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Monte Carlo Study of Some Classification-Based Ridge Parameter Estimators

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Ridge estimator in linear regression model requires a ridge parameter, K, of which many have been proposed. In this study, estimators based on Dorugade (2014) and Adnan et al. (2014) were classified into different forms and various types using the idea of Lukman and Ayinde (2015). Some new ridge estimators were proposed. Results shows that the proposed estimators based on Adnan et al. (2014) perform generally better than the existing ones.

Keywords: linear regression model, multicollinearity, ridge estimator, mean square error

Introduction

The parameter estimates obtained through the use of the Ordinary Least Squares (OLS) estimator have optimal performance when there is no violation of any of the assumptions of the classical linear regression model. One of the most basic of these assumptions is that explanatory variables are independent. Multicollinearity refers to the presence of strong or perfect linear relationships among the explanatory variables. Multicollinearity is an inherent phenomenon in most economic relationships due to the nature of economic magnitude (Koutsoyiannis, 2003). When there is a perfect relationship among the explanatory variables, the regression coefficients of the OLS estimator are indeterminate, and the standard error of the estimates becomes very large. Also, when there are strong relationships among the explanatory variables, the regression estimates are determinate but possesses large standard error (Koutsoyiannis, 2003).

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Generally, the performance of OLS estimator is unsatisfactory when there is multicollinearity (Koutsoyiannis, 2003). Several techniques have been suggested in the literature to handle this problem. Massy (1965) introduced the principal component regression to eliminate the model instability and reduce the variances of the regression coefficients. Wold (1966) developed the partial least square to deal with the problem of multicollinearity. Hoerl and Kennard (1970) proposed the ridge estimator for dealing with multicollinearity in a regression model, which modifies the OLS to allow biased estimation of the regression coefficients. This study is limited to the application of the ridge regression estimator in handling the problem of multicollinearity. Ridge estimator is defined as:

$$\hat{\beta}_R = \left(X'X + \mathrm{KI}\right)^{-1} X'Y \tag{1}$$

where K is a non-negative constant known as ridge parameter and I denotes an identity matrix. When K equals zero, (1) returns to OLS estimator; this is defined as follows:

$$\hat{\beta}_{OLS} = \left(X'X\right)^{-1} X'Y \tag{2}$$

The corresponding mean square error (MSE) of (1) and (2) are defined respectively as:

$$MSE\left(\hat{\beta}_R\right) = \hat{\sigma}^2 \sum_{i=1}^{p} \frac{\lambda_i}{\left(\lambda_i + \hat{K}\right)^2} + \hat{K}^2 \sum_{i=1}^{p} \frac{\hat{\beta}_i^2}{\left(\lambda_i + \hat{K}\right)^2} \tag{3}$$

$$MSE\left(\hat{\beta}_{OLS}\right) = \hat{\sigma}^2 \sum_{i=1}^{p} \frac{1}{\lambda_i} \tag{4}$$

where λ_1, λ_2, …, λ_p are the eigenvalues of $X'X$, $\hat{K}$ is the estimator of the ridge parameter K and $\hat{\beta}_i$ is the i^{th} element of the vector $\hat{\beta}$.

Although this estimator is biased, it gives a smaller mean squared error when compared to the OLS estimator for a positive value of K (Hoerl and Kennard, 1970). The use of the estimator depends largely on the ridge parameter, K. Several methods for estimating this ridge parameter have been proposed by different authors, as follows: Hoerl and Kennard (1970); McDonald and

Galarneau (1975); Lawless and Wang (1976); Hocking et al. (1976); Wichern and Churchill (1978); Gibbons (1981); Nordberg (1982); Kibria (2003), Khalaf and Shukur (2005), Alkhamisi et al. (2006), Muniz and Kibria (2009), Mansson et al. (2010), Dorugade (2014) and recently, Lukman and Ayinde (2015). The purpose of this study is to classify the ridge parameters proposed by Dorugade (2014) and Adnan et al. (2014) into different forms and various types. A simulation study is conducted and the performances of the estimators is examined via mean square error (MSE).

Model and Estimators

A linear regression model can be expressed in matrix form as:

$$Y = \mathrm{X}\beta + \mathrm{U} \tag{5}$$

where X is an $n \times p$ matrix with full rank, Y is a $n \times 1$ vector of dependent variable, β is a $p \times 1$ vector of unknown parameters, and U is the error term such that E(U) = 0 and E(UU′) = $\sigma^2 \mathrm{I}_n$. The Ordinary Least Square (OLS) estimator of β is defined in (2): Model (5) can be written in canonical form. Suppose there exists an orthogonal matrix Q such that $X'\mathrm{Q}X = \Lambda$, where $\Lambda = \mathrm{diag}(\lambda_1, \lambda_2, \ldots, \lambda_p)$ and $\lambda_1, \lambda_2, \ldots, \lambda_p$ are the eigenvalues of $X'X$. Substituting $\alpha = \mathrm{Q}'\beta$, model (5) can be written as:

$$Y = \mathrm{Z}\alpha + \mathrm{U} \tag{6}$$

where $Z'Z = \Lambda$.

Therefore, the ridge estimator of α can be defined as:

$$\hat{\alpha}_R = \left(Z'\mathrm{Z} + \mathrm{KI}\right)^{-1} Z'Y \tag{7}$$

The corresponding mean square error (MSE) is defined as:

$$MSE\left(\hat{\alpha}_R\right) = \hat{\sigma}^2 \sum_{i=1}^{p} \frac{\lambda_i}{\left(\lambda_i + \hat{\mathrm{K}}\right)^2} + \hat{\mathrm{K}}^2 \sum_{i=1}^{p} \frac{\hat{\alpha}_i^2}{\left(\lambda_i + \hat{\mathrm{K}}\right)^2} \tag{8}$$

where $\hat{\alpha}_i$ is the i^{th} element of the vector $\alpha = Q'\beta$. Hoerl and Kennard (1970) defined the value of the ridge parameter K that minimizes the mean square error as:

$$\hat{\mathrm{K}}_i = \frac{\hat{\sigma}^2}{\hat{\alpha}_i^2}, \text{ where } \hat{\sigma}^2 = \frac{\sum_{i=1}^{n} e_1^2}{n-p} \tag{9}$$

Hoerl and Kennard (1970) proposed

$$\hat{\mathrm{K}}_{\mathrm{HK}i} = \frac{\hat{\sigma}^2}{\hat{\alpha}_i^2}.$$

They suggested estimating ridge parameter by taking the maximum (Fixed Maximum) of α_i^2 such that the estimator of K is:

$$\hat{\mathrm{k}}_{\mathrm{HK}}^{\mathrm{FM}} = \frac{\hat{\sigma}^2}{\max\left(\hat{\alpha}_i^2\right)} \tag{10}$$

Hoerl et al. (1975) proposed a different estimator of K by taking the Harmonic Mean of the ridge parameter $\mathrm{K}_{\mathrm{HK}i}$. This estimator is given as:

$$\hat{\mathrm{K}}_{\mathrm{HK}}^{\mathrm{HM}} = \frac{p\hat{\sigma}^2}{\sum_{i=1}^{p} \alpha_i^2} \tag{11}$$

Kibria (2003) proposed some new estimators of K by taking the geometric mean, arithmetic mean and median ($p \geq 3$) of the ridge parameter $\mathrm{K}_{\mathrm{HK}i}$. These estimators are respectively defined as:

$$\hat{\mathrm{K}}_{\mathrm{HK}}^{\mathrm{GM}} = \frac{\hat{\sigma}^2}{\left(\prod_{i=1}^{p} \hat{\alpha}_i^2\right)^{\frac{1}{p}}} \tag{12}$$

$$\hat{\mathrm{K}}_{\mathrm{HK}}^{\mathrm{AM}} = \frac{\hat{\sigma}^2}{p} \sum_{i=1}^{p} \frac{1}{\hat{\alpha}_i^2} \tag{13}$$

$$\hat{\mathrm{K}}_{\mathrm{HK}}^{\mathrm{M}} = \mathrm{Median}\left(\frac{\hat{\sigma}^2}{\hat{\alpha}_i^2}\right) \tag{14}$$

Furthermore, Muniz and Kibria (2009) proposed some estimators of K in the form of the square root of the geometric mean of $\mathrm{K}_{\mathrm{HK}_i}$ and its reciprocal, the median of the square root of $\mathrm{K}_{\mathrm{HK}_i}$ and its reciprocal, and varying maximum of the square root of $\mathrm{K}_{\mathrm{HK}_i}$ and its reciprocal. These estimators are respectively defined as:

$$\hat{\mathrm{K}}_{\mathrm{HK}}^{\mathrm{GMSR}} = \sqrt{\frac{\hat{\sigma}^2}{\left(\prod_{i=1}^{p}\hat{\alpha}_i^2\right)^{\frac{1}{p}}}} \tag{15}$$

$$\hat{\mathrm{K}}_{\mathrm{HK}}^{\mathrm{GMRSR}} = \frac{1}{\sqrt{\frac{\hat{\sigma}^2}{\left(\prod_{i=1}^{p}\hat{\alpha}_i^2\right)^{\frac{1}{p}}}}} \tag{16}$$

$$\hat{\mathrm{K}}_{\mathrm{HK}}^{\mathrm{MSR}} = \mathrm{Median}\left(\sqrt{\frac{\hat{\sigma}^2}{\hat{\alpha}_i^2}}\right) \tag{17}$$

$$\hat{\mathrm{K}}_{\mathrm{HK}}^{\mathrm{MRSR}} = \mathrm{Median}\left(\frac{1}{\sqrt{\frac{\hat{\sigma}^2}{\hat{\alpha}_i^2}}}\right) \tag{18}$$

$$\hat{\mathrm{K}}_{\mathrm{HK}}^{\mathrm{VMSR}} = \max\left(\sqrt{\frac{\hat{\sigma}^2}{\hat{\alpha}_i^2}}\right) \tag{19}$$

$$\hat{\mathrm{K}}_{\mathrm{HK}}^{\mathrm{VMRSR}} = \max\left(\frac{1}{\sqrt{\frac{\hat{\sigma}^2}{\hat{\alpha}_i^2}}}\right) \tag{20}$$

Dorugade (2014) suggested the modification of the generalized ridge parameter in (9) by multiplying the denominator with $\lambda_{max}/2$. The estimator is defined as:

$$\hat{k}_D = \frac{2\sigma^2}{\lambda_{max}\hat{\alpha}_i^2} \tag{21}$$

where λ_{max} is the maximum eigenvalue of $X'X$.

Following Kibria (2003), Dorugade (2014) suggested the following ordinary ridge regression for the ridge parameter in (21).

$$\hat{K}_D^{HM} = \frac{2p\hat{\sigma}^2}{\lambda_{max}\sum_{i=1}^{p}\hat{\alpha}_i^2} \tag{22}$$

$$\hat{K}_D^{M} = \text{Median}\left(\frac{2\hat{\sigma}^2}{\lambda_{max}\hat{\alpha}_i^2}\right) \tag{23}$$

$$\hat{K}_D^{GM} = \frac{2\hat{\sigma}^2}{\lambda_{max}\left(\prod_{i=1}^{p}\hat{\alpha}_i^2\right)^{\frac{1}{p}}} \tag{24}$$

$$\hat{K}_{HK}^{AM} = \frac{2\hat{\sigma}^2}{\lambda_{max}p}\sum_{i=1}^{p}\frac{1}{\hat{\alpha}_i^2} \tag{25}$$

Following Dorugade (2014), Adnan et al. (2014) proposed some ridge parameters:

$$\hat{K}_{N1}^{HM} = \frac{\sqrt{5}p\hat{\sigma}^2}{\lambda_{max}\sum_{i=1}^{p}\hat{\alpha}_i^2} \tag{26}$$

$$\hat{K}_{N2}^{HM} = \frac{p\hat{\sigma}^2}{\sqrt{\lambda_{max}}\sum_{i=1}^{p}\hat{\alpha}_i^2} \tag{27}$$

$$\hat{K}_{N3}^{HM} = \frac{2p\hat{\sigma}^2}{\sum_{i=1}^{p}\left(\lambda_i^{\frac{1}{4}}\right)\sum_{i=1}^{p}\hat{\alpha}_i^2} \tag{28}$$

$$\hat{K}_{N4}^{HM} = \frac{2p\hat{\sigma}^2}{\sqrt{\sum_{i=1}^{p}\lambda_i}\sum_{i=1}^{p}\hat{\alpha}_i^2} \tag{29}$$

The proposed ridge estimators by Dorugade (2014) and Adnan et al. (2014) are classified into different forms and various types.

Ridge Parameter Proposed by Dorugade (2014)

Dorugade (2014) proposed the ridge parameter

$$\hat{K}_{D_i} = \frac{2\hat{\sigma}^2}{\lambda_{max}\hat{\alpha}_i^2}.$$

Its estimators in the light of different forms and various types are summarized in Table 1.

Table 1. Summary of Different Forms and Various Types for $\hat{K}_{D_i} = 2\hat{\sigma}^2/\lambda_{max}\hat{\alpha}_i^2$

Forms	Types of K			
	Original	*Reciprocal*	*Square Root*	*Reciprocal Square Root*
Fixed Maximum	$\hat{K}_D^{FMO} = \frac{2\hat{\sigma}^2}{\lambda_{max}\max\left(\hat{\alpha}_i^2\right)}$	$\hat{K}_{HK}^{FMR} = \frac{1}{\hat{K}_D^{FMO}}$	$\hat{K}_{HK}^{FMSR} = \sqrt{\hat{K}_D^{FMO}}$	$\hat{K}_D^{FMRSR} = \frac{1}{\sqrt{\hat{K}_D^{FMO}}}$
Varying Maximum	$\hat{K}_D^{VMO} = \max\left(\frac{2\hat{\sigma}^2}{\lambda_{max}\hat{\alpha}_i^2}\right)$	$\hat{K}_D^{VMR} = \max\left(\frac{1}{\hat{K}_{D_i}}\right)$	$\hat{K}_D^{VMSR} = \max\left(\sqrt{\hat{K}_{D_i}}\right)$	$\hat{K}_D^{VMRSR} = \max\left(\frac{1}{\sqrt{\hat{K}_{D_i}}}\right)$
Arithmetic Mean	$\hat{K}_D^{AMO} = \frac{2\hat{\sigma}^2}{\lambda_{max}p}\sum_{i=1}^{p}\frac{1}{\hat{\alpha}_i^2}$ *	$\hat{K}_D^{R} = \frac{1}{\hat{K}_D^{AMO}}$	$\hat{K}_D^{AMSR} = \sqrt{\hat{K}_D^{AMO}}$	$\hat{K}_D^{AMRSR} = \frac{1}{\sqrt{\hat{K}_D^{AMO}}}$
Harmonic Mean	$\hat{K}_D^{HMO} = \frac{2p\hat{\sigma}^2}{\lambda_{max}\sum_{i=1}^{p}\hat{\alpha}_i^2}$ *	$\hat{K}_D^{HMR} = \frac{1}{\hat{K}_D^{HMO}}$	$\hat{K}_D^{HMSR} = \sqrt{\hat{K}_D^{HMO}}$	$\hat{K}_D^{HMRSR} = \frac{1}{\sqrt{\hat{K}_D^{HMO}}}$
Geometric Mean	$\hat{K}_D^{GMO} = \frac{2\hat{\sigma}^2}{\lambda_{max}\left(\prod_{i=1}^{p}\hat{\alpha}_i^2\right)^{\frac{1}{p}}}$ *	$\hat{K}_D^{GMR} = \frac{1}{\hat{K}_D^{GMO}}$	$\hat{K}_D^{GMSR} = \sqrt{\hat{K}_D^{GMO}}$	$\hat{K}_D^{GMRSR} = \frac{1}{\sqrt{\hat{K}_D^{GMO}}}$
Median	$\hat{K}_D^{MO} = \text{median}\left(\frac{2\hat{\sigma}^2}{\lambda_{max}\hat{\alpha}_i^2}\right)$ *	$\hat{K}_D^{MR} = \text{median}\left(\frac{1}{\hat{K}_{D_i}}\right)$	$\hat{K}_D^{MSR} = \text{median}\left(\sqrt{\hat{K}_{D_i}}\right)$	$\hat{K}_D^{MRSR} = \text{median}\left(\frac{1}{\sqrt{\hat{K}_{D_i}}}\right)$

Notes: * Dorugade (2014); all others are proposed estimators

Ridge Parameter Proposed by Adnan et al. (2014)

Adnan et al. (2014) proposed the ridge parameter

$$\hat{K}_{AYA_i} = \frac{2\hat{\sigma}^2}{\sqrt{\sum_{i=1}^{p}\lambda_i\hat{\alpha}_i^2}}.$$

Its estimators in the light of different forms and various types are summarized in Table 2.

Table 2. Summary of Different Forms and Various Types for $\hat{K}_{AYA_i} = 2\hat{\sigma}^2 / \sqrt{\sum_{i=1}^{p}\lambda_i\hat{\alpha}_i^2}$

Forms	Types of K: *Original*	*Reciprocal*	*Square Root*	*Reciprocal Square Root*
Fixed Maximum	$\hat{K}_{AYA}^{FMO} = \frac{2\hat{\sigma}^2}{\sqrt{\sum_{i=1}^{p}\lambda_i}\max(\hat{\alpha}_i^2)}$	$\hat{K}_{AYA}^{FMR} = \frac{1}{\hat{K}_{AYA}^{FMO}}$	$\hat{K}_{AYA}^{FMSR} = \sqrt{\hat{K}_{AYA}^{FMO}}$	$\hat{K}_{AYA}^{FMRSR} = \frac{1}{\sqrt{\hat{K}_{AYA}^{FMO}}}$
Varying Maximum	$\hat{K}_{AYA}^{VMO} = \max\left(\frac{2\hat{\sigma}^2}{\sqrt{\sum_{i=1}^{p}\lambda_i\hat{\alpha}_i^2}}\right)$	$\hat{K}_{AYA}^{VMR} = \max\left(\frac{1}{\hat{K}_{AYA}}\right)$	$\hat{K}_{AYA}^{VMSR} = \max\left(\sqrt{\hat{K}_{AYA}}\right)$	$\hat{K}_{AYA}^{VMRSR} = \max\left(\frac{1}{\sqrt{\hat{K}_{AYA}}}\right)$
Arithmetic Mean	$\hat{K}_{AYA}^{AMO} = \frac{2\hat{\sigma}^2}{\sqrt{\sum_{i=1}^{p}\lambda_i}\,p}\sum_{i=1}^{p}\frac{1}{\hat{\alpha}_i^2}$	$\hat{K}_{AYA}^{AMR} = \frac{1}{\hat{K}_{AYA}^{AMO}}$	$\hat{K}_{AYA}^{AMSR} = \sqrt{\hat{K}_{AYA}^{AMO}}$	$\hat{K}_{AYA}^{AMRSR} = \frac{1}{\sqrt{\hat{K}_{AYA}^{AMO}}}$
Harmonic Mean	$\hat{K}_{AYA}^{HMO} = \frac{2p\hat{\sigma}^2}{\sqrt{\sum_{i=1}^{p}\lambda_i\sum_{i=1}^{p}\hat{\alpha}_i^2}}$ *	$\hat{K}_{AYA}^{HMR} = \frac{1}{\hat{K}_{AYA}^{HMO}}$	$\hat{K}_{AYA}^{HMSR} = \sqrt{\hat{K}_{AYA}^{HMO}}$	$\hat{K}_{AYA}^{HMRSR} = \frac{1}{\sqrt{\hat{K}_{AYA}^{HMO}}}$
Geometric Mean	$\hat{K}_{AYA}^{GMO} = \frac{2\hat{\sigma}^2}{\sqrt{\sum_{i=1}^{p}\lambda_i}\left(\prod_{i=1}^{p}\hat{\alpha}_i^2\right)^{\frac{1}{p}}}$	$\hat{K}_{AYA}^{GMR} = \frac{1}{\hat{K}_{AYA}^{GMO}}$	$\hat{K}_{AYA}^{GMSR} = \sqrt{\hat{K}_{AYA}^{GMO}}$	$\hat{K}_{AYA}^{GMRSR} = \frac{1}{\sqrt{\hat{K}_{AYA}^{GMO}}}$
Median	$\hat{K}_{AYA}^{MO} = \mathrm{median}\left(\frac{2\hat{\sigma}^2}{\sqrt{\sum_{i=1}^{p}\lambda_i\hat{\alpha}_i^2}}\right)$	$\hat{K}_{AYA}^{MR} = \mathrm{median}\left(\frac{1}{\hat{K}_{AYA}}\right)$	$\hat{K}_{AYA}^{MSR} = \mathrm{median}\left(\sqrt{\hat{K}_{AYA}}\right)$	$\hat{K}_{AYA}^{MRSR} = \mathrm{median}\left(\frac{1}{\sqrt{\hat{K}_{AYA}}}\right)$

Notes: ***** Adnan et al. (2014); all others are proposed estimators

The ridge parameter estimators in Table 1 and 2 were examined and evaluated in this study.

Monte Carlo Simulation

The considered regression model is of the form:

$$Y_t = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \cdots + B_p X_{ip} + U_t \tag{30}$$

where $t = 1, 2, \ldots, n$; $p = 3, 7$.

The error term U_t was generated to be normally distributed with mean zero and variance σ^2, $U_t \sim N(0, \sigma^2)$. In this study, σ were taken to be 0.5, 1 and 5.

β_0 was taken to be identically zero. When $p = 3$, the values of β were chosen to be $\beta = (0.8, 0.1, 0.6)'$. When $p = 7$, the values of β were chosen to be $\beta = (0.4, 0.1, 0.6, 0.2, 0.25, 0.3, 0.53)'$. The parameter values were chosen such that $\beta'\beta = 1$ which is a common restriction in simulation studies of this type (Muniz and Kibria, 2009). We varied the sample sizes between 10, 20, 30, 40 and 50. Following McDonald and Galarneau (1975), Wichern and Churchill (1978), Gibbons (1981), Kibria (2003), Muniz and Kibria (2009), Lukman and Ayinde (2015), the explanatory variables were generated using the following equation:

$$X_{ij} = \left(1-\rho^2\right)^{\frac{1}{2}} Z_{ij} + \rho Z_{ip},\ i = 1, 2, 3, \ldots, n,\ j = 1, 2, \ldots, p. \tag{31}$$

where Z_{ij} is independent standard normal distribution with mean zero and unit variance, ρ is the correlation between any two explanatory variables and p is the number of explanatory variables. The number of explanatory variable (p) is taken to be three (3) and seven (7). The value of ρ is taken as 0.95, 0.99 respectively. Three different values of σ, 0.5, 1 and 5, were also used. The experiment is replicated 1,000 times. The ridge parameter estimators are evaluated using mean square error (MSE).

Results

The results of the simulation are presented in Table 3 and 4. These tables provide the results of the estimated mean square error of the ridge parameter when the number of regressors is three (3) and seven (7) respectively. The mean square error increases as the multicollinearity level increases. Across each multicollinearity level, the mean square error decreases as the sample sizes increase from 10 to 50, while increasing the number of regressors increases the estimated MSE. However, it is observed that the ridge estimators based on K_{AYA} performed consistently better than K_D. Occasionally, this method performs better than K_{AYA}. For instance, estimators $\hat{K}_D^{VMSR}$ and $\hat{K}_D^{AMSR}$ perform consistently well over estimators based on K_{AYA} especially when the number of regressors increases to seven (7), and when the number of regressors is three (3), especially when $n \leq 20$. This can be seen in Figure 1 and 2. The following ridge parameter estimators based on K_{AYA}: $\hat{K}_{AYA}^{FMSR}$, $\hat{K}_{AYA}^{HMO}$, $\hat{K}_{AYA}^{FMO}$, $\hat{K}_{AYA}^{HMSR}$, $\hat{K}_{AYA}^{GMO}$, $\hat{K}_{AYA}^{MO}$, and $\hat{K}_{AYA}^{GMSR}$ performed best when compared to others. All but $\hat{K}_{AYA}^{HMO}$ are proposed in this study. When $p = 3$, $\hat{K}_{AYA}^{FMSR}$ performs better than the existing ridge parameter $\hat{K}_{AYA}^{HMO}$, while $\hat{K}_{AYA}^{HMO}$ performs better than $\hat{K}_{AYA}^{FMSR}$ when $p = 7$. The estimators considered best in this study have the least MSE when compared to others. The proposed estimators perform better than the existing estimators based on K_D.

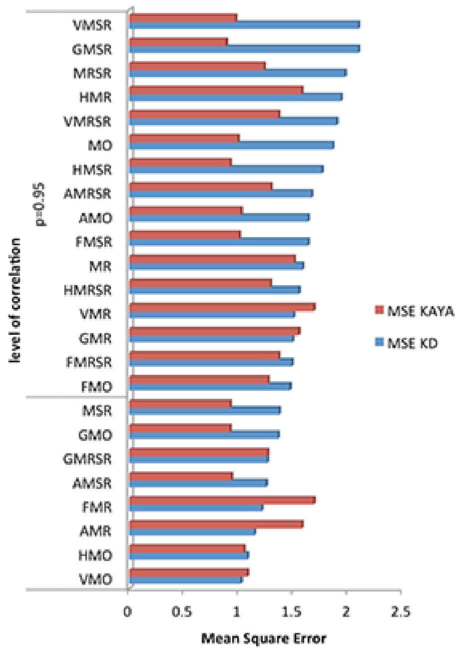

Figure 1. Graphical Illustration when $n = 20$, $\sigma^2 = 0.25$, $p = 3$

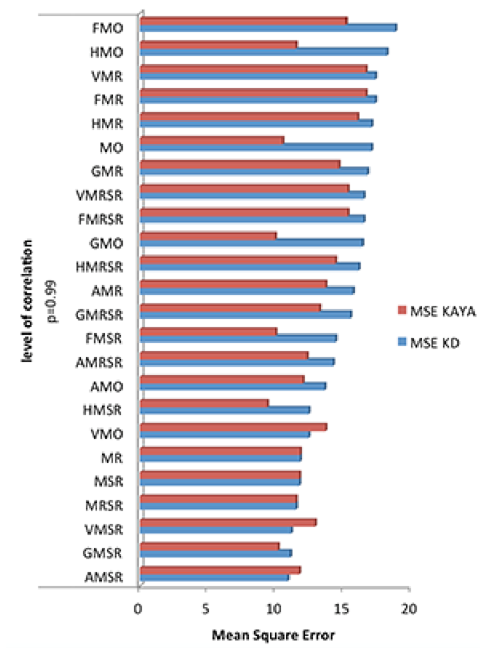

Figure 2. Graphical Illustration when $n = 50$, $\sigma^2 = 0.25$, $p = 7$

Table 3. Estimated Mean Square Error of ridge parameter when p = 3

	$p = 3, \sigma = 0.5, \rho = 0.95$									
	$n = 10$		$n = 20$		$n = 30$		$n = 40$		$n = 50$	
Methods	$\hat{K}_D$	$\hat{K}_{AYA}$	$\hat{K}_D$	$\hat{K}_{AYA}$	$\hat{K}_D$	$\hat{K}_{AYA}$	$\hat{K}_D$	$\hat{K}_{AYA}$	$\hat{K}_D$	$\hat{K}_{AYA}$
FMO	3.501	2.297	1.757	1.265	1.633	1.224	0.877	0.750	0.680	0.611
FMR	4.470	4.049	2.091	1.683	2.149	1.722	1.191	0.795	1.017	0.631
FMSR	2.340	1.757	1.369	1.004	1.309	0.941	0.789	0.627	0.635	0.532
FMRSR	3.904	3.550	1.630	1.363	1.690	1.390	0.805	0.595	0.649	0.456
VMO	2.374	2.263	1.248	1.078	1.235	1.039	0.705	0.572	0.581	0.471
VMR	4.470	4.049	2.091	1.683	2.149	1.722	1.191	0.795	1.017	0.631
VMSR	2.004	2.289	1.019	0.969	1.000	0.948	0.607	0.493	0.514	0.407
VMRSR	3.904	3.550	1.630	1.363	1.690	1.390	0.805	0.595	0.649	0.456
AMO	2.589	2.033	1.358	1.019	1.337	0.987	0.755	0.571	0.611	0.477
AMR	4.092	3.659	1.856	1.575	1.889	1.532	1.016	0.760	0.843	0.592
AMSR	1.995	2.088	1.079	0.932	1.054	0.906	0.652	0.506	0.546	0.426
AMRSR	3.532	3.107	1.465	1.293	1.490	1.242	0.725	0.630	0.572	0.500
HMO	3.184	1.863	1.665	1.045	1.579	1.027	0.865	0.666	0.674	0.559
HMR	4.322	3.889	1.968	1.573	2.037	1.606	1.102	0.724	0.938	0.568
HMSR	2.098	1.723	1.259	0.920	1.215	0.868	0.759	0.575	0.618	0.494
HMRSR	3.789	3.380	1.548	1.287	1.614	1.302	0.754	0.574	0.602	0.448
GMO	2.782	1.762	1.483	0.919	1.447	0.903	0.821	0.562	0.651	0.476
GMR	4.206	3.747	1.892	1.546	1.956	1.535	1.043	0.730	0.882	0.571
GMSR	1.963	1.855	1.142	0.883	1.109	0.845	0.709	0.521	0.588	0.447
GMRSR	3.665	3.215	1.489	1.260	1.546	1.242	0.727	0.592	0.577	0.468
MO	2.930	1.856	1.581	0.992	1.508	0.965	0.837	0.618	0.658	0.508
MR	4.243	3.710	1.931	1.505	2.007	1.532	1.105	0.732	0.938	0.586
MSR	2.045	1.910	1.209	0.920	1.162	0.874	0.733	0.549	0.600	0.467
MRSR	3.675	3.192	1.499	1.230	1.567	1.230	0.754	0.580	0.605	0.461

Table 3, continued.

	p = 3, σ = 0.5, ρ = 0.99									
	n = 10		n = 20		n = 30		n = 40		n = 50	
Methods	$\hat{K}_D$	$\hat{K}_{AYA}$	$\hat{K}_D$	$\hat{K}_{AYA}$	$\hat{K}_D$	$\hat{K}_{AYA}$	$\hat{K}_D$	$\hat{K}_{AYA}$	$\hat{K}_D$	$\hat{K}_{AYA}$
FMO	18.092	11.590	8.556	5.451	8.606	5.586	4.642	3.313	3.604	2.739
FMR	23.328	22.984	9.951	9.565	10.616	10.215	5.488	5.093	4.354	3.988
FMSR	9.438	9.998	4.895	4.092	5.062	4.119	3.310	2.287	2.815	1.905
FMRSR	22.714	22.414	9.400	9.056	10.082	9.728	5.018	4.634	3.932	3.534
VMO	11.597	16.282	5.494	6.071	6.106	6.470	3.466	2.787	2.872	2.165
VMR	23.328	22.984	9.951	9.565	10.616	10.215	5.488	5.093	4.354	3.988
VMSR	14.796	18.237	5.274	6.626	5.578	7.151	2.623	2.900	2.174	2.099
VMRSR	22.714	22.414	9.400	9.056	10.082	9.728	5.018	4.634	3.932	3.534
AMO	11.942	13.339	6.064	5.189	6.616	5.595	3.748	2.659	3.087	2.144
AMR	22.669	21.913	9.565	8.967	10.164	9.469	5.173	4.703	4.038	3.562
AMSR	13.197	16.753	4.928	5.998	5.217	6.536	2.692	2.669	2.292	1.972
AMRSR	22.012	21.079	8.921	8.200	9.590	8.753	4.695	4.075	3.603	2.997
HMO	15.724	9.207	7.667	4.343	7.982	4.490	4.469	2.739	3.517	2.309
HMR	23.199	22.808	9.808	9.399	10.482	10.073	5.361	4.965	4.240	3.868
HMSR	9.301	11.554	4.433	4.382	4.564	4.508	2.997	2.214	2.595	1.760
HMRSR	22.602	22.215	9.291	8.846	9.992	9.529	4.936	4.450	3.855	3.358
GMO	12.880	9.977	6.567	4.300	6.973	4.520	4.091	2.440	3.318	2.024
GMR	23.011	22.525	9.658	9.191	10.324	9.846	5.248	4.809	4.153	3.710
GMSR	11.006	14.465	4.495	5.213	4.643	5.625	2.745	2.381	2.387	1.792
GMRSR	22.407	21.806	9.128	8.529	9.840	9.186	4.823	4.231	3.753	3.156
MO	13.018	11.525	6.768	4.807	7.190	5.032	4.263	2.569	3.396	2.121
MR	22.980	22.395	9.662	9.091	10.321	9.762	5.264	4.760	4.171	3.693
MSR	11.634	14.736	4.766	5.443	4.932	5.955	2.864	2.499	2.464	1.856
MRSR	22.373	21.717	9.106	8.456	9.822	9.103	4.822	4.177	3.758	3.124

Table 3, continued.

	$p = 3, \sigma = 1, \rho = 0.95$									
	$n = 10$		$n = 20$		$n = 30$		$n = 40$		$n = 50$	
Methods	$\hat{K}_D$	$\hat{K}_{AYA}$	$\hat{K}_D$	$\hat{K}_{AYA}$	$\hat{K}_D$	$\hat{K}_{AYA}$	$\hat{K}_D$	$\hat{K}_{AYA}$	$\hat{K}_D$	$\hat{K}_{AYA}$
FMO	3.730	2.432	1.811	1.306	1.675	1.249	0.894	0.763	0.690	0.620
FMR	4.650	4.224	2.228	1.806	2.151	1.726	1.190	0.794	1.015	0.630
FMSR	2.488	1.864	1.413	1.039	1.340	0.960	0.804	0.638	0.644	0.539
FMRSR	4.077	3.716	1.747	1.457	1.695	1.396	0.805	0.597	0.649	0.459
VMO	2.516	2.394	1.291	1.127	1.259	1.052	0.720	0.582	0.584	0.472
VMR	4.650	4.224	2.228	1.806	2.151	1.726	1.190	0.794	1.015	0.630
VMSR	2.127	2.426	1.063	1.021	1.020	0.961	0.617	0.499	0.520	0.410
VMRSR	4.077	3.716	1.747	1.457	1.695	1.396	0.805	0.597	0.649	0.459
AMO	2.749	2.150	1.407	1.060	1.367	1.001	0.771	0.582	0.616	0.478
AMR	4.309	3.860	1.975	1.663	1.900	1.548	1.020	0.768	0.843	0.598
AMSR	2.117	2.216	1.123	0.977	1.078	0.920	0.663	0.513	0.553	0.430
AMRSR	3.725	3.288	1.558	1.361	1.496	1.257	0.730	0.639	0.574	0.506
HMO	3.386	1.969	1.716	1.080	1.619	1.045	0.882	0.676	0.685	0.566
HMR	4.497	4.059	2.102	1.686	2.040	1.611	1.101	0.725	0.937	0.569
HMSR	2.229	1.829	1.299	0.954	1.243	0.885	0.773	0.584	0.627	0.501
HMRSR	3.956	3.539	1.658	1.372	1.618	1.309	0.754	0.579	0.602	0.452
GMO	2.945	1.868	1.535	0.961	1.483	0.922	0.835	0.570	0.662	0.481
GMR	4.395	3.936	2.018	1.642	1.956	1.541	1.044	0.735	0.882	0.574
GMSR	2.081	1.970	1.184	0.923	1.135	0.862	0.721	0.528	0.597	0.453
GMRSR	3.837	3.381	1.589	1.335	1.549	1.250	0.729	0.599	0.578	0.473
MO	3.110	1.966	1.634	1.028	1.543	0.982	0.853	0.628	0.668	0.515
MR	4.411	3.853	2.059	1.602	2.010	1.537	1.104	0.734	0.937	0.587
MSR	2.173	2.036	1.250	0.958	1.188	0.890	0.746	0.557	0.609	0.473
MRSR	3.834	3.333	1.601	1.305	1.571	1.238	0.755	0.586	0.606	0.465

Table 3, continued.

	***p* = 3, *σ* = 1, *ρ* = 0.99**									
	***n* = 10**		***n* = 20**		***n* = 30**		***n* = 40**		***n* = 50**	
Methods	$\hat{K}_D$	$\hat{K}_{AYA}$	$\hat{K}_D$	$\hat{K}_{AYA}$	$\hat{K}_D$	$\hat{K}_{AYA}$	$\hat{K}_D$	$\hat{K}_{AYA}$	$\hat{K}_D$	$\hat{K}_{AYA}$
FMO	19.320	12.313	8.834	5.679	8.819	5.696	4.733	3.366	3.659	2.771
FMR	24.239	23.888	10.731	10.337	10.647	10.247	5.495	5.099	4.357	3.991
FMSR	10.025	10.572	5.094	4.299	5.168	4.187	3.368	2.318	2.854	1.926
FMRSR	23.617	23.307	10.161	9.794	10.115	9.758	5.025	4.638	3.936	3.537
VMO	12.249	17.173	5.723	6.502	6.221	6.549	3.509	2.806	2.902	2.180
VMR	24.239	23.888	10.731	10.337	10.647	10.247	5.495	5.099	4.357	3.991
VMSR	15.584	19.109	5.605	7.122	5.650	7.209	2.648	2.914	2.191	2.112
VMRSR	23.617	23.307	10.161	9.794	10.115	9.758	5.025	4.638	3.936	3.537
AMO	12.630	14.109	6.275	5.487	6.741	5.673	3.806	2.676	3.120	2.161
AMR	23.633	22.899	10.276	9.604	10.198	9.528	5.186	4.710	4.041	3.567
AMSR	13.939	17.597	5.190	6.413	5.290	6.596	2.723	2.684	2.312	1.985
AMRSR	22.911	21.970	9.631	8.816	9.639	8.791	4.702	4.083	3.607	3.001
HMO	16.754	9.756	7.941	4.546	8.168	4.569	4.553	2.776	3.569	2.333
HMR	24.107	23.707	10.588	10.161	10.513	10.104	5.367	4.970	4.243	3.871
HMSR	9.865	12.191	4.630	4.626	4.652	4.572	3.047	2.241	2.629	1.778
HMRSR	23.501	23.099	10.045	9.567	10.024	9.558	4.942	4.453	3.859	3.361
GMO	13.612	10.593	6.819	4.498	7.099	4.591	4.167	2.466	3.365	2.040
GMR	23.903	23.382	10.441	9.922	10.361	9.899	5.259	4.815	4.160	3.715
GMSR	11.654	15.236	4.708	5.533	4.714	5.684	2.786	2.401	2.416	1.806
GMRSR	23.293	22.663	9.872	9.222	9.875	9.221	4.832	4.237	3.758	3.161
MO	13.811	12.216	7.015	5.072	7.339	5.106	4.338	2.598	3.442	2.143
MR	23.880	23.264	10.423	9.789	10.353	9.795	5.270	4.763	4.176	3.698
MSR	12.330	15.539	5.007	5.781	5.009	6.012	2.905	2.521	2.494	1.873
MRSR	23.259	22.573	9.839	9.130	9.853	9.130	4.827	4.180	3.762	3.129

Table 3, continued.

	$p = 3$, $\sigma = 5$, $\rho = 0.95$									
	$n = 10$		$n = 20$		$n = 30$		$n = 40$		$n = 50$	
Methods	$\hat{K}_D$	$\hat{K}_{AYA}$	$\hat{K}_D$	$\hat{K}_{AYA}$	$\hat{K}_D$	$\hat{K}_{AYA}$	$\hat{K}_D$	$\hat{K}_{AYA}$	$\hat{K}_D$	$\hat{K}_{AYA}$
FMO	11.368	7.438	3.629	2.802	2.943	1.823	1.443	1.109	1.019	0.857
FMR	10.621	10.033	6.060	5.043	2.220	1.935	1.172	0.872	0.997	0.690
FMSR	7.609	5.767	2.910	2.281	2.232	1.451	1.265	0.940	0.937	0.750
FMRSR	9.739	9.054	4.878	3.849	1.820	1.663	0.835	0.762	0.672	0.589
VMO	7.360	8.088	2.773	2.920	1.880	1.335	1.014	0.712	0.805	0.580
VMR	10.621	10.033	6.060	5.043	2.220	1.935	1.172	0.872	0.997	0.690
VMSR	6.811	7.703	2.385	2.638	1.505	1.251	0.884	0.641	0.719	0.528
VMRSR	9.739	9.054	4.878	3.849	1.820	1.663	0.835	0.762	0.672	0.589
AMO	7.511	7.398	3.017	2.465	2.125	1.309	1.123	0.732	0.870	0.604
AMR	10.043	9.853	5.115	3.460	2.193	2.206	1.145	1.113	0.912	0.821
AMSR	6.579	7.210	2.448	2.377	1.644	1.242	0.974	0.680	0.777	0.565
AMRSR	8.675	8.335	3.880	2.948	1.791	1.824	0.889	0.973	0.683	0.731
HMO	9.971	5.816	3.471	2.400	2.752	1.446	1.406	0.928	1.006	0.752
HMR	10.380	9.682	5.817	4.507	2.124	1.926	1.091	0.891	0.927	0.689
HMSR	6.650	5.618	2.704	2.158	2.021	1.297	1.201	0.840	0.906	0.685
HMRSR	9.452	8.641	4.562	3.485	1.760	1.643	0.803	0.797	0.640	0.615
GMO	7.658	6.150	3.220	2.209	2.420	1.247	1.293	0.749	0.956	0.619
GMR	9.931	9.221	5.533	3.973	2.107	2.036	1.080	1.007	0.892	0.757
GMSR	6.115	6.311	2.529	2.169	1.803	1.220	1.099	0.736	0.852	0.609
GMRSR	8.890	8.085	4.239	3.182	1.739	1.690	0.824	0.877	0.644	0.669
MO	8.041	6.399	3.326	2.325	2.588	1.354	1.338	0.844	0.975	0.683
MR	9.931	8.925	5.641	4.182	2.105	1.882	1.116	0.942	0.941	0.721
MSR	6.330	6.458	2.613	2.193	1.913	1.285	1.146	0.788	0.877	0.646
MRSR	8.852	7.938	4.375	3.302	1.716	1.602	0.824	0.833	0.655	0.639

Table 3, continued.

	$p = 3$, $\sigma = 5$, $\rho = 0.99$									
	$n = 10$		$n = 20$		$n = 30$		$n = 40$		$n = 50$	
Methods	$\hat{K}_D$	$\hat{K}_{AYA}$	$\hat{K}_D$	$\hat{K}_{AYA}$	$\hat{K}_D$	$\hat{K}_{AYA}$	$\hat{K}_D$	$\hat{K}_{AYA}$	$\hat{K}_D$	$\hat{K}_{AYA}$
FMO	59.692	37.770	18.672	13.998	15.022	8.319	7.487	4.664	5.332	3.620
FMR	53.657	53.218	32.462	31.863	11.276	10.893	5.636	5.244	4.434	4.088
FMSR	30.381	30.342	12.064	11.318	7.969	5.753	4.993	3.111	3.975	2.487
FMRSR	52.809	52.248	31.367	30.231	10.761	10.376	5.175	4.763	4.032	3.644
VMO	38.646	46.837	14.345	22.905	8.345	7.902	4.911	3.312	3.799	2.541
VMR	53.657	53.218	32.462	31.863	11.276	10.893	5.636	5.244	4.434	4.088
VMSR	42.916	47.776	17.149	23.077	7.080	8.247	3.447	3.340	2.766	2.443
VMRSR	52.809	52.248	31.367	30.231	10.761	10.376	5.175	4.763	4.032	3.644
AMO	37.226	42.827	13.573	17.671	9.686	7.071	5.585	3.234	4.197	2.570
AMR	53.188	53.091	31.009	26.815	10.952	10.668	5.464	5.240	4.288	4.035
AMSR	40.230	45.767	14.585	20.156	6.903	7.713	3.689	3.163	2.993	2.351
AMRSR	49.835	48.112	28.792	25.131	10.257	9.549	4.890	4.452	3.789	3.374
HMO	50.356	29.122	17.437	11.743	13.310	6.311	7.033	3.630	5.128	2.922
HMR	53.471	52.979	32.281	31.474	11.141	10.762	5.512	5.110	4.328	3.984
HMSR	29.248	33.662	11.305	12.463	6.897	5.957	4.396	2.882	3.602	2.239
HMRSR	52.585	51.797	31.043	29.510	10.654	10.172	5.076	4.586	3.949	3.490
GMO	35.454	34.552	15.197	11.493	11.040	6.049	6.262	3.140	4.669	2.489
GMR	52.980	51.557	31.959	30.408	10.994	10.583	5.417	5.054	4.261	3.947
GMSR	34.408	41.170	11.755	15.542	6.585	6.913	3.917	2.952	3.220	2.206
GMRSR	51.759	50.074	30.358	28.030	10.483	9.835	4.959	4.430	3.852	3.358
MO	37.649	36.864	15.980	12.632	11.137	6.681	6.483	3.275	4.891	2.644
MR	53.025	51.491	32.038	30.644	10.985	10.394	5.437	4.922	4.260	3.795
MSR	35.513	41.459	11.932	14.442	6.907	7.242	4.045	3.064	3.376	2.305
MRSR	51.761	50.035	30.626	28.647	10.446	9.710	4.952	4.354	3.838	3.263

Table 4. Estimated Mean Square Error of ridge parameter when p = 7

	$p = 7, \sigma = 0.5, \rho = 0.95$									
	***n* = 10**		***n* = 20**		***n* = 30**		***n* = 40**		***n* = 50**	
Methods	$\hat{K}_D$	$\hat{K}_{AYA}$	$\hat{K}_D$	$\hat{K}_{AYA}$	$\hat{K}_D$	$\hat{K}_{AYA}$	$\hat{K}_D$	$\hat{K}_{AYA}$	$\hat{K}_D$	$\hat{K}_{AYA}$
FMO	76.120	47.232	7.028	5.439	4.881	4.079	2.493	2.267	2.262	2.018
FMR	81.940	81.594	6.749	6.071	5.540	4.870	2.855	2.186	2.544	1.960
FMSR	36.541	45.788	5.381	3.873	4.109	3.051	2.316	1.900	2.072	1.655
FMRSR	81.165	80.643	5.940	5.289	4.789	4.137	2.192	1.660	1.960	1.584
VMO	66.682	76.133	4.525	5.406	3.600	4.332	1.837	2.072	1.625	1.918
VMR	81.940	81.594	6.749	6.071	5.540	4.870	2.855	2.186	2.544	1.960
VMSR	72.487	76.644	4.136	4.789	3.209	3.754	1.597	1.636	1.439	1.530
VMRSR	81.165	80.643	5.940	5.289	4.789	4.137	2.192	1.660	1.960	1.584
AMO	58.101	71.296	4.656	4.586	3.658	3.671	1.996	1.733	1.750	1.601
AMR	78.493	74.787	5.534	5.222	4.306	3.761	2.071	1.971	1.842	1.835
AMSR	68.811	74.331	3.983	4.278	3.119	3.342	1.736	1.504	1.528	1.405
AMRSR	78.005	74.811	4.786	4.485	3.667	3.320	1.661	1.773	1.535	1.637
HMO	57.198	33.682	6.594	4.036	4.731	3.217	2.463	1.946	2.226	1.666
HMR	81.751	81.162	6.439	5.582	5.263	4.403	2.644	1.833	2.335	1.712
HMSR	43.538	57.236	4.637	3.498	3.687	2.736	2.199	1.669	1.939	1.445
HMRSR	80.853	80.051	5.639	4.879	4.515	3.768	1.978	1.534	1.790	1.492
GMO	42.525	58.346	5.592	3.572	4.256	2.806	2.355	1.537	2.079	1.331
GMR	81.271	79.981	6.073	4.937	4.957	3.766	2.405	1.683	2.099	1.588
GMSR	60.770	69.328	4.072	3.645	3.258	2.818	2.023	1.475	1.743	1.321
GMRSR	80.170	78.605	5.239	4.455	4.146	3.378	1.767	1.534	1.616	1.456
MO	45.003	60.574	5.992	3.791	4.438	2.983	2.397	1.702	2.122	1.437
MR	77.079	77.064	4.594	4.616	3.422	3.432	1.983	1.987	1.858	1.862
MSR	74.023	73.944	4.031	4.027	3.163	3.162	1.347	1.347	1.310	1.310
MRSR	76.542	76.518	4.168	4.172	3.130	3.132	1.741	1.744	1.611	1.613

Table 4, continued.

	p = 7, σ = 0.5, ρ = 0.99									
	n = 10		n = 20		n = 30		n = 40		n = 50	
Methods	$\hat{K}_D$	$\hat{K}_{AYA}$	$\hat{K}_D$	$\hat{K}_{AYA}$	$\hat{K}_D$	$\hat{K}_{AYA}$	$\hat{K}_D$	$\hat{K}_{AYA}$	$\hat{K}_D$	$\hat{K}_{AYA}$
FMO	447.805	273.075	37.946	28.276	27.117	21.641	13.239	11.463	12.476	10.244
FMR	485.223	485.064	34.950	34.397	29.708	29.129	14.293	13.683	13.301	12.705
FMSR	258.034	346.812	22.494	18.038	18.429	14.253	10.843	7.742	9.695	6.900
FMRSR	484.613	484.223	34.080	33.385	28.880	28.145	13.521	12.635	12.567	11.742
VMO	419.503	465.165	23.743	28.568	20.208	24.458	9.430	10.457	9.084	10.228
VMR	485.223	485.064	34.950	34.397	29.708	29.129	14.293	13.683	13.301	12.705
VMSR	459.749	473.060	24.503	28.718	20.411	24.204	8.218	9.863	8.292	9.733
VMRSR	484.613	484.223	34.080	33.385	28.880	28.145	13.521	12.635	12.567	11.742
AMO	368.598	442.992	24.438	24.674	20.076	21.319	10.316	8.959	9.659	8.950
AMR	482.205	476.562	32.652	30.640	27.796	24.643	12.890	10.899	11.882	10.120
AMSR	447.205	465.894	22.355	26.311	18.496	22.178	8.094	8.673	7.963	8.840
AMRSR	481.236	477.172	31.482	28.781	26.510	23.579	11.709	9.718	10.819	9.212
HMO	327.951	196.248	34.716	20.103	25.809	16.181	12.964	9.262	12.106	7.938
HMR	485.147	484.810	34.688	33.978	29.441	28.742	14.020	13.272	13.029	12.282
HMSR	331.301	401.383	19.395	19.438	15.862	15.458	9.612	6.985	8.450	6.648
HMRSR	484.361	483.812	33.801	32.646	28.625	27.393	13.264	11.887	12.313	11.078
GMO	258.937	377.570	28.176	19.582	22.283	16.053	12.173	7.586	11.000	7.152
GMR	484.816	484.036	34.340	32.952	29.120	27.714	13.791	12.336	12.782	11.289
GMSR	418.634	449.401	19.796	22.825	15.970	18.870	8.533	7.266	7.694	7.517
GMRSR	483.824	482.511	33.300	31.360	28.138	26.069	12.861	10.917	11.877	10.176
MO	286.980	388.382	29.404	20.143	23.176	16.568	12.488	8.068	11.506	7.514
MR	481.859	481.850	29.872	29.865	23.813	23.805	9.406	9.406	8.879	8.879
MSR	464.829	464.318	26.449	26.407	22.389	22.372	8.670	8.663	8.806	8.802
MRSR	480.163	480.143	29.045	29.028	23.410	23.399	9.042	9.038	8.746	8.743

Table 4, continued.

	p = 7, σ = 1, ρ = 0.95									
	n = 10		n = 20		n = 30		n = 40		n = 50	
Methods	$\hat{K}_D$	$\hat{K}_{AYA}$	$\hat{K}_D$	$\hat{K}_{AYA}$	$\hat{K}_D$	$\hat{K}_{AYA}$	$\hat{K}_D$	$\hat{K}_{AYA}$	$\hat{K}_D$	$\hat{K}_{AYA}$
FMO	81.355	51.963	7.225	5.692	5.002	4.184	2.546	2.311	2.298	2.048
FMR	101.096	100.762	7.808	7.122	5.690	5.012	2.866	2.192	2.559	1.972
FMSR	41.344	54.271	5.599	4.132	4.213	3.129	2.364	1.935	2.103	1.678
FMRSR	100.314	99.768	6.965	6.277	4.927	4.254	2.200	1.670	1.973	1.598
VMO	82.129	94.427	5.134	6.301	3.688	4.469	1.879	2.077	1.656	1.959
VMR	101.096	100.762	7.808	7.122	5.690	5.012	2.866	2.192	2.559	1.972
VMSR	90.071	95.087	4.777	5.626	3.295	3.870	1.631	1.651	1.456	1.559
VMRSR	100.314	99.768	6.965	6.277	4.927	4.254	2.200	1.670	1.973	1.598
AMO	70.252	88.388	5.085	5.329	3.746	3.782	2.048	1.756	1.766	1.639
AMR	97.638	93.084	6.270	5.636	4.394	3.901	2.088	2.008	1.877	1.876
AMSR	85.460	92.346	4.485	5.009	3.208	3.439	1.775	1.528	1.538	1.428
AMRSR	96.866	93.247	5.506	4.960	3.742	3.418	1.678	1.805	1.570	1.676
HMO	62.106	37.820	6.801	4.291	4.849	3.298	2.515	1.980	2.260	1.688
HMR	100.913	100.326	7.485	6.594	5.411	4.528	2.655	1.841	2.350	1.727
HMSR	51.275	69.899	4.858	3.835	3.779	2.802	2.244	1.699	1.968	1.464
HMRSR	99.984	99.135	6.639	5.797	4.644	3.870	1.985	1.549	1.803	1.508
GMO	49.184	72.188	5.809	4.021	4.356	2.872	2.402	1.560	2.106	1.341
GMR	100.412	99.036	7.093	5.768	5.099	3.863	2.417	1.704	2.110	1.612
GMSR	75.548	86.521	4.371	4.189	3.333	2.884	2.062	1.498	1.765	1.334
GMRSR	99.227	97.505	6.178	5.224	4.262	3.462	1.778	1.557	1.629	1.477
MO	52.630	75.436	6.197	4.179	4.547	3.056	2.447	1.731	2.155	1.459
MR	95.434	95.380	5.109	5.127	3.494	3.504	2.026	2.031	1.887	1.892
MSR	92.275	92.184	4.751	4.746	3.246	3.244	1.363	1.363	1.326	1.326
MRSR	95.045	95.014	4.754	4.754	3.200	3.202	1.777	1.779	1.635	1.638

Table 4, continued.

	$p = 7$, $\sigma = 1$, $\rho = 0.99$									
	$n = 10$		$n = 20$		$n = 30$		$n = 40$		$n = 50$	
Methods	$\hat{K}_D$	$\hat{K}_{AYA}$	$\hat{K}_D$	$\hat{K}_{AYA}$	$\hat{K}_D$	$\hat{K}_{AYA}$	$\hat{K}_D$	$\hat{K}_{AYA}$	$\hat{K}_D$	$\hat{K}_{AYA}$
FMO	479.139	301.137	39.063	29.750	27.804	22.235	13.521	11.688	12.671	10.387
FMR	601.450	601.301	40.901	40.362	30.487	29.903	14.363	13.750	13.399	12.801
FMSR	303.518	422.962	23.865	20.055	18.910	14.607	11.059	7.873	9.835	6.988
FMRSR	600.849	600.446	40.020	39.319	29.651	28.898	13.588	12.689	12.663	11.829
VMO	521.305	578.507	27.350	33.856	20.911	25.040	9.655	10.539	9.117	10.329
VMR	601.450	601.301	40.901	40.362	30.487	29.903	14.363	13.750	13.399	12.801
VMSR	572.441	587.779	29.099	34.101	20.959	24.853	8.356	9.942	8.340	9.813
VMRSR	600.849	600.446	40.020	39.319	29.651	28.898	13.588	12.689	12.663	11.829
AMO	456.594	552.428	26.595	29.195	20.724	21.906	10.561	9.080	9.685	9.035
AMR	599.015	594.110	39.025	36.016	28.368	25.098	12.893	10.930	11.870	10.240
AMSR	557.880	579.699	26.220	31.327	19.004	22.800	8.244	8.765	8.007	8.906
AMRSR	597.876	593.860	37.571	34.529	27.070	24.034	11.731	9.725	10.879	9.258
HMO	357.282	220.799	35.927	21.611	26.467	16.608	13.235	9.418	12.293	8.041
HMR	601.379	601.052	40.645	39.929	30.218	29.511	14.089	13.332	13.127	12.373
HMSR	402.765	496.974	21.078	22.475	16.253	15.805	9.793	7.083	8.569	6.727
HMRSR	600.587	600.009	39.730	38.547	29.390	28.126	13.327	11.932	12.407	11.160
GMO	307.081	470.214	29.384	22.461	22.805	16.390	12.417	7.694	11.182	7.217
GMR	601.046	600.232	40.284	38.886	29.891	28.429	13.860	12.374	12.885	11.378
GMSR	523.625	560.779	22.637	27.212	16.298	19.295	8.679	7.345	7.796	7.590
GMRSR	600.000	598.580	39.196	37.154	28.882	26.745	12.917	10.954	11.972	10.257
MO	344.556	486.174	30.481	23.214	23.721	16.948	12.738	8.169	11.673	7.610
MR	597.746	597.734	35.219	35.201	24.426	24.419	9.471	9.472	8.965	8.966
MSR	578.893	578.320	31.713	31.665	22.965	22.948	8.725	8.718	8.889	8.885
MRSR	595.905	595.883	34.454	34.434	24.003	23.992	9.094	9.090	8.820	8.817

Table 4, continued.

	***p* = 7, *σ* = 5, *ρ* = 0.95**									
	***n* = 10**		***n* = 20**		***n* = 30**		***n* = 40**		***n* = 50**	
Methods	$\hat{K}_D$	$\hat{K}_{AYA}$	$\hat{K}_D$	$\hat{K}_{AYA}$	$\hat{K}_D$	$\hat{K}_{AYA}$	$\hat{K}_D$	$\hat{K}_{AYA}$	$\hat{K}_D$	$\hat{K}_{AYA}$
FMO	245.847	206.324	13.648	12.477	8.925	7.687	4.247	3.636	3.406	2.937
FMR	702.157	701.896	38.766	37.977	10.527	9.586	3.195	2.433	3.243	2.535
FMSR	200.188	334.411	11.878	10.816	7.614	5.710	3.873	3.014	3.086	2.405
FMRSR	701.135	700.018	37.386	35.751	9.385	8.006	2.435	2.084	2.536	2.116
VMO	610.902	681.797	23.315	33.177	6.684	8.515	2.612	2.460	2.307	2.557
VMR	702.157	701.896	38.766	37.977	10.527	9.586	3.195	2.433	3.243	2.535
VMSR	655.825	681.263	24.043	30.872	5.936	7.314	2.321	2.025	2.045	2.081
VMRSR	701.135	700.018	37.386	35.751	9.385	8.006	2.435	2.084	2.536	2.116
AMO	507.349	652.570	17.667	28.221	6.640	7.108	3.127	2.187	2.559	2.192
AMR	671.962	627.505	30.695	20.513	7.973	6.736	2.732	3.339	2.492	2.740
AMSR	627.882	668.286	19.588	27.354	5.634	6.357	2.707	2.008	2.217	1.947
AMRSR	678.095	654.138	28.502	21.283	6.708	5.920	2.343	2.966	2.130	2.427
HMO	219.097	173.903	13.235	10.792	8.651	5.956	4.166	2.932	3.339	2.376
HMR	702.008	701.327	38.330	36.735	10.155	8.469	2.951	2.289	3.006	2.263
HMSR	307.207	481.565	11.101	12.850	6.747	4.953	3.636	2.568	2.877	2.085
HMRSR	700.344	698.117	36.401	33.452	8.766	7.010	2.225	2.161	2.320	2.041
GMO	272.739	544.532	11.967	18.062	7.653	5.074	3.925	2.192	3.104	1.884
GMR	700.807	695.054	37.026	30.008	9.481	6.656	2.678	2.595	2.697	2.241
GMSR	555.757	634.786	13.407	21.094	5.773	5.074	3.309	2.187	2.580	1.882
GMRSR	696.388	688.388	33.710	27.725	7.809	5.970	2.106	2.400	2.122	2.072
MO	314.950	575.882	12.144	18.589	7.980	5.216	4.041	2.530	3.195	2.061
MR	672.820	672.433	17.880	17.781	6.061	6.084	3.521	3.530	2.814	2.821
MSR	668.530	668.190	27.900	27.872	6.048	6.045	1.790	1.791	1.802	1.802
MRSR	674.310	674.097	20.956	20.872	5.432	5.436	2.985	2.991	2.395	2.399

Table 4, continued.

	$p = 7, \sigma = 5, \rho = 0.99$									
	$n = 10$		$n = 20$		$n = 30$		$n = 40$		$n = 50$	
Methods	$\hat{K}_D$	$\hat{K}_{AYA}$	$\hat{K}_D$	$\hat{K}_{AYA}$	$\hat{K}_D$	$\hat{K}_{AYA}$	$\hat{K}_D$	$\hat{K}_{AYA}$	$\hat{K}_D$	$\hat{K}_{AYA}$
FMO	1465.681	1221.250	74.342	67.324	49.754	41.369	22.547	18.404	18.808	15.170
FMR	4242.117	4242.026	218.709	218.292	55.081	54.422	16.355	15.626	17.341	16.648
FMSR	1810.703	2897.433	59.291	71.107	34.486	25.768	17.728	11.619	14.416	9.990
FMRSR	4241.490	4240.837	217.652	216.573	54.035	52.742	15.463	14.173	16.487	15.313
VMO	3993.568	4193.263	140.741	198.687	36.040	46.002	13.257	12.360	12.430	13.682
VMR	4242.117	4242.026	218.709	218.292	55.081	54.422	16.355	15.626	17.341	16.648
VMSR	4149.675	4204.280	173.098	199.007	36.755	45.257	10.669	11.592	11.141	12.899
VMRSR	4241.490	4240.837	217.652	216.573	54.035	52.742	15.463	14.173	16.487	15.313
AMO	3570.762	4095.430	100.596	173.300	35.184	39.306	15.756	11.148	13.609	12.012
AMR	4212.933	4143.105	210.922	190.956	50.930	43.785	15.140	14.313	15.691	13.685
AMSR	4079.126	4172.108	150.398	186.934	32.406	40.952	11.345	10.514	10.893	11.728
AMRSR	4225.677	4199.721	208.595	194.082	48.556	41.904	13.481	12.148	14.228	12.291
HMO	1297.400	1030.908	71.579	57.889	47.574	30.452	21.850	13.783	18.180	11.491
HMR	4242.073	4241.811	218.503	217.683	54.787	53.806	16.070	14.931	17.057	16.032
HMSR	2733.910	3548.233	64.883	106.353	28.803	26.953	15.284	9.776	12.436	9.368
HMRSR	4241.023	4239.830	217.053	214.838	53.558	51.219	15.076	13.201	16.126	14.403
GMO	1907.678	3525.788	64.386	116.466	40.055	28.037	19.950	10.384	16.380	9.948
GMR	4241.579	4239.498	217.808	214.219	54.328	51.297	15.782	13.679	16.767	14.639
GMSR	3848.564	4067.859	112.552	163.774	27.263	33.591	13.008	9.372	11.073	10.181
GMRSR	4238.586	4232.948	214.991	208.642	52.448	48.033	14.490	12.179	15.517	13.231
MO	2077.145	3612.882	66.849	123.869	40.641	28.986	20.697	11.191	17.048	10.500
MR	4230.358	4230.299	197.452	197.302	41.674	41.646	11.876	11.896	11.794	11.799
MSR	4154.180	4152.100	190.071	189.885	41.694	41.662	10.327	10.321	11.743	11.739
MRSR	4222.197	4222.106	196.988	196.881	41.815	41.785	10.818	10.821	11.499	11.496

Conclusion

In this study, ridge parameters proposed by Dorugade (2014) and Adnan et al. (2014) are classified into different forms and various types following the idea of Lukman and Ayinde (2015), and some new ridge parameters are proposed. The performances of these estimators are evaluated through Monte Carlo Simulation, where levels of multicollinearity, sample sizes, number of regressors and error variances have been varied. The performance evaluation was done using the mean square error. The proposed estimators generally have the least minimum square error when compared to others.

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