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Fuzzy Hybrid NSGA-II for Multi-Objective Reliability Decision-Making with Various Membership Functions

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Abstract: **Practically, reliability-based system design is modeled with flexibility and adaptability to the human decision-making process. Fuzzy set theory is a suitable technique for modeling and analyzing the system design in a better way. Generally, linear membership function is used to model the problem because of its simplicity. However, shapes of other membership functions such as quadratic, parabolic and hyperbolic can be considered for making empirical justification or assumption. Moreover, multi-objective optimization problems (MOOPs) are suggested to be solved using multi-objective evolutionary algorithms (MOEAs). The main reason for using MOEAs is to provide multiple trade-offs in one simulation run. However, it is not possible to generate the entire Pareto-optimal set in many complex applications. Keeping these views in mind, fuzzy hybrid non-dominated sorting genetic algorithm-II is proposed for multi-objective reliability-based system design problem with various membership functions. Non-dominated sorting genetic algorithm-II (NSGA-II) is one of the elitist MOEAs which has less computational complexity, elitist strategy, and parameter-less sharing approach. A fuzzy-based local search strategy is suggested to update the Pareto-optimal solutions from an NSGA-II simulation run and clustering technique gives a handful of solutions from a practical standpoint. The conflicting objectives such as maximization of system reliability and minimization of system cost are considered simultaneously in a numerical example of the over-speed protection system. A comparative analysis among these membership functions has been performed. Finally, the best compromise solution in each membership is obtained by the fuzzy ranking method.**

Keywords: System reliability, NSGA-II, Pareto-optimal front (POF), Membership function, Local search, Clustering

I. INTRODUCTION

Reliability is characterized as a measure of performance of the system. In many practical situations where reliability enhancement is involved, decision-making is complicated due to the presence of several other mutually conflicting objectives such as cost, weight, volume, etc. Moreover, various kinds uncertainty such as expert's information character, qualitative statements, lack of evidence, incompleteness and unreliability of input information, vagueness, etc., are not inevitable in the decision-making of reliability [1, 2]. Fuzzy set theory [3] is a suitable technique for handling such types of uncertainty. Park [4] suggested that fuzzy non-linear optimization technique is a superior way of analyzing system reliability. Dhingra [5], Rao and Dhingra [6] presented a fuzzy optimization technique to solve multi-objective reliability-redundancy allocation problem (MORRAP). Huang [1] developed fuzzy multi-objective optimization decision-making of series system. Ravi et al. [7] modeled the reliability of complex systems as a fuzzy MOOP where apart from the system reliability, system cost, weight, and volume are all considered as fuzzy goals/objectives. Huang et al. [8] proposed an interactive fuzzy multi-objective optimization method for engineering design. Mahapatra and Roy [9] suggested fuzzy multi-objective mathematical programming on reliability optimization model. An MOOP gives a set of optimal solutions (popularly known as Pareto-optimal solutions) instead of a single solution. Generally, the weighted sum approach [10] is used to solve an MOOP. In this approach, an MOOP is converted into a single-objective optimization problem (SOOP) by assigning a particular weight to each of the objective functions. When such methods are applied, it needs to be applied many times. But it does not suit to the ideal multi-objective optimization procedure which is suggested by Deb [10]. The goals of an MOOP are given as follows:

i. to discover a set of solutions as close as possible to the Pareto-optimal front;
ii. to maintain the diversity in the solutions set as far as possible.

The first goal follows the optimization task similar to an SOOP while the second goal is entirely specific to multi-objective optimization and it shows adequate exploration without loss of any valuable information in the search space. Due to this reason, a number of MOEAs such as Multi-Objective Genetic Algorithm (MOGA) [11], Niched Pareto Genetic Algorithm

(NPGA) [12], Non-dominated Sorting Genetic Algorithm (NSGA) [13], Strength Pareto Evolutionary Algorithm (SPEA) [14], Pareto Archived Evolution Strategy (PAES) [15], SPEA2 [16], PESA-II [17], NSGA-II [18], Multi-Objective Evolutionary Algorithm based on Decomposition (MOEA/D) [19] etc., are developed to search the Pareto-optimal front in a single simulation run. Many applications of MOEA are given in the different branches of science and technology like one of the uses MOEA can be seen in the designing of protein-protein interaction network [20]. NSGA-II is one of the popular MOEAs which gives better convergence and maintains good diversity in the solutions set. It is known its some special features such as elitist strategy, parameter-less sharing approach, crowding distance, classifying the solutions into the fronts, and low computational requirements. Apart from this, simulation results of the constrained NSGA-II show its superiority on several non-linear problems [18]. That is why NSGA-II has been applied to many engineering design problems including reliability-based system design successfully. Of these, Salazar et al. [21], Wang et al. [22], Kishore et al. [23, 24], Safari [25], Sharifi et al. [26], Kumar and Yadav [27, 30], etc., have successfully demonstrated. This paper presents a hybrid NSGA-II technique for fuzzy multi-objective reliability-based system design problem. Various membership functions are used in the decision-making of reliability to discuss the over-all system design process. The rest of the paper is organized as follows. Section II gives a brief description of NSGA-II. Section III gives the mathematical model of the problem. Section IV gives the proposed methodology in step by step. Section V gives the computational results with its discussions and Section VI gives the conclusions.

II. NON-DOMINATED SORTING GENETIC ALGORITHM-II (NSGA-II)

Non-dominated sorting genetic algorithm (NSGA) was initially proposed by Srinivas and Deb [13]. It faces some difficulty regarding the computational complexity, non-elitist approach and highly dependent on the parameters of fitness sharing. Deb et al. [18] developed a new algorithm by resolving these issues and named it as NSGA-II. It has some new features like "fast non-dominated sorting, crowding distance and comparison operator".

A fast-non-dominated sorting approach gives worst-case computational complexity of $o(k(2N)^2)$. This approach searches iteratively non-dominated solutions into different fronts. First, for each solution i in the population, the algorithm calculates two entities:

i. n_i, the number of solutions dominating i,

ii. S_i, a set of solutions dominated by i.

If $n_i = 0$ then it belongs to the 1st front. For each member j of the set S_i, the value n_j is reduced by one. If any n_j is reduced to zero during this stage, the corresponding member j is put in the 2nd front. This process is continued to each member in the 2nd front to identify 3rd front and so on. Moreover, NSGA-II gives the concept of crowding-distance (see Figure 1) with the worst-case computational complexity of $o(k(2N)\log(2N))$. It replaces the fitness sharing approach that requires a sharing parameter to be set by the user. The crowding-distance value (CD_i) is given by

$$CD_i = \sum_{p=1}^{k} d_{ip}\ , \text{ where } d_{ip} = \frac{f_p^{i+1} - f_p^{i-1}}{f_p^{\max} - f_p^{\min}} \qquad (1)$$

Here, f_p^{i+1} and f_p^{i-1} are the p^{th} objective function of the $(i+1)^{th}$ and $(i-1)^{th}$ individual (solution) respectively; $f_p^{\max}$ and $f_p^{\min}$ denote the maximum and minimum values of the p^{th} objective function.

According to Deb et al. [18] "A higher value of crowding-distance gives the lesser crowded region and vice versa". So, the crowding-distance picks the solutions located in less-crowded regions after the fast-non-dominated sorting procedure. It is extended up to the entire POF in order to maintain the diversity in the solutions set. Finally, NSGA-II uses an elitist strategy with the worst-case computational complexity of $o((2N)\log(2N))$.

Figure 2 shows an evaluation cycle of the NSGA-II algorithm. For handling the constraints, the binary tournament selection method is used. According to Deb et al. [18], "a search space (decision space) is divided by the constraints into two regions – feasible and infeasible and a solution α is called as a *constrained-dominate* to a solution β if

i. α is feasible and β is infeasible.

ii. α and β are infeasible, but α has a smaller overall constraint violation.

iii. α and β are feasible, but α dominates β".

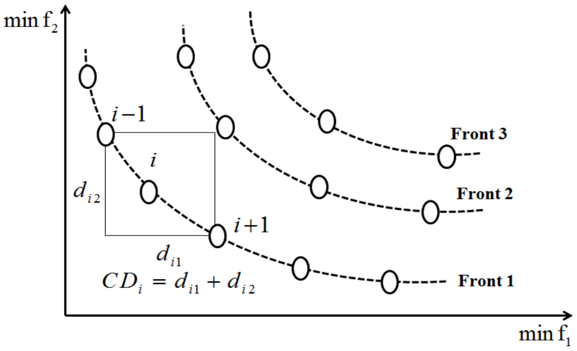


Figure 1. Crowding distance and Fitness evaluation

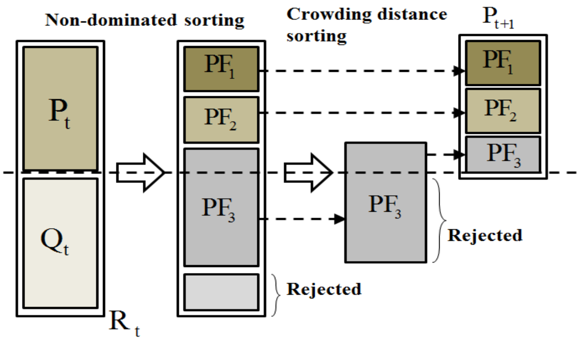


Figure 2. Main Loop of NSGA-II [19]

The Pseudo code of the NSGA-II is given as:

```
Initialize randomly population $P_0$
Compute the fitness values of individuals in $P_0$
Perform non-dominated sorting on $P_0$
Apply binary tournament selection on $P_0$
Generate child population $Q_0$
Apply recombination and mutation
**While** $t < t_{\max}$ (max no. of generation) **do**
    Generate $R_t = P_t \bigcup$
    Perform non-dominated sorting on $R_t$
    Copy individuals from non-dominated fronts $P_{t+1}$
    Apply binary tournament selection on $P_{t+1}$
    Generate child population $Q_{t+1}$
    Apply recombination and mutation
**End while**
Return
```

III. MATHEMATICAL MODEL OF THE PROBLEM

This paper considers a 4-stage over-speed protection system for a gas turbine [5, 6]. Here, the electrical and mechanical systems continuously provide detection. If an over-speed occurs, then 4-control valves (V1-V4) get closed and the fuel supply gets interrupted. Each component has a constant failure rate in the system. A schematic representation of the over-speed protection system is shown in Figure 3.

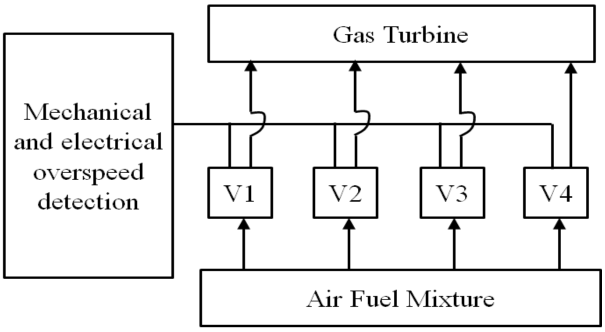


Figure 3. A four-stage the Overspeed protection system

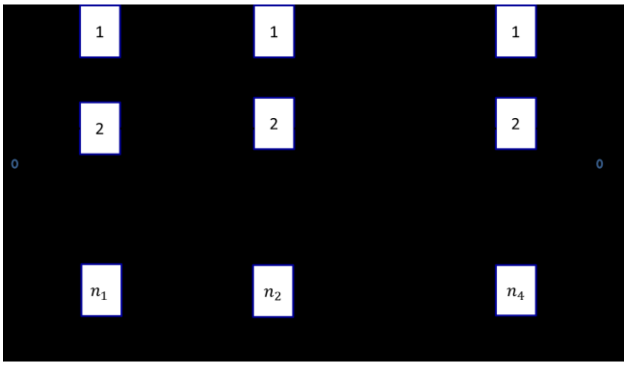


Figure 4. Series-parallel system configurations

This system is a series-parallel system configuration having several parallels and identical components arrayed at each stage. It is shown in Figure 4. Here, two-fold design variables are required in order to determine the goals. Let R_S & C_S be the reliability and cost of the system. First, the design variable is given as r_j is the reliability of j^{th} component at the stage j; second n_j is the redundant component at the stage j. Moreover, several design constraints such as the minimum requirement for the system reliability R, overall cost of the system C, the total permissible volume of the system $V_{\lim}$, and maximum allowable weight of the system $W_{\lim}$ are considered. The objective is to determine the optimal design variables $(r_j, n_j), j = 1,2,3,4$ in the MOOP defined as:

$$Max.R_S(r,n) = \prod_{j=1}^{4}\left[1-(1-r_j)^{n_j}\right] \tag{2}$$

$$Min.C_S(r,n) = \sum_{j=1}^{4} C_j(r_j)\left[n_j + \exp\left({n_j}/{4}\right)\right]$$

$$subject\,to\,V_S = \sum_{j=1}^{4} v_j n_j^2 \le V_{\lim}$$

$$W_S = \sum_{j=1}^{4} w_j n_j \exp\left(n_j/4\right) \le W_{\lim}$$

$$\prod_{j=1}^{4}\left[1-(1-r_j)^{n_j}\right] \ge R$$

$$\sum_{j=1}^{4} C_j(r_j)\left[n_j + \exp\left(n_j/4\right)\right] \le C$$

$$0.5 \le r_j \le 1-10^{-6}, 1 \le n_j \le 10, j = 1,2,3,4; r_j \in \square \quad , 2,...,10$$

Each component of the system has a constant failure rate λ_j that follows an exponential distribution [5]. The reliability of each component is given as:

$$r_j(T) = \int_T^{\infty} \lambda_j e^{-\lambda_j T} dT = e^{-\lambda_j T} \tag{3}$$

It is also mentioned that "$C_j(r_j)$ is assumed to be an increasing function of r_j (conversely, a decreasing function of the component failure rate) in the form" [5]:

$$C_j(r_j) = \frac{\gamma_j}{\lambda_j^{\delta_j}} \tag{4}$$

From (3) and (4), we have

$$C_j(r_j) = \gamma_j\left[{-T}/{\ln(r_j)}\right]^{\delta_j} \tag{5}$$

where the parameters δ_j and γ_j are the physical features (shaping and scaling factor) of the cost-reliability curve of each component in the j^{th} subsystem or stage; T is the active operational time; the factor $\exp\left({n_j}/{4}\right)$ is responsible for the additional cost due to the interconnection between the parallel components. The design data of the numerical example are given in Table 1.

IV. PROPOSED METHODOLOGY

This section describes the proposed methodology in step by step as follows.

Step 1. Find the Pseudo weights.

For any solution X in the set of optimal solutions obtained by the NSGA-II, the Pseudo weight vector for system reliability and system cost are calculated as:

$$w_{R_S}(X)=\frac{(R_S-R_S^l)/(R_S^u-R_S^l)}{(R_S-R_S^l)/(R_S^u-R_S^l)+(C_S^u-C_S)/(C_S^u-C_S^l)} \tag{6}$$

$$w_{C_S}(X)=\frac{(C_S^u-C_S)/(C_S^u-C_S^l)}{(R_S-R_S^l)/(R_S^u-R_S^l)+(C_S^u-C_S)/(C_S^u-C_S^l)} \tag{7}$$

where R_S^u and R_S^l are the upper and lower limits on R_S; C_S^u and C_S^l are the upper and lower limits on C_S; $h_1(R_S)$ is a monotonically increasing function of R_S; $h_2(C_S)$ is a monotonically decreasing function of C_S. These values are determined by the decision-maker (DM) according to the given situation.

Step 2. Construct the membership function.

The first objective in the MOOP (2) is R_S which values lie in [0, 1]. It can be transformed into a fuzzy region of satisfaction $\mu_{\tilde{R}_S}$ as per the degree of satisfaction α_R. Similarly, the second objective of (2) is C_S which values lie in [0, ∞). It also can be transformed into a fuzzy region of satisfaction $\mu_{\tilde{C}_S}$ as per the degree of satisfaction α_C. The shapes of various membership functions of $\mu_{\tilde{R}_S}$ and $\mu_{\tilde{C}_S}$ are shown respectively as in Figures 5 and 6.

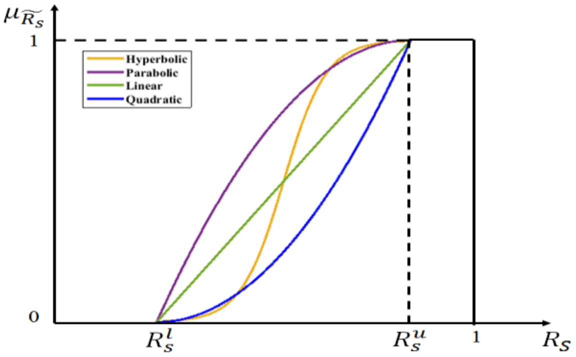


Figure 5. Monotonically increasing various membership functions for system reliability

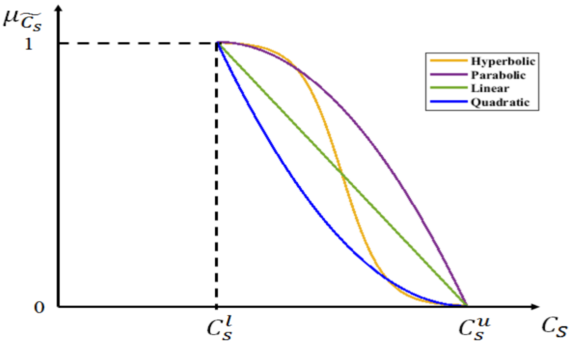


Figure 6. Monotonically decreasing various membership functions for system cost

It is defined as follows.

$$\mu_{\tilde{R}_S}=\begin{cases}0, & R_S\le R_S^l\\ h_1(R_S), & R_S^l\le R_S\le R_S^u\\ 1, & R_S\ge R_S^u\end{cases} \tag{8}$$

$$\mu_{\tilde{C}_S}=\begin{cases}1, & C_S\le C_S^l\\ h_2(C_S), & C_S^l\le C_S\le C_S^u\\ 0, & C_S\ge C_S^u\end{cases} \tag{9}$$

i. Linear [27]:

$$\mu_{\tilde{R}_S}=\begin{cases}0, & R_S\le R_S^l\\ \dfrac{\left(R_S-R_S^l\right)}{\left(R_S^u-R_S^l\right)}, & R_S^l\le R_S\le R_S^u\\ 1, & R_S\ge R_S^u\end{cases} \tag{10}$$

$$\mu_{\tilde{C}_S}=\begin{cases}1, & C_S\le C_S^l\\ \dfrac{\left(C_S^u-C_S\right)}{\left(C_S^u-C_S^l\right)}, & C_S^l\le C_S\le C_S^u\\ 0, & C_S\ge C_S^u\end{cases} \tag{11}$$

ii. Quadratic [28]:

$$\mu_{\tilde{R}_S}=\begin{cases}0, & R_S\le R_S^l\\ \left(\dfrac{R_S-R_S^l}{R_S^u-R_S^l}\right)^2, & R_S^l\le R_S\le R_S^u\\ 1, & R_S\ge R_S^u\end{cases} \tag{12}$$

$$\mu_{\tilde{C}_S}=\begin{cases}1, & C_S\le C_S^l\\ \left(\dfrac{C_S^u-C_S}{C_S^u-C_S^l}\right)^2, & C_S^l\le C_S\le C_S^u\\ 0, & C_S\ge C_S^u\end{cases} \tag{13}$$

iii. Parabolic [29]:

$$\mu_{\tilde{R}_S}=\begin{cases}0, & R_S\le R_S^l\\ 1-\left(\dfrac{R_S^u-R_S}{R_S^u-R_S^l}\right)^2, & R_S^l\le R_S\le R_S^u\\ 1, & R_S\ge R_S^u\end{cases} \tag{14}$$

$$\mu_{\tilde{C}_S}=\begin{cases}1, & C_S\le C_S^l\\ 1-\left(\dfrac{C_S-C_S^l}{C_S^u-C_S^l}\right)^2, & C_S^l\le C_S\le C_S^u\\ 0, & C_S\ge C_S^u\end{cases} \tag{15}$$

iv. Hyperbolic [5], [29]:

$$\mu_{\tilde{R}_S}=\begin{cases}0, & R_S\le R_S^l\\ \dfrac{1}{2}\tanh\left(\left(R_S-\dfrac{\left(R_S^l+R_S^u\right)}{2}\right)\alpha_1\right)+\dfrac{1}{2}, & R_S^l\le R_S\le R_S^u\\ 1, & R_S\ge R_S^u\end{cases} \tag{16}$$

$$\mu_{\tilde{C}_S} = \begin{cases} 1, & C_S \le C_S^l \\ \frac{1}{2}\tanh\left(\left(\frac{C_S^l + C_S^u}{2} - C_S\right)\alpha_2\right) + \frac{1}{2}, & C_S^l \le C_S \le C_S^u \\ 0, & C_S \ge C_S^u \end{cases} \tag{17}$$

$$\alpha_1 = \left(\frac{6}{R_S^u - R_S^l}\right);\ \alpha_2 = \left(\frac{6}{C_S^u - C_S^l}\right) \tag{18}$$

Step 3. Start the local search.

The fuzzy-based local search at each solution vector X is given by

$$Maximize\left(1 \wedge \frac{\alpha_R(X)}{w_{R_S}}\right) \wedge \left(1 \wedge \frac{\alpha_C(X)}{w_{C_S}}\right) \tag{19}$$

$$subject\ to\ \alpha_R(X) = \mu_{\tilde{R}_S(X)}$$

$$\alpha_C(X) = \mu_{\tilde{C}_S(X)}$$

$$X_L \le X \le X_U$$

where $\wedge$ denotes the aggregate operator known as the minimum operator. The local search comprises a non-domination check.

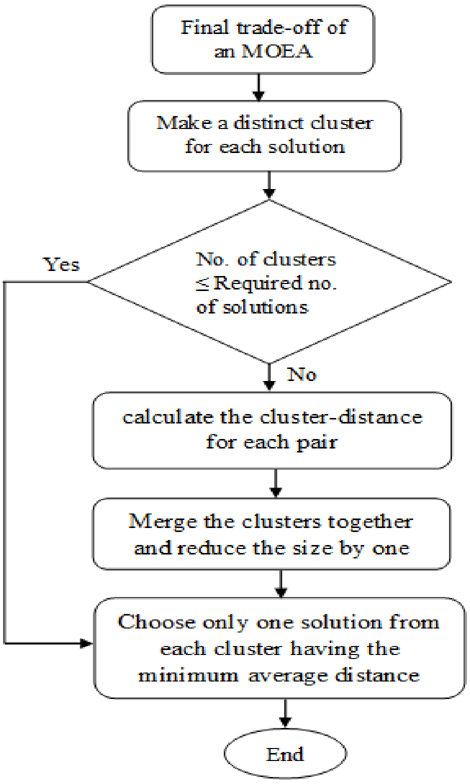


Figure 7. Flow diagram of the clustering technique

Step 4. Clustering.

Let P_{t+1} be the set of non-dominated solutions of size N. The clustering algorithm (see Figure 7) is given as follows:
Step i. Initially, each solution belongs to a distinct cluster or $C_i = \{i\}$ so that $C = \{C_1, C_2, ..., C_N\}$.
Step ii. If $N \le N'$ (required no. of solutions), go to Step v. Otherwise, go to Step iii.

Step iii. For each pair of clusters (there are $\binom{|C|}{2}$ of them), calculate the cluster-distance by using the following formula:

$$D_{12} = \left(1/|C_1||C_2|\right)\sum_{i \in C_1, j \in C_2} D(i,j), \tag{20}$$

where $|C_1|$ and $|C_2|$ number of solutions in the cluster set C_1 and C_2 respectively, and $D(i,j)$ is the Euclidean distance between two solutions i and j. Find the pair (i_1, i_2) which corresponds to the minimum cluster-distance.
Step iv. Merge the clusters C_{i_1} and C_{i_2} together. This deducts the size of C by one. Go to Step ii.
Step v. Select only one solution from each cluster and remove others. The solution having the minimum average distance from other solutions in the cluster represents a solution to that cluster. The proposed fuzzy hybrid NSGA-II model is shown in Figure 8.

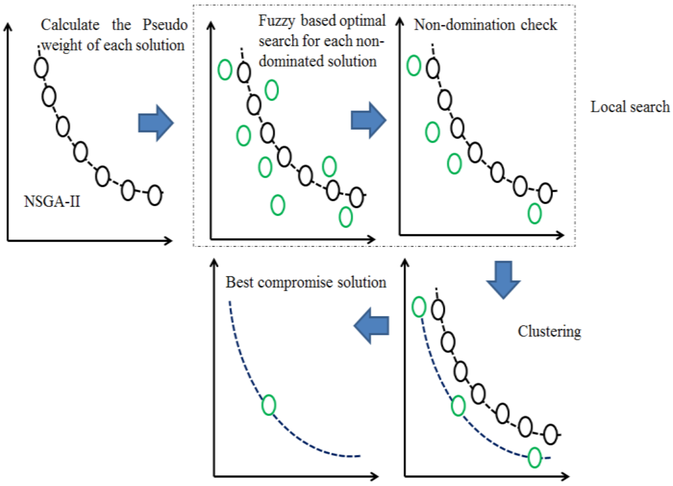


Figure 8. The proposed model of the fuzzy hybrid NSGA-II

Step 5. The best trade-off or compromise solution.

Fuzzy ranking method [31] ranks the solutions as per their degree of satisfaction levels and it is defined as:

$$\mu_{best} = \max_{P}\left[\min\left\{\mu_{\tilde{R}_S}, \mu_{\tilde{C}_S}\right\}\right] \tag{20}$$

where P is the number of obtained Pareto-optimal solutions.

V. COMPUTATIONAL RESULTS AND DISCUSSIONS

This section gives the computational results obtained by the proposed approach. The integer variable n_j is initially treated as a real variable and it is then converted to the nearest integer value. Figure 9 shows a complete picture of the solutions space obtained by NSGA-II after a single simulation run. All the fronts are shown in it. In order to find the end-points on each objective, a heuristic method namely cuckoo search (CS) [32] is used. CS is a population-based algorithm which has fewer parameters to be tuned and better convergence than other heuristic methods like genetic algorithm (GA), and particle swarm optimization (PSO) [32]. The population size (host nests), the probability of abandoned worst nests in CS are taken as 30 and 0.25 respectively. In order to remove stochastic discrepancy, there are made 25 independent runs.

Table 2 gives the upper and lower limits on R_S and C_S obtained by the heuristic methods. The parameter settings for the NSGA-II are given as: Population size N = 40, Maximum no. generations ($t_{\max}$) = 80, Crossover rate = 0.8, Mutation rate = 1/no. of design variables, Crossover index = 10, Mutation index = 40.

Table 1. Design data of the given problem

Stage	$10^5\gamma_j$	δ_j	v_j	w_j
1	1.0	1.5	1	6
2	2.3	1.5	2	6
3	0.3	1.5	3	8
4	2.3	1.5	2	7
R	C	$V_{\lim}$	$W_{\lim}$	T
0.75	400	250	500	1000h

Table 2. The optimal values of the objectives obtained by the heuristic methods

Heuristic Methods	$Max.R_S$	$Min.C_S$
CS	0.99994	20.30
GA	0.99810	20.62
PSO	0.99961	20.71

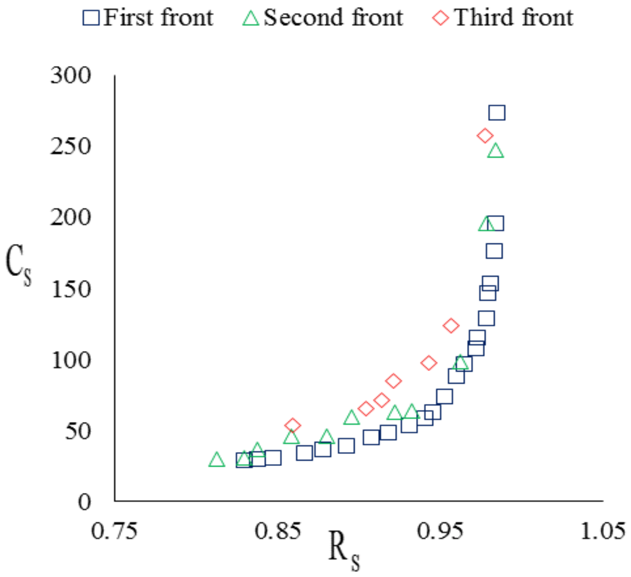


Figure 9. A complete picture of solutions space obtained by NSGA-II in one simulation run

Figure 10 shows the simulation results of the proposed fuzzy hybrid NSGA-II using various membership functions. Here, bold markers present clustered solutions. Figure 11 shows the convergence characteristics of various membership functions in order to achieve maximum satisfaction levels. The final front obtained by various membership functions is shown in Figure 12. It is shown in the objective space with the non-fuzzy approach in the original NSGA-II.

Figures 13 and 14 are shown as comparative analysis among the various membership functions in the form of maximum satisfaction and hypervolume metric [33] respectively.

Finally, the best "trade-off" or "compromise solution" is given to each of the membership functions in Table 3. All the works are implemented in Dev C++ and run in MS window environment on the PC which has intel coreTM i3 Duo processor with 2.40 GHz and 2GB RAM.

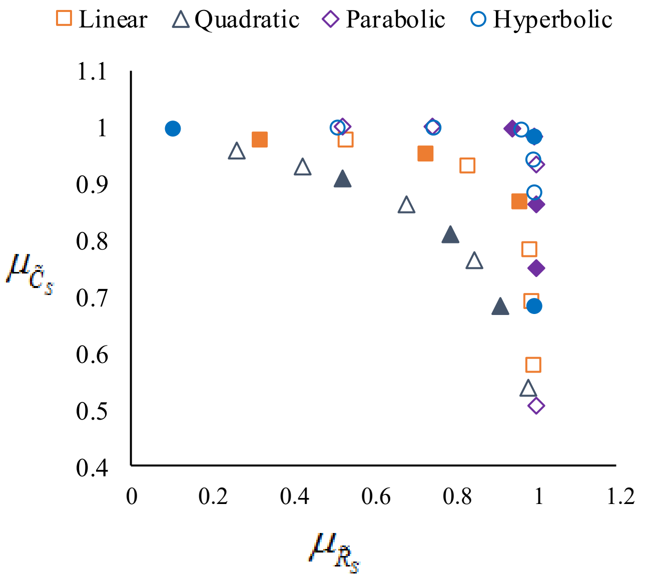


Figure 10. The Pareto-optimal solutions obtained by the various membership functions in the membership grades

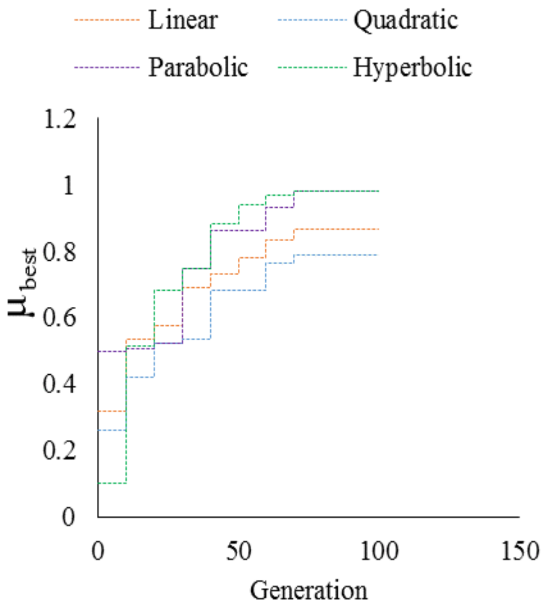
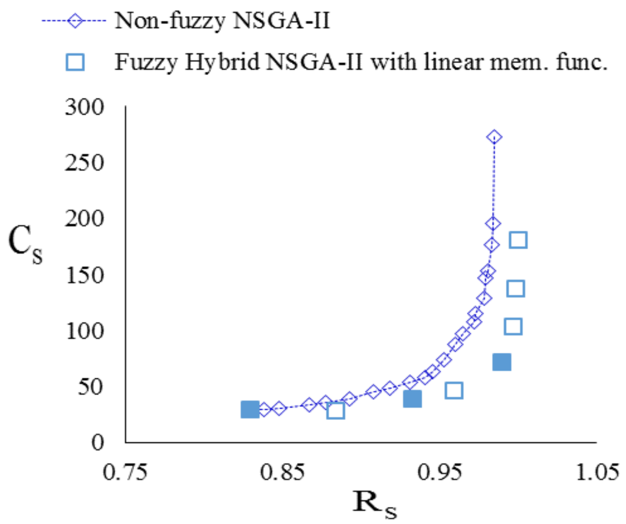
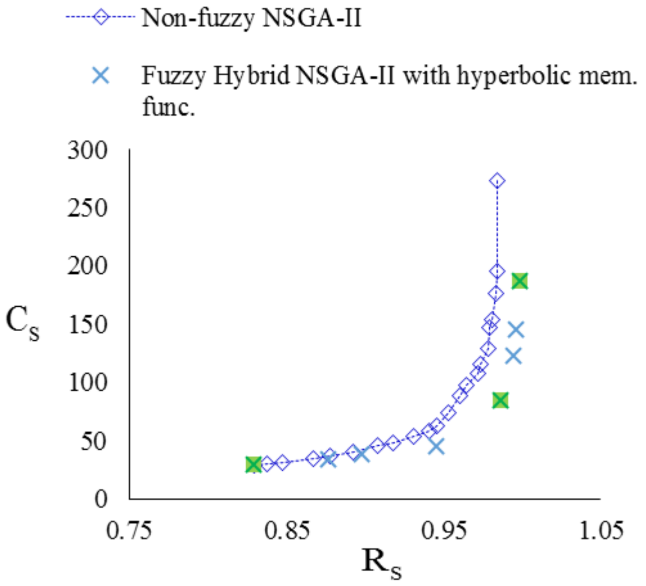
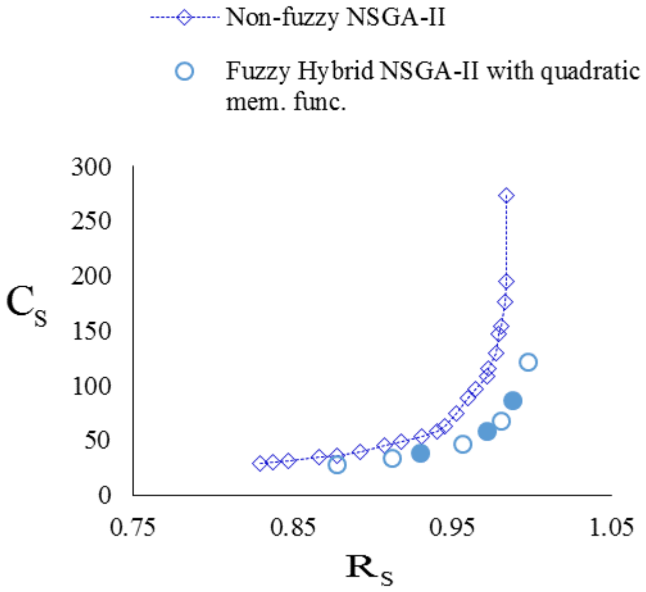


Figure 11. Convergence characteristics of various membership functions







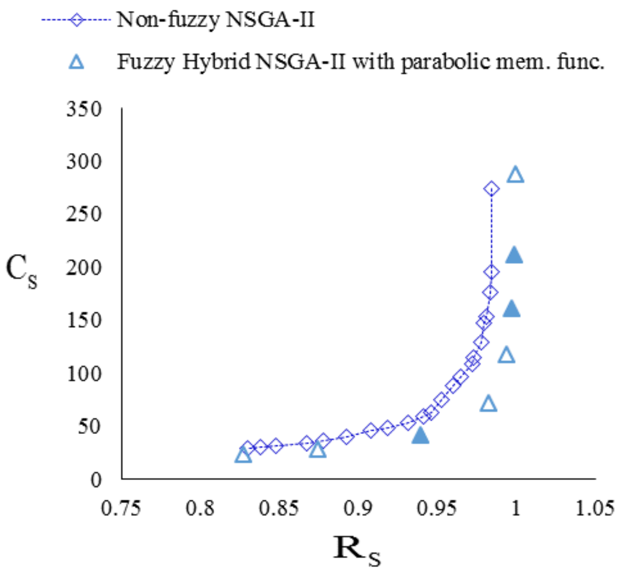


Figure 12. Simulation results of the proposed fuzzy hybrid NSGA-II in the objective space using the various membership functions

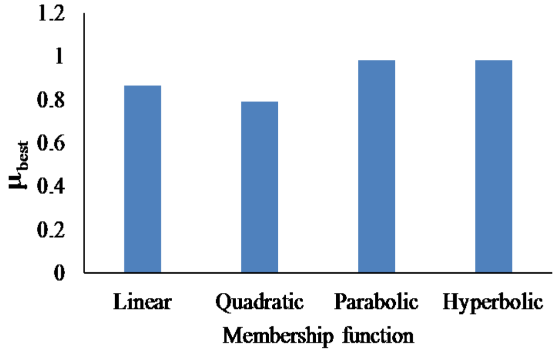


Figure 13. Maximum satisfaction levels achieved by various membership functions

Figure 14. A hypervolume metric comparison on the basis of various membership functions

Table 3. The optimal solution on the basis of various membership functions

	Linear	Quadratic	Parabolic	Hyperbolic
r_1	0.7184	0.7173	0.749	0.7684
n_1	5	4	4	4
r_2	0.6784	0.6853	0.7195	0.7722
n_2	5	4	4	4
r_3	0.7947	0.7761	0.8087	0.8029
n_3	4	4	4	4
r_4	0.6819	0.6878	0.7217	0.7129
n_4	5	4	4	4
R_S	0.9898	0.9721	0.9826	0.9862
C_S	71.52	58.46	72.29	83.9
W_S	418.57	293.57	293.57	293.57
V_S	173	128	128	128
w_{R_S}	0.5184	0.5058	0.5058	0.5058
w_{C_S}	0.4816	0.4942	0.4942	0.4942
α_R	0.9596	0.7897	0.9952	0.9952
α_C	0.8651	0.8091	0.9813	0.9818

VI. CONCLUSIONS

This paper presented fuzzy hybrid NSGA-II based decision-making and applied to the real word problem of reliability-based system design. The mutually conflicting objectives such as system reliability and system cost are considered with several other design constraints such as the minimum requirement of the system reliability, maximum allowable of system weight, volume, and cost. The proposed hybrid technique gives a small set of diverse solutions to practical purposes. The impact of various membership functions has been studied. These membership functions give the flexibility to the DM in the decision-making, where the non-fuzzy approach is unable to produce. It is observed that the best optimal solution obtained by the hyperbolic membership function gives the maximum satisfaction or achievement level to the DM. Linear membership function does not give any biases towards the objectives. So, it has the highest hypervolume metric. On the other hand, quadratic membership function gives the minimum satisfaction level towards the goals of the objectives while parabolic gives more than linear and quadratic but, slightly less than hyperbolic. A DM can adopt the membership function according to his/her own interest.

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