A result On Rings of type *(p,q,r) = (r,q,p)*

K. Subhashini[1], G.Venkatarami Reddy[2], K.Raghavendra[3]

[1.] Professor, Department of Humanities & Basic Sciences, subhashini.gprec@gmail.com
[2,3] Assistant Professor,Department of Humanities & Basic Sciences
2.venkataramireddy.bs@gprec.ac.in ; 3kraghavendra.bs@gprec.ac.in
G.Pulla Reddy Engineering College, Kurnool-518002, Andhra Pradesh, INDIA

Abstract

A non-associative ring with *(p,q,r) = (r,q,p)* is called as an antiflexible ring. Some other interesting non-associative rings are Assosymmetric, Accessible, Alternative, Novikov and flexible rigs. By nature, Nucleus N_u is an associator whereas Center C_e is a commutator. Moreover nucleus is not equal to center in any non-associative rings. Let us take a non-associative 2 divisible simple ring A with *(p,q,r) = (r,q,p)* which is neither associative nor commutative. In this paper it will be established first that " Nucleus of A commutes with every associator of A" . With the help of this property, a well known another property " Every associator is in the nucleus of Antiflexible ring" will be obtained and finally $N_u = C_e$ will be proved in simple antiflexible rings

I. INTRODUCTION:

A ring A is said to be antflexible ring if it satisfies the following two identities:

$$D(p,q,r) = (p,q,r) - (r,q,p) = 0 \qquad \text{.......... (1)}$$

And

$$(p,p,p) = 0 \qquad \text{..........(2)}$$

Identity (1) is antiflexible whereas (p,q,p) = 0 is flexible. Jordan and Lie rings are flexible.

Linearization of (2) with the aid of (1), one can reach the identity

$$E(p,q,r) = \sum (x,y,z) = (p,q,r) + (q,r,p) + (r,p,q) = 0 \qquad \text{.........(3)}$$

Where the associator (p,q,r) = pq.r– p.qr for all the elements p,q,r of A. A ring A to be a 2-divisible ring if *2x = 0* implies *x = 0* for all x in *A*. Let *I* be a subset of a ring *A*, *I* becomes an ideal of a ring, if *I* is a subgroup and the products *xa*, *ax* belong to *I*, $\forall x \in I$ and $\forall a \in A$. This paper is systematically presented in five sections. Section II gives a brief survey of research on non-associative rings which are related to the rings of type *(p,q,r) = (r,q,p)*. The necessary equations like Jacobi and Techmullar identities are presented in the Methodology section. In this section Notation and definitions of Nucleus and Center are mentioned. Lemmas and main theorem are proved in the Experimental section and finally the conclusion section is followed.

II. LITERATURE WORK

In [3] , the authors assumed Weak Novikov identity (s,p,qr) = q(s,p,r) in semi prime Flexible rings and further they have proved that those rings are associative. Strong novikov identity p*(qr)* = *q(pr)* was derived in prime antiflexible rings when weak Novikov identity holds [5].In a simple Assosymmetric rings, Nucleus coincides with center was established in [2]. In [4] same result was asserted when a ring is Prime assosymmetric with idempotent *e*. The important identity ((p,q,r), s,t) = 0 was proved in [1]. Same result was extended to middle nucleus and right nucleus by many researchers in non-associtive rings.

III. METHODOLOGY

Throughout this paper A will be referred to as a 2 divisible antiflexible ring. For any non associative ring the nucleus Nu and the center Ce are given by

Nu = { n : n € R, (n,A,A) = (A,n,A) = (A,A,n) = 0} ,

Ce = { c : c € A, (c,A) = 0}

The following two identities hold good in any arbitrary ring

M(s,p,q,r) = (sp,q,r) – (s,pq,r) + (s,p,qr) – s(p,q,r) – (s,p,q)r = 0 ....(4)

(pq,r) = p(q,r) + (p,r)q + (p,q,r) + (r,p,q) – (r,p,q) ....(5)

In antiflexible rings, when (1) and (3) are used, equation (5) becomes

(pq,r) = p(q,r) + (p,r)q – 2 (q,r,p) ....(6)

IV. EXPERIMENTAL RESULTS

Lemma 1 : In a 2 divisible antiflexible ring Nucleus of A commutes with every

associator of A i.e (Nu,(A,A,A)) = 0

Proof :Let us consider the different combinations of (4)

M(s,p,q,r) – M(p,q,s,r) + M(q,r,s,p) – M(r,s,p,q) = 0 gives

(s,(p,q,r)) – (p,(q,r,s)) + (q,(r,s,p)) – (r,(s,p,q)) = 0 ....(7)

For n € Nu, put s = *n* in (7), we get

(n,(p,q,r)) = 0 which is equivalent to (Nu,(A,A,A)) = 0.

Lemma 2 : : In a 2 divisible antiflexible ring, (A,Nu).(A,A,A) = 0

Proof : Commute the identity (4) with the element *n* of Nu and by using Lemma1, one can obtain

((s,p,q) r,n) = - (s(p,q,r), n) ....(8)

Replace p,q by arbitrary associators x,y of A and r by n in (6) and using lemma1,

It will be (xy,n) = 0. If x = (a,b,c) then ((a,b,c) y,n) = 0. Apply this to equation (8), one can get

(a (b,c,y), n) = 0 ....(9)

Apply (6) to (9) then apply lemma1 and n € N gives

(a,n) (b,c,y) = 0 ....(10)

Let I be the set generated by ((A,A,A),A,A). Since (A,A,A) ⊂ N and from equations (3) and (4) it is clear that I be the ideal generated by ((A,A,A),A,A). If I be the ideal generated by the set (A,N) then from (10) we have I ⊆ N.

Theorem: If A is a two divisible simple antiflexible ring then either Nu = Ce or associative.

Proof : If y is an arbitrary associator, then from (10)

(a,n) I = 0.

Since A is simple either I= 0 or I= A.

If I = 0 which implies ⇒ (b,c,y) = (A,A,(A,A,A)) = 0. then. From (1), it gives ((A,A,A),A,A)= 0. Associators are in the left nucleus. Hence it is known from [1] that R is associative.

If I= A which implies (a,n) I= 0. This implies (I,Nu) = 0. Hence n € C. Therefore Nu = Ce.

V. CONCLUSION

Nucleus is not equal to the center in any non-associative ring. But an equality relation between N_u and C_e was brought in a simple antiflexible ring. Without any additional assumptions, the main result was proved. This result will be proved in some other non-associative rings if simplicity is replaced by prime. Because of the non-associative property, this type of rings are used in designing s-boxes in cryptosystem

REFERENCES

1. Chen-Te-Yen, Simple rings of characteristic not 2 with associators in the Left Nucleus are Associative, Tamkang Journal of mathematics, Vol.33, No. 1, Spring 2002, pp 93-95.
   DOI: https://doi.org/10.5556/j.tkjm.33.2002.309
2. D.Bharathi and M.Munirathnam, Simple Assosymmetric Rings, Asian European Journal of Mathematics, Vol. 6, No. 4 (2013) 1350047-1-4.
3. E.Kleinfeld and Harry F.Smith ,SemiprimeflexbleWeakly Novikov rings are associative, Communications in Algebra, 23(13), 5077-5083 (1995).
4. K.Jayalakshmi and K.Madhavi Devi, Nucleus Equals Center in Assosymmetric Rings, Int.J.Modern Engineering Research (IJMER), Vol. 3, Issue. 3, May-June 2013 pp 1815-1816
5. M.Hema Prasad and D.Bharathi, Antiflexible rings with weak Novikov identity, Int.J. mathematics Trends and Technology, Vol. 14, No. 2, Oct 2014 "