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Shape description based on fusion of Curvature and Fourier descriptors

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Abstract: **In this paper we intend to introduce a shape descriptor for planar closed curves invariants under geometric transformations. The proposed descriptor is a fusion between the curvature and Fourier descriptors. The curvature properties provide an apparently powerful cue to the underlying structure of the curve and captures completely the structure of planar curve. In addition, it is stable and complete. Fourier descriptors are powerful features for the recognition of two-dimensional connected shapes and is supported by the well-developed and well-understood Fourier theory. We used the Fourier descriptors proposed by Ghorbel, this set of invariants is also stable and complete. Experiments are conducted on different datasets such as MPEG-7,Kimia-99, Kimia-216 and our Brachiopods data set presented to illustrate the performance of the proposed descriptor.**

Keywords: Classification, Completeness, Stable descriptor, Curvature, Fourier descriptors, CSS.

I. Introduction

Many types of descriptions have been used to recognize image patterns in a number of applications. However, The choice of a description method will largely depend on the desired application. A good number of shape descriptors, which are prevalent in the literature are categorized into two groups: contour based shape descriptors and region-based shape descriptors. Contour-based shape descriptors use only boundary information , they cannot depict shape interior content[1],[2]. Of the contour-based descriptors we cite Fourier descriptors [3], [23], [25] which have been largely used. In [11] the authors propose a method to define Fourier descriptors for broken shapes, meaning shapes that can have more than one contour. Curvature approaches [21], have also been used. In [21] a shape is described in a scale space by the maximum of the curvature. In [20] the authors presented a pattern description approach based on the multi-scale analysis of the contour of planar objects. Wavelet descriptors can be used to describe a given object shape by wavelet descriptors (WD) [26].

In region based shape descriptors, all pixels surrounded by the shape boundary are taken into consideration to yield the shape descriptor. Frequently referred to methods are based on moment theory to describe shape [4]. These include Complex moment [5], orthogonal polynomial moments: Zernike [6]. In [7] we present a contour-region based shape descriptor for Brachiopods classification by using a combinations of Fourier descriptors and R-transform extracted from Radon transform.

The object descriptions could be also divided into two basic groups: local and global. The first group represents the local feature of the objects. The contour or region of the shape is broken down into sub-sections or sub-regions to generate feature to represent the shape. The second group contains characteristics representing the general qualities of the visible surface.

In this paper, we propose a contour based descriptor, it's a combination between a local and global descriptor. We used the curvature as local descriptor. Local features can consist of a specific shape of a small section of the boundary, and fully capture the structure of a plane curve. Also, it is stable and complete.

We choose the Fourier descriptors, which are considered to be auspicious as global descriptor because they are based on a solid theoretical foundation. The advantages of FD over many other shape descriptors are (i) simple to compute; (ii) each descriptor has a specific physical significance; (iii) simple to do normalization, making shape matching a simple task; (iv) geometrical invariance properties. We used the Fourier descriptors proposed by Ghorbel in [17], [18], this set of invariants is stable and complete. Stability criterion means that when two invariant representations have a small difference, the objects they represent should also have a small shape difference. Completeness of the description guaranteed that if two shapes have the same invariant representation

then the shapes they represent should also be similar.
For edge detection we have used the Chan and Vese model implemented by the Level set method [8].
We tested and compared the proposed descriptor with CSS method, Curvature and Ghorbel descriptors using standard databases like MPEG-7 and Kimia-216. After we applied the proposed descriptor for classification of fossils species and especially Brachiopods. There are many reasons for fossils classification (i)they are an aid to memory because it is impossible to remember the characteristics of a large number of different things unless we can group them into categories, whose members share many characteristics; (ii) greatly improve our predictive powers;(iii)classification systems improve our ability to explain relationships among things. For biologists, this is especially important when we attempt to reconstruct the evolutionary pathways that have produced the diversity of organisms living today. Several authors tried to classify fossil species [12], [13]. In [15] we presented a descriptor for Brachiopods classification it is a fusion between local and global descriptors, and we cite [14], the authors used the Outline Shape Analysis technique for describing Ammonite shape. It aims at approaching the shape by a trigonometric function defined by the sum of sine and cosine terms. This latter is decomposed in either a series of harmonic amplitudes and phase angles or a series of Fourier coefficients, serving as variables for quantitative analysis. This technique was applied to other invertebrate groups such as Trilobites [16], bivalves and Ostracodes [9].
The rest of this paper is organized as follows. In Section 2, we present the Region-based active Contour. Also, we describe the Chan and Vese model in terms of level set functions and compute the associated Euler-Lagrange equation. The smoothing of the contours and parametrization are recalled in Section 3. The Curvature Scale Space and the proposed descriptor are explained in Section 4 and 5. Finally, experimental results are given in Section 6.

II. Overview of Region-based active Contour

The region-based active contours model is based on the minimization of a well defined energy to segment the image[19]. Let I an image and Ω his domain. Chan and Vese in [8] proposed to minimize the following energy:

$$\begin{aligned} E(c_1,c_2,\phi) &= \mu \int_\Omega \mid \nabla H(\phi(x,y)) \mid dxdy \\ &+ \nu \int_\Omega H(\phi(x,y))dxdy \\ &+ \lambda_1 \int_\Omega \mid I(x,y)-c_1 \mid^2 H(\phi(x,y))dxdy \\ &+ \lambda_2 \int_\Omega \mid I(x,y)-c_2 \mid^2 (1-H(\phi(x,y)))dxdy \end{aligned} \quad (1)$$

where $\mu \geq 0$, $\nu \geq 0$, $\lambda_1 > 0$, $\lambda_2 > 0$ are fixed parameters and H is the Heaviside function. and ϕ is the level set function associated to the contour evolving.The constants c_1 and c_2 are the averages of I in $\phi > 0$ and $\phi < 0$ respectively.

The discretized evolution equation is:

$$\frac{\phi^{n+1}_{(i,j)} - \phi^n_{(i,j)}}{\Delta t} = \delta_\epsilon(\phi^n_{(i,j)})(\mu div(\frac{\nabla \phi^n_{(i,j)}}{\mid \nabla \phi^n_{(i,j)} \mid}) - \nu$$
$$-\lambda_1(I(i,j)-c_1(\phi^n_{(i,j)}))^2 + \lambda_2(I(i,j)-c_2(\phi^n_{(i,j)}))^2) \quad (2)$$

where $\delta_\epsilon(z)$ is a regular form of $\delta(z)$.

III. Smoothing Contours and Parametrization

A. Smoothing Contours

The principal cause for smoothing is to remove the noise, which can be product in the contours during edge detection step. The basic idea of smoothing is very simple. We passe through the point by point in the contour. For each data point we yield a new value that is depending of the original value at that point and surrounding data point. There are various methods for smoothing like Gaussian kernel and B-spline. The kernel of smoothing determines the function's form that is used to take the average of the neighboring points. The Gaussian kernel is defined in 1-D as:

$$G_\sigma(t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{t^2}{2\sigma^2}} \quad (3)$$

with σ determines the width of the Gaussian kernel which controls the degree of smoothing and has to be chosen suitably. A large value of σ will remove all small details of the contour curvature, while a small value will permit false concavities and convexities to remain in the contour.
We propose here to use B-spline for smoothing curves. Really, it's well known that these functions have good smoothing quality and are robust relatively to multiple derivatives and rounding errors. Let $C(u)$ be the position vector along the curve as a function of the parameter u. The B-spline curve, $C(u)$, is defined as:

$$C(u) = \sum_{i=0}^{k} B_{(i,p)} P_j \quad (4)$$

where P_j is a control point, u is a parameter, and $B_{(i,p)}$ are the normalized B-spline basis functions of order p.

B. Parametrization

The contours Parametrization step provide all the computation results. For a given contour there are many types of parameter. A Parametrization function will be chosen which are suitable for the intended invariance. Here, we are interested in the parametrization invariant under the similarity transformation. To obtain a parametrization invariant under the similarity transformation we used the arc length parametrization, defined by the Euclidian norm which possesses an intrinsic link with the notion of similarity.
Suppose we have the coordinates of the contour $C(t) = (x(t), y(t))$, where t is a real value called the parameter of the contour. To reparametrize the contour by the arc length we need to calculate:

$$s(l) = \frac{1}{L} \int_0^l \mid C(t)' \mid dt \quad (5)$$

where L denotes the curve length.
On the other hand, to obtain a parametrization invariant under affine transformation it's necessary to use affine arc parametrization.

IV. Curvature Scale Space

The curvature scale space (CSS) approach was introduced by Mokhtarian et al in [22] as a shape representation for planar curves. This method is based on finding points of inflection (i.e, curvature zero-crossings) on the curve at varying levels of detail. The curvature k of a planar curve, at a point on the curve, is defined as the instantaneous rate of change of the slope of the tangent at that point with respect to arc length, and it can be expressed as follows:

$$k(s,\sigma) = \frac{X_s(s,\sigma)Y_{ss}(s,\sigma) - X_{ss}(s,\sigma)Y_s(s,\sigma)}{(X_s(s,\sigma)^2 + Y_s(s,\sigma)^2)^{3/2}} \quad (6)$$

where $X_s(s,\sigma)$ and $X_{ss}(s,\sigma)$ correspond to the first and second derivatives of $x(s)$, $Y_s(s,\sigma)$ and $Y_{ss}(s,\sigma)$ correspond to the first and second derivatives of $y(s)$. If we determine the locations of curvature zero crossings of every C_σ during evolution we can display the resulting points in (s,σ) plane where s is the normalized arc length and σ is the width of the Gaussian kernel. The result of this process can represent the CSS image of the curve. CSS is used to describe a contour. It is also an invariant with respect to the similarities of group. CSS is stable with respect to errors due to noise. In addition, it is complete [22]. Mokhtarian et al used the maxima of CSS to index shapes. This choice is justified by the fact that the maxima of the CSS are the most significant local point on the body of the CSS image. The locations of maxima are not readily available and must be extracted from the CSS image using Algorithm (1) as shown in Figure (1).

Algorithm 1 Extraction of the CSS maxima

Require: CSS image
  stp is a constant used to increment the variable σ during the construction of the CSS Image
  for $i = \sigma_{max}$ to 1 **do**
    if a zero crossing is found at point (i, s_j) **then**
      check the neighboring points $(i + stp; s_{j-1})$; $(i + stp; s_j)$ et $(i + stp; s_{j+1})$
      if the three neighbors above are not zero-crossings **then**
        (i, s_j) is a candidate for the maxima of the CSS
        find all the other candidates to the maxima as $\sigma = i$
      end if
      For each candidate (i, s_j), check the neighboring candidate
      if (i, s_k) is a neighbor candidate **then**
        if $|j - k| > 5$ **then**
          (i, s_j) is a maxima of CSS
        else
          $(i, (j + k)/2)$ is a maxima of CSS
        end if
      end if
    end if
    $i = i - stp$
  end for

V. The proposed descriptor

A. Overview of the Fourier descriptors methods

Let $x(t)$ and $y(t)$ be the coordinates of the contour, Since boundary is a closed curve and when the two-dimensional plane is considered as a complex plane, the cartesian coordinates of the contour are represented in the complex plane by $z(t) = x(t) + j.y(t)$. The discrete Fourier coefficients of $z(t)$ is defined as follow:

$$c_k = \sum_{k=0}^{N-1} z(t) exp(-j2\pi kt/N) \quad (7)$$

with $t \in [0, N-1]$, and N is a number of points in the contour.
Applying the inverse Fourier transform, we can recreate a contour from its Fourier descriptors. Initially, the first set of invariants was constructed by taking the modulus of Fourier descriptors.

$$\forall k \in [0, N-1], \;\; I_k = |c_k|, \quad (8)$$

In [27] the authors used this method in their comparative study. The first coefficient is the only one depends on position of the shape. To obtain translation invariance the coefficient c_0 can be discarded. Scale invariance is achieved by dividing all the Fourier coefficients by the absolute value of the second coefficient. To obtain a rotation and shift invariant descriptor, the phase can completely ignored and the only use the absolute value of the Fourier coefficients.
The acquired Fourier descriptors is used to discriminate between simple-shaped objects [24]. But, this set of invariants is not complete in the sense defined by Crimmins in [10], A set of descriptors is said to be complete if the following property is verified:
two objects have the same shape if and only if they have the same set of invariants.
Completness allows to retrieve shapes from their invariants, up to an Euclidean transformation. In fact, the set of descriptors defined in equation (8) is not complete since we can find objects with different shapes but with the same magnitude for their Fourier coefficients.
To resolve the problem of completeness, Crimins proposed in [10] a complete set of invariant, defined as:

$$\begin{aligned} I_{k_0} &= |c_{k_0}|, \;\; for \;\; k_0 \;\; such \;\; that \;\; c_{k_0} \neq 0 \\ I_{k_1} &= |c_{k_1}|, \;\; for \;\; k_1 \neq k_0 \;\; such \;\; that \;\; c_{k_1} \neq 0 \\ I_k &= c_k^{(k_0-k_1)} \, c_{k_0}^{(k_1-k)} \, c_{k_1}^{(k-k_0)}, \;\; \forall k \neq k_0, k_1 \end{aligned} \quad (9)$$

However, this set is not stable, because a slight modification of the invariants can induce a remarkable distortion of shape. To overcome this problem, Ghorbel in [17] presented a complete and stable set of invariant Fourier descriptors, defined as (the reader can consult chapter II page 53-71 in [18] for more details):

$$\begin{aligned} I_{k_0} &= |c_{k_0}|, \;\; for \;\; k_0 \;\; such \;\; that \;\; c_{k_0} \neq 0 \\ I_{k_1} &= |c_{k_1}|, \;\; for \;\; k_1 \neq k_0 \;\; such \;\; that \;\; c_{k_1} \neq 0 \\ I_k &= \frac{c_k^{(k_0-k_1)} \, c_{k_0}^{(k_1-k)} \, c_{k_1}^{(k-k_0)}}{|c_{k_1}|^{(k-k_0-p)} \;\; |c_{k_0}|^{(k_1-k-q)}}, \;\; \forall k \neq k_0, k_1 \end{aligned} \quad (10)$$

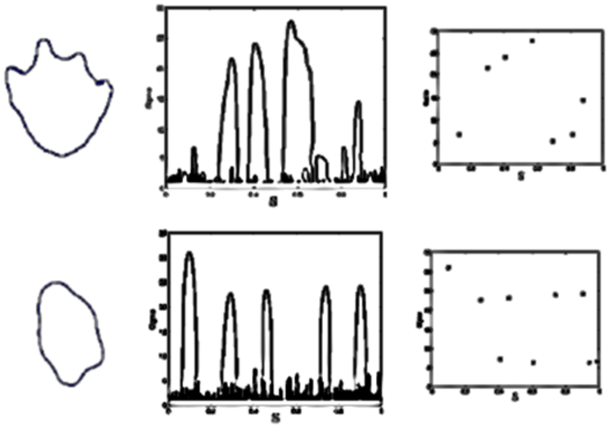

Figure. 1: CSS image and its maxima, left: Contour, middle: normalised CSS image, right: normalised maxima of CSS images.

with $p, q > 0$.

B. The proposed feature descriptor

The proposed feature descriptors is based on the fusion of local and global feature of the contours of objects. Local features are usually on the boundary of an object or represent a distinguishable small area of a region. Global features can be obtained for either a region by considering all points within a region, or only for those points on the boundary of a region. In each case, the intent is to find descriptors that are obtained by considering all points, their locations, intensity characteristics, and spatial relations.
We used the curvature as a local feature thanks to the arc length parameterization. It has been often noted that curvature properties provide a powerful indicator to the underlying structure of the curve and captures completely the structure of a plane curve. The curvature function is an invariant property of the curve itself. Therefore, it's insensitive to changes in the coordinate system like rotation. When the curve changes size the local relationships between the curvature of different parts of the curve are conserved. This offers that the relative topological structure of the curvature function can achieve a scale invariant. In addition, it is stable and complete. Stable, meaning a slight modification of the invariants can induce a slight distortion of shape. Complete that is two objects have the same shape if and only if they have the same set of invariants. Completness allows to retrieve shapes from their invariants.
The global feature is based on Fourier descriptors, we used the Fourier descriptor proposed by Ghorbel equation(10). This set of invariants is also stable and complete. The proposed feature descriptor has the following form:

$$X = (k(s_1), k(s_2), ..., k(s_N), I_0, I_1, ..., I_n) \tag{11}$$

where $n \in [0, N-1]$ and N is a number of points in the contour.

VI. Experimental results

Following the preprocessing stage, the contour is detected using region based active contours method, specifically chan and vese model implemented with the Level Set method, the parameters are fixed at $(\mu = 1, \nu = 0, \lambda_1 = 1 \; and \; \lambda_2 = 1)$, this model provides satisfactory results for the contour detection in an image (see section 2). Every object is represented by the x and y coordinates of its boundary points. The number of these points varies from 600 to 1200 for images in the databases. Due to the boundary discrete representation and quantization errors, false local concavities and convexities along a contour are formed. This noisy nature of binary contours must be taken into account to obtain reliable calculation of contour curvature. Therefore, a cubic B-spline is used to smooth the contour points to reduce the noise effect. After, we parameterized the contours by the arc length method.
When the pretreatment step is completed, The curvature for each point on the smoothed curve is computed and normalized between -1 and 1 . Then, we calculate the set of invariants Fourier descriptors proposed by Ghorbel equation(10) also normalized between -1 and 1. This set of invariants is stable and complete. We use the FFT algorithm for reducing the computing time. For efficient shape description, Zhang and Lu in [27] have found that 10 Fourier descriptors features are sufficient to represent shape. In our study, the number of invariants Ghorbel descriptors is calculated in advance based on experimentation reconstruction performed on the entire database. We proceed by an inversion invariants over the entire of datasets using equation (12) and compare the original contour with the contour reconstructed in the sense of the norm 2 with truncation (see [18] for more details).

$$c_k = I_k^{\frac{1}{k_0-k_1}} I_{k_0}^{\frac{-p}{(k_0-k_1)(k_0-k_1+p)}} I_{k_1}^{\frac{-q}{(k_0-k_1)(k_1-k_0+q)}} \tag{12}$$
$$exp(\frac{2i\pi}{k_0-k_1}(k(\theta_0-\theta_1) - k_1\theta_0 + k_0\theta_1))$$

$$C_r(t) = FFT^{-1}(c_k)$$

where k_0 and k_1 are constants fixed when computing Ghorbel descriptors I_k, c_k is the Fourier coefficients after inversion of Ghorbel invariants, θ_0 and θ_1 are the arguments of c_{k_0} and c_{k_1} respectively. FFT^{-1} is the inverse discrete Fourier transform, $C_r(t)$ is the reconstructed Contour. The number p of invariants Ghorbel descriptors retained will be one that

Table 1: Recongition Rate for dataset.

Methods	MPEG7-Shape-1-Part B	Kimia-99	Kimia-216
CSS	90	92	93
Curvature	63	65	68
Radial Function	91	93	95
Ghorbel descriptor	93	95	95
Proposed Method	95	97	98

Table 2: Recongition Rate for dataset.

Methods	Computing time (seconds)
CSS	2.508904
Curvature	0.171818
Radial Function	0.000806
Ghorbel descriptor	0.051111
Proposed Method	0.2292

allows a reconstruction with an error less than 5 per cent. After several tests on our Brachiopods database, we found that p=16 is sufficient to represent shape with an error less than 5 per cent as show in figure (2).
Once p is found, we sample equidistantly the p arc length of the curvature function. We take the values of normalized curvature at these p points of the curve. This reduces the number of local descriptors to $p < N$. We choose the point that has the highest value of the curvature as a starting point.
For evaluation of the proposed descriptor, we provide the recognition rates of the proposed algorithm and compare it with the state of the art shape recognition methods like Curvature,Radial function, Ghorbel and CSS descriptors. The experiments was conducted on three popular benchmarks : MPEG7-CE-Shape-1-Part B, Kimia-99 and Kimia-216 (see figures (3), (4), (5)).
The search process in the system using CSS as a descriptor begins with a pre-selection phase during which we rejected all shape not having the same number of maximum points of the CSS that the request shape. The normalization relative to the arc length ensures invariance in translation and scale. The rotational invariance is also provided by a lateral shift of the maximum of CSS. For two shapes represented by their descriptors, the similarity between the two shapes is measured by the city-block distance between the two feature vectors of the shapes. Therefore, the online matching is efficient and simple. We used the recognition rate to compare the discrimination of the five descriptors. We applied several groups of queries for each data set. Each group contain images that do not belong to the database, they are the images of database that have met a change as rotation and scale.The table 1 illustrates the average recognition rate for each dataset.

For more evaluation of the proposed invariants we compared the five descriptor for classification of Brachiopods. We dispose a database of Brachiopods that contains 800 binary images, 20 are original others are distorted copies of the image database, which they are subject to rotations and scaling, as shown in figure (6).
We applied request the seven groups of Brachiopods each group contains 100 images of Brachiopods. The first two groups contain images of the database. Others contains images that have undergone changes such as rotation and scale. The results of this comparison for the classification of Brachiopods show that the system using the proposed descriptor gives the greatest recognition rate comparable to rates obtained by four other methods as shown in figure(7).
The table 2 illustrate the CPU time required for each method. The four methods are executed on a processor machine Intel(R) Core (TM) i5-6300U 2.5 Ghz with 4GB of RAM.

VII. CONCLUSIONS

In this paper, we have presented a contour based descriptor for shape classification relatively to the planar transformation. The proposed descriptor is based on the fusion of Curvature and Fourier descriptors. The curvature presents the local information of the curve and provides a powerful cue to underlying the structure of the shape. In addition, it is stable and complete. To enhance the ability of the proposed descriptor to capture global shape information, We used the stable and complete set of invariants Fourier descriptors proposed by Ghorbel as a global information of the contour. Experiments are carried out on several popular shapes benchmarks, including MPEG-7, Kimia-99,Kimia-216 and our Brachiopods database. The results indicate that the proposed algorithm works well compared to many existing methods, particularly under planar transformation.

References

[1] S.Elghoul and F.Ghorbel, Fast global SA(2,R) shape registration based on an invertible invariant descriptor, Signal Processing Image Communication, November 2020.

[2] E.Ghorbel, F.Ghorbel, I.sakli and s. Mhiri Fast Blinding based on invertible and stable invariants, International Conference on Pattern Recognition ICPR, Milan 2020.

[3] Y. Ait Khouya , M. Ait Oussous, F. Ghorbel (2021) Describing Brachiopods Shape Based on Fusion of Curvature and Fourier Descriptors. In: Abraham A. et al. (eds) Proceedings of the 12th International Conference on Soft Computing and Pattern Recognition (SoCPaR 2020). SoCPaR 2020. Advances in Intelligent Systems and Computing, vol 1383. Springer, Cham.

[4] R. Bailey and M. Srinath, *Orthogonal moment features for use with parametric and non-parametric classifiers*, IEEE Trans. Pattern Anal.Mach. Intell, **18(4)** (1996) 389–399.

[5] F. Ghorbel, S. Derrode, R. Mezhoud, T. Bannour and S. Dhahbi, Image reconstruction from a complete set of similarity invariants extracted from complex moments,Pattern Recognition Letters 27 (2006) 1361–1369.

[6] A. Khotanzad, Y.H. Hong,. Invariant image recognition by Zernike moments. IEEE Trans. Pattern Anal. Mach. Intell. 12 (5) (1990), 489–497.

[7] Y. Ait Khouya and F. Ghorbel, Brachiopods classification based on fusion of contour and region based descriptors, Int. J. Advanced Intelligence Paradigms, Vol. 14, Nos. 1/2 (2019) 140–156.

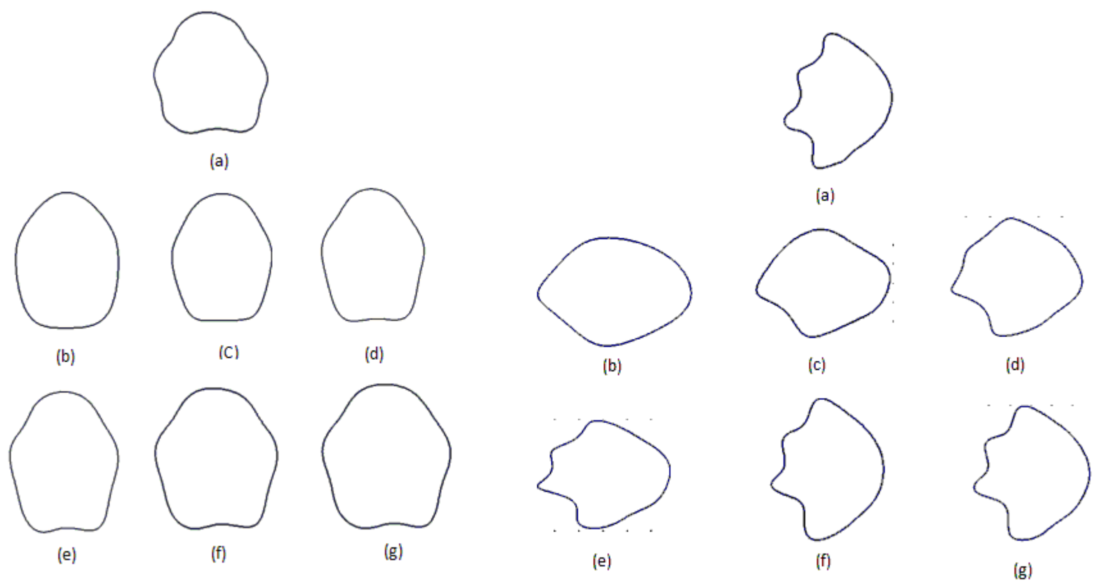

Figure. 2: (a) original Contour; (b),(c), (d), (e), (f), and (g) are reconstructed contour with 8, 10, 12, 14, 16, and 18 of Ghorbel Coefficients respectively.

Figure. 3: Kimia-99 dataset, each row shows a different class.

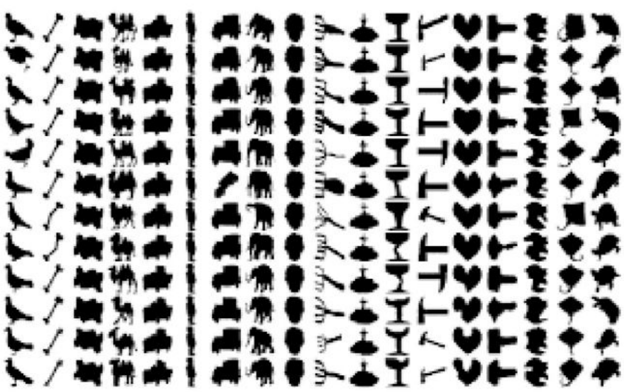

Figure. 4: Kimia-216 dataset, where each column shows a different object.

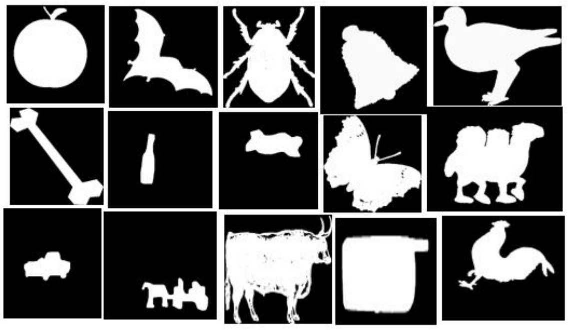

Figure. 5: Example of MPEG7-CE-Shape-1-Part B dataset.

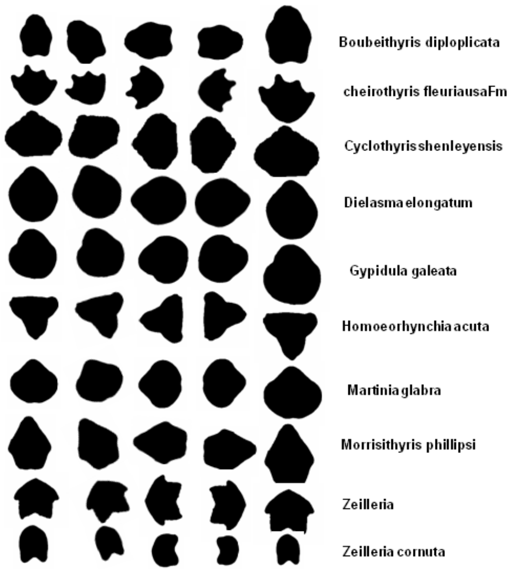

Figure. 6: Exemple of Brachiopods database.

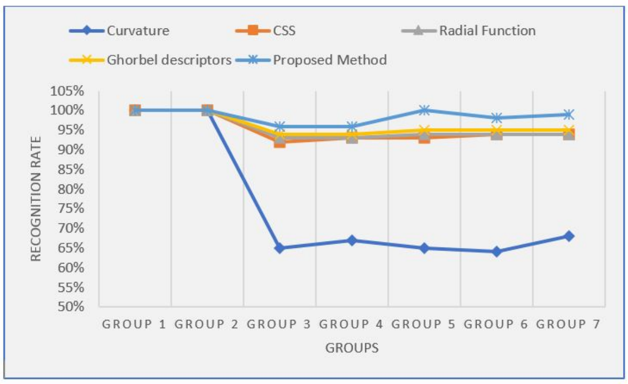

Figure. 7: Recognition rate for Groups.

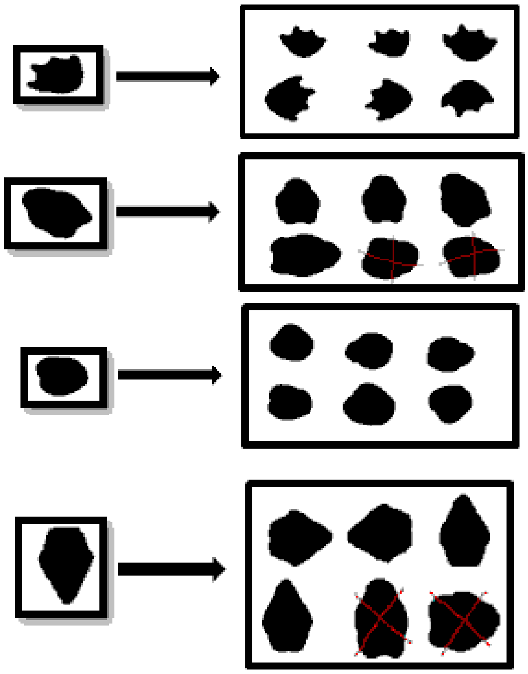

Figure. 8: Retrieval result of arbitrary queries on our database using the proposed descriptor.

[8] T. Chan and L. Vese, *Active contours without edges*, IEEE Transactions on Image Processing, **10(2)** (2001) 266–277.

[9] J. Crampton, *Elliptic fourier shape analysis of fossil bivalves: some practical considerations*, Lethaia, **28(2)** (1995) 179–186.

[10] T.R. Crimmins, *A complete set of fourier descriptors for twodimensional shapes*, IEEE trans. on Systems, Man, and Cybernetics, **12** (1982) 848–855.

[11] C. Dalitz, C. Brandt, S. Goebbels, D. Kolanus, *Fourier descriptors for broken shapes*, EURASIP Journal on Advances in Signal Processing, 2013, 2013:161.

[12] Y. Ait Khouya and F. Ghorbel, *Brachiopods classification based on fusion of contour and region based descriptors* International Journal of Advanced Intelligence Paradigms (IJAIP), **14, No. 1/2**, 2019, pp.140 - 156.

[13] Y. Ait Khouya and F. Ghorbel, *Description shape based on B-spline approximation and contour based shape descriptors:Application to Brachiopods classification* Journal of Network and Innovative Computing, **(7)** (2019) pp. 025–031.

[14] K. El Hariri and A. Bachnou, *Describing ammonite shape using fourier analysis*, Journal of African Earth Sciences, **39** (2004) 347–352.

[15] Y. Ait khouya, F.Ghorbel and N. Alaa, *Brachiopods classification based on fusion of global and local complete and stable descriptors*, 12th ACS/IEEE International Conference on Computer Systems and Applications, Marrakech, Morocco, November 17-20, (2015).

[16] M. Foote, *Perimeter-based fourier analysis: a new morphometric method applied to the trolobite cranidium*, Journal of Paleontology, **63(6)**, 880–885, (1989) .

[17] F. Ghorbel, *Stability of invariant fourier descriptors and its inference in the shape classification*, in 11th Int. Conf. in Pattern Recognition, The Hague, The Netherlands, 30 August-3 September, (1992) .

[18] F. Ghorbel, *Invariants de formes et de mouvement Onze cas du 1D au 4D et de l'euclidien aux projectifs*, Arts Pi, Tunis, 2013.

[19] S. Jehan-Besson, M. Barlaud and G. Aubert, *Dream2s: Deformable regions driven by an eulerian accurate minimization method for image and video segmentation*, International Journal of Computer Vision, **53(1)** (2003) 45–70.

[20] K. Kpalma and J. Ronsin, *Multiscale contour description for pattern recognition*, Pattern Recognition Letters, **27(13** (2006) 1545–1559.

[21] F. Mokhtarian and S. Abbasi, *Shape similarity retrieval under affine transforms*, Pattern Recognition , **10(2)** (2002) 31–41.

[22] F. Mokhtarian and A.K. Mackworth, *A theory of multiscale, curvaturebased shap representation for planar curves*, IEEE PAMI , **14** (1992) 789–805.

[23] E. Persoon and K. Fu, *Shape discrimination using fourier descriptors*, IEEE Trans. Syst. Man Cyb , **7(3)** (1977) 170–179.

[24] E. Persoon and K. Fu, *Shape discrimination using fourier descriptors*, IEEE trans. On Pattern Analysis and Machine Intelligence , **8(3)** (1986) 388–397.

[25] Y. Rui and A. She, *A modified fourier descriptor for shape matching in mars*,mage Databases and Multimedia Search , **8** (1998) 165–180.

[26] Q. M. Tieng and W.W. Boles, *Recognition of 2d object contours using the wavelet transform zero crossing representation*, IEEE Trans. on PAMI, **19(8)** (1997) 910–916.

[27] D.S. Zhang, G. Lu, *A comparative study of Fourier descriptors for shape representation and retrieval*, in: Proceedings of the Fifth Asian Conference on Computer Vision (ACCV02), Melbourne, Australia, January 22-25, 2002, pp. 646–651.