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Some remarks and correction on a new distribution, Gamma Lindley, of which the Lindley distribution is a particular case, are given pertaining to its parameter space.

Keywords: Lindley distribution, Gamma Lindley distribution

Introduction

A new distribution was proposed by Nedjar and Zeghdoudi (2016a, 2016b) called the Gamma Lindley distribution (GaL). It is a mixture of a gamma ($2, \theta$) and one-parameter Lindley distribution. There are numerous examples of using a mixture of two known distributions is to generate a new distribution. For example, Shanker, Sharma, and Shanker (2013) used a mixture of exponential (θ) and gamma ($2, \theta$) to create a two-parameter Lindley distribution. Zakerzadeh and Dolati (2010) used a gamma (α, θ) and gamma ($\alpha + 1, \theta$) to create a generalized Lindley distribution. Zeghdoudi and Nedjar (2016, 2017) introduced another new distributions, called the pseudo-Lindley distribution, which is based on mixtures of gamma ($2, \theta$) and exponential (θ) distributions. Here, a mixture of a gamma ($2, \theta$) and a one-parameter Lindley distribution is used to generate a Gamma Lindley distribution, which is useful in modeling lifetime data and survival analysis and actuarial science.

Nedjar and Zeghdoudi (2016a, 2016b) developed various properties of the distribution, such as the probability density function (pdf), cumulative distribution function (cdf), survival and hazard rate function, moment generating function (mgf), mean, variance, and quantile functions, Lorenz curve, and some results on stochastic orderings. Plots of the pdf and cdf for some parameter values were also given, along with maximum likelihood estimates and moment estimates. However, there the parameter space was incorrect. The density function of the random variable X was given by:

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$$f_{\mathrm{GaL}}(x;\theta,\beta)=\theta^2\left(\left(\beta+\beta\theta-\theta\right)x+1\right)\frac{e^{-\theta x}}{\beta(1+\theta)}$$

Unfortunately, $f_{\mathrm{GaL}}(x;\theta,\beta)$ is not a proper pdf, because each of them can be negative for some values of the parameters $\theta > 0$, $\beta > 0$. For example, see Figures 1 and 2.

To obtain a proper pdf for $f_{\mathrm{GaL}}(x;\theta,\beta)$, modify the parameter space to be $\theta > 0$, $\beta > (\theta / (1+\theta))$. Now it can be shown that the proper $f_{\mathrm{GaL}}(x;\theta,\beta)$, where $\theta > 0$ and $\beta > (\theta / (1+\theta))$ are, in fact, general cases of a two-parameter Lindley distribution with pdf

$$f_{\mathrm{TPL}}(x;\theta,\alpha)=\theta^2(\alpha x+1)\frac{e^{-\theta x}}{\alpha+\theta},\quad x,\theta,\alpha>0$$

Taking $\alpha = \beta + \beta\theta - \theta$ in the pdf of the Gamma Lindley distribution leads to the pdf of the two-parameter Lindley distribution.

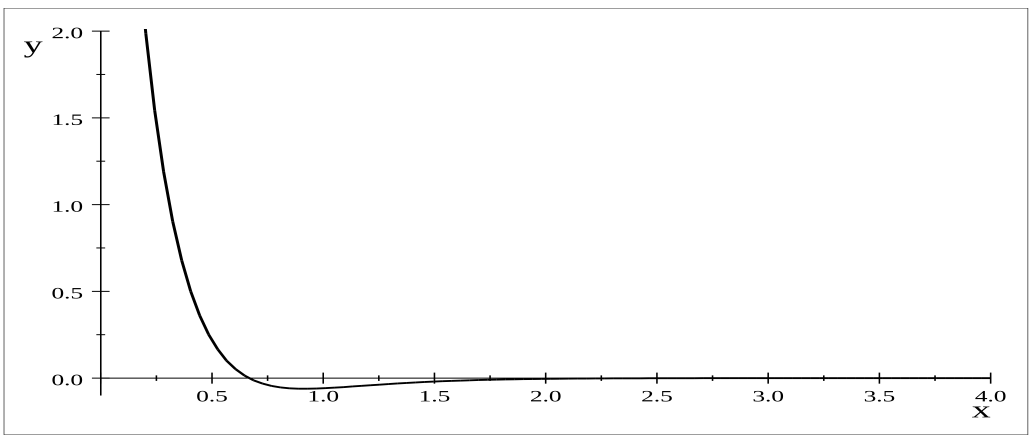

Figure 1. $(\theta, \beta) = (4, 0.5)$ with $\beta = 0.5 < (\theta / (1+\theta)) = (4/5)$

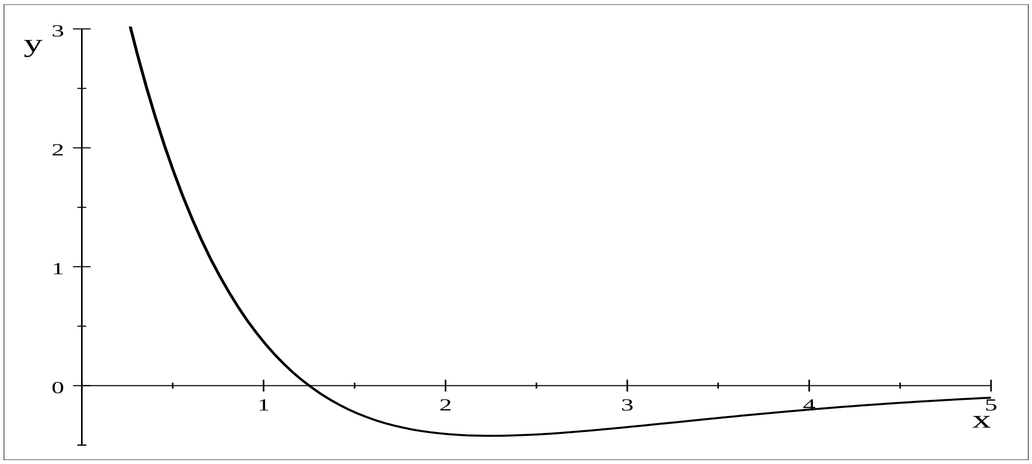

Figure 2. (θ, β) = (1, 0.1) with $\beta = 0.1 < (\theta / (1 + \theta)) = (1 / 2)$

Conclusion

This new distribution might attract wider sets of applications in actuarial science, finance, medicine, and engineering. The reliability behavior of the Gamma Lindley distribution allows an improved performance for lifetime data modeling, and the hazard rate function can have various shapes, so this approach is more realistic and provides a greater degree of flexibility. Also, a new version of a compound Poisson distribution named the Poisson gamma Lindley (PGaL) distribution may be obtained by compounding the Poisson and Gamma Lindley distributions, which is applicable to the collective risk model by considering the proposed distribution as primary distribution and exponential and Erlang as secondary distributions.

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