Journal of Modern Applied Statistical Methods

Volume 17 | Issue 1 Article 29

12-10-2018

A Proficient Two-Stage Stratified Randomized Response Strategy

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Journal of Modern Applied Statistical Methods
May 2018, Vol. 17, No. 1, eP2520
doi: 10.22237/jmasm/1544453468

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ISSN 1538 − 9472

A Proficient Two-Stage Stratified Randomized Response Strategy

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A stratified randomized response model based on R. Singh, Singh, Mangat, and Tracy (1995) improved two-stage randomized response strategy is proposed. It has an optimal allocation and large gain in precision. Conditions are obtained under which the proposed model is more efficient than R. Singh et al. (1995) and H. P. Singh and Tarray (2015) models. Numerical illustrations are also given in support of the present study.

Keywords: Randomized response technique, stratified random sampling, dichotomous population, Sensitive attribute, estimation of proportion, optimum allocation

Introduction

Warner (1965) introduced a randomized response (RR) model to estimate proportion for sensitive attributes including sexual orientation, criminal activity, child abuse, suicidal tendency in teenagers, all cases of AIDS, abortion, or drug addiction. Greenberg, Abul-Ela, Simmons, and Horvitz (1969) envisaged an unrelated question randomized response model using Warner's sensitive question and unrelated question. This technique has generated much interest in the statistical literature since the publication of Warner's RR model. The RR model has been studied by many authors, such as Chaudhuri and Muherjee (1988), Ryu, Hong, and Lee (1993), Tracy and Mangat (1996), S. Singh (2003), and Kim and Elam (2007).

Hong, Yum, and Lee (1994) suggested a stratified RR technique using a proportional allocation. Kim and Warde (2004) presented a stratified RR technique based on Warner's (1965) model that has an optimal allocation. Kim and Elam (2005) have applied Kim and Warde's (2004) stratified Warner's RR model to the two-stage model of Mangat and Singh (1990). Kim, Tebbs, and An (2006) proposed a Bayesian version of the Mangat (1994) model. Kim and Warde (2005) have

doi: 10.22237/jmasm/1544453468 | Accepted: April 12, 2017; Published: December 10, 2018.
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suggested a mixed randomized response model using simple random sampling which rectifies the privacy problem and extended the proposed model to stratified sampling. Lee, Uhm, and Kim (2011) have extended the work of Land, Singh, and Sedory (2011) in stratified sampling using a Poisson distribution.

Recently, H. P. Singh and Tarray (2015) have applied a stratified RR model to the Tracy and Osahan (1999) two-stage model and derived the probability θ_i of "Yes" answer in stratum i for this procedure as:

$$\theta_i = T_i\pi_{Si} + (1-T_i)\left[P_i\pi_{Si} + \frac{(1-P_i)}{2}\right], \quad \text{for } i = 1, 2, \ldots, k \tag{1}$$

where π_{Si} is the proportion of people with the sensitive trait in a stratum i. The unbiased estimator $\hat{\pi}_{ST}$ of a sensitive proportion estimate of $\pi_S = \sum_{i=1}^{k} w_i\pi_{Si}$ is given by

$$\hat{\pi}_{ST} = \sum_{i=1}^{k} w_i\hat{\pi}_{Si} = \sum_{i=1}^{k} w_i\left[\frac{\hat{\theta}_i - (1-T_i)(1-P_i)/2}{T_i + P_i(1-T_i)}\right] \tag{2}$$

where $w_i = (N_i / N)$ for $i = 1, 2,\ldots, k$, so that $w = \sum_{i=1}^{k} w_i = 1$; N is the number of units in the entire population; N_i is the total number of units in the stratum i; and $\hat{\theta}_i$, the proportion of "Yes" answers obtained from the n_i sampled respondents using simple random sampling with replacement (SRSWR) of stratum i, is a point estimate of θ_i in (1).

The minimal variance of the estimator $\hat{\pi}_{ST}$ under optimal allocation is given by

$$V(\hat{\pi}_{ST}) = \frac{1}{n}\left[\sum_{i=1}^{k} w_i\left\{\pi_{Si}(1-\pi_{Si}) + \frac{(1-T_i)(1-P_i)\{2-(1-T_i)(1-P_i)\}}{4\{T_i + P_i(1-T_i)\}^2}\right\}^{1/2}\right]^2 \tag{3}$$

In this paper, we have suggested a new stratified randomized response model based on the R. Singh et al. (1995) two-stage RR model. It is demonstrated that the estimator resulting from the proposed model is more efficient than those of its competent models under the condition presented in the case of completely truthful reporting.

Suggested Model

The population is divided in to k strata and a sample is selected by SRSWR in each stratum. To get the full benefit from stratification, we assume that the number of units in each stratum is known. In the first stage of the survey interview, an individual respondent in the sample of stratum i is instructed to use the randomization device R_{1i}, which consists of a sensitive question (S) card with probability T_i and a "Go to randomization device R_{2i} in the second stage" direction card with probability $(1 - T_i)$. The respondents in the second stage of the stratum i are instructed to use the randomization device R_{2i} which uses three statements: (i) "I possess the sensitive attribute A;" (ii) "Yes;" and (iii) "No;" with probabilities P_i, $(1 - P_i)\alpha_i$, and $(1 - P_i)(1 - \alpha_i)$, respectively, where $\alpha_i \in [0, 1]$. Let n_i denote the number of units in the sample from stratum i and n denote the total number of units in sample from all stratum so that $n = \sum_{i=1}^{k} n_i$. Thus, when respondents report truthfully, the probability of a "Yes" answer in stratum i for this procedure is given by

$$\theta_{\alpha i} = T_i \pi_{Si} + (1 - T_i)\left[P_i \pi_{Si} + \alpha_i (1 - P_i)\right], \quad \text{for } i = 1, 2, \ldots, k \tag{4}$$

where $\theta_{\alpha i}$ is the proportion of "Yes" responses and π_{Si} is the proportion of respondents with the sensitive trait in the population from stratum i.

If $\hat{\theta}_{\alpha i}$ denotes the estimate of the proportion of "Yes" answers in a stratum i, the relation (4) yields

$$\hat{\pi}_{\alpha Si} = \frac{\hat{\theta}_{\alpha i} - (1 - T_i)(1 - P_i)\alpha_i}{T_i + P_i(1 - T_i)}, \quad \text{for } i = 1, 2, \ldots, k \tag{5}$$

as an unbiased estimator of the proportion π_{Si} of respondents with the sensitive trait in the population from stratum i.

The variance of the estimator $\hat{\pi}_{ST}$ is given by

$$\mathrm{V}\left(\hat{\pi}_{\alpha \mathrm{S}i}\right)=\left[\frac{\pi_{\mathrm{S}i}\left(1-\pi_{\mathrm{S}i}\right)}{n_i}+\frac{\left(1-T_i\right)\left(1-P_i\right)\left[\pi_{\mathrm{S}i}\left\{T_i+P_i\left(1-T_i\right)\right\}\left(1-2\alpha_i\right)+\alpha_i\left\{1-\left(1-T_i\right)\left(1-P_i\right)\alpha_i\right\}\right]}{n_i\left[T_i+\left(1-T_i\right)P_i\right]^2}\right] \quad (6)$$

Because the selection in different strata are made independently, the estimator for individual strata can be added together to obtain an estimator for the whole population. The maximum likelihood estimator of π_{S}, the proportion of respondents with the sensitive trait, is

$$\hat{\pi}_{\mathrm{S}}=\sum_{i=1}^{k} w_i \hat{\pi}_{\alpha \mathrm{S}i}=\sum_{i=1}^{k} w_i\left[\frac{\hat{\theta}_{\alpha i}-\left(1-T_i\right)\left(1-P_i\right)\alpha_i}{T_i+P_i\left(1-T_i\right)}\right] \quad (7)$$

where N denotes the number of units in the entire population, N_i is the total number of units in the stratum i, and $w_i = (N_i / N)$ for $i = 1, 2,\ldots, k$, so $w=\sum_{i=1}^{k} w_i=1$. The proposed estimator $\hat{\pi}_{\mathrm{S}}$ is an unbiased estimator for the population proportion π_{S}.

Theorem 1. The variance of the estimator $\hat{\pi}_{\mathrm{S}}$ is

$$\mathrm{V}\left(\hat{\pi}_{\mathrm{S}}\right)=\sum_{i=1}^{k}\frac{w_i^2}{n_i}\left[\frac{\pi_{\mathrm{S}i}\left(1-\pi_{\mathrm{S}i}\right)}{n_i}+\frac{\left(1-T_i\right)\left(1-P_i\right)\left[\pi_{\mathrm{S}i}\left\{T_i+P_i\left(1-T_i\right)\right\}\left(1-2\alpha_i\right)+\alpha_i\left\{1-\left(1-T_i\right)\left(1-P_i\right)\alpha_i\right\}\right]}{n_i\left[T_i+\left(1-T_i\right)P_i\right]^2}\right] \quad (8)$$

Proof. This follows from taking the variance of (7) and corollary 1 in section 5.5 of Cochran (1977).

In practice, information on $\pi_{\mathrm{S}i}$ is usually not known. But if prior information on $\pi_{\mathrm{S}i}$ is available from past experience then it helps to establish the following optimal allocation formula:

Theorem 2. The optimal allocation of n to n_1, $n_2,\ldots, n_{k-1}$, and n_k to derive the minimum variance of $\hat{\pi}_{\mathrm{S}}$ subject to $n=\sum_{i=1}^{k} n_i$ is approximately given by

$$\frac{n_i}{n} = \frac{w_i\left[\pi_{Si}\left(1-\pi_{Si}\right) + \frac{(1-T_i)(1-P_i)\left[\pi_{Si}\left\{T_i+P_i(1-T_i)\right\}(1-2\alpha_i)+\alpha_i\left\{1-(1-T_i)(1-P_i)\alpha_i\right\}\right]}{n_i\left[T_i+(1-T_i)P_i\right]^2}\right]^{\frac{1}{2}}}{\sum_{i=1}^{k} w_i\left[\pi_{Si}\left(1-\pi_{Si}\right) + \frac{(1-T_i)(1-P_i)\left[\pi_{Si}\left\{T_i+P_i(1-T_i)\right\}(1-2\alpha_i)+\alpha_i\left\{1-(1-T_i)(1-P_i)\alpha_i\right\}\right]}{n_i\left[T_i+(1-T_i)P_i\right]^2}\right]^{\frac{1}{2}}} \quad (9)$$

Proof. Follows from section 5.5 of Cochran (1977).

The minimal variance of the estimator $\hat{\pi}_S$ is given by

$$V\left(\hat{\pi}_S\right) = \frac{1}{n}\left[\sum_{i=1}^{k} w_i\left\{\frac{\pi_{Si}\left(1-\pi_{Si}\right)}{n_i} + \frac{\pi_{Si}\left\{T_i+P_i(1-T_i)\right\}(1-2\alpha_i)(1-T_i)(1-P_i)}{n_i\left[T_i+(1-T_i)P_i\right]^2} + \frac{\alpha_i\left\{1-(1-T_i)(1-P_i)\alpha_i\right\}(1-T_i)(1-P_i)}{n_i\left[T_i+(1-T_i)P_i\right]^2}\right\}^{\frac{1}{2}}\right]^2 \quad (10)$$

By substituting $(n_i - 1)$ for n_i in (8), the unbiased minimal variance of the estimator $\hat{\pi}_S$ can be obtained.

Remark 1. The proposed procedure is equivalent to the H. P. Singh and Tarray (2015) procedure for $\alpha = 1 / 2$.

Remark 2. For any given pair (P_i, T_i), the variance $V(\hat{\pi}_{\alpha Si})$ at (π_{Si}, α_i) and $[(1 - \pi_{Si}), (1 - \alpha_i)]$ has equal value. This means that $V(\hat{\pi}_{\alpha Si})$ is symmetric about the point (1/2, 1/2) along any line in the (π_{Si}, α_i) plane passing through this point; see Corollary 2.3 in R. Singh et al. (1995, p. 268).

Relative Efficiency

An efficiency comparison of the proposed stratified randomized response technique and the two-stage randomized response techniques that were presented by R. Singh et al. (1995) will be conducted by comparing variances.

Theorem 3. Suppose there are two strata in the population, $n = n_1 + n_2$, $P = P_1 = P_2$ (P is the probability of selecting the sensitive question in the second stage), $T = T_1 = T_2$ (T is the probability of selecting the sensitive question in the first stage), and $\hat{\pi}_{\mathrm{S}} = w_1\hat{\pi}_{\alpha\mathrm{S1}} + w_2\hat{\pi}_{\alpha\mathrm{S2}}$. The proposed estimator $\hat{\pi}_{\mathrm{S}}$ is more efficient than the R. Singh et al. (1995) estimator $\hat{\pi}_{\mathrm{SSMT}}$ (say), where $\pi_{\mathrm{S1}} \neq \pi_{\mathrm{S2}}$, under the following conditions:

$$\left[\left(\pi_{\mathrm{S1}} - \pi_{\mathrm{S2}}\right)^2 + \left(\sqrt{A_1} - \sqrt{A_2}\right)^2\right] > \left\{1 + \left(w_1 w_2\right)^{-1}\right\} B \tag{11}$$

where

$$A_1 = \left[\pi_{\mathrm{S1}}\left(1-\pi_{\mathrm{S1}}\right) + A\pi_{\mathrm{S1}} + B\right], A_2 = \left[\pi_{\mathrm{S2}}\left(1-\pi_{\mathrm{S2}}\right) + A\pi_{\mathrm{S2}} + B\right]$$
$$A = \frac{(1-T)(1-P)\{T+P(1-T)\}(1-2\alpha)}{\{T+P(1-T)\}^2}$$
$$B = \frac{(1-T)(1-P)\alpha\left[1-(1-T)(1-P)\alpha\right]}{\{T+P(1-T)\}^2}$$

The following theorem exhibits that the proposed estimator $\hat{\pi}_{\mathrm{S}}$ is more efficient than the H. P. Singh and Tarray (2015) estimator $\hat{\pi}_{\mathrm{ST}}$:

Theorem 4. Assume that there are two strata in the population, $n = n_1 + n_2$, $P = P_1 = P_2$, and $T = T_1 = T_2$. The proposed estimator $\hat{\pi}_{\mathrm{S}}$ will be always more efficient than the H. P. Singh and Tarray (2015) estimator $\hat{\pi}_{\mathrm{ST}}$ (where $\pi_{\mathrm{S1}} \neq \pi_{\mathrm{S2}}$) under the following condition:

$$\text{Either} \quad \alpha < \left[\frac{1}{(1-T)(1-P)} - \frac{1}{2} \right] \quad \text{or} \quad \alpha > \left[\frac{1}{(1-T)(1-P)} - \frac{1}{2} \right] \tag{12}$$

Empirical Study

To see the tangible idea about the performance of the proposed estimator $\hat{\pi}_{\mathrm{S}}$ over the H. P. Singh and Tarray (2015) estimator $\hat{\pi}_{\mathrm{ST}}$, compute the percent relative efficiency (PRE) of $\hat{\pi}_{\mathrm{S}}$ with respect to $\hat{\pi}_{\mathrm{ST}}$ for two strata, i.e. $k = 2$, $P = P_1 = P_2$, and $T = T_1 = T_2$, by using the formula

$$\begin{aligned} \mathrm{PRE}\left(\hat{\pi}_{\mathrm{S}}, \hat{\pi}_{\mathrm{ST}}\right) &= \frac{\mathrm{V}\left(\hat{\pi}_{\mathrm{ST}}\right)}{\mathrm{V}\left(\hat{\pi}_{\mathrm{S}}\right)} \times 100 \\ &= \frac{\left[w_1 \sqrt{(G_1 + H)} + w_2 \sqrt{(G_2 + H)} \right]^2}{\left[w_1 \sqrt{(G_1 + H_1)} + w_2 \sqrt{(G_2 + H_2)} \right]^2} \times 100 \end{aligned} \tag{13}$$

where

$$G_1 = \pi_{\mathrm{S1}}\left(1-\pi_{\mathrm{S1}}\right), G_2 = \pi_{\mathrm{S2}}\left(1-\pi_{\mathrm{S2}}\right)$$

$$H = \frac{(1-T_i)(1-P_i)\left\{2-(1-T)(1-P)\right\}}{4\left\{T + P(1-T)\right\}^2}$$

$$H_1 = \frac{(1-T)(1-P)\left[\pi_{\mathrm{S1}}\left\{T + P(1-T)\right\}(1-2\alpha) + \alpha\left\{1-(1-T)(1-P)\alpha\right\}\right]}{\left[T + (1-T)P\right]^2}$$

$$H_2 = \frac{(1-T)(1-P)\left[\pi_{\mathrm{S2}}\left\{T + P(1-T)\right\}(1-2\alpha) + \alpha\left\{1-(1-T)(1-P)\alpha\right\}\right]}{\left[T + (1-T)P\right]^2}$$

The computed PRE($\hat{\pi}_{\mathrm{S}}$, $\hat{\pi}_{\mathrm{ST}}$) for $n = 1000$, $\alpha = 0\ (0.1)\ 0.4$, $T = 0.1\ (0.4)\ 0.9$, $P = 0.6$, 0.8; and for $\pi_{\mathrm{S1}} = 0.28$, $\pi_{\mathrm{S2}} = 0.33$ and different values of w_1 and w_2 such that $\pi_{\mathrm{S}} \leq 0.5$. Findings are shown in Table 1.

Table 1. The relative efficiency of the proposed estimator $\hat{\pi}_S$ with respect to the H. P. Singh and Tarray (2015) estimator $\hat{\pi}_{ST}$, i.e. RE($\hat{\pi}_S$, $\hat{\pi}_{ST}$) = V($\hat{\pi}_{ST}$ / $\hat{\pi}_S$), when n = 1000, T = 0.1, 0.4, 0.9, π_{S1} = 0.28, π_{S2} = 0.33, and different values of w_1 and w_2

							P				
π_{S1}	π_{S2}	w_1	w_2	π_S	α	T	**0.30**	**0.33**	**0.36**	**0.39**	**0.42**
0.28	0.33	0.9	0.1	0.285	0.0	0.6	145.05	141.75	138.58	135.60	132.80
0.28	0.33	0.9	0.1	0.285	0.0	0.7	128.99	127.10	125.29	123.55	121.88
0.28	0.33	0.9	0.1	0.285	0.0	0.8	118.54	115.59	114.66	113.76	112.89
0.28	0.33	0.8	0.2	0.290	0.1	0.6	140.19	137.56	134.56	131.56	129.53
0.28	0.33	0.8	0.2	0.290	0.1	0.7	125.53	123.89	122.32	120.82	119.37
0.28	0.33	0.8	0.2	0.290	0.1	0.8	114.28	113.47	112.67	111.90	111.50
0.28	0.33	0.7	0.3	0.295	0.4	0.6	129.61	127.61	125.70	123.90	122.18
0.28	0.33	0.7	0.3	0.295	0.4	0.7	117.43	116.34	115.29	114.29	113.32
0.28	0.33	0.7	0.3	0.295	0.4	0.8	108.64	108.14	107.65	107.18	106.72
0.28	0.33	0.6	0.4	0.300	0.0	0.6	144.20	140.91	137.82	134.90	132.15
0.28	0.33	0.6	0.4	0.300	0.0	0.7	128.39	126.54	124.76	123.05	121.41
0.28	0.33	0.6	0.4	0.300	0.0	0.8	116.16	115.23	114.33	113.44	112.59
0.28	0.33	0.5	0.5	0.305	0.1	0.6	139.46	136.61	133.91	131.37	128.96
0.28	0.33	0.5	0.5	0.305	0.1	0.7	125.02	123.41	121.87	120.38	118.96
0.28	0.33	0.5	0.5	0.305	0.1	0.8	113.97	113.17	112.39	111.63	110.90
0.28	0.33	0.4	0.6	0.310	0.4	0.6	129.11	127.13	125.25	123.47	121.78
0.28	0.33	0.4	0.6	0.310	0.4	0.7	117.10	116.03	115.00	114.01	113.05
0.28	0.33	0.4	0.6	0.310	0.4	0.8	108.46	107.96	107.49	107.02	106.13
0.28	0.33	0.3	0.7	0.315	0.0	0.6	143.37	140.13	137.09	134.22	131.51
0.28	0.33	0.3	0.7	0.315	0.0	0.7	127.81	125.99	124.24	122.56	120.95
0.28	0.33	0.3	0.7	0.315	0.0	0.8	115.80	114.89	114.00	113.14	112.30
0.28	0.33	0.2	0.8	0.320	0.1	0.6	138.74	135.93	133.28	130.77	128.40
0.28	0.33	0.2	0.8	0.320	0.1	0.7	124.52	122.94	121.42	119.96	118.56
0.28	0.33	0.2	0.8	0.320	0.1	0.8	113.66	112.87	112.11	111.37	110.65
0.28	0.33	0.1	0.9	0.325	0.4	0.6	128.62	126.66	124.81	123.05	121.38
0.28	0.33	0.1	0.9	0.325	0.4	0.7	116.78	115.72	114.71	113.73	112.79
0.28	0.33	0.1	0.9	0.325	0.4	0.8	108.28	107.80	107.32	106.87	106.43

Seven values of P and three different values of α were set to verify the relative efficiency of the suggested estimator $\hat{\pi}_S$ with respect to the H. P. Singh and Tarray (2015) estimator $\hat{\pi}_{ST}$. Shown in Table 1, the value of percent relative efficiency PRE($\hat{\pi}_S$, $\hat{\pi}_{ST}$) decreases as the values of P and α increase.

The values of PRE($\hat{\pi}_S$, $\hat{\pi}_{ST}$) are greater than 100 for all values of π_{S1}, π_{S2}, w_1, w_2, P, α, and T considered here. The proposed estimator $\hat{\pi}_S$ is more efficient than the H. P. Singh and Tarray (2015) estimator $\hat{\pi}_{ST}$ in the case of the two strata in the population. It is further noted that, depending on the values of π_{S1}, π_{S2}, w_1, and w_2, and keeping the cooperation from the respondents in view, the value of α as near to "Zero" or "Unity" as possible should be taken as an indication of a more efficient strategy.

Acknowledgements

The authors are thankful to the Editor-in-Chief and to the anonymous learned referee for their valuable suggestions regarding improvement of the paper.

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