frac{1}{m} \sum_{t=1}^{m} \left| \hat{\sigma}_t^2 - \sigma_t^2 \right| \tag{10}$$

$$MAPFE = 100\sum_{t=1}^{m}\left|\frac{\hat{\sigma}_t^2 - \sigma_t^2}{\sigma_t^2}\right| \tag{11}$$

The Theil inequality is given as,

$$TI = \frac{\sqrt{\frac{1}{m}\sum_{t=1}^{m}\left(\hat{\sigma}_t^2 - \sigma_t^2\right)^2}}{\sqrt{\frac{1}{m}\sum_{t=1}^{m}\hat{\sigma}_t^2} + \sqrt{\frac{1}{m}\sum_{t=1}^{m}\sigma_t^2}} \tag{12}$$

The inequality coefficient is time invariant and always lies between 0 and unity. The smaller these forecast evaluation criteria, the better the candidate model represents well the data.

Monte Carlo Experiment, Results and Discussion

The Monte Carlo experiment is set up using the Data Generating Processes (DGPs) in (13)-(16) below. The AR (1) DGP in (12) is the mean equation, with $\phi_0 = 0.15$ and $\phi_1 = 0.5$, setting the process at the stationarity level.

$$y_t = 0.15 + 0.5y_{t-1} + \varepsilon_t \tag{13}$$

The error distribution $\varepsilon_t = \sigma_t z_t$, $z_t \sim N(0, 1)$ for each of the variance equations,

$$\sigma_t^2 = 0.02 + 0.25\varepsilon_{t-1}^2 + 0.60\sigma_{t-1}^2 \tag{14}$$

$$\log\sigma_t^2 = 0.02 + 0.25\left|\frac{\varepsilon_{t-1}}{\sigma_{t-1}}\right| + 0.60\log\sigma_{t-1}^2 - 0.10\left(\frac{\varepsilon_{t-1}}{\sigma_{t-1}}\right) \tag{15}$$

$$\sigma_t^{1.2} = 0.02 + 0.25\left(\left|\varepsilon_{t-1}\right| + 0.10\varepsilon_{t-1}\right)^{1.2} + 0.60\log\sigma_{t-1}^{1.2} \tag{16}$$

representing GARCH (1,1), EGARCH (1,1) and APARCH (1,1) models, respectively. The parameters of the models were generated by arbitrarily fixing values for them making sure the parameters of the ARCH and GARCH terms are in stationarity range, and this realizes positive definite stationary non-explosive conditional variance. These parameter values are fixed in the three models. The AR (1) DGP is combined with each variance equations in (14), (15) and (16) giving AR (1)-GARCH (1,1), AR (1)-EGARCH (1,1) and AR (1)-APARCH (1,1) DGPs, respectively. The asymmetric parameter in EGARCH and APARCH models are fixed at $\gamma_1 = -0.10$ and the power parameter in APARCH model fixed at 1.2. The misspecification of each model is considered and the behaviour of the realized conditional variance is examined using the model and forecast evaluation criteria. Sample sizes are $N = 2000$, 4000 and 6000 each with 25% of samples as in-sample forecasts.

The results of the Monte Carlo experiment are presented here as Scenarios 1-3, where both parameter and forecasts evaluation estimates are given.

Scenario 1: When the true model is GARCH

Table 1. Model parameter estimates

Sample size	Estimated Model	$\hat{\phi}_0$ (0.15)	$\hat{\phi}_1$ (0.50)	$\hat{w}$ (0.02)	$\hat{\alpha}_1$ (0.25)	$\hat{\beta}_1$ (0.60)
2000	GARCH	0.1480	0.4839	0.0169	0.2110	0.6596
4000	GARCH	0.1517	0.4724	0.0173	0.2049	0.6590
6000	GARCH	0.1475	0.4750	0.0180	0.2052	0.6503
2000	EGARCH	0.1480	0.4839	0.0169	0.2110	0.6596
4000	EGARCH	0.1517	0.4724	0.0173	0.2049	0.6590
6000	EGARCH	0.1475	0.4750	0.0180	0.2052	0.6503
2000	APARCH	0.1464	0.4875	0.0405	0.1988	0.0184
4000	APARCH	0.1488	0.4741	0.0412	0.1947	0.0633
6000	APARCH	NA	NA	NA	NA	NA

The results presented in Scenario 1 is when GARCH simulated series is used to estimate EGARCH, APARCH as well as GARCH model and the parameter and in-sample forecasts estimates presented in Tables 1 and 2, respectively. The parameter estimates for the three models are very close to the real values but these are not consistent with sample sizes. This is expected since we do not expect the least squares estimates to be consistent in the presence of serial correlation and heteroscedasticity of the residuals. We also noted the similarity in the results obtained for GARCH and EGARCH models, across the sample sizes. The APARCH estimation posed serious problem at very high sample sizes due to tendencies of the simulator to realize some non-positive volatility and the power estimates of these cannot be obtained.

Table 2. Forecast evaluation estimates

Sample size	Estimated Model	RMSFE	MAFE	MAPFE	Theil
2000	GARCH	0.000271	0.011228	48.09666	0.0289
4000	GARCH	0.000174	0.010415	44.88254	0.0257
6000	GARCH	0.000139	0.010104	49.58554	0.0287
2000	EGARCH	0.002067	0.085482	617.9674	0.1772
4000	EGARCH	0.001301	0.078142	645.6404	0.1688
6000	EGARCH	0.001165	0.084255	639.2407	0.1798
2000	APARCH	0.000520	0.021537	94.48200	0.1077
4000	APARCH	0.000344	0.020411	93.54733	0.0974
6000	APARCH	NA	NA	NA	NA

From the forecasts evaluation results in Table 2 of Scenario 1, the estimates obtained for GARCH and EGARCH models are different. Actually the RMSFE and MAFE for the models across different sample sizes are very low but the MAPFE vary significantly. The RMSFE, MAFE, MAPFE and Theil inequality coefficient for the GARCH models are the lowest, followed by that of APARCH models. This is expected since the DGP is GARCH. The MAPFE estimates vary significantly, about 50% for GARCH, 600% for EGARCH and 90% for APARCH models. It is clear to see that GARCH model forecasts are better than EGARCH and APARCH model forecasts in terms of RMSPE and Theil inequality when GARCH model is the DGP.

Scenario 2: When the true model is EGARCH

Table 3. Model parameter estimates

Sample size	Estimated Model	$\hat{\phi}_0$ (0.15)	$\hat{\phi}_1$ (0.50)	$\hat{w}$ (0.02)	$\hat{\alpha}_1$ (0.25)	$\hat{\beta}_1$ (0.60)
2000	GARCH	0.1320	0.4998	0.2037	0.0078	0.7817
4000	GARCH	0.1441	0.5067	0.4191	0.0058	0.5268
6000	GARCH	0.1212	0.4737	0.1355	-0.0101	0.8559
2000	EGARCH	0.1320	0.4998	0.2037	0.0078	0.7817
4000	EGARCH	0.1293	0.4806	0.1915	-0.0094	0.7933
6000	EGARCH	0.1212	0.4737	0.1355	-0.0101	0.8559
2000	APARCH	0.1679	0.4765	0.5952	-0.0823	1.0000
4000	APARCH	NA	NA	NA	NA	NA
6000	APARCH	NA	NA	NA	NA	NA

In Scenario 2 of Table 3, the true series follows EGARCH process. The parameter estimates are not consistent with sample sizes. Here, both the estimates of the mean and variance equations are very far from the real values. Even with EGARCH DGP to estimate EGARCH model, the estimates seem not to improve when compared with that of the misspecified GARCH model. Estimates of APARCH model for all the sampled points in the simulations could not be computed except for sample 2000, the estimation was very slow at samples 4000 and 6000 and the estimation process crashed unexpectedly.

Table 4. Forecast evaluation estimates

Sample size	Estimated Model	RMSFE	MAFE	MAPFE	Theil
2000	GARCH	0.009880	0.412570	79.28039	0.2282
4000	GARCH	0.006603	0.389932	69.61645	0.2152
6000	GARCH	0.005286	0.381419	69.51897	0.2158
2000	EGARCH	0.028755	1.224082	210.8467	0.4251
4000	EGARCH	0.010901	0.642821	138.9766	0.3230
6000	EGARCH	0.006889	0.495946	107.6174	0.2687
2000	APARCH	0.065849	2.763889	372.1500	0.9589
4000	APARCH	NA	NA	NA	NA
6000	APARCH	NA	NA	NA	NA

In Table 4 of Scenario 2, forecast estimates for the three models are different, with estimated GARCH models presenting better forecasts than the estimated EGARCH and APARCH model at corresponding sample sizes.

Scenario 3: When the true model is APARCH

Table 5. Model parameter estimates

Sample size	Estimated Model	$\hat{\phi}_0$ (0.15)	$\hat{\phi}_1$ (0.50)	$\hat{w}$ (0.02)	$\hat{\alpha}_1$ (0.25)	$\hat{\beta}_1$ (0.60)
2000	GARCH	0.1514	0.4795	0.0037	0.2240	0.6188
4000	GARCH	0.1459	0.5028	0.0046	0.2567	0.5399
6000	GARCH	0.1526	0.4721	0.0039	0.2110	0.6157
2000	EGARCH	0.1514	0.4795	0.0037	0.2240	0.6188
4000	EGARCH	0.1438	0.5052	0.0046	0.2514	0.5526
6000	EGARCH	0.1526	0.4721	0.0039	0.2110	0.6157
2000	APARCH	0.1476	0.5164	0.0260	0.2753	-0.0012
4000	APARCH	0.1395	0.5345	0.0188	0.2371	-0.0800
6000	APARCH	*NA*	*NA*	*NA*	*NA*	*NA*

Scenario 3, Table 5 presents the case where APARCH series is assumed. The APARCH model is more complex in structure than the GARCH and EGARCH models, therefore estimating APARCH model from the series at very high sample size posed serious problems. For samples 2000 and 4000, estimates of parameters were computed.

Table 6. Forecast evaluation estimates

Sample size	Estimated Model	RMSFE	MAFE	MAPFE	Theil
2000	GARCH	8.20E-06	0.000341	54.08614	0.0053
4000	GARCH	5.98E-06	0.000401	56.20084	0.0057
6000	GARCH	4.25E-06	0.000307	54.36182	0.0054
2000	EGARCH	0.000347	0.014360	3191.426	0.0925
4000	EGARCH	0.000480	0.028411	5535.569	0.1344
6000	EGARCH	0.000187	0.013544	31268.40	0.0910
2000	APARCH	2.33E-05	0.000955	85.89381	0.0182
4000	APARCH	9.13E-06	0.000538	94.78748	0.0181
6000	APARCH	NA	NA	NA	NA

In Scenario 3, Table 6, the simulated forecasts for GARCH, EGARCH and APARCH models from APARCH DGP are presented. Closer look still showed that GARCH forecasts are the best in terms of forecast evaluation criteria. Followed after GARCH is the APARCH model and EGARCH is the least.

Conclusion

The misspecification of some GARCH models were considered using parameter and forecast evaluation estimates as criteria. It was found that a correctly specified EGARCH and APARCH models actually, in the real sense, did not give better parameter estimates and forecasts when compared with that of GARCH model. These results are not consistent with sample sizes. The results obtained in this paper therefore support the seminal work of Hansen and Lunde (2005) titled: "A Forecast Comparison of Volatility Models: Does Anything Beat a GARCH(1,1)", which was their argument with Andersen and Bollerslev (1998). Great care should be taken wherever volatility model are being specified for assets returns, since misspecified model could cause great loss in model information criteria and forecasts. This work, therefore re-popularize the use of symmetric GARCH (1,1) model of Bollerslev (1986) and Taylor (1986) in empirical analysis and simulations.

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