Journal of Network and Innovative Computing.
ISSN 2160-2174 Volume 7 (2019) pp. 025–031
© MIR Labs, www.mirlabs.net/jnic/index.html

Description shape based on B-spline approximation and contour based shape descriptors:Application to Brachiopods classification

Youssef Ait khouya[1] **and Faouzi Ghorbel**[2]

[1]Qualifying High School, Alhidaya Islamiya, Safi,
Ministry of National Education, Morocco
youssefaitkhouya@gmail.com

[2]GRIFT Research Group, CRISTAL Laboratory,
National School of Computer Sciences University of Manouba, Tunisia
faouzi.ghorbel@ensi.rnu.tn

Abstract: **The fossils classification has a great importance in palaeontological studies. On the one hand, they make it possible to understand the biodiversity in its morphological dimension. On the other hand, they show the morphological transformations suffered during the biological evolution. In this paper, we present a comparative study between some of contour based shape descriptors after applying a global B-spline approximation to the contours. Specifically, we studied Curvature, Fourier descriptors, Wavelet descriptors, Radial function and Fourier of Radial function. Applied B-spline approximation we can reduce the number of points in the contours and gives a smooth curve. We used this technique for fossils classification, especially Brachiopods. The five shape descriptors are evaluated against each other using the City block distance and the recognition rate.**

Keywords: B-spline, Shape descriptors, Curvature,Fourier descriptors, Wavelet descriptors, Radial function

I. Introduction

Shape descriptors are important task in many pattern recognition application [1], [2], [3]. The goal of the shape descriptors is to uniquely depict the object shape in a large image database. A good number of shape descriptors, which are prevalent in the literature are largely categorized into two groups: contour based shape descriptors and region based shape descriptors. Contour-based shape descriptors use only boundary information, they cannot depict shape interior content. Of the contour based descriptors we cite Fourier descriptors [24], [34], [36] which have been largely used. In [16] the authors propose a method to define Fourier descriptors for broken shapes, meaning shapes that can have more than one contour. Curvature approaches [27], [28], [26], [38],[39] have also been used. In [26] a shape is described in a scale space by the maximum of the curvature. In [25] the authors presented a pattern description approach based on the multi-scale analysis of the contour of planar objects. Wavelet descriptors can be used to describe a given object shape by wavelet descriptors (WD) [37], [40].In [4] we presented a descriptor for shape classification it is a fusion between local and global complete and stable descriptors, we use Curvature as local descriptor and Ghorbel descriptors as a global descriptors. In region based techniques, all pixels surrounded by the shape boundary are taken into consideration to yield the shape descriptor. Frequently referred to methods are based on moment theory to describe shape [9], [10]. These include geometric moments, Legendre moments, Zernike moments and pseudo Zernike moments. In this paper we suggest to make a comparison between some of contour based shape descriptors like Curvature, Fourier descriptors, Wavelet descriptors, Radial function and Fourier of Radial function after applying a global B-spline approximation to the contours. The B-spline is largely used to describe a complex curve [19], [20]. We applied this technique for Brachiopods classification. The fossils classification has a great importance in palaeontological field. There are many reasons for fossils classification such as: (i) We can remember the characteristics of a large number of different things only if they are grouped into categories whose members share given characteristics ; (ii) To make better our predictive powers;(iii) Thanks to classification systems we are able to account for the ways apparently different things are related to each other. According for biologists the importance of the classification systems resides especially in reconstructing the evolutionary pathways that have produced the diversity of organisms living today. Several authors tried to classify fossil species [5], [14].

In [7],[8] we present a method for Brachiopod classification based on the contour and we cite [17], the authors used the Outline Shape Analysis technique for describing Ammonite shape. It aims at approaching the shape by a trigonometric function defined by the sum of sine and cosine terms. This latter is decomposed in either a series of harmonic amplitudes and phase angles or a series of Fourier coefficients, serving as variables for quantitative analysis. This technique was applied to other invertebrate groups such as Trilobites [18], bivalves [13].

The region-based active contour and especially the Chan and Vese model implemented by the Level set method [11] is used here for edge detection. The rest of the paper is organized as follows:
In Section 2, we recall the Region-based active Contour. Also, we formulate the Chan and Vese model in terms of level set functions and compute the associated Euler-Lagrange equation. In section 3 we present the principle of B-spline approximation. The parametrization step remind in section 4. In Section 5, the five shape descriptors are described. In Section 6, the shape based descriptors are compared and evaluated, and The paper is concluded in Section 7.

II. Overview of Region-based active Contour

The basic idea in active contour models or snakes is to evolve a curve, subject to constraints from a given image, in order to detect objects in that image. Starting with a curve around the object to be detected, the curve moves toward its interior normal and has to stop on the boundary of the object. Active contours based on region information are part of a very active area of research since the years 90 in Computer Vision. Originally, the work of [44] present a method of competitive regions in a mixed environment that is both Bayesian and minimizing the minimum description length criterion. In parallel, the work of D. Mumford and J. Shah realized in [30] have opened another branch, more geometric in the field of segmentation based on regions. On the other hand, many papers have dealt with region-based approaches using the level set framework, including the deformable regions by Jehan-Besson et al [23], Dind et al [12] and the geodesic active regions by Paragios and Deriche [32], benefiting of adaptive topology at the expense of computational cost.
The region-based active contours model is based on the minimization of a well defined energy to segment the image. Let I an image and Ω his domain. Chan and Vese in [11] proposed to minimize the following energy:

$$\begin{aligned} E(c_1, c_2, \phi) &= \mu \int_\Omega \mid \nabla H(\phi(x,y)) \mid dxdy \\ &+ \nu \int_\Omega H(\phi(x,y)) dxdy \\ &+ \lambda_1 \int_\Omega \mid I(x,y) - c_1 \mid^2 H(\phi(x,y)) dxdy \\ &+ \lambda_2 \int_\Omega \mid I(x,y) - c_2 \mid^2 (1 - H(\phi(x,y))) dxdy \end{aligned} \quad (1)$$

where $\mu \geq 0$, $\nu \geq 0$, $\lambda_1 > 0$, $\lambda_2 > 0$ are fixed parameters and H is the Heaviside function defined as:

$$H(z) = \begin{cases} 1 & \text{if } z \geq 0 \\ 0 & \text{if } z < 0 \end{cases}$$

and ϕ is the level set function associated to the contour evolving.The constants c_1 and c_2 are the averages of I in $\phi > 0$ and $\phi < 0$ respectively. So they are easily computed as:

$$c_1(\phi) = \frac{\int_\Omega I(x,y) H(\phi(x,y)) dxdy}{\int_\Omega H(\phi(x,y)} \quad (2)$$

$$c_2(\phi) = \frac{\int_\Omega I(x,y)(1 - H(\phi(x,y))) dxdy}{\int_\Omega (1 - H(\phi(x,y))} \quad (3)$$

The discretized evolution equation is:

$$\frac{\phi^{n+1}_{(i,j)} - \phi^n_{(i,j)}}{\Delta t} = \delta_\epsilon(\phi^n_{(i,j)})(\mu div(\frac{\nabla \phi^n_{(i,j)}}{\mid \nabla \phi^n_{(i,j)} \mid}) - \nu$$
$$-\lambda_1 (I(i,j) - c_1(\phi^n_{(i,j)}))^2 + \lambda_2 (I(i,j) - c_2(\phi^n_{(i,j)}))^2) \quad (4)$$

where $\delta_\epsilon(z)$ is a regular form of $\delta(z)$.

III. B-spline approximation

B-splines are piecewise polynomial curves that are controlled by a set of points called the control points. This technique is widely used to represent a complex curve [19], [20]. The B-spline curve is a generalization of the Bezier curve. Let $C(u)$ be the position vector along the curve as a function of the parameter u. The B-spline curve, $C(u)$, is defined as:

$$C(u) = \sum_{i=0}^{h} B_{i,p}(u) P_j \quad (5)$$

where P_j is a control point, u is a parameter, and $B_{i,p}$ are the normalized B-spline basis functions of order p defined recursively as follows:

$$\begin{aligned} B_{(i,0)}(u) &= \begin{cases} 1 & \text{if } u_i \leq u < u_{i+1} \\ 0 & \text{otherwise} \end{cases} \\ B_{(i,p)}(u) &= \frac{u - u_i}{u_{i+p} - u_i} B_{(i,p-1)}(u) \\ &+ \frac{u_{i+p+1} - u}{u_{i+p+1} - u_{i+1}} B_{(i+1,p-1)}(u) \end{aligned} \quad (6)$$

where u_i is known as a knot. In the approximation the B-spline curve does not have to pass through all data points except the first and last data points. A number of the B-spline control points would reflect the goodness of the approximation. Using least-square minimization, we compute the control points P_j, $j = 0, ...n$ of a B-spline curve $C(t)$ by minimizing the least-squares error defined as:

$$E(P_1, P_2, ..., P_n) = \sum_{k=1}^{m} |C(\overline{u_i}) - P_k| \quad (7)$$

Where m is the number of data points and $\overline{u_i}$ is a parametric values of the points.

IV. Parametrization

The contours Parametrization step provide all the computation results. For a given contour there are many types of parameter. A Parametrization function will be chosen which are suitable for the intended invariance. Here, we are interested in the parametrization invariant under the similarity transformation. To obtain a parametrization invariant under the similarity transformation we used the arc length parametrization, defined by the Euclidian norm which possesses an intrinsic link with the notion of similarity.
Suppose we have the coordinates of the contour $C(t) = (x(t), y(t))$, where t is a real value called the parameter of

the contour. To reparametrize the contour by the arc length we need to calculate:

$$s(l) = \frac{1}{L}\int_0^l \mid C(t)' \mid dt \quad (8)$$

where L denotes the curve length.
On the other hand, to obtain a parametrization invariant under affine transformation it's necessary to use an affine arc parametrization.

V. Shape descriptors

In this section, we present and explain the principle of the five descriptors compared.

A. Curvature

The curvature of a curve is defined as follows:
Let C be a smooth curve with position vector $\overrightarrow{r(s)}$ where s is the arc length parameter. The curvature k of C is defined to be:

$$k(s) = \| \frac{d\overrightarrow{T}}{ds} \| \quad (9)$$

where $\overrightarrow{T}$ is the unit tangent vector. The notion of curvature measures how sharply a curve bends. The curvature is equal to 0 for a straight line, have a small value when the curves bend is very little and to be high for curves which bend sharply.
Consider a parametric vector equation for a curve:
C(s)=(x(s),y(s))
The formula for computing the curvature function can be expressed as

$$k(s) = \frac{\dot{x}(s)\ddot{y}(s) - \ddot{x}(s)\dot{y}(s)}{(\dot{x}^2(s) + \dot{y}^2(s))^{3/2}} \quad (10)$$

where $\dot{x}$ and $\ddot{x}$ are the first and second derivation of $x(s)$,

B. Radial function

The Radial function is a function defined on a Euclidean space R^n that represents the distance from the digitized boundary of the object to its centroid as a function of distance along the boundary. The centroid point (x_c, y_c) is computed using the following formula:

$$x_c = \frac{1}{N}\sum_{i=1}^{N} x_i \quad (11)$$

$$y_c = \frac{1}{N}\sum_{i=1}^{N} y_i \quad (12)$$

where (x_i, y_i) is the discreet contour and N is the number of the boundary points. The Radial function $\rho(i), i = 1, 2, ...N$, are calculated starting from an fixed position of the boundary using as distance measure on a Euclidean norm from the centroid to the boundary points as:

$$\rho(i) = \sqrt{(x_i - x_c)^2 + (y_i - y_c)^2} \quad (13)$$

Since the radial function is only dependent on the location of the centroid and the points on the boundary, it is invariant to the translation due to the subtraction of the centroid from boundary coordinates, and if the starting point is fixed the function is invariant to the rotation, then the function of the original and rotated shapes will be identical. This function alone is not invariant to a change in starting point or scaling. In order to obtain a descriptor that is invariant to translation, rotation, scaling, and change of starting point we apply the Fourier transformation, and then take the magnitude of these normalized coefficients defined as:

$$V = (\frac{|F\rho(1)|}{|F\rho(0)|}, \frac{|F\rho(2)|}{|F\rho(0)|}, ..., \frac{|F\rho(N)|}{|F\rho(0)|}) \quad (14)$$

where F is the discrete one-dimensional Fourier transform.

C. Fourier descriptors

The Fourier Descriptor (FD) is a powerful tool for shape analysis and has been successfully applied to many shape representation applications. Let $x(t)$ and $y(t)$ be the coordinates of the contour, Since boundary is a closed curve and when the two-dimensional plane is considered as a complex plane, the cartesian coordinates of the contour are represented in the complex plane by $z(t) = x(t) + j.y(t)$. The discrete Fourier coefficients of $z(t)$ is defined as follow:

$$c_k = \sum_{k=0}^{N-1} z(t) exp(-j2\pi kt/N) \quad (15)$$

with $t \in [0, N-1]$, and N is a number of points in the contour.
Applying the inverse Fourier transform, we can recreate a contour from its Fourier descriptors. Initially, the first set of invariants was constructed by taking the modulus of Fourier descriptors.

$$\forall k \in [0, N-1], \;\; I_k = |c_k|, \quad (16)$$

In [43] the authors used this method in their comparative study. The first coefficient is the only one depends on position of the shape. To obtain translation invariance the coefficient c_0 can be discarded. Scale invariance is achieved by dividing all the Fourier coefficients by the absolute value of the second coefficient. To obtain a rotation and shift invariant descriptor, the phase can completely ignored and the only use the absolute value of the Fourier coefficients.
The acquired Fourier descriptors is used to discriminate between simple-shaped objects [35], [41]. But, this set of invariants is not complete in the sense defined by Crimmins in [15], A set of descriptors is said to be complete if the following property is verified:
two objects have the same shape if and only if they have the same set of invariants.
Completness allows to retrieve shapes from their invariants, up to an Euclidean transformation. In fact, the set of descriptors defined in equation (16) is not complete since we can find objects with different shapes but with the same magnitude for their Fourier coefficients.
To resolve the problem of completeness, Crimins proposed in [15] a complete set of invariant, defined as:

$$\begin{aligned} I_{k_0} &= |c_{k_0}|, \;\; for \;\; k_0 \;\; such \;\; that \;\; c_{k_0} \neq 0 \\ I_{k_1} &= |c_{k_1}|, \;\; for \;\; k_1 \neq k_0 \;\; such \;\; that \;\; c_{k_1} \neq 0 \quad (17) \\ I_k &= c_k^{(k_0-k_1)} \; c_{k_0}^{(k_1-k)} \; c_{k_1}^{(k-k_0)}, \;\; \forall k \neq k_0, k_1 \end{aligned}$$

However, this set is not stable, i.e. a slight modification of the invariants can induce a remarkable distortion of shape. To overcome this problem, Ghorbel in [21] presented a complete and stable set of invariant Fourier descriptors, defined as (the reader can consult chapter II page 53-71 in [22] for more details):

$$\begin{aligned} I_{k_0} &= |c_{k_0}|, \ for\ k_0\ such\ that\ c_{k_0} \neq 0 \\ I_{k_1} &= |c_{k_1}|, \ for\ k_1 \neq k_0\ such\ that\ c_{k_1} \neq 0 \\ I_k &= \frac{c_k^{(k_0-k_1)}\, c_{k_0}^{(k_1-k)}\, c_{k_1}^{(k-k_0)}}{|c_{k_1}|^{(k-k_0-p)}\, |c_{k_0}|^{(k_1-k-q)}}, \ \forall k \neq k_0, k_1 \end{aligned} \quad (18)$$

with $p, q > 0$.

D. Wavelet descriptors

Wavelet transforms performing the decomposition of the signal on different levels, it's a multiresolution transformation [29]. We use here the discrete wavelet transform, representing the original contour on many levels. Suppose that the contours $C = (x(t); y(t))$ are parameterized by the arc length s, by applying the wavelet transformation on the contours, we obtain:

$$\begin{pmatrix} x(s) \\ y(s) \end{pmatrix} \cong \begin{pmatrix} x_a(s) \\ y_a(s) \end{pmatrix} + \sum_{m=k}^{M} \begin{pmatrix} x_{dm}(s) \\ y_{dm}(s) \end{pmatrix} \quad (19)$$

where $x_a(s)$ and $y_a(s)$ are the approximated residual signals are expressed through the scaling functions ϕ_{mn} as follow: $x_a(s)$=$\sum_n a_n \phi_{Mn}$ and $y_a(s)$=$\sum_n c_n \phi_{Mn}$.
The $x_{dm}(s)$ and $y_{dm}(s)$ are the detailed signals corresponding to the point of the sequence. Where the subscript M is the maximum level of decomposition and n is the translation index. The detailed signals are expressed through the wavelet functions ψ_{mn} in the form $x_{dm}(s) = \sum_n r_{mn} \psi_{mn}$ and $y_{dm}(s) = \sum_n d_{mn}$ where the subscripts $m \in [1...M]$ is the succeeding levels of decomposition.
Applying the wavelets to the contours representations we have to select the maximum number of levels, representative for the task and normalize their coefficients to achieve invariances under translation, scale and rotation. In this study we use the wavelet descriptors proposed by Kimcheng Kith et al in [33] based on the distance to the center of gravity as a signature. They propose the **Algorithm(1)**.
The choice of decomposition level is one of the key factors for the wavelet decomposition. The same authors present a method to choose the level of decomposition using the signature energy. As the energy of the contour represnts information contained in this contour, and more the energy is high, better the contour is represented.

Algorithm 1 :WD extraction

Require: The coordinates of the points of the contour
Ensure: Descriptor based on the Contour invariant to similarity transformation

1. Parametrize the contour by the normalized arc length
2. Re-sample the contour uniformly in $N = 2^n$ points
3. Fixing the starting point on the contour
4. Calculate the contour signature (distance to center of gravity)
5. Application of the discrete wavelet transformation with decimation DWT
6. The descriptor vector is a vector whose elements are the contour approximation coefficients at level M.

VI. The results of numerical experiments

For our experimentation, We dispose a database of Brachiopods containing 20 classes and each class contains 20 images. Samples of the Brachiopods database are depicted in figure(1). Following the preprocessing stage, the contour is detected using region based active contours method, specifically chan and vese model implemented with the Level Set method, this model provides satisfactory results for the contour detection in an image (see section 2). Every object is represented by the x and y coordinates of its boundary points. The number of these points varies from 600 to 1200 for images in our databases. We have re-sampled the contour in $N = 2^{10} = 1024$ points. When the pretreatment step is completed, we approximate the contours using global B-spline approximation, in order to reduce the number of points in the contour. Many spline function are available in the literature but here we use a cubic B-spline because it'is the most popular in engineering application and also is a smoothest function, is used to smooth the contour points to reduce the noise effect which can be product in edge detection step. We choose a point which has the furthest distance as starting point at the contour point such that its distance to the centroid is maximal. The value of decomposition level M is fixed to 3. We realized a comparative study of Curvature, Fourier descriptors, Wavelet descriptors, Radial function and Fourier of Radial function for classification of Brachiopods. In the representation of the shape we have used 20 samples for each descriptor for efficient shape description.
For two shapes represented by their descriptors, the similarity between the two shapes is measured by the city-block distance between the two feature vectors of the shapes. Therefore, the online matching is efficient and simple. We used the recognition rate to compare the discrimination of the descriptors. We applied request the seven groups each group contains 100 images of Brachiopods. The first two groups contain images of the database. Others contain images that do not belong to the database, they are the images of database that have met a change as rotation and scale. The results of the comparison are shown in figure(2). The table(1) present a comparison of the run time of the three descriptors executed on a processor machine Intel Core 2 Duo 2GHz with 2GB of RAM.

VII. Conclusions

The aim of this paper is to present a comparison of some of contour based shape descriptors like Curvature, Fourier descriptors, Wavelet descriptors, Radial function and Fourier of Radial function. after applying a global B-spline approx-

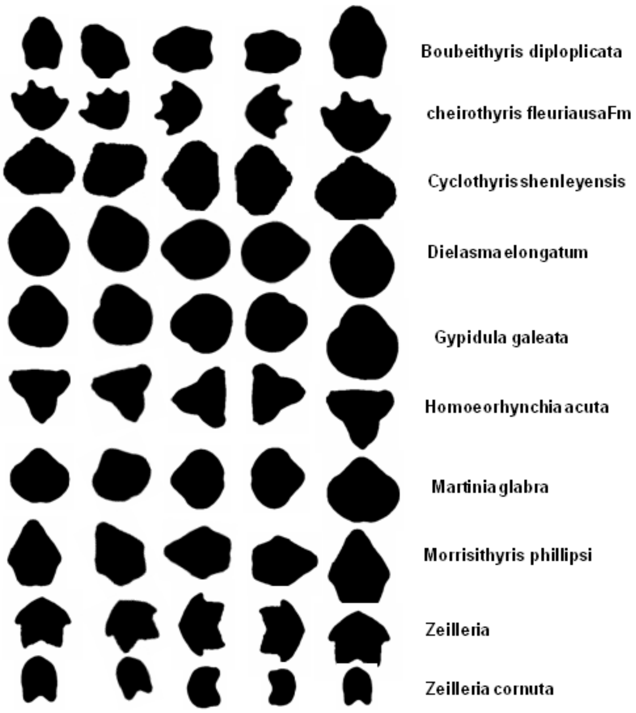

Figure. 1: Samples of the Brachiopods database.

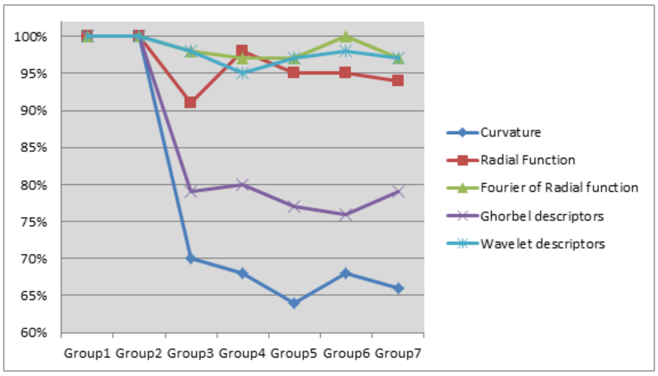

Figure. 2: Recognition rate of Brachiopods groups.

imation to the contours. The raisons for applying a global B-spline approximation are : We can smooth and reduce the number of points in the contours. We used the recognition rate to measure the discrimination of the descriptors and the City block distance as a classifier. We applied this technique for Fossils classification, especially the Brachiopod. The results of this comparison show that the Fourier of radial function and Wavelet descriptors give the greatest recognition rate followed of the radial function and ultimately the Fourier descriptors and Curvature, and the computation time is very small. Therefore, this method is very useful in many computer vision applications.

Table 1: Comparison of the run time of the five descriptors.

Descriptors	**Run time(ms)**
Radial function	0.031
Curvatures	7.3
Fourier of radial function	0.056
Fourier descriptors(Ghorbel)	0.444
Wavelet descriptors	53

References

[1] J. B. Florindoa, G. Landini and O. M. Bruno, *Three-dimensional connectivity index for texture recognition*, Pattern Recognition Letters, **84**, pp. 239-244, 2016.

[2] C. Liua, W. Wanga , Q. Zhaoa , X. Shenb and M. Konana, *A new feature selection method based on a validity index of feature subset*, Pattern Recognition Letters **92**, pp. 1-8, 2017.

[3] Sujata Dash, *Hill-climber Based Fuzzy-Rough Feature Extraction with an Application to Cancer Classification*, Journal of Network and Innovative Computing **2**, pp. 081-089, 2014.

[4] Y. Ait khouya, F.Ghorbel and N. Alaa, *Brachiopods classification based on fusion of global and local complete and stable descriptors*, 12th ACS/IEEE International Conference on Computer Systems and Applications, Marrakech, Morocco, November 17-20, 2015.

[5] Y. Ait Khouya, M. Ait oussous and N. Alaa, Use the Radon transform for describing Brachiopode shape, Applied Mathematical Sciences, **7(15)**, pp. 701-711, 2013.

[6] A. Bachnou, *Modlisation des contours ferms: anmorph outil mis au point pour maitriser les composantes des profils latraux des ostracodes. perspective d'application systmatique*, Gobis, **32(5)**, pp.733-742, 1999.

[7] Y. Ait khouya and F.Ghorbel, *Shape classification using B-spline approximation and contour based shape descriptors*, 9thInternational Conference on Soft Computing and Pattern Recognition, Marrakech, Morocco, December 11-13, 2017.

[8] Y. Ait khouya and F.Ghorbel, *Shape classification using B-spline approximation and contour based shape descriptors*, Advances in Intelligent Systems and Computing, Springer, **737**, pp. 60-69, 2018.

[9] R. Bailey and M. Srinath, *Orthogonal moment features for use with parametric and non-parametric classifiers*, IEEE Trans. Pattern Anal.Mach. Intell, **18(4)** pp.389-399, 1996.

[10] S. Belkasim, M. Shridar and M. Ahmadi, *Orthogonal moment features for use with parametric and non-parametric classifiers*, IEEE Trans. Pattern Anal.Mach. Intell, **24**, pp. 1117-1138, 1991.

[11] T. Chan and L. Vese, *Active contours without edges*, IEEE Transactions on Image Processing, **10(2)**, pp. 266-277, 2001.

[12] K. Ding , L. Xiao , G. Weng, *Active contours driven by local pre-fitting energy for fast image segmentation*, **104**, pp. 29-36, 2018

[13] J. Crampton, *Elliptic fourier shape analysis of fossil bivalves: some practical considerations*, Lethaia, **28(2)**, pp. 179-186, 1995.

[14] Y. Ait khouya and N. Alaa, *A new method for classification of Brachiopods based on the Radon transformation*, International Journal of Computer Science, **8 (5)**, pp.376–381, 2011.

[15] T.R. Crimmins, *A complete set of fourier descriptors for twodimensional shapes*, IEEE trans. on Systems, Man, and Cybernetics, **12**, pp. 848–855, 1982.

[16] C. Dalitz, C. Brandt, S. Goebbels, D. Kolanus, *Fourier descriptors for broken shapes*, EURASIP Journal on Advances in Signal Processing, **2013**, 2013:161.

[17] K. El Hariri and A. Bachnou, *Describing ammonite shape using fourier analysis*, Journal of African Earth Sciences, **39**, pp. 347-352, 2004.

[18] M. Foote, *Perimeter-based fourier analysis: a new morphometric method applied to the trolobite cranidium*, Journal of Paleontology, **63(6)**, pp. 880–885, 1989.

[19] F.S.Cohen, Z.Huang, and Z.Yang, *Invariant Matching and Identification of Curves using B-Splines Curve Representation*, IEEE Trans. Image Proc., **4(1)** pp. 1-10, 1995.

[20] G. E. Farin, *Curves and Surfaces for CAGD: A Practical Guide, Morgan Kaufmann; 5th edition*, Oct, 2001.

[21] F. Ghorbel, *Stability of invariant fourier descriptors and its inference in the shape classification*, in 11th Int. Conf. in Pattern Recognition, The Hague, The Netherlands, 30 August-3 September, 1992.

[22] F. Ghorbel, *Invariants de formes et de mouvement Onze cas du 1D au 4D et de l'euclidien aux projectifs*, Arts Pi, Tunis, 2013.

[23] S. Jehan-Besson, M. Barlaud and G. Aubert, *Dream2s: Deformable regions driven by an eulerian accurate minimization method for image and video segmentation*, International Journal of Computer Vision, **53(1)**, pp.45-70, 2003.

[24] H. Kauppinen, T. Seppnen and M. Pietikinen, *An experimental comparison of autoregressive and fourier-based descriptors in 2d shape classification*, IEEE Trans. Pattern Anal. Mach. Intell, **17(2)**, pp. 201-207, 1995.

[25] K. Kpalma and J. Ronsin, *Multiscale contour description for pattern recognition*, Pattern Recognition Letters, **27(13**, pp. 1545-1559, 2006.

[26] F. Mokhtarian and S. Abbasi, *Shape similarity retrieval under affine transforms*, Pattern Recognition , **10(2)**, pp. 31-41, 2002.

[27] F. Mokhtarian and A.K. Mackworth, *Scale-based description and recognition of planar curves and two-dimensional shapes*, IEEE Transactions on Pattern Analysis and Machine Intelligence , **8(1)** pp. 34-43, 1986.

[28] F. Mokhtarian and A.K. Mackworth, *A theory of multiscale, curvaturebased shap representation for planar curves*, IIEEE PAMI , **14**, pp. 789-805, 1992.

[29] S. *Mallat, Wavelet Tour of Signal Processing*, Academic Press, New York, 1998.

[30] D. Mumford and J. Shah, *Optimal approximation by piecewise smooth functions and associated variational problems*, Commun. Pure Appl. Math , **42**, pp. 577-685, 1989.

[31] S. Nagabhushana, *Computer Vision and Image Processing*, New Age International (P) Limited, 2005.

[32] N. Paragios and R. Deriche, *Geodesic active regions and level set methods for supervised texture segmentation*, International Journal of Computer Vision , **46(3)**, pp. 223-247, 2002.

[33] K.Kith, B. J.Van Wyk and M. A.Van Wyk, *The normalized wavelet descriptor for shape retrieval*. International journal of wavelets, multiresolution and information processing, **6(01)**, pp. 25-36, 2008.

[34] E. Persoon and K. Fu, *Shape discrimination using fourier descriptors*, IEEE Trans. Syst. Man Cyb , **7(3)**, pp. 170-179, 1977.

[35] E. Persoon and K. Fu, *Shape discrimination using fourier descriptors*, IEEE trans. On Pattern Analysis and Machine Intelligence , **8(3)**, pp. 388-397, 1986.

[36] Y. Rui and A. She, *A modified fourier descriptor for shape matching in mars*,mage Databases and Multimedia Search , **8**, pp. 165-180, 1998.

[37] Q. M. Tieng and W.W. Boles, *Recognition of 2d object contours using the wavelet transform zero crossing representation*, IEEE Trans. on PAMI, **19(8)**, pp. 910-916, 1997.

[38] C. Urdiales, A. Bandera and F. Sandoval, *Non-parametric planar shape representation based on adaptive curvature functions*, Pattern Recogn, **35**, pp. 43-53, 2002.

[39] B. Mokhtari, K. Eddine Melkemi, D. Michelucci and S. Foufou, *A 3D shape matching and retrieval approach based on fusion of curvature and geometric diffusion features*,International Journal of Computer Applications in Technology, **55(2)**, pp.79-91, 2017.

[40] H.S. Yang, S.U. Lee and K.M. Lee, *Recognition of 2d object contours using starting-point-independent wavelet coefficient matching*, Journal of Visual Communication and Image Representation, **9(2)**, pp. 171-181, 1998.

[41] C. Zahn and R. Roskies, *Fourier descriptors for plane closed curves*, Transactions on Computers, **21(3)**, pp. 269-281, 1972.

[42] D.S. Zhang, G. Lu, *A comparative study of Fourier descriptors for shape representation and retrieval*, in:Proceedings of the Fifth Asian Conference on Computer Vision (ACCV02), Melbourne, Australia, January 22-25, pp. 646-651, 2002.

[43] D.Zhang and G. Lu, *Study and evaluation of different fourier methods for image retrieval*, Image Vis. Comput, **123**, pp. 33-49, 2005.

[44] S. Zhu and A. Yull, *Region competition : Unifying snakes, region growing, and bayes/mdl for multiband image segmentation*, IEEE Transaction on Pattern Analysis and Machine Intelligence, **18**, pp. 884-900, 1996.