Use of RDTM (Reduced Differential Transform Method) in solution of fingero-instability phenomenon with effect of magnetic field in porous media

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Abstract:

This paper studies the efficiency of the Reduced Differential Transform Method (RDTM) in simulating fingering instability in the flow of two immiscible fluids water and oil through a homogeneous porous medium under the influence of a vertically downward variable magnetic field. The problem is of direct relevance to secondary oil recovery processes, where two of the dominant mechanisms, fingering and imbibition, play very important roles in influencing the displacement efficiency of the fluid. The use of a variable magnetic field is thought to increase the velocity of the injected conductive fluid, thus affecting the flow dynamics. The result of mathematical formulation is followed by a non-linear partial differential equation (PDE), the reduced differential transform method (RDTM) is, used to suitable initial and boundary conditions. RDTM is demonstrated to be a very good analytical method that offers a basic but accurate method for the solution of hard nonlinear systems without resort to computationally intensive numerical methods. The study offers quantitative results as well as graphical schemes to illustrate the physical behavior of the fluid system under the influence of the magnetic field. The physical implications of the solution are explained along with numerical data and graphical representations.

Keywords:

fingero-instability, Magnetic field, Two Phase flow, Reduced differential transform method (RDTM).

Introduction:

Using previous studies the fingero-instability phenomenon in a horizontal direction and by using magnetic field effect with gravitational force, has been incorporated. Many researchers have researched the phenomenon from various perspectives, in the past few decades only a limited number of them have taken into account the magnetic field effect, Different authors have historically treated the fingero-imbibition phenomenon in different manners. Mehta et. Al.(1977) [1],[2] was among the early contributors who used the matched asymptotic expansion method to solve the governing equations for fingero-imbibition under a magnetic field. His research gave valuable information regarding the behavior of the wetting phase saturation and its spatial distribution, which is the average area covered by schematic fingers. These results paved the way for more sophisticated mathematical models and simulation aids.

Swaroop and Mehta (2002) [3] works further by using the finite element method, their numerical solution provided detailed insight into localized behaviors but also emphasized the requirement of complementary analytical methods for capturing larger trends and parametric dependencies.

Vyas et al. (2011) [4] offered another insight by considering horizontal flow arrangements. They derived a power series solution for magnetic field-influenced saturation of the injected phase and determined that the application of a magnetic field increases the levels of saturation of the conductive fluid compared to the non-magnetic case under the same conditions. Their findings confirmed the positive contribution of magnetic fields towards enhanced displacement efficiency yet only applied to horizontal cases.

V. Gohil and R. Meher have done the work in heterogenous saturation profile using Homotopy analysis method [5]. K. K. Patel et al. have studied the counter-current model using Homotopy series methodology [6]. X. You et al. have studied the low permeability reservoir by using HAM [7].

Continuing this, presently we focus on the consideration with the magnetic field effect in a vertical downward direction for the fingero-instability phenomenon in homogeneous porous medium. The purpose is to determine the solution for the fingero-instability phenomenon on the saturation of the injected fluid (water) in the cross-sectional area of the fingers of average size under the combined effects of gravitational and magnetic field effects, using the reduced differential transform method (RDTM).

Here one dimension fluid flow with the injected fluid is water while the native fluid is oil in homogeneous medium, the small part as cylindrical porous matrix as common interface ($z = 0$) for the saturation of injected fluid (water).

Let us choose a length L vertical finite cylindrical piece of homogeneous porous matrix which is fully permeated with oil. An adjacent formation of water containing conductive magnetic particles, as shown in Figure (1) tagged as imbibition face (z = 0). For 1 dimensional, It has been considered as fingers of rectangular shape as shown [8], [9] in figure 2, figure (2 - a) and its moderate length is considered as shown in figure (2 - b).

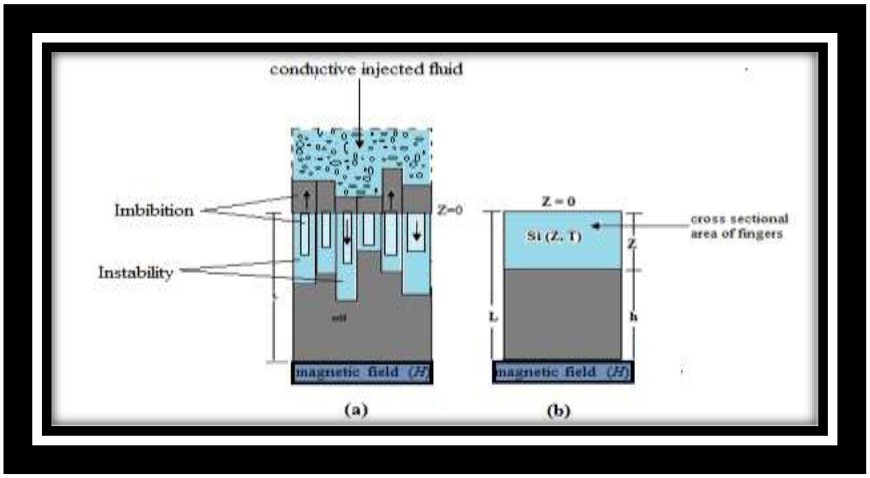

Figure 1: fingero-instability in vertically downward porous media added magnetic field

Under effect of “g” and “H” the natural flow of water with magnetic field into the medium, displaces the oil in a downward direction. Also, due to external injecting force, fingers will occur by water and shoots through the oil formation. Saturation of the medium initiated by "P_c", "g" and with "H" effect gives rise to displacement process in which water displaces oil [10].

Mathematical Formulations:

Under “g” and “H” effects [11],[12], the velocities is given by is as follows:

$$V_i = -\left(\frac{K_i}{\mu_i}\right) K \left(\frac{\partial P_i}{\partial z} + \rho_i\, g\right) + \frac{\mu H}{4\pi}\frac{\partial H}{\partial z} \tag{1}$$

$$V_n = -\left(\frac{K_n}{\mu_n}\right) K \left(\frac{\partial P_n}{\partial z} + \rho_n\, g\right) \tag{2}$$

Nomenclature:

K = permeability of given medium.	μ_i□kinematic viscosity of water.
□□□ Permeability of H.	ρ_n= density of oil.
K_i= relative permeability of water	μ_n= kinematic viscosity of oil.
K_n =relative permeability of oil.	g = acceleration due to gravity.

ρ_i= density of water	V_i= injected fluid (water) and V_n= native fluid (oil)
Pc =Capillary pressure	H =magnetic field

The Continuity equation [13] is as below:

$$P\frac{\partial S_i}{\partial t}+\frac{\partial V_i}{\partial z}=0 \tag{3}$$

Where, S_i = saturation of water, P = porosity of the medium.

For the capillary pressure (Pc) [14] ,

$$P_c(S_i)=P_n-P_i \tag{4}$$

Standard relations for relative permeability of water and oil, given [12] as follows:

$$k_i = S_i , k_n = 1 - \alpha S_i \tag{5}$$

Another relation [15] is,

$$Pc = -\beta Si \text{ , Where } \beta \text{ is constant} \tag{6}$$

with external magnetic field effect, the size of fingers can be extend up to the end of porous matrix at $z=L$,in that previous fingers are developed of very small size up to depth $z=l$ which is very near to common interface.

Also

$$V_i+V_n=0 \tag{7}$$

From equation (1) and (2) and (7),

$$\left(\frac{K_i}{\mu_i}\right)K\left(\frac{\partial P_i}{\partial z}+ \rho_i g\right)+\left(\frac{K_n}{\mu_n}\right)K\left(\frac{\partial P_n}{\partial z}+ \rho_n g\right)-\frac{\mu H}{4\pi}\frac{\partial H}{\partial z}=0 \tag{8}$$

Using equation (4) and (8),

$$\frac{\partial P_i}{\partial z}\left(\frac{K_i}{\mu_i}+\frac{K_n}{\mu_n}\right)+\frac{K_n}{\mu_n}\frac{\partial P_c}{\partial z}=-\left(\frac{K_i}{\mu_i}\rho_i+\frac{K_n}{\mu_n}\rho_n\right)g+\frac{\mu H}{4\pi K}\frac{\partial H}{\partial z} \tag{9}$$

Hence,

$$\frac{\partial \mathrm{Pi}}{\partial z} = -\left[\frac{\left(\frac{K_i}{\mu_i}\rho_i + \frac{K_n}{\mu_n}\rho_n\right)g - \frac{\mu H}{4\pi K}\frac{\partial H}{\partial z} + \frac{K_n}{\mu_n}\frac{\partial P_c}{\partial z}}{\frac{K_i}{\mu_i} + \frac{K_n}{\mu_n}}\right] \quad (10)$$

From equation (1) and (10),

$$V_i = -\left(\frac{\mathrm{Ki}}{\mu \mathrm{i}}\right) K\left[\frac{-\left(\frac{K_i}{\mu_i}\rho_i + \frac{K_n}{\mu_n}\rho_n\right)g + \frac{\mu H}{4\pi K}\frac{\partial H}{\partial z} - \frac{K_n}{\mu_n}\frac{\partial P_c}{\partial z} + \rho_i\, g}{\frac{K_i}{\mu_i} + \frac{K_n}{\mu_n}}\right] + \frac{\mu H}{4\pi}\,\frac{\partial H}{\partial z}$$

$$V_i = -\left(\frac{K_i}{\mu_i}\right) K\left[\frac{\left(\frac{K_n}{\mu_n}(\rho_i - \rho_n)g\right) + \frac{\mu H}{4\pi K}\frac{\partial H}{\partial z} - \frac{\mathrm{Kn}}{\mu \mathrm{n}}\frac{\partial \mathrm{Pc}}{\partial z}}{\frac{K_i}{\mu_i} + \frac{K_n}{\mu_n}}\right] + \frac{\mu H}{4\pi}\,\frac{\partial H}{\partial z} \quad (11)$$

From equation (3) and (11), we get

$$P\frac{\partial S_i}{\partial t} + \frac{\partial}{\partial z}\left[K\left(\frac{\frac{K_i}{\mu_i}\frac{K_n}{\mu_n}}{\frac{K_i}{\mu_i} + \frac{K_n}{\mu_n}}\right)\frac{\partial P_c}{\partial S_i}.\frac{\partial S_i}{\partial z}\right] - \frac{\partial}{\partial z}\left[K(\rho_i - \rho_n)g\left(\frac{\frac{K_i}{\mu_i}\frac{K_n}{\mu_n}}{\frac{K_i}{\mu_i} + \frac{K_n}{\mu_n}}\right)\right] + \frac{\partial}{\partial z}\left[\left(1 - \frac{K_i}{\mu_i}\right)\frac{\mu H}{4\pi}\,\frac{\partial H}{\partial z}\right] \quad (12)$$

Due to one dimensional fingero-instability phenomenon under the magnetic field effect, n = -1 [16],

$$H = \lambda z \quad (13)$$

and

$$P\frac{\partial S_i}{\partial t} + \frac{\partial}{\partial z}\left[K\left(\frac{\frac{K_i}{\mu_i}\frac{K_n}{\mu_n}}{\frac{K_i}{\mu_i} + \frac{K_n}{\mu_n}}\right)\frac{\partial P_c}{\partial S_i}.\frac{\partial S_i}{\partial z}\right] - \frac{\partial}{\partial z}\left[K(\rho_i - \rho_n)g\left(\frac{\frac{K_i}{\mu_i}\frac{K_n}{\mu_n}}{\frac{K_i}{\mu_i} + \frac{K_n}{\mu_n}}\right)\right] + \frac{\partial}{\partial z}\left[\left(1 - \frac{K_i}{\mu_i}\right)\frac{\mu\lambda^2}{4\pi}\right] = 0 \quad (14)$$

Scheidegger [17] has related,

$$\left[\frac{\frac{K_i}{\mu_i}\frac{K_n}{\mu_n}}{\frac{K_i}{\mu_i} + \frac{K_n}{\mu_n}}\right] \approx \frac{K_n}{\mu_n} = \frac{1 - \alpha S_i}{\mu_n}, K_i = S_i \quad (15)$$

Putting values of equation (6) and (14) in equation (14), we have

$$P\frac{\partial S_i}{\partial t} = \frac{K\beta}{\mu_n}\frac{\partial}{\partial z}\left[(1 - \propto S_i)\frac{\partial S_i}{\partial z}\right] + \frac{K(\rho_i - \rho_n)g}{\mu_n}\frac{\partial}{\partial z}(1 - \propto Si) + \frac{\mu\lambda^2}{4\pi\mu_i}\frac{\partial S_i}{\partial z} \quad (16)$$

Using given conditions,

$S_i(z,0) = S_0(z)$, when $t = 0$ and $z > 0$ (17)

$S_i(0,t) = S_{i0}(t), t > 0$ (18)

$S_i(L,t) = S_{i1}(t), t > 0$ (19)

$S_{i0}(t)$ *and* $S_{i1}(t)$ are the saturations at z = 0 and $z = L$ respectively.

Put, $S = 1-\propto S_i$ in equation (16), we get

$$P\frac{\partial S_i}{\partial t} = \frac{K\beta}{\mu_n}\frac{\partial}{\partial z}\left[S\frac{\partial S}{\partial z}\right] - \frac{K\alpha(\rho_i-\rho_n)g}{\mu_n}\frac{\partial S}{\partial z} - \frac{\mu\lambda^2}{4\pi\mu_i}\frac{\partial S}{\partial z} \quad (20)$$

By choosing dimensionless variables [18,19,20],

$$T = \frac{K\beta t}{L^2 P\mu n}, Z = \frac{z}{L} \; where\; 0 \le Z \le 1 \; and\; 0 \le T \le 1$$

Then equation (20) becomes,

$$\frac{\partial S}{\partial T} = \frac{\partial}{\partial z}\left[S\frac{\partial S}{\partial z}\right] - M\frac{\partial S}{\partial z} - N\frac{\partial S}{\partial z} \quad (21)$$

$$\text{where, } M = \frac{L\alpha(\rho_i - \rho_n)g}{\beta}, N = \frac{\mu\lambda^2 LP\mu_n}{4\pi K\beta\mu_i}$$

$$\frac{\partial S}{\partial T} = \frac{\partial}{\partial z}\left[S\frac{\partial S}{\partial z}\right] - A\frac{\partial S}{\partial z} \quad (22)$$

Where, A=M+N with gravitational effect and Magnetic effect with,

$S(Z,0) = 1 - S_0(Z)$ *when* $T = 0, 0 \le Z \le 1$ (23)

$S(0,T) = 1 - S_{i0}(T),\; 0 \le T \le 1$ (24)

$S(L,T) = 1 - S_{i1}(T), 0 \le T \le 1$ (25)

The problem is solved by using RDTM.

Reduced differential transform method (RDTM) :

Consider the function of 2 variables $u(x,t)$ and suppose that it can be expressed as a product of 2 single variable functions. So $u(x,t) = f(x).g(t)$.

Based on the properties of one-dimensional differential transform the function $u(x,t)$ can be represented as follows:

$$u(x,t) = \left(\sum_{i=0}^{\infty} F(i)\, x^i\right)\left(\sum_{j=0}^{\infty} G(j)\, t^j\right) = \sum_{k=0}^{\infty} U_k(x)\, t^k \tag{26}$$

Where, $U_k(x)$ is called t-dimensional spectrum function of $u(x,t)$.

If function $u(x,t)$ is analytic and differentiable continuously with respect to time 't' in the domain of interest, then

$$U_k(x) = \frac{1}{k!}\left[\frac{\partial^k}{\partial x^k} u(x,t)\right]_{t=0} \tag{27}$$

Hence the differential inverse transform of $U_k(x)$ is defined as follows:

$$u(x,t) = \sum_{k=0}^{\infty} U_k(x) t^k \tag{28}$$

From equation (27) and (28)

$$u(x,t) = \sum_{k=0}^{\infty} \frac{1}{k!}\left[\frac{\partial^k}{\partial x^k} u(x,t)\right]_{t=0} t^k \tag{29}$$

Using RDTM on equation (22),

$$(k+1)S_{i(k+1)}(Z) = \left[\sum_{r=0}^{k} (S_{ir})(S_{i(k-r)})_{ZZ}\right] + ((S_{ik})_Z)^2 + A(S_{ik})_Z \tag{30}$$

Taking $S_0(Z) = \frac{1-e^z}{1-e}$ for any $t > 0$.

$$\therefore S_{i0}(Z) = 1 - \left(\frac{1-e^z}{1-e}\right) \tag{31}$$

Now let $k = 0$, then put the initial condition, so

$$S_{i1}(Z) = [(S_{i0})(S_{i0})_{ZZ}] + ((S_{i0})_Z)^2 + A(S_{i0})_Z$$

$$S_{i1}(Z) = \left[\left(1 - \left(\frac{1-e^z}{1-e}\right)\right)\left(\frac{e^z}{1-e}\right)\right] + \left(\frac{e^z}{1-e}\right)^2 + A\left(\frac{e^z}{1-e}\right) \tag{32}$$

$$S_{i1}(Z) = \left(\frac{e^z}{1-e}\right) + \left(\frac{-e^z}{1-e}\right)\left(\frac{1-e^z}{1-e}\right) + \left(\frac{e^{2z}}{(1-e)^2}\right) + A\left(\frac{e^z}{1-e}\right)$$

$$S_{i1}(Z) = \frac{(A+1)(1-e)e^z - e^z + (2)e^{2z}}{(1-e)^2}$$

For second iteration k=1,

$$2\, S_{i2}(Z) = \left[(S_{i0})(S_{i1})_{ZZ} + (S_{i1})\left(S_{i(1-1)}\right)_{ZZ}\right] + ((S_{i1})_Z)^2 + A(S_{i1})_Z$$

$$2\, S_{i2}(Z) = \left[\left(1-\left(\frac{1-e^z}{1-e}\right)\right)\left(\frac{(A+1)(1-e)e^z-e^z+(2)e^{2z}}{(1-e)^2}\right) + \left(\left(\frac{(A+1)(1-e)e^z-e^z+(8)e^{2z}}{(1-e)^2}\right)+\left(\frac{e^z}{1-e}\right)\right)\right] + \left(\left(\frac{(A+1)(1-e)e^z-e^z+(4)e^{2z}}{(1-e)^2}\right)\right)^2 + A\left(\frac{(A+1)(1-e)e^z-e^z+(4)e^{2z}}{(1-e)^2}\right) \tag{33}$$

In this way we can generate other polynomials,

Now by inverse differential transform, $u(x,t) = \sum_{k=0}^{\infty} U_k(x)t^k$

$$\therefore S_i(Z,T) = S_{i0}(Z).T^0 + S_{i1}(Z).T^1 + S_{i2}(Z).T^2 + \cdots$$

$$S_i(Z,T) = 1-\left(\frac{1-e^z}{1-e}\right).T^0 + \left[\frac{(A+1)(1-e)e^z-e^z+(2)e^{2z}}{(1-e)^2}\right].T^1 + \left[\left(1-\left(\frac{1-e^z}{1-e}\right)\right)\left(\frac{(A+1)(1-e)e^z-e^z+(2)e^{2z}}{(1-e)^2}\right) + \left(\left(\frac{(A+1)(1-e)e^z-e^z+(8)e^{2z}}{(1-e)^2}\right)+\left(\frac{e^z}{1-e}\right)\right)\right] + \left(\left(\frac{(A+1)(1-e)e^z-e^z+(4)e^{2z}}{(1-e)^2}\right)\right)^2 + A\left(\frac{(A+1)(1-e)e^z-e^z+(4)e^{2z}}{(1-e)^2}\right).\frac{T^2}{2} + \cdots \tag{34}$$

Assuming some standard values as follows:

L= 1 m,g=9.8 $\frac{m}{s^2}$, ρn=0.3 $\frac{kg}{m^3}$, ρi=0.1 $\frac{kg}{m^3}$, β=1 ,α= 1.11,μ=0.1 $\frac{kg.mA^2}{s^2}$,λ=0.1 A ,P=0.5,μn=10 $\frac{m^2}{s}$,μi=0.29 $\frac{m^2}{s}$,K= 0.001 $\frac{kg.mA^2}{s^2}$,$\Longrightarrow A\approx 5.673$ $\frac{kg.m^2}{s^2}$.

Tabular and graphical presentations:

Tabular presentations of solution (34) have been obtained by using MATLAB coding.

Time (T) →	5×10^{-1}	6×10^{-1}	7×10^{-1}	8×10^{-1}	9×10^{-1}
Depth (Z) ↓	Saturation of injected water (Si)				
0	0.0501	0.0600	0.0704	0.0801	0.0903
10^{-1}	0.0554	0.0663	0.0772	0.0881	0.0990
2×10^{-1}	0.0696	0.0832	0.0968	0.1104	0.1240
3×10^{-1}	0.0926	0.1107	0.1288	0.1469	0.1650

4×10^{-1}	0.1244	0.1488	0.1732	0.1976	0.2220
5×10^{-1}	0.1650	0.1975	0.2300	0.2625	0.2950
6×10^{-1}	0.2144	0.2568	0.2992	0.3416	0.3840
7×10^{-1}	0.2726	0.3267	0.3808	0.4349	0.4890
8×10^{-1}	0.3396	0.4072	0.4748	0.5424	0.6100
9×10^{-1}	0.4154	0.4983	0.5812	0.6641	0.7470
1.0	0.4996	0.5999	0.6995	0.7998	0.9000

Table: 1 Si for different depth Z for fixed time T > 0 with magnetic field effect

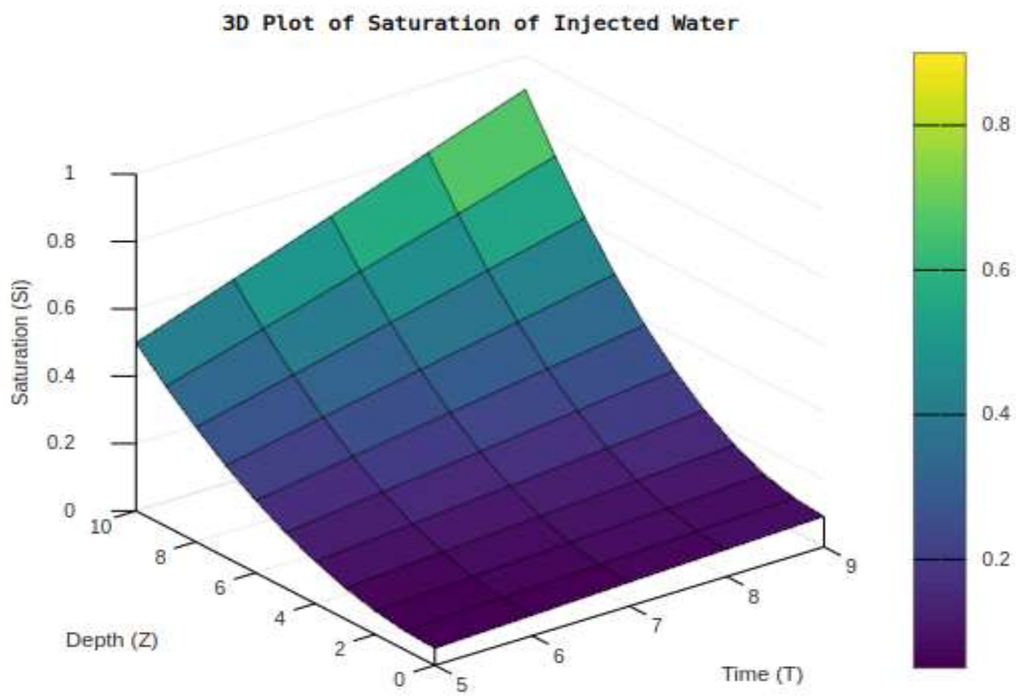

Figure 3: 3D plot of saturation of injected fluid with different time and depth

- This graph in Figure 3 shows the 3D plot of saturation of injected fluid with different time $(T = 0.5, 0.6, 0.7, 0.8, 0.9)$ and depth $(Z = 0,0.1,0.2,0.3,0.4, 0.5, 0.6, 0.7, 0.8, 0.9,1)$.It can be seen that saturation increases with the depth for any $T > 0$.
- Figure 4 shows the graph of saturation of injected fluid (S_i) vs depth (Z) for different time $(T = 0.5)$.
- Figure 5 shows the graph of saturation of injected fluid (S_i) vs depth (Z) for different time $(T = 0.6)$.

- Figure 6 shows the graph of saturation of injected fluid (S_i) vs depth (Z) for different time $(T = 0.7)$.
- Figure 7 shows the graph of saturation of injected fluid (S_i) vs depth (Z) for different time $(T = 0.8)$.
- Figure 8 shows the graph of saturation of injected fluid (S_i) vs depth (Z) for different time $(T = 0.9)$.
- Figure 9 shows the graph of saturation of injected fluid with different time $(T = 0.5, 0.6, 0.7, 0.8, 0.9)$ and depth $(Z = 0,0.1,0.2,0.3,0.4, 0.5, 0.6, 0.7, 0.8, 0.9,1)$.

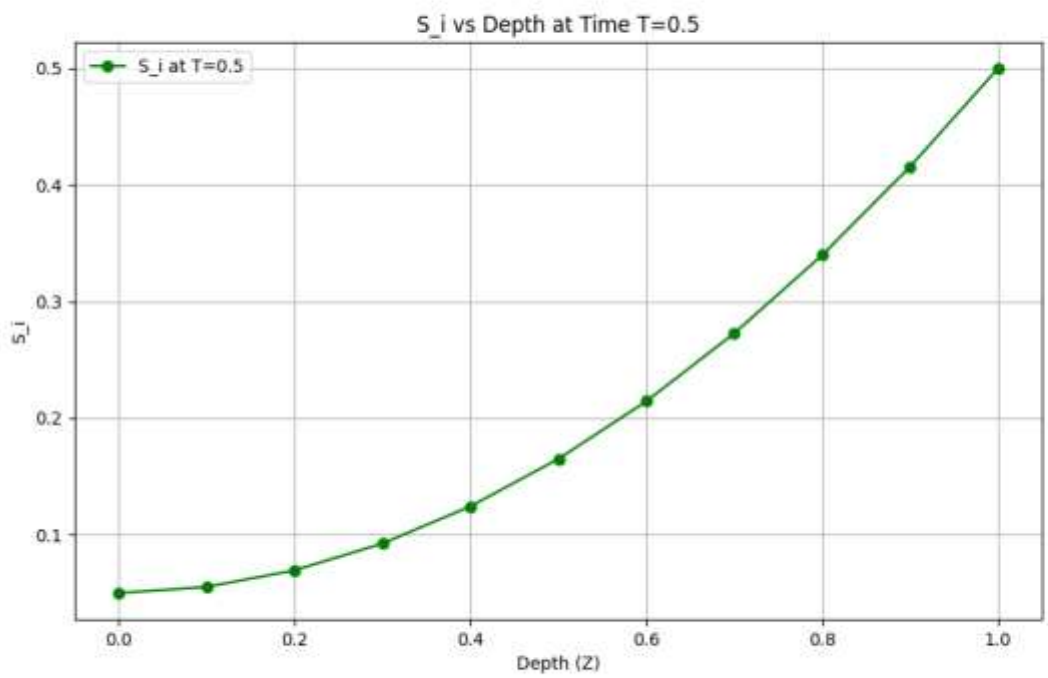

Figure 4: S_i *vs depth* Z *for* $T = 0.5$

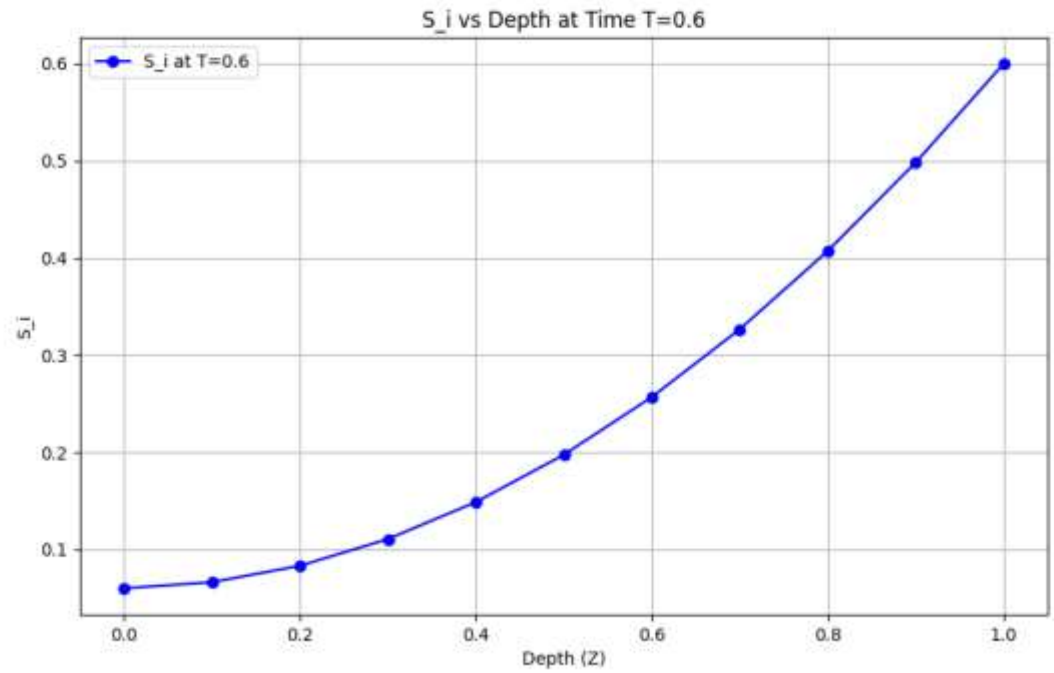

Figure 5: S_i *vs depth Z for* $T = 0.6$

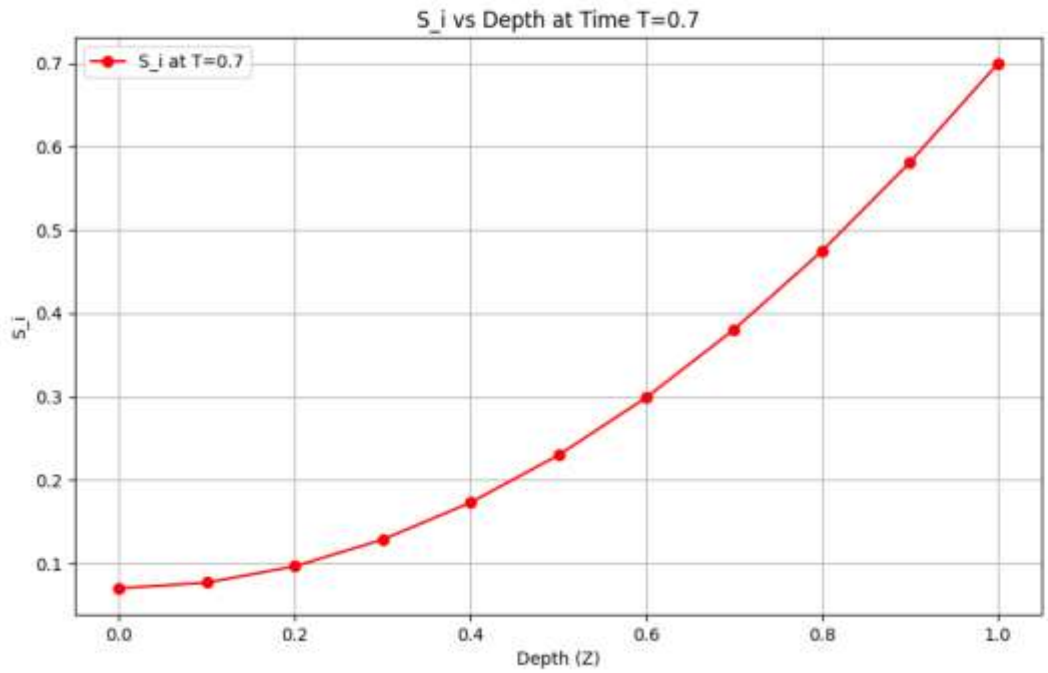

Figure 6: S_i *vs depth Z for* $T = 0.7$

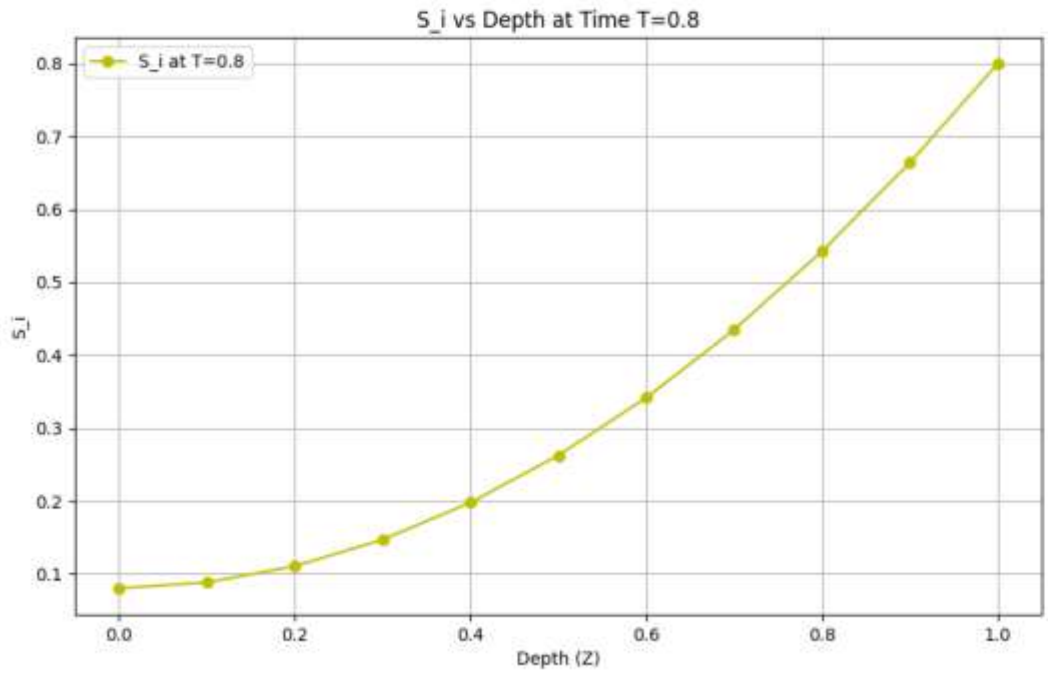

Figure 7: S_i *vs* Z *for* $T = 0.8$

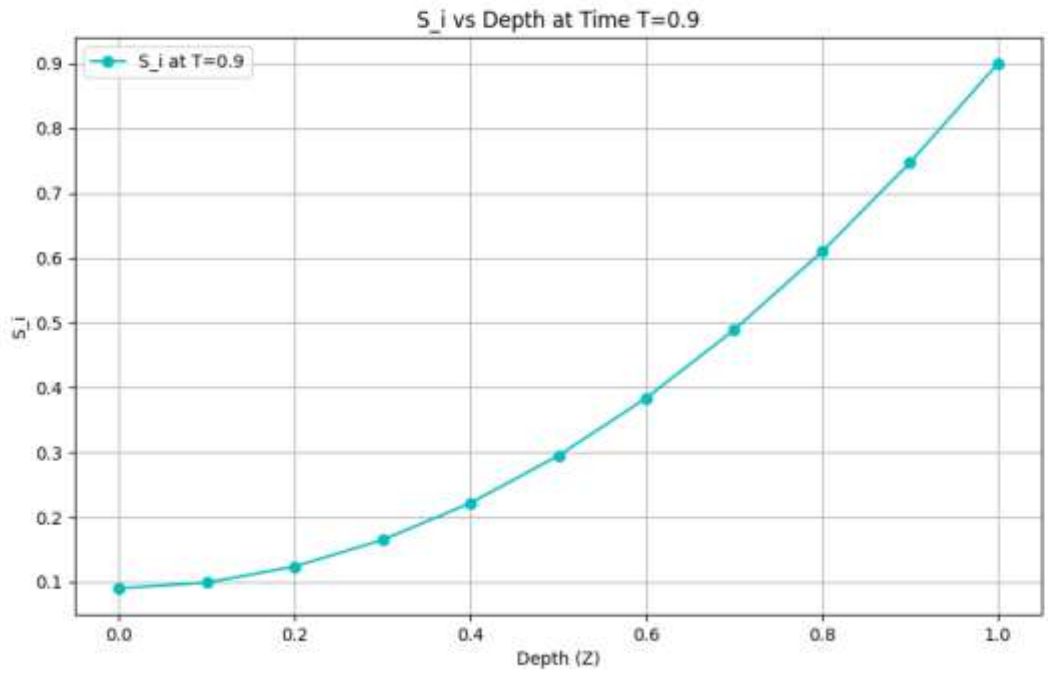

Figure 8: S_i *vs* $depth\ Z$ *for* $T = 0.9$

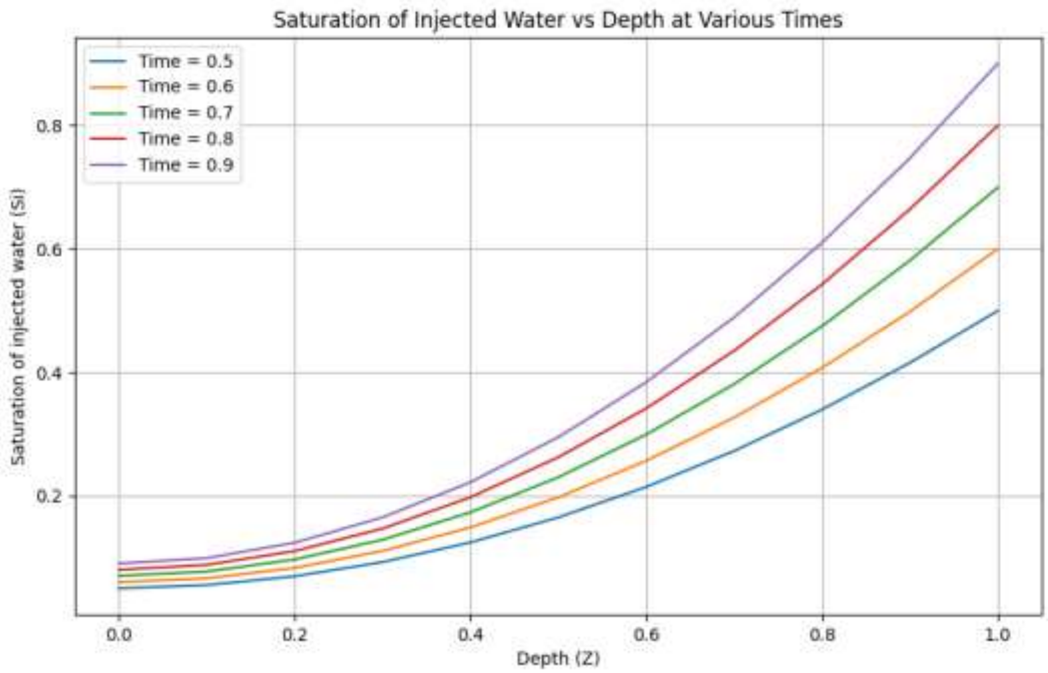

Figure 9: S_i *vs depth Z for different time* $(T = 0.5, 0.6, 0.7, 0.8, 0.9)$

Convergence analysis:

From equation (34) we get the series solution as a non-linear PDE then the convergence of the power series in 't' can be seen as following theorem [24],[25].

Theorem:

If $\emptyset_k(x,t) = U(i,k)\,(t - t_0)^k$, then the series solution $\sum_{k=0}^{\infty} \emptyset_k(x,t)$ stated in above equation $\forall\, k \in N \cup \{0\}$ follows as,

It is convergent if $\exists\; 0 < \lambda < 1$ such that $\|\emptyset_{k+1}(x,t)\| \leq \|\emptyset_k(x,t)\|$.
It is divergent if $\exists\; \lambda > 1$ such that$\|\emptyset_{k+1}(x,t)\| \geq \|\emptyset_k(x,t)\|$.

Conclusion:

From the numerical and graphical results obtained in this research, one can see that the imposition of a magnetic field has a significant effect on saturation behavior of injected fluid. That is, as the magnetic field is imposed, the injected fluid's saturation is increased, showing an obvious interaction between magnetic phenomena and fluid dynamics in porous media. This result indicates that magnetic fields are indeed capable of being used effectively to optimize fluid injection procedures, which could be useful in many engineering and environmental applications.

Application of the Reduced Differential Transform Method (RDTM) has been very effective and computationally inexpensive to solve the problem's governing equations. Compared to traditional numerical approaches such as the Finite Difference Method (FDM) or Finite Volume Method (FVM), RDTM minimizes the solution process without compromising precision and meeting the system's boundary and initial conditions as stated in equation (34). This underscores the applicability of the method as a reliable substitute for solving complex nonlinear partial differential equations.

Also, the graphical representation of saturation depending on depth at different instants of time $(T = 0.5, 0.6, 0.7, 0.8, 0.9)$ demonstrates a monotonous trend: saturation increases with the depth for any $T > 0$. This effect, as it is shown by Figures 3 to 9, speaks for itself about the dynamic nature of fluid motion under magnetic fields and indicates the temporal evolution of saturation profiles in the system under study.

In general, the cooperative application of magnetic fields and RDTM is a promising direction for further research in augmented fluid flow through porous media. For forthcoming studies, various ranges of parameters can be investigated, multiphase flow systems can be considered, and experimental data can be used to validate the results to enhance the practical relevance of the introduced technique.

Conflict of Interest:

The authors declare that they have no conflict of interest.

Author Contributions:

The author contributed to the conceptualization, mathematical modeling, analytical derivations, computational simulations, data interpretation, and manuscript preparation of this research work. All aspects of the study, including literature review, problem formulation, implementation of the Reduced differential transform method (RDTM), result analysis, and manuscript drafting, were performed independently by the author.

Data Availability Statement:

The data supporting the findings of this study are available within the article itself. Any additional numerical data, computational codes, or simulation files generated during the current study are available from the corresponding author upon reasonable request.

Funding Statement:

The author declares that no financial support, grants, or funding from any external agency, commercial, or non-commercial source was received for conducting this research and preparing this manuscript.

Acknowledgement:

The author sincerely acknowledges and expresses heartfelt gratitude to all the anonymous reviewers for their valuable suggestions and constructive feedback, which significantly improved the quality of this paper. Special thanks to my family for their continuous encouragement, insightful discussions, and technical support, which greatly helped in enhancing the quality and clarity of the present work.

References:

1) Introduction to fluid flow through porous media. Lambert academic publishing (2014).
2) Modeling Groundwater Flow and Contaminant Transport, by Chang et.al.
3) An overview of immiscible fingering in porous media by BH kueper et.al. (1988).
4) Mathematical modelling for phenomenon influence of the magnetic field during different nano floodings by CW Sahabandu et.al. (2017).
5) Gohil, V. P., and Ramakanta Meher. "Effect of viscous fluid on the counter-current imbibition phenomenon in two-phase fluid flow through heterogeneous porous media with magnetic field." Iranian Journal of Science and Technology, Transactions A: Science 43 (2019): 1799-1810.
6) Patel, Kajal K., M. N. Mehta, and Twinkle R. Singh. "A homotopy series solution to a nonlinear partial differential equation arising from a mathematical model of the counter-current imbibition phenomenon in a heterogeneous porous medium." European Journal of Mechanics-B/Fluids 60 (2016): 119-126.
7) You X, Li S, Kang L, Cheng L. A Study of the Non-Linear Seepage Problem in Porous Media via the Homotopy Analysis Method. Energies. 2023; 16(5):2175. https://doi.org/10.3390/en16052175
8) Meher R. et. Al. Adomain decomposition approach to phenomena in double phase flow through porous media. Int. J. of Appl. Math and Mech., 6 (9) (2010), 34-36.
9) Mathematical model of phenomenon arises during secondary oil recovery process by A.K.Parikh et.al.
10) Mehta M. N., Asymptotic expansion of fluid flow through porous media. Ph.D. thesis South Gujarat University, Surat, India. (1977).
11) Muskat M., The Flow of Homogeneous Fluids Through Porous Media. McGraw-Hill, New York, (1937).
12) Semi Analytic-Numerical Solution of Imbibition Phenomenon in Homogeneous Porous Medium Using Hybrid Differential Transform Finite Difference Method by Aruna Sharma and Amit K. Parikh(2023).
13) Scheidegger A. E., The Physics of Flow through Porous Media, University of Toronto press. (1960).

14) The Statistical behavior of instabilities in displacement process in porous media. Canadian J. Physics (1961) by Scheidegger A. E., Johnson E. F.
15) Swaroop A, et.al. A Numerical Simulation of the phenomenon in porous media with magnetic field effect using finite element method. Proc. Indian Journal of Petroleum Geology. 11, 2, (2002), 55-64.
16) Snap-off during imbibition in porous media: Mechanisms, influencing factors, and impacts by G Li et.al.(2023).
17) Vyas N. B., et.al. Power seris solution of double phase instability phenomena under magnetic field effect. Int. J. of Appl. Math and Mech., (2011),
18) Handbook of Partial Differential Equations: Exact Solutions [in Russian]. Mezhdunarodnaya Programma Obrazovaniya, Moscow (1996).
19) Mathematical Modeling and Analysis of Fingero- Imbibition Phenomenon in Homogeneous Porous Medium with Magnetic Field Effect in Vertical Downward Direction by A.K. Parikh ,et.al. IJLTEMAS, 2014.
20) Study on fingero imbibition phenomena in inclined porous media by optimal homotopy analysis method by Shreekant pathak et. Al. ASEJ, 2018.
21) Phenomenon with magnetic field effect in two-phase flow through porous medium by M.A. patel et. Al. IJCMSE Vol 12(03), 2023.
22) Numerical solution of Countercurrent Imbibition in Inclined Homogeneous Porous Medium by using Polynomial based Differential Quadrature Method by Jishan Shaikh et. Al. PJAEE, 17 (9) (2020).
23) Application of Polynomial based differential quadrature method in double phase (Oil-Water) flow problem during secondary oil recovery process by Jishan Shaikh et. Al. , Indian Journal of Applied Research (IJAR) (2019).
24) The use of RDTM (reduced differential transform method) for solving partial differential equations with variable coefficients by B. A. Taha (2011).
25) Reduced differential transform method for solving parabolic-like and hyperbolic-like equations by N. Taghizadeh ,S. R. Moosavi Noori (2016).