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PENDANT EQUITABLE DOMINATION IN GRAPHS

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Abstract

Let G be any graph. An equitable dominating set S in G is called a Pendant equitable dominating set, if $\langle S \rangle$ contains atleast one Pendant equitable vertex. The least cardinality of the pendant equitable dominating set in G is called the Pendant equitable domination number of G, denoted by $\gamma_{pee}(G)$. And also we define Pendant equitable edge domination in graph. In this paper we initiate a study of pendant equitable domination and Pendant equitable edge domination in graph and compute exact value for some well known standard graphs.

Keywords : Equitable Domination number, Pendant Equitable Domination number, Pendant Equitable edge Domination number .

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1. Introduction

Let G be any graph. The paired equitable domination was an interesting topic introduced by Meenakshi [8]. Motivated by this concept we are now introducing the concept of Pendant Equitable Domination in graphs.

Let $G = (V, E)$ be any graph, The order and size of graph G are denoted by n and m respectively. The minimum and maximum of the degree among the vertices of G is denoted by $\delta(G)$ and $\Delta(G)$ respectively. A graph G is said to be regular if $\delta(G) = \Delta(G)$. A vertex of degree zero is called an isolated vertex and a vertex of degree one is called a pendant vertex. for graph terminology, we refer to [3], [4]. For each vertex $v \in V$, the open neighborhood and closed neighborhood of v are denoted by $N(v)$ and $N[v] = N(v) \cup \{v\}$ respectively. If $S \subseteq V$, then $N(S) = \bigcup_{v \in S} N(v)$ and $N[S] = N(S) \cup S$.

Definition 1.1. [5] A subset S of V (G) is a dominating set of G if each vertex u ∈ V − S is adjacent to atleast one vertex in S. The least cardinality of a dominating set in G is called the domination number of G and is usually denoted by γ(G).

Definition 1.2. [9] A dominating set S in G is called a pendant dominating set, if ⟨S⟩ contains at least one pendant vertex. The minimum cardinality of a pendant dominating set is called the pendant domination number, denoted by $\gamma_{pe}(G)$.

Definition 1.3. [6] A dominating set S in G is called a paired dominating set, if ⟨S⟩ has a perfect matching. The minimum cardinality of a paired dominating set is called the paired domination number denoted by $\gamma_{pr}(G)$.

Definition 1.4. [2] A subset S of V (G) is called an equitable dominating set, if for every v ∈ (V − S) there exists a vertex u ∈ S such that uv ∈ E(G) and |deg(u) − deg(v)| ≤ 1.The minimum cardinality of such an equitable dominating set is called equitable domination number of G and is denoted by $\gamma_e(G)$

If $u \in V$ such that $|deg(u) - deg(v)| \geq 2$ for every $v \in N(u)$ then u is in every equitable dominating set such points are called equitable isolates. I_e denotes the set of all equitable isolates. The equitable neighborhood of u denoted by $N_e(u)$ is defined as $N_e(u) = \{v \in V: |v \in N(u), |deg(u) - deg(v)| \leq 1\}$. The maximum and minimum equitable degree of a point in G are denoted by $\Delta_e(G)$ and $\delta_e(G)$ that is $\Delta_e(G) = max_{u \in V(G)}|N_e(u)|$ and $\delta_e(G) = min_{u \in V(G)}|N_e(u)|$. The open equitable neighborhood and closed equitable neighborhood of v are denoted by $N_e(v)$ and $N_e[v] = N_e(v) \cup \{v\}$ respectively. If $S \subseteq V$ then $N_e(S) = \cup_{v \in S} N_e(v)$ and $N_e[S] = N_e(S) \cup S$.

Any graph G with atleast one bridge is called a bridged graph. The n-Barbell graph is the simple graph obtained by connecting two copies of a complete graph K_n by a bridge. The n-Pan graph is the graph obtained by joining a cycle graph C_n to a singleton graph K_1 with a bridge. The ladder graph is a Cartesian product of P_2 and P_n where P_n is a path graph.

We recall the following results required for our study:

Theorem 1.1. [10] A dominating set S of a graph G is a minimal pendant dominating set if and only if for every $u \in S$, one of the following condition holds,

1. u is either an isolate or a pendant vertex of S.
2. Each vertex of $S - \{u\}$ belongs to some cycle in G.
3. There exists a vertex $v \in V - S$ for which $N(v) \cap S = \{u\}$.

By the graph $G = (V, E)$ For any $e \in E$. The open neighborhood and closed neighborhood of e are denoted by $N(e)$ and $N[e] = N(e) \cup \{e\}$ respectively. If $X \subseteq E$, then $N(X) = \cup_{e \in X} N(e)$ and $N[X] = N(X) \cup X$. If $X \subseteq E$ and $e_1 \in X$, then the private neighbor set of e_1 with respect to X is given by $pn[e_1, X] = \{e_2 : N[e_2] \cap X = \{e_1\}\}$. The degree of an edge $e = uv$ of G is defined by $dege = degu + degv - 2$. $\delta'(G)$ $(\Delta'(G))$ is the minimum (maximum) degree among the edges of G. Let $X \subseteq E$, a graph $G - X$ is obtained from the graph G by removing the edges of X. Let H be a subgraph of G and $e \in E$ denotes the distance from e to H. For a graph G, its line graph denoted by L(G) is defined to be a graph whose vertex set will be the edge set of G and two vertices in L(G) are adjacent if corresponding edges are adjacent in G. The dominating set of G and its line graph L(G) are related in the sense that vertices dominated in L(G) represents edges dominated in G [7].

Definition 1.5. [1] A subset X of E (G) is called an equitable edge dominating set if for every f ∈ (E − X) is adjacent to atleast one edge f′ ∈ X such that

$|deg(f) - deg(f')| \leq 1$.The minimum cardinality of such an equitable edge dominating set is called equitable edge domination number of G and is denoted by $\gamma'_e(G)$.

If for any $f \in X$ such that $|deg(f) - deg(f')| \geq 2$ for every $f' \in N(f)$ then f is in every equitable edge dominating set such edges are called equitable isolates. I'_e denotes the set of all equitable edge isolates. The equitable neighborhood of f denoted by $N'_e(f)$ is defined as $N'_e(f) = \{f \in E : f' \in N'_e(f), |deg(f) - deg(f')| \leq 1\}$. The maximum and minimum equitable edge degree of an edge in G are denoted by $\Delta'_e(G)$ and $\delta'_e(G)$ that is $\Delta'_e(G)= \max_{f\in E(G)}|N'_e(f)|$ and $\delta'_e(G)=\min_{f\in E(G)}|N'_e(f)|$. The open equitable neighborhood and closed equitable neighborhood of f are denoted by $N'_e(f)$ and $N'_e[f] = N'_e(f) \cup \{f\}$ respectively. If $X \subseteq E$ then $N'_e(X) = \cup_{f\in X} N'_e(f)$ and $N'_e[X] = N'_e(X) \cup X$.

Definition 1.6. [9] An edge dominating set X of a graph G is called pendant edge dominating set , if the induced subgraph $\langle X \rangle$ contains atleast one pendant edge. The minimum cardinality of a pendant edge dominating set of G is called pendant edge domination number, denoted by $\gamma'_{pe}(G)$.

2. The Pendant Equitable Domination number of a Graph

Definition 2 .1. Let S be an equitable dominating set in G. Then S is called a pendant equitable dominating set if $\langle S \rangle$ contains atleast one pendant vertex. The pendant equitable dominating set of minimum cardinality is called the pendant equitable domination number, denoted by $\gamma_{pee}(G)$. Any pendant equitable dominating set of cardinality $\gamma_{pee}(G)$ is called a γ_{pee}-set.

Example 2.1

1. For a graph G of order n with $\Delta_e(G) = n - 1$, $\gamma_{pee}(G) = 2$.
2. For a star graph $K_{1,2}$, $\gamma_{pee}(K_{1,2}) = 2$

Observation 2.1. The new domination parameter is not defined for a totally disconnected graph, Hence, throught this paper , by a graph we assume that G has atleast one edge.

Observation 2.2. If there exist's a γ_e -set S of G such that $\langle S \rangle$ has an isolated vertex , then $\gamma_{pee}(G) = \gamma_e(G)$ or $\gamma_e(G) + 1$

Theorem 2.1. Let G be a path with $n \geq 2$ verices, Then

$$\gamma_{pee}(G) = \begin{cases} \frac{n}{3} + 1 & if\ n \equiv 0(mod 3) \\ \left\lceil \frac{n}{3} \right\rceil, & if\ n \equiv 1(mod 3) \\ \left\lceil \frac{n}{3} \right\rceil + 1 & if\ n \equiv 2(mod 3) \end{cases}$$

Proof. Let G be a path and let $V(G) = \{v_1, v_2, ..., v_n\}$. We consider the following possible cases here:

Case 1: Suppose $n \equiv 0\ (mod\ 3)$. Then $n = 3k$, for some integer $k > 0$. Then the set $S = \{v_1, v_{3i-1} | 1 \leq i \leq k\}$ will be a pendant equitable dominating set of G. Hence,

$\gamma_{pee}(G) \leq |S|$. i.e., $\gamma_{pee}(G) \leq \frac{n}{3} + 1$. On the other hand, we have $\gamma_e(G) = \frac{n}{3}$ and any least dominating set of G contains only vertices of degree zero.

Thus $\gamma_{pee}(G) \geq \frac{n}{3} + 1$. Therefore, $\gamma_{pee}(G) = \frac{n}{3} + 1$.

Case 2: Suppose $n \equiv 1(mod\ 3)$. Then it is easy to check that any $\gamma_e -$ set in G contains a pendant equitable vertex. Hence any $\gamma_e -$ set in G itself a pendant equitable dominating set in G. Therefore, $\gamma_{pee}(G) = \gamma_e(G) = \left\lceil \frac{n}{3} \right\rceil$.

Case 3: Proof of this case is analogous to Case 1.

Theorem 2.2. Let G be a cycle with $n \geq 3$ verices, Then

$$\gamma_{pee}(G) = \begin{cases} \frac{n}{3} + 1 & if\ n \equiv 0(mod3) \\ \left\lceil \frac{n}{3} \right\rceil, & if\ n \equiv 1(mod3) \\ \left\lceil \frac{n}{3} \right\rceil + 1 & if\ n \equiv 2(mod3) \end{cases}$$

Theorem 2.3. For a wheel graph of $n \geq 6$, $\gamma_{pee}(G) = 2 + \left\lceil \frac{n-2}{3} \right\rceil$.

Theorem 2.4. Let G be a Barbel graph , Then $\gamma_{pee}(G) = 2$.

Theorem 2.5. Let G be a Pan graph, Then $\gamma_{pee}(G) = 3 + \left\lceil \frac{n-4}{3} \right\rceil$.

Theorem 2.6. For a Ladder graph of $n \geq 4$, $\gamma_{pee}(G) = 2 + \left\lceil \frac{n-1}{3} \right\rceil$.

Theorem 2.7. Let G be a disconnected graph with components $G_1, G_2, \ldots G_m$. Then

$$\gamma_{pee}(G) = min_{1 \leq i \leq m}\{\gamma_{pee}(G_i) + \sum_{j=1, j\neq i}^{m} \gamma_e(G_j)\}$$

Proof. We prove this result by using mathematical induction. Since G is disconnected , $m \geq 2$. Suppose $m = 2$. Then $G = G_1 \cup G_2$. Let S_1, S_2 be the γ_{pee}- sets of G_1 and G_2 respectively. Then $S_1 \cup S_2'$ and $S_2 \cup S_1'$ are pendant equitable dominating sets in G, where S_i' denotes the γ_e- set of G_i , $i = 1,2$.
Therefore $\gamma_{pee}(G) \leq min\{\gamma_{pee}(G_1) + \gamma_e(G_2),\ \gamma_{pee}(G_2) + \gamma_e(G_1)\}$.

On the other hand, Let S be any pendant equitable dominating set in G. Then S has to dominate both $V(G_1)$ and $V(G_2)$ and $\langle S \rangle$ should contain atleast one pendant equitable vertex. Moreover , the set S should contain pendant equitable dominating set of G_1 or G_2. Otherwise $\langle S \rangle$ contains no pendant equitable vertex which is a contradiction. This contradiction shows that $|S| \geq min\{\gamma_{pee}(G_1) + \gamma_e(G_2),\ \gamma_{pee}(G_2) + \gamma_e(G_1)\}$. Hence $|S| = min\{\gamma_{pee}(G_1) + \gamma_e(G_2),\ \gamma_{pee}(G_2) + \gamma_e(G_1)\}$ proving the result for m = 2.

Next, Suppose $m \geq 3$ and assume that the result is true for $m = k - 1$. Let G be any graph with components $G_1, G_2, \ldots G_{k-1}, G_k$. Let G' be a graph with $k - 1$ components ,say $G_1, G_2, \ldots G_{k-1}$. Then from the induction hypothesis we have
$\gamma_{pee}(G') = min_{1 \leq i \leq k-1}\{\gamma_{pee}(G_i) + \sum_{j=1, j\neq i}^{m} \gamma_e(G_j)\}$. Now we have $G = G' \cup G_m$. Now from the case $m = 2$, we obtain that $\gamma_{pee}(G) = min_{1 \leq i \leq k}\{\gamma_{pee}(G_i) + \sum_{j=1, j\neq i}^{m} \gamma_e(G_j)\}$.

Therefore the result for m = k and hence true for any positive integer m. Thus we have $\gamma_{pee}(G) = min_{1\le i\le r}\{\gamma_{pee}(G_i) + \sum_{j=1, j\ne i}^{m} \gamma_e(G_j)\}$.

Theorem 2.8. An equitable dominating set S is a minimal pendant equitable dominating set if and only if for each vertex $u \in S$ one of the following conditions holds.

1. u is either an equitable isolate or a pendant equitable vertex of S.
2. each equitable vertex of $S - \{u\}$ belongs to some cycle in G.
3. There exists a vertex $v \in V - S$ for which $N_e(v) \cap S = \{u\}$.

Proof. Let S be a minimal pendant equitable dominating set of G. Then for every vertex $u \in S$, The set $S - \{u\}$ is not a pendant equitable dominating set in G. So, we have following two possible cases

Case 1: Suppose $S - \{u\}$ is not an equitable dominating set of G. Then from theorem 1.1, it follows that either u is an equitable isolated vertex of S or there exists a vertex $v \in V - S$ such that $N_e(v) \cap S = \{u\}$.

Case 2: Suppose $S - \{u\}$ is an equitable dominating set , contains no pendant equitable vertex. Then each vertex of $S - \{u\}$ is either an isolated vertex of S or has degree atleast 2. If all vertices of $S - \{u\}$ are isolated vertices of S, Then u will be the pendant equitable vertex of S. For if each vertex has degree atleast two, Then each vertex of $S - \{u\}$ belongs to some cycle in G.

Conversely, assume that S is a pendant equitable dominating set satisfying the above stated three conditions. For the purpose of contradiction, assume S is not a minimal pendant equitable dominating set. Then there is a vertex $u \in S$ such that $S - \{u\}$ is also a pendant equitable dominating set. Hence, u must be adjacent to at least one vertex $v \in S - \{u\}$, so $\{u\}$ is not an isolate of S and if v is pendant equitable vertex of $S - \{u\}$, Then v is not a pendant equitable vertex of S. Hence condition (1) fails to hold. Clearly, condition (2) does not hold, since $S - \{u\}$ contains pendant equitable vertex. Finally, every vertex in $V - S$ must be adjacent to atleast one vertex in $S - \{u\}$.so the condition (3) fails to hold. Hence, none of the above conditions holds, Which is a contradiction to our assumption. So, this contradiction proves that atleast one of the condition should hold.

Corollary 2.1. If S is a pendant equitable dominating set of G, Which is minimal with respect to pendant equitable domination, then there exist's a vertex $v \in S$ such that $S - \{v\}$ is a minimal pendant equitable dominating set of G.

Proof. Let G be any connected graph and $S \subseteq V(G)$ be a minimal equitable dominating set of G. Then $V - S$ is also an equitable dominating set of G. But, generally this is not true in the case of pendant equitable dominating set. Next theorem gives the condition under which complement of a pendant equitable dominating set is an equitable dominating set.

Theorem 2.9. Let G be a graph with $n \ge 3$ vertices. Then complement $V - S$ of any pendant equitable dominating set is a pendant equitable dominating set , If S contains no induced path P_3.

Proof. Suppose S is any pendant equitable dominating set in G. If S contains no induced path P_3 , then every vertex in G will be either a vertex of S adjacent to some vertex in S.

Therefore, $V - S$ will be a pendant equitable dominating set in G.

Theorem 2.10. Let G be any graph Then $\gamma_{pee}(G) = \gamma_e(G)$ if and only if G contains a $\gamma_e -$ set, which is neither an independent set in G nor each vertex of S has degree 0 or belongs to some cycle in S.

Proof. Let G be any graph. If G is acyclic then we are done. Assume that G is a cyclic graph and $\gamma_{pee}(G) = \gamma_e(G)$. On contrary, Suppose that every $\gamma_e -$ set S in G is either independent or each vertex of S has degree zero or belongs to cycle in S, then $\gamma_{pee} -$ set will be obtained by adding one vertex $u \in V - S$ to a $\gamma_e -$ set in G. Hence $\gamma_e(G) < \gamma_{pee}(G)$, a contradiction. Conversely , if every $\gamma_e -$ set in G fails to satisfy the above stated conditions, then $\langle S \rangle$ must contain atleast one pendant equitable vertex. Therefore S itself a pendant equitable dominating set and so $\gamma_{pee}(G) = \gamma_e(G)$.

3. Bounds for $\gamma_{pee}(G)$

Proposition 3.1. Let G be any graph with n vertices. Then $2 \le \gamma_e(G) \le \gamma_{pee}(G) \le n$. Further $\gamma_{pee}(G) = 2$ if and only if G is a complete graph contains an edge of degree at least $n - 2$.

Proof. Let G be any graph with n vertices. Then inequalities are obvious. Suppose $\gamma_{pee}(G) = 2$. Let $S = \{u, v\}$ be a $\gamma_{pee} - set$ in G. Then u and v must be adjacent and every vertex in $V - S$ must be adjacent to either u or v. Thus, the degree of the edge $e = uv$ must be atleast $n - 2$. Converse is obvious.

Proposition 3.2. Let G be any graph with n vertices. Then $\gamma_{pee}(G) = n$ if and only if G is a path $P_{2.}$

Theorem 3.1. Let G be a connected graph of order n. Then $\gamma_{pee}(G) = n - 1$ if and only if G is one of the graphs P_3 , K_3 .

Proof. First , Assume $\gamma_{pee}(G) = n - 1$. Suppose there exist's two adjacent vertices u and v in G of degree atleast two. Then the set $S = V - \{u, v\}$ is an equitable dominating set in G. Suppose S contains no edge, then S should have exactly one vertex. Since otherwise $\gamma_{pee}(G) \le n - 2$. If $S = \{w\}$, Then $G \cong K_3$. Suppose S contains an edge, then S will be a pendant equitable dominating set in G. Therefore $\gamma_{pee}(G) \le n - 2$, a contradiction. Hence either u or v must be a pendant vertex in G and so $G \cong K_{1,n-1}$. But we have $\gamma_{pee}(K_{1,n-1}) = 2$, from which it follows that $n = 3$ showing that $G \cong P_3$.

Let $\mathbb{G}$ be the collection of graphs of following types. A cycle , path and a complete graph each of order 4 and a path , cycle of order 5.

Theorem 3.2. Let G be a connected graph of order n. Then $\gamma_{pee}(G) = n - 2$ if and only if $G \in \mathbb{G}$.

Proof. Suppose $\gamma_{pee}(G) = n - 2$ and S is a $\gamma_{pee} - set$, then $\langle V - S \rangle$ either K_2 or $\overline{K_2}$.

We first prove that $n \le 5$. Clearly $V - S$ will be an equitable dominating set of G. Now if $\langle V - S \rangle = K_2$, then $V - S$ is itself a pendant equitable dominating set of G and hence $n \le 5$. Assume $\langle V - S \rangle = \overline{K_2}$ and assume $V(\overline{K_2}) = \{u, v\}$. Suppose u and v have atleast two neighbors in S. Then the set $S' = (S - N_e(u, v)) \cup \{u, v\}$ is an equitable dominating set in G of cardinality less than $n - 3$. Hence G contains a pendant equitable dominating set of size less than $n-2$, a contradiction. Therefore each vertex in $V-S$ has atmost one neighbor in S, from which it follows that $|S| \leq 3$ and consequently $n \leq 5$. Thus G must be one among the graphs in $\mathbb{G}$. Converse is obvious.

Theorem 3.3. Let G be any graph, Then $\left\lceil \frac{n}{1+\Delta_e(G)} \right\rceil \leq \gamma_{pee}(G) \leq n - \Delta_e(G) + 1$.

Proof. Let G be any graph. Since any vertex in G can dominate at most $\Delta_e(G)+1$ vertices including itself, it follows that $\left\lceil \frac{n}{1+\Delta_e(G)} \right\rceil \leq \gamma_{pee}(G)$. On the other hand, let u be a vertex of maximum degree $\Delta_e(G)$ in G. Then for any vertex $v \in N_e(u)$, the set $(V - N_e(u)) \cup \{v\}$ will be a pendant equitable dominating set in G. Therefore $\gamma_{pee}(G) \leq n - \Delta_e(G) + 1$.

Theorem 3.4. For any graph G such that $G \text{ and } \bar{G}$ have no equitable isolates. Then $\gamma_{pee}(G) + \gamma_{pee}(\bar{G}) \leq n+2$.

Proof. Since $G \text{ and } \bar{G}$ have no equitable isolates, Then from [11] , we have $\gamma_e(G) \leq \left\lfloor \frac{n}{2} \right\rfloor$ and $\gamma_e(\bar{G}) \leq \left\lfloor \frac{n}{2} \right\rfloor$. Since $\gamma_{pee}(G) \leq \gamma_e(G) + 1$ always, it follows that $\gamma_{pee}(G) \leq \left\lfloor \frac{n}{2} \right\rfloor + 1$ and $\gamma_{pee}(\bar{G}) \leq \left\lfloor \frac{n}{2} \right\rfloor + 1$. Therefore $\gamma_{pee}(G) + \gamma_{pee}(\bar{G}) \leq 2\left\lfloor \frac{n}{2} \right\rfloor + 2 \leq n+2$. Hence , For any graph G have no equitable isolates , we have $\gamma_{pee}(G) + \gamma_{pee}(\bar{G}) \leq n+2$.

4. The Pendant Equitable Edge Domination number of a Graph

Consider a graph $G = (V, E)$ of order n and size m. A set F be an equitable edge dominating set in G. Then F is called a pendant equitable edge dominating set (PEED set, in short) if each edge not in F is incident to some edge in F and edge induced subgraph $\langle F \rangle$ contains an edge of degree one. The minimum number of edges in a PEED set of G is called the PEED number, denoted by $\gamma'_{pee}(G)$.

Example 4.1. consider a diamond graph as shown in figure 1. Clearly $F = \{(xy), (xz)\}$ will be an equitable edge dominating set of G. Let $e = xy$, degree of e in F is $deg e = deg x + deg y - 2 = 1$. Hence $\gamma'_{pee}(G) = 2$.

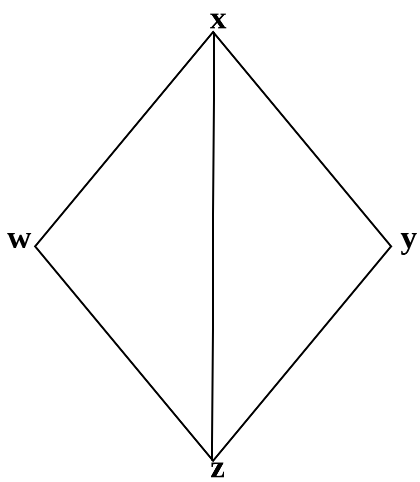

Figure 1. Diamond graph

Remark 4.1. From the above definition it is clear that any PEED set must have cardinality at least two and the parameter is defined only for graphs containing P_3. In other words

$\gamma'_{pee}(G)$ is defined only for connected graphs of order at least three. For other graphs , we define $\gamma'_{pee}(G) = 0$.

Further, it is easy to note that $E(G)$ is a unique maximum PEED set, if graphs contains a pendant edge or else there may be more than one PEED set for a given graph of cardinality $k < m$. A complete graph K_n contains m such PEED set of maximum cardinality. Throughout this article, for our convenience, by a graph we mean a connected graph of order atleast three, unless otherwise stated.

Proposition 4.1.

1. Let G be a cycle of order $n \geq 3, then\ \gamma'_{pee}(G) = \left\lceil \frac{n+2}{3} \right\rceil$.

2. Let G be a star graph $K_{1,n-1}. for\ n \geq 3$, Then $\gamma'_{pee}(G) = 2$.

3. Let G be a pan graph. Then $\gamma'_{pee}(G) = 2 + \left\lceil \frac{n-4}{3} \right\rceil$.

4. Let G be a path of order $n \geq 3$,Then $\gamma'_{pee}(G) = \left\lceil \frac{n+1}{3} \right\rceil$.

5. For a complete graph of order $n \geq 3$, Then $\gamma'_{pee}(G) = 2$.

6. For a complete bipartite graph $K_{m,n}(m, n \geq 2)$, Then $\gamma'_{pee}(G) = min\{m, n\}$.

7. For any two connected graphs G and H $\gamma'_{pee}(G \cup H) = \gamma'_{pee}(G) + \gamma'_{pee}(H)$.

From the definition of line graph of a graph it is clear that PED set of $L(G)$ will be the PEED set of G.We have the following characterization for minimal PEED set of G.

Theorem 4.1. An equitable edge dominating set F is minimal PEED set if and only if for each edge $e \in F$ one of the following condition holds.

1. e is either an equitable isolate edge or a pendant equitable edge of F.
2. Each equitable edge of $F - \{e\}$ belongs to some cycle in G.
3. There exists an edge $f \in E - F$ for which $N_e(f) \cap F = \{e\}$.

Let F be an equitable edge dominating set of G. Then $E - F$ is also an equitable edge dominating set, the same is not true for PEED set. For instance, consider a path P_5 of order 5. Then for any PEED set F of P_5, its complement $E(P_5) - F$ will be a disconnected graph which contains no pendant equitable edge. Hence, edge complement of PEED set is not a PEED set.

Theorem 4.2. For any graph G of size m, we have $\gamma'_{pee}(G) \leq \left\lceil \frac{m}{2} \right\rceil + 1$. Equality holds if G is either a path $P_3, P_4\ or\ C_4$ or union of these graphs.

We shall have Nardhaus-Gaddum type results from the above theorem, which stated in the next result.

Theorem 4.3. For any connected graph of order m, we have

1. $\gamma'_{pee}(G) + \gamma'_{pee}(\bar{G}) \leq 2\left(\left\lfloor\frac{m}{2}\right\rfloor + 1\right)$.
2. $\gamma'_{pee}(G).\gamma'_{pee}(\bar{G}) \leq 2\left(\left\lfloor\frac{m}{2}\right\rfloor + 1\right)^2 + m + 1$.

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