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Let $\{\mathbf{X}_i;\ 1 \le i \le n\}$ be independent and identically distributed random vectors (i.i.d.r.v.) with a p-variate normal distribution with mean vector $\boldsymbol{\theta}$ and dispersion matrix $\boldsymbol{\Sigma}$, where $\boldsymbol{\Sigma}$ is assumed to be positive definite (p.d.). Partition $\boldsymbol{\theta}$ and $\boldsymbol{\Sigma}$ as

$$\boldsymbol{\theta} = \begin{pmatrix} \boldsymbol{\theta}_1 \\ \boldsymbol{\theta}_2 \end{pmatrix} \text{ and } \boldsymbol{\Sigma} = \begin{bmatrix} \boldsymbol{\Sigma}_{11} & \boldsymbol{\Sigma}_{12} \\ \boldsymbol{\Sigma}_{21} & \boldsymbol{\Sigma}_{22} \end{bmatrix},$$

where $\boldsymbol{\theta}_1{:}p_1\times 1$, $\boldsymbol{\theta}_2{:}p_2\times 1$, $\boldsymbol{\Sigma}_{11}{:}p_1\times p_1$, $\boldsymbol{\Sigma}_{22}{:}p_2\times p_2$, $p_1+p_2 = p$, $0 < p_2 < p$. The problem of interest is to test

$$\begin{gathered} H_0 : \boldsymbol{\theta}_1 = \mathbf{0},\ \boldsymbol{\theta}_2 = \mathbf{0},\ \boldsymbol{\Sigma} \text{ unspecified} \\ \text{versus} \\ H_1 : \boldsymbol{\theta}_1 \ne \mathbf{0},\ \boldsymbol{\theta}_2 = \mathbf{0},\ \boldsymbol{\Sigma} \text{ unspecified.} \end{gathered} \tag{1}$$

For every n (≥ 2), let

$$\bar{\mathbf{X}} = n^{-1}\sum_{i=1}^{n}\mathbf{X}_i \text{ and } \mathbf{S} = \sum_{i=1}^{n}\left(\mathbf{X}_i - \bar{\mathbf{X}}\right)\left(\mathbf{X}_i - \bar{\mathbf{X}}\right)',$$

and express Hotelling's T^2-statistic as

$$T^2 = n(n-1)\bar{\mathbf{X}}'\mathbf{S}^{-1}\bar{\mathbf{X}}.$$

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Partition $\bar{\mathbf{X}}$ and $\mathbf{S}$ the same as $\boldsymbol{\theta}$ and $\boldsymbol{\Sigma}$, respectively, and define

$$\bar{\mathbf{X}}_{1:2} = \bar{\mathbf{X}}_1 - \mathbf{S}_{12}\mathbf{S}_{22}^{-1}\bar{\mathbf{X}}_2,$$
$$\mathbf{S}_{11:2} = \mathbf{S}_{11} - \mathbf{S}_{12}\mathbf{S}_{22}^{-1}\mathbf{S}_{21}.$$

For the problem (1), Rao (1946, 1949) proposed two test statistics which are of the forms

$$W = \frac{n(n-1)\bar{\mathbf{X}}_{1:2}'\mathbf{S}_{11:2}^{-1}\bar{\mathbf{X}}_{1:2}}{1+n(n-1)\bar{\mathbf{X}}_2'\mathbf{S}_{22}^{-1}\bar{\mathbf{X}}_2}$$

and

$$U = n(n-1)\bar{\mathbf{X}}_{1:2}'\mathbf{S}_{11:2}^{-1}\bar{\mathbf{X}}_{1:2}$$

respectively.

Tsai (2003) generalized the Stein (1956) approach to show that Rao's U-test is admissible for the problem (1), which seems contradictory to Marden and Perlman (1980) who proved that Rao's U-test is inadmissible for the problem

$$\begin{gathered} H_0 : \boldsymbol{\theta}_1 = \mathbf{0}, \boldsymbol{\Sigma} \text{ unspecified} \\ \text{versus} \\ H_1 : \boldsymbol{\theta}_1 \neq \mathbf{0}, \boldsymbol{\Sigma} \text{ unspecified.} \end{gathered} \tag{2}$$

There is no contradiction between the two results, because the parameter spaces for the two problems are different. Rao's parameter space is $\Theta_R = \{(\boldsymbol{\theta}_1;\mathbf{0};\boldsymbol{\Sigma})\}$ and MP's is $\Theta_{MP} = \{(\boldsymbol{\theta}_1;\boldsymbol{\theta}_2;\boldsymbol{\Sigma})\}$. Rao's parameter space is smaller than MP's. It is possible that a test is admissible for a smaller parameter space while inadmissible for a larger parameter space. There may exist a test ϕ, such that $\beta_\phi(\boldsymbol{\theta}) \leq \beta_U(\boldsymbol{\theta})$, for all $\boldsymbol{\theta} \in \Theta_R$, $\beta_U(\boldsymbol{\theta}) \leq \beta_\phi(\boldsymbol{\theta})$, for all $\boldsymbol{\theta} \in \Theta_{MP}\backslash\Theta_R$, and $\beta_U(\boldsymbol{\theta}) < \beta_\phi(\boldsymbol{\theta})$, for some $\boldsymbol{\theta} \in \Theta_{MP}\backslash\Theta_R$, where $\beta_\phi(\boldsymbol{\theta})$ denotes the power of the test ϕ at the parameter $\boldsymbol{\theta}$.

Marden and Perlman (1980) proved the admissibility of Hotelling's T^2-test for the problem (2), while Tsai (2003) proved the inadmissibility of Hotelling's T^2-test for the problem (1). The inadmissibility result of Tsai (2003) does not contradict to the admissibility result of Marden and Perlman (1980). If ψ is a test controlling T^2 for the problem (1), that is, $\beta_{T^2}(\boldsymbol{\theta}) \leq \beta_\psi(\boldsymbol{\theta})$, for all $\boldsymbol{\theta} \in \Theta_R$ and $\beta_{T^2}(\boldsymbol{\theta}) < \beta_\psi(\boldsymbol{\theta})$, for some $\boldsymbol{\theta} \in \Theta_R$, we must have $\beta_{T^2}(\boldsymbol{\theta}) > \beta_\psi(\boldsymbol{\theta})$, for some $\boldsymbol{\theta} \in \Theta_{MP} \backslash \Theta_R$. Hence, it does not control T^2 for the problem (2).

Although the results of Tsai (2003) and Marden and Perlman (1980) are not contradictory, the results of Tsai (2003) were obtained for an accurate model of Rao.

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