A PRODUCTION INVENTORY MODEL FOR DETERIORATING ITEMS WITH PRICE & GREEN SENSITIVE DEMAND AND ALLOWABLE SHORTAGES UNDER TWO-LEVEL TRADE CREDIT POLICY

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Received:15.04.2024, Revised: 21.05.2024, Accepted: 27.05.2024

Abstract:
This paper considers production inventory model for deteriorating items with allowable shortages under two level trade credit policy. In the present time, green technology is the humming term in the business world. It is challenging for the business manager to seize the market by offering the most incredible green quality at a reasonable price in a specific economy. Here demand depends on linear selling prices and nonlinear green level. The effect of the reliability on the production system has been studied. In this two-level trade credit policy manufacturer offers a full trade credit period to the retailer, and the retailer provides a partial trade credit to the customers. The main objective of this study is to obtain the optimal cycle length for maximum total average profit through genetic algorithm approach. Finally, to illustrate the model and to show the effectiveness of the proposed approach, a numerical example is provided. A sensitivity analysis of the optimal solution with respect to the parameters of the system is carried out.

Keywords: Inventory, Selling price and green level dependent demand, Trade credit policy, Reliability, Deterioration, Genetic Algorithm.

1. Introduction:

In the modern age, inventory model having shortages is most important for all business purpose to obtain the classical economic production quantity (EPQ). Production house try to produce more reliable products in the current, highly competitive marketing environment. As a result, they frequently check to see if the products they make are perfect or not. In this paper, I have described this type production problem, and the impact of system dependability has been covered. In this work I consider that the production is going on a constant production rate. When the storage capacity attains its maximum level, then the production will stop and it will again start when inventory level reaches the level of maximum allowable shortages.

It is obvious that, in production base economic quantity management, the business is totally depending on demand and supply of goods. So demand is one of the important parameter. Green retailing is a green management strategy that emphasizes providing products and services to maintaining environmental safety for the customer. It includes several methods and procedures for distribution and sales that do not cause damage to the environment. Greenness is the most important issue in the current business scenario. Here, I assumed a deteriorated green product whose demand is dependent on price and nonlinear green level.

In the field of inventory management, deterioration of a fresh good is a major headache. This is also an important issue to a production base system during the stock period. In our daily life deterioration of goods is a common factor. To overcome this real phenomenon, many researchers/scientists have involved to discuss about the deterioration, which indicates the damage/rejection of an item. In general most of goods such as fruits, foods, vegetables, medicines, etc. have a time span to keep the fresh quality, i.e. no deterioration occurs during that span. This property of a good is called non-instantaneous deterioration.

In most of the stock-out inventory systems, either all the demand is back-ordered, in which all customers wait until their demands are satisfied; or the whole, demand is lost. However, in many realistic situations, during the stock-out period, the longer the waiting time is, the smaller the backlogging rate would be. For instance, for fashionable commodities, high-tech products with short life cycle, the willingness for a customer to wait for backlogging is diminishing with the length of the waiting time.

Now-a-days, the trade credit policy is one of the powerful tools in order to control demands, improve sales and promote commodities. Each member of supply chain who wants to purchase obtains full or partial trade credit based on the powerful decision-making right. In practice, to increase sales and market share, the manufacturer offers the retailer a permissible delay period and the retailer in turn provides a trade credit to its customers. If the payment is made within the trade credit period, there is usually no interest levied on the outstanding amount. As a result, the retailer has the option to sell the goods, deposit the money so it can earn interest. If the retailer does not pay within the trade credit period on previously negotiated terms and circumstances, the manufacturer might charge an interest rate. Retailers will benefit financially because they can receive interest on the income generated during the allowable wait period.

So, in this paper I try to develop a production base economic model for deteriorating items with allowable shortages under two level trade credit policy. Here I assume that the production rate is constant under reliability. Also I assume a deteriorated green product whose demand is dependent on price and nonlinear green level. The objective of this model is to find the optimal cycle length to maximize the total average profit.

2. Literature Review:

A short life cycle degrading product recycle in green supply chain inventory control system is provided by Chung and Wee [1]. Dem and Singh [2] take into account a multi-item integrated production inventory system on the idea of a vendor-managed strategy with a green manufacturing approach. Yadav et al. [3] provide a green supply chain management of the automotive sector using an inventory model with distribution centres utilising Particle Swarm Optimization. A group of researchers have considered inventory control systems with green manufacturing approach in their research such as Huang et al. [4], Saxena et al. [5], Sana [6] and others.

A neutrosophic economic order quantity (EOQ) model was developed by Mohanta et al. [7] under the concept that consumer demand is sensitive to retail price and marketing efforts. Yang et al. [8] gives a supply chain for deteriorating items with multi-retailer and price-sensitive demand. Giri et al. [9] proposed an inventory model in which green sensitive consumer demand has been taken with revenue sharing contract. Yadav and Khanna[10] considered expiration date of the perishable product with price dependent demand. Singh & Ummeferva[11] considered green inventory model with price and green sensitive demand under reliability. Mahapatra et al. [12] provides an inventory model for products that decay, with demand that is time and reliability dependent and partial backorder. Adak & Mahaptra [13] find the impact of reliability on multi-item inventory system with shortages and partial backlog incorporating time dependent demand and deterioration.

In today's competitive business transactions, it is found that the retailers are allowed some credit period before they settle the account with the wholesaler. Goyal [14] first studied an EOQ model under the conditions of permissible delay in payments. Huang [15] is the pioneer of an EOQ model along with a two-level trade credit period policy. Later on, Teng and Chang [16] investigated an EPQ model with two-level trade credit periods. Kumar et al. [17] proposed an inventory model on preservation technology with trade credits under demand rate dependent on advertisement, time and selling price. Das [18] developed an EPQ model for a deteriorating item under permissible delay in payment. Singh et al. [19] studied a two-echelon supply chain, which included a producer and a supplier with optimal trade-credit policies. Ummeferva et al. [20] developed two-level trade credit policy approach under greening degree dependent demand and reliability.

Table 1:

Summary of related literature for production inventory models:

Authors	Demand	Trade Credit	Shortage	Detei oration	Reliability
Mahapatra et al. (2017)	time& reliabilitydependent	No	Partial backorder	Yes	Yes
Das (2019)	Stock-dependent	trade credit	No	Yes	No
Adak &Mahapatra (2020)	time dependent	No	Partial backlogging	Yes	Yes
Kumar et al. (2020)	Advertisement, time and selling price dependent	Full trade credit	No	Yes	Yes
Yadav & Khanna (2021)	expiration date and price dependent	No	No	No	No
Singh & Ummeferva (2022)	price and green sensitive	No	No	Yes	No
Ummeferva et al. (2023)	green quality and selling price dependent	two-level trade credit	No	Yes	Yes
This Paper	linear selling prices and nonlinear green level	two-level trade credit	fully backlogging	Yes	Yes

3. Notations and Assumptions:

The following notations and assumptions are employed throughout this paper so as to develop the inventory model.

3.1 Notations:

P =Constant production rate per unit time.

$q(t) =$ Inventory level of the item at any time $t(>0)$.

W = Maximum shortages level.

C_3 = Ordering cost per order.

c_h = Holding cost per unit time.

c_p = Production cost per unit item.

c_s = Shortage cost per unit item.

θ = The constant deterioration rate, where 0< θ <1.

t_1 = Time at which shortages reach its maximum level and production starts.

t_2 = Time at which shortages are met.

t_3 = Time at which inventory level reaches its maximum level.

T = Duration of complete cycle where inventory level vanishes.

M = Full trade credit period of the retailer provided by the manufacturer, where $M > t_1$.

N = Partial trade credit period of the customer provided by the retailer, where $N > t_1$.

γ = Some percentage of total purchase price to be paid by the customer to the retailer, where 0< γ <1.

I_e = Interest earned per dollar per unit time by the retailer.

I_p = Interest paid per dollar per unit time by the retailer.

r = Reliability rate of produced items (%).

C_H = Total holding cost.

C_S = Total shortage cost.

C_P = Total production cost.

D_T = Total deteriorated items throughout the process.

P_I = Total amount of produced item.

s = Sales revenue per unit item.

S_T = Total selling price.

P_T = Retailer's total interest payable per cycle.

E_T = Retailer's total interest earned per cycle.

TP = Total profit.

TAP = The total average profit.

3.2 Assumptions:

i) Production rate is known and constant.

ii) The time horizon of the inventory system is infinite.

iii) Shortages are allowed and fully backlogged.

iv) Demand is price and green sensitive dependent.

v) Constant fraction of on-hand inventory gets deteriorated per unit time.

vi) There is no deterioration during the time $[t_1,\ t_2]$.

vii) The manufacturer produces the items with reliability.

viii) In this two-level trade credit policy manufacturer offers a full trade credit period to the retailer, and the retailer provides a partial trade credit to the customers.

4. Model Formulation:

In this model, I have considered a manufacturing system in which the demand rate $D(s,g)$ is dependent on greening level of the manufactured goods and selling price and is of the form: $D(s,g) = a - b.s + c.g^{\lambda}$, where a,b,c,λ >0 and value of green level (g) is measured within the interval (0, 1).

In the development of the mathematical formulation, let us assume that the shortage reaches its maximum level W at time $t = t_1$ and to make up shortages, production cycle starts at $t = t_1$. The shortages met at time $t = t_2$ and production cycle continue up to $t = t_3$, where the inventory level reaches its maximum level and then production stops. After that inventory level decline due to demand and deterioration and at time $t = T$ it becomes zero.

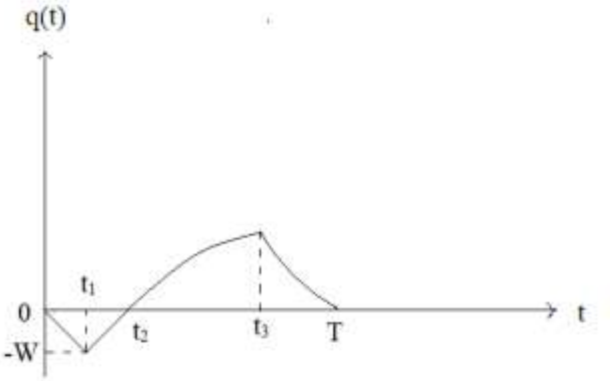

Fig. Graphical representation of a production inventory model under two-level trade credit policy.

The differential equations describing the inventory level $q(t)$ are given by,

$$\frac{dq(t)}{dt}=\begin{cases}-D(s,g), & 0\le t\le t_1\\ rP-D(s,g), & t_1\le t\le t_2\\ rP-D(s,g)-\theta.q(t), & t_2\le t\le t_3\\ -D(s,g)-\theta.q(t), & t_3\le t\le T\end{cases} \tag{1}$$

With the boundary conditions $q(t_1)=-W,\ q(t_2)=0,\ q(T)=0$.

The solutions of the differential equations in (1) are represented by

$$q(t)=\begin{cases}D(s,g).(t_1-t)-W, & 0\le t\le t_1\\ \{rP-D(s,g)\}.(t-t_2), & t_1\le t\le t_2\\ \dfrac{rP-D(s,g)}{\theta}\left[1-e^{-\theta(t-t_2)}\right], & t_2\le t\le t_3\\ \dfrac{D(s,g)}{\theta}\left[e^{\theta(T-t)}-1\right], & t_3\le t\le T\end{cases} \tag{2}$$

Using continuity condition on the equations in (2) at $t=t_1$ we have,

$$t_2=t_1+\frac{W}{rP-D(s,g)} \tag{3}$$

Using continuity condition on the equation in (2) at $t=t_3$ we get,

$$t_3=\frac{1}{\theta}.\log\left|\frac{\{rP-D(s,g)\}.e^{\theta t_2}+D(s,g).e^{\theta T}}{rP}\right| \tag{4}$$

Total Holding Cost (C_H) is given by

$$\begin{aligned}C_H&=c_h\int_{t_2}^{T}q(t)\,dt=c_h\left[\int_{t_2}^{t_3}q(t)\,dt+\int_{t_3}^{T}q(t)\,dt\right]\\ &=c_h\left[\frac{rP-D(s,g)}{\theta}\int_{t_2}^{t_3}\left\{1-e^{-\theta(t-t_2)}\right\}dt+\frac{D(s,g)}{\theta}\int_{t_3}^{T}\left\{e^{\theta(T-t)}-1\right\}dt\right]\\ &=c_h\left[\frac{rP-D(s,g)}{\theta}\left\{(t_3-t_2)+\frac{1}{\theta}\left(e^{-\theta(t_3-t_2)}-1\right)\right\}+\frac{D(s,g)}{\theta}\left\{\frac{1}{\theta}\left(e^{\theta(T-t_3)}-1\right)-(T-t_3)\right\}\right]\end{aligned} \tag{5}$$

Total amount of produced item (P_I) is given by

$$P_I = \int_{t_1}^{t_3} rP\,dt = rP(t_3 - t_1) \tag{6}$$

Total Production Cost, $C_P = c_p.rP(t_3 - t_1)$ (7)

The amount of inventory items deteriorated during the time period $(0,T)$ is given by

$$D_T = \theta\left[\int_{t_2}^{t_3} q(t)\,dt + \int_{t_3}^{T} q(t)\,dt\right]$$
$$= \left[\{rP - D(s,g)\}.\left\{(t_3 - t_2) + \frac{1}{\theta}(e^{-\theta(t_3-t_2)} - 1)\right\} + D(s,g).\left\{\frac{1}{\theta}(e^{\theta(T-t_3)} - 1) - (T - t_3)\right\}\right] \tag{8}$$

Total Shortages Cost (C_S) is given by

$$C_S = c_s\left[\int_0^{t_1} q(t)\,dt + \int_{t_1}^{t_2} q(t)\,dt\right] \tag{9}$$
$$= \frac{c_s}{2}\left[\{D(s,g).t_1 - 2W\}.t_1 - \{rP - D(s,g)\}.(t_2 - t_1)^2\right]$$

Total selling price, $S_T = s(P_I - D_T)$ (10)

For calculating the total profit of the retailer, the interest paid and earned must be calculated as well. For this purpose, the following cases are considered.

Case I: $N < M$

There are five sub cases:

Sub case 1: $t_1 < N < M \le t_2$

Since manufacturer offers full trade credit to the retailer and retailer offers partial trade credit to the customer, then Retailer's interest payable per cycle is given by

$$P_T = c_p I_p \int_M^T q(t)\,dt$$
$$= c_p I_p \left[\int_M^{t_2} q(t)\,dt + \int_{t_2}^{t_3} q(t)\,dt + \int_{t_3}^{T} q(t)\,dt\right] \tag{11}$$
$$= c_p I_p \left[\{rP - D(s,g)\}\left\{\frac{t_2^2 - M^2}{2} - t_2(t_2 - M)\right\}\right.$$
$$\left. + \frac{rP - D(s,g)}{\theta}\left\{(t_3 - t_2) + \frac{1}{\theta}\left(e^{-\theta(t_3-t_2)} - 1\right)\right\} + \frac{D(s,g)}{\theta}\left\{\frac{1}{\theta}\left(e^{\theta(T-t_3)} - 1\right) - (T - t_3)\right\}\right]$$

Retailer's interest earned per cycle is given by

$$E_T = sI_e \int_N^M D(s,g).t\,dt + sI_e\gamma \int_{t_1}^{N} D(s,g).t\,dt$$
$$= \frac{sI_e D(s,g)}{2}\left[(M^2 - N^2) + \gamma(N^2 - t_1^2)\right] \tag{12}$$

Sub case 2: $t_2 < M \le t_3$

Retailer's interest payable per cycle is given by

$$
\begin{aligned}
P_T &= c_p I_p \int_M^T q(t)\,dt \\
&= c_p I_p \left[\int_M^{t_3} q(t)\,dt + \int_{t_3}^T q(t)\,dt \right] \\
&= c_p I_p \left[\frac{rP - D(s,g)}{\theta} \left\{ (t_3 - M) + \frac{1}{\theta}\left(e^{-\theta(t_3 - t_2)} - e^{-\theta(M - t_2)} \right) \right\} + \frac{D(s,g)}{\theta} \left\{ \frac{1}{\theta}\left(e^{\theta(T - t_3)} - 1 \right) - (T - t_3) \right\} \right]
\end{aligned}
\tag{13}
$$

Retailer's interest earned per cycle is given by

$$
\begin{aligned}
E_T &= sI_e \int_N^M D(s,g).t\,dt + sI_e\gamma \int_{t_1}^N D(s,g).t\,dt \\
&= \frac{sI_e D(s,g)}{2} \left[\left(M^2 - N^2 \right) + \gamma \left(N^2 - t_1^2 \right) \right]
\end{aligned}
\tag{14}
$$

Sub case 3: $t_3 < M \le T$

Retai ler's interest payable per cycle is given by

$$
\begin{aligned}
P_T &= c_p I_p \int_M^T q(t)\,dt \\
&= c_p I_p \left[\frac{D(s,g)}{\theta} \left\{ \frac{1}{\theta}\left(e^{\theta(T - M)} - 1 \right) - (T - M) \right\} \right]
\end{aligned}
\tag{15}
$$

Retailer's interest earned per cycle is given by

$$
\begin{aligned}
E_T &= sI_e \int_N^M D(s,g).t\,dt + sI_e\gamma \int_{t_1}^N D(s,g).t\,dt \\
&= \frac{sI_e D(s,g)}{2} \left[\left(M^2 - N^2 \right) + \gamma \left(N^2 - t_1^2 \right) \right]
\end{aligned}
\tag{16}
$$

Sub case 4: $N < T \le M$

In this case there is no interest payable.

Retailer's interest earned per cycle is given by

$$
\begin{aligned}
E_T &= sI_e \int_N^T D(s,g).t\,dt + sI_e (M - T) \int_T^M D(s,g)dt + sI_e\gamma \int_{t_1}^N D(s,g).t\,dt \\
&= \frac{sI_e D(s,g)}{2} \left[\left(T^2 - N^2 \right) + 2(M - T)^2 + \gamma \left(N^2 - t_1^2 \right) \right]
\end{aligned}
\tag{17}
$$

Sub case 5: $T \le N < M$

In this case there is no interest payable.

Retailer's interest earned per cycle is given by

$$E_T = sI_e\gamma\int_{t_1}^{T} D(s,g).t\,dt + sI_e\gamma(N-T)\int_{T}^{N} D(s,g)dt + sI_e(M-N)\int_{N}^{M} D(s,g)dt$$
$$= \frac{sI_e D(s,g)}{2}\left[2(M-N)^2 + 2\gamma(N-T)^2 + \gamma\left(T^2 - t_1^2\right)\right] \quad (18)$$

Case II: $M < N$

There are four sub cases:

Sub case 6: $M < N \le t_2$

Retailer's interest payable per cycle is given by

$$P_T = c_p I_p \int_{M}^{T} q(t)\,dt$$
$$= c_p I_p \left[\int_{M}^{t_2} q(t)\,dt + \int_{t_2}^{t_3} q(t)\,dt + \int_{t_3}^{T} q(t)\,dt\right]$$
$$= c_p I_p \left[\{rP - D(s,g)\}\left\{\frac{t_2^2 - M^2}{2} - t_2(t_2 - M)\right\}\right.$$
$$\left. + \frac{rP - D(s,g)}{\theta}\left\{(t_3 - t_2) + \frac{1}{\theta}\left(e^{-\theta(t_3 - t_2)} - 1\right)\right\} + \frac{D(s,g)}{\theta}\left\{\frac{1}{\theta}\left(e^{\theta(T-t_3)} - 1\right) - (T - t_3)\right\}\right] \quad (19)$$

Retailer's interest earned per cycle is given by

$$E_T = sI_e\gamma\int_{t_1}^{M} D(s,g).t\,dt$$
$$= \frac{sI_e\gamma D(s,g)}{2}(M^2 - t_1^2) \quad (20)$$

Sub case 7: $t_2 < M < N \le t_3$

Retailer's interest payable per cycle is given by

$$P_T = c_p I_p \int_{M}^{T} q(t)\,dt$$
$$= c_p I_p \left[\int_{M}^{t_3} q(t)\,dt + \int_{t_3}^{T} q(t)\,dt\right]$$
$$= c_p I_p \left[\frac{rP - D(s,g)}{\theta}\left\{(t_3 - M) + \frac{1}{\theta}\left(e^{-\theta(t_3 - t_2)} - e^{-\theta(M - t_2)}\right)\right\} + \frac{D(s,g)}{\theta}\left\{\frac{1}{\theta}\left(e^{\theta(T-t_3)} - 1\right) - (T - t_3)\right\}\right] \quad (21)$$

Retailer's interest earned per cycle is given by

$$E_T = sI_e\gamma\int_{t_1}^{M} D(s,g).t\,dt$$
$$= \frac{sI_e\gamma D(s,g)}{2}(M^2 - t_1^2) \quad (22)$$

Sub case 8: $t_3 < M < N \le T$

Retai ler's interest payable per cycle is given by

$$P_T = c_p I_p \int_M^T q(t)\,dt$$
$$= c_p I_p \left[\frac{D(s,g)}{\theta} \left\{ \frac{1}{\theta}\left(e^{\theta(T-M)} - 1\right) - \left(T - M\right) \right\} \right] \quad (23)$$

Retailer's interest earned per cycle is given by

$$E_T = sI_e\gamma \int_{t_1}^{M} D(s,g).t\,dt$$
$$= \frac{sI_e\gamma D(s,g)}{2}(M^2 - t_1^2) \quad (24)$$

Sub case 9: $T \le M < N$

In this case there is no interest payable.

Retailer's interest earned per cycle is given by

$$E_T = sI_e\gamma \int_{t_1}^{T} D(s,g).t\,dt + sI_e\gamma(M-T)\int_{T}^{M} D(s,g)\,dt$$
$$= \frac{sI_e\gamma D(s,g)}{2}\left[(T^2 - t_1^2) + 2(M-T)^2\right] \quad (25)$$

Therefore, the retailer's total relevant profit for each sub case is obtained as follows:

$$[TP]_k = S_T - C_3 - C_H - C_S - C_P - P_T + E_T \quad (26)$$

Where k= 1, 2, 3, 4, 5, 6, 7, 8, 9.

Therefore, the retailer's total average profit per unit time for each sub case is obtained as follows:

$$[TAP]_k = \frac{[TP]_k}{T} \quad (27)$$

Where k= 1, 2, 3, 4, 5, 6, 7, 8, 9.

So, the above problem can be formulated as,

$$Maximize\ [TAP]_k, \qquad k = 1,2,3,4,5,6,7,8,9.$$

5. Solution Procedure:

Genetic Algorithm (GA)

The discovery of genetic algorithms (GA) by Holland [21] is further described by Goldberg [22]. GA is a randomized global search technique that solves problems imitating processes observed from natural evolution. GA continually exploits new and better solutions without any pre-assumptions such as continuity and unimodality. GA has been successfully adopted in many complex optimization problems and shows its merits over traditional optimization methods, especially when the system under study has multiple local optimal solutions. A GA normally starts with a set of potential solutions (called initial population) of the decision making problem under consideration. Individual solutions are called chromosomes. Crossover and mutation operations happen among the potential solutions to get a new set of solutions and it continues until terminating conditions are encountered. Michalewicz [23] proposed a GA named contractive mapping genetic algorithm (CMGA) and proved the asymptotic convergence of the algorithm by Banach's fixed-point theorem. In CMGA, movement from old population to new takes place only when average fitness of new population is better than the old one. The algorithm is presented below. In the algorithm, p_c; p_m are probability of crossover and probability of mutation, respectively, T is the generation counter, and $P(T)$ is the population of potential solutions for generation T. M is iteration counter in each generation to improve $P(T)$ and M_0 is the upper limit of M. Initializing $(P(1))$ function generates the initial population $P(1)$ (initial guess of solution set). Objective function value due to each solution is taken as fitness of the solution. Evaluating $(P(T))$ function evaluates fitness of each member of $P(T)$.

GA algorithm

1. Set generation counter T =1, iteration counter in each generation $M = 0$.
2. Initialize probability of crossover p_c, probability of mutation p_m, upper limit of iteration counter M_0, population size N.
3. Initialize ($P(T)$).
4. Evaluate ($P(T)$)
5. While ($M < M_0$).
6. Select N solutions from $P(T)$ for mating pool using roulette-wheel selection process Michalewicz [23]. Let this set be $P^{'}(T)$.
7. Select solutions from $P^{'}(T)$, for crossover depending on p_c.
8. Make crossover on selected solutions.
9. Select solutions from $P^{'}(T)$, for mutation depending on p_m.
10. Make mutation on selected solutions for mutation to get population $P_1(T)$.
11. Evaluate ($P_1(T)$).
12. Set M =M + 1.
13. If average fitness of $P_1(T)$ > average fitness of $P(T)$, then
14. Set $P(T+1)$ = $P_1(T)$.
15. Set $T = T+1$.
16. Set $M = 0$.
17. End if
18. End while
19. Output: Best solution of $P(T)$.
20. End algorithm.

The above model is solved by using GA approach. Our GA consists of parameters, population size = 50, probability of crossover = 0.6, probability of mutation = 0.2, and maximum generation = 50. A real number presentation is used here. In this representation, each chromosome X is a string of n numbers of GA, which denote the decision variable. For each chromosome X, every gene, which represents the independent variables, is randomly generated between their boundaries until it is feasible. In this GA, arithmetic crossover and random mutation are applied to generate new off springs.

6. Numerical Analysis:

The optimal total average profit of the above said two level trade credit policy production model with constant production rate for deteriorating items having price and green sensitive dependent demand has been treated with numerical data. An example is presented to illustrate the effect of the model developed here with the numerical data.
Here C_3 =60, c_h =0.25, c_p =6.5, P =225, s =8.5, c_s =0.15, γ =0.01, W =80, θ =0.1, a =140, b =0.5, c =3, λ =0.08, g =0.90, I_p =0.2, I_e =0.15, r =0.9 in appropriate units.

Now according to the proposed computation procedure (GA) the results listed in the following tables 1, 2, 3, 4 & 5 are obtained for Case-I of sub cases 1, 2, 3, 4 & 5 respectively and the results listed in the following tables 6, 7, 8 & 9 are obtained for Case-II of sub cases 6, 7, 8, & 9 respectively.

Case I: $N < M$

Table-1: Optimal solutions for illustrated example of sub case 1.

M	N	t_1	T	$[TAP]_1$
2.6	2.5	1.35	3.40	203.6584
2.7	2.5	1.46	3.41	210.1084
2.8	2.5	1.56	3.46	217.4018
2.7	2.4	1.46	3.27	222.8888
2.7	2.5	1.46	3.41	210.1084
2.7	2.6	1.46	3.59	197.3174

Table-2: Optimal solutions for illustrated example of sub case 2.

M	N	t_1	T	$[TAP]_2$
3.8	3.5	0.02	4.81	272.2584
3.9	3.5	0.21	4.85	282.6246
4.0	3.5	0.19	4.98	292.1078
3.9	3.4	0.21	4.85	295.0856
3.9	3.5	0.21	4.85	282.6246
3.9	3.6	0.21	4.85	269.8024

Table-3: Optimal solutions for illustrated example of sub case 3.

M	N	t_1	T	$[TAP]_3$
4.1	3.7	0.01	4.12	335.3328
4.2	3.7	0.01	4.21	349.2383
4.3	3.7	0.01	4.32	361.9789
4.2	3.6	0.01	4.21	364.4196
4.2	3.7	0.01	4.21	349.2383
4.2	3.8	0.01	4.21	333.6409

Table-4: Optimal solutions for illustrated example of sub case 4.

M	N	t_1	T	$[TAP]_4$
6.4	5.2	0.02	6.37	410.5184
6.5	5.2	0.15	6.49	423.9857
6.6	5.2	0.18	6.52	427.5379
6.5	5.1	0.03	6.45	434.5824
6.5	5.2	0.15	6.49	423.9857
6.5	5.3	0.18	6.49	409.4030

Table-5: Optimal solutions for illustrated example of sub case 5.

M	N	t_1	T	$[TAP]_5$
6.4	5.2	0.05	1.36	554.3473
6.5	5.2	0.05	1.36	586.8609

6.6	5.2	0.05	1.36	621.9756
6.5	5.1	0.03	1.31	638.6233
6.5	5.2	0.05	1.36	586.8609
6.5	5.3	0.05	1.34	559.7065

Case II: $M < N$

Table-6: Optimal solutions for illustrated example of sub case 6.

M	N	t_1	T	$[TAP]_6$
3.4	4.2	2.96	5.81	127.9298
3.5	4.2	2.96	5.81	128.5692
3.6	4.2	2.95	5.75	129.0151
3.5	4.1	2.85	5.67	131.0342
3.5	4.2	2.96	5.81	128.5692
3.5	4.3	3.05	5.92	126.7205

Table-7: Optimal solutions for illustrated example of sub case 7.

M	N	t_1	T	$[TAP]_7$
4.4	5.5	0.97	6.73	148.9733
4.5	5.5	0.97	6.71	152.6283
4.6	5.5	0.37	6.88	155.2483
4.5	5.4	0.32	6.79	156.3688
4.5	5.5	0.97	6.71	152.6283
4.5	5.6	1.11	6.81	147.0559

Table-8: Optimal solutions for illustrated example of sub case 8.

M	N	t_1	T	$[TAP]_8$
4.6	5.3	0.01	5.31	234.5869
4.7	5.3	0.04	5.33	235.5543
4.8	5.3	0.01	5.34	238.0188
4.7	5.2	0.10	5.22	238.5098
4.7	5.3	0.04	5.33	235.5543
4.7	5.4	0.01	5.42	232.3593

Table-9: Optimal solutions for illustrated example of sub case 9.

M	N	t_1	T	$[TAP]_9$
2.4	2.7	0.01	1.29	361.5651
2.5	2.7	0.01	1.28	362.5652
2.6	2.7	0.01	1.29	363.0132
2.5	2.6	0.01	1.28	362.5652
2.5	2.7	0.01	1.28	362.5652

2.5	2.8	0.01	1.28	362.5652

From the above tables, it is observed that, for fixed N as M increases retailer's total average profit also increases and these observations are realistic and also for fixed M as N increases retailer's total average profit decreases and these observations are also realistic.

6.1: Sensitivity Analysis:

For the given numerical example mentioned in section 6, sensitivity analyses are performed to study the effect of changes of different values of the demand parameters a,b,c and deterioration parameter θ on maximum retailer's total average profit of the system. It is observed that as demand parameter a increases when b,c,θ are fixed, retailer's total average profit increases, as demand parameter c increases when a,b,θ are fixed, retailer's total average profit also increases, as demand parameter b increases when a,c,θ are fixed, retailer's total average profit decreases and also as deterioration parameter θ increases when a,b,c are fixed, retailer's total average profit decreases. All these observations agree with the reality.

Table-10: The sensitivity analysis for sub case 1 when $M = 2.7 \; and \; N = 2.5$

$b=0.5, c=3, \theta=0.1$				$a=140, c=3, \theta=0.1$			
a	t_1	T	$[TAP]_1$	b	t_1	T	$[TAP]_1$
135	1.54	3.41	196.1894	0.4	1.43	3.48	213.5866
140	1.46	3.41	210.1083	0.5	1.46	3.41	210.1083
145	1.35	3.41	226.9873	0.6	1.49	3.59	206.3563
$a=140, b=0.5, \theta=0.1$				$a=140, b=0.5, c=3$			
c	t_1	T	$[TAP]_1$	θ	t_1	T	$[TAP]_1$
2	1.49	3.41	206.3346	0.075	1.46	3.46	210.6424
3	1.46	3.41	210.1083	0.100	1.46	3.41	210.1083
4	1.43	3.48	213.8510	0.125	1.46	3.36	209.6297

Table-11: The sensitivity analysis for sub case 2 when $M = 3.9 \; and \; N = 3.5$

$b=0.5, c=3, \theta=0.1$				$a=140, c=3, \theta=0.1$			
a	t_1	T	$[TAP]_2$	b	t_1	T	$[TAP]_2$
135	0.19	4.99	257.9602	0.4	0.02	4.89	288.2073
140	0.21	4.85	282.6246	0.5	0.21	4.85	282.6246
145	0.10	4.76	307.9549	0.6	0.14	4.89	278.9268
$a=140, b=0.5, \theta=0.1$				$a=140, b=0.5, c=3$			
c	t_1	T	$[TAP]_2$	θ	t_1	T	$[TAP]_2$
2	0.02	4.94	278.3454	0.075	0.02	4.92	289.4289
3	0.21	4.85	282.6246	0.100	0.21	4.85	282.6246
4	0.02	4.88	289.0778	0.125	0.02	4.88	277.3939

Table-12: The sensitivity analysis for sub case 3 when $M = 4.2 \; and \; N = 3.7$

$b=0.5, c=3, \theta=0.1$				$a=140, c=3, \theta=0.1$			
a	t_1	T	$[TAP]_3$	b	t_1	T	$[TAP]_3$
135	0.05	4.23	329.6113	0.4	0.03	4.23	350.6048

140	0.01	4.21	349.2383	0.5	0.01	4.21	349.2383
145	0.02	4.22	366.1220	0.6	0.03	4.22	345.0883
$a=140, b=0.5, \theta=0.1$				$a=140, b=0.5, c=3$			
c	t_1	T	$[TAP]_3$	θ	t_1	T	$[TAP]_3$
2	0.03	4.22	344.5942	0.075	0.03	4.22	354.7092
3	0.01	4.21	349.2383	0.100	0.01	4.21	349.2383
4	0.03	4.23	351.1021	0.125	0.03	4.23	341.0997

Table-13: The sensitivity analysis for sub case 4 when $M=6.5 \, and \, N=5.2$

$b=0.5, c=3, \theta=0.1$				$a=140, c=3, \theta=0.1$			
a	t_1	T	$[TAP]_4$	b	t_1	T	$[TAP]_4$
135	0.05	6.45	396.7027	0.4	0.18	6.45	424.5322
140	0.15	6.49	423.9857	0.5	0.15	6.49	423.9857
145	0.02	6.37	434.6898	0.6	0.15	6.49	419.9519
$a=140, b=0.5, \theta=0.1$				$a=140, b=0.5, c=3$			
c	t_1	T	$[TAP]_4$	θ	t_1	T	$[TAP]_4$
2	0.15	6.49	419.2813	0.075	0.15	6.49	436.0351
3	0.15	6.49	423.9857	0.100	0.15	6.49	423.9857
4	0.18	6.45	424.1976	0.125	0.18	6.45	408.2264

Table-14: The sensitivity analysis for sub case 5 when $M=6.5 \, and \, N=5.2$

$b=0.5, c=3, \theta=0.1$				$a=140, c=3, \theta=0.1$			
a	t_1	T	$[TAP]_5$	b	t_1	T	$[TAP]_5$
135	0.05	1.27	583.2573	0.4	0.03	1.36	593.9822
140	0.05	1.36	586.8609	0.5	0.05	1.36	586.8609
145	0.03	1.47	588.6893	0.6	0.05	1.31	581.9003
$a=140, b=0.5, \theta=0.1$				$a=140, b=0.5, c=3$			
c	t_1	T	$[TAP]_5$	θ	t_1	T	$[TAP]_5$
2	0.05	1.31	581.3585	0.075	0.05	1.36	587.8689
3	0.05	1.36	586.8609	0.100	0.05	1.36	586.8609
4	0.03	1.36	594.5213	0.125	0.05	1.36	585.8529

Table-15: The sensitivity analysis for sub case 6 when $M=3.5 \, and \, N=4.2$

$b=0.5, c=3, \theta=0.1$				$a=140, c=3, \theta=0.1$			
a	t_1	T	$[TAP]_6$	b	t_1	T	$[TAP]_6$
135	3.05	5.75	118.5519	0.4	2.93	5.80	130.7000
140	2.96	5.81	128.5692	0.5	2.96	5.81	128.5692
145	2.85	5.97	140.0948	0.6	2.99	5.77	126.4674
$a=140, b=0.5, \theta=0.1$				$a=140, b=0.5, c=3$			
c	t_1	T	$[TAP]_6$	θ	t_1	T	$[TAP]_6$
2	2.99	5.77	126.2316	0.075	2.96	5.94	130.0613
3	2.96	5.81	128.5692	0.100	2.96	5.81	128.5692
4	2.93	5.80	130.9484	0.125	2.95	5.71	127.4383

Table-16: **The sensitivity analysis for sub case 7 when** $M = 4.5\, and\ N = 5.5$

$b=0.5, c=3, \theta=0.1$				$a=140, c=3, \theta=0.1$			
a	t_1	T	$[TAP]_7$	b	t_1	T	$[TAP]_7$
135	1.28	6.71	139.7456	0.4	0.87	6.71	156.6116
140	0.97	6.71	152.6283	0.5	0.97	6.71	152.6283
145	0.32	6.73	177.2216	0.6	0.99	6.73	148.9079
$a=140, b=0.5, \theta=0.1$				$a=140, b=0.5, c=3$			
c	t_1	T	$[TAP]_7$	θ	t_1	T	$[TAP]_7$
2	1.07	6.71	148.2928	0.075	0.67	6.87	158.3541
3	0.97	6.71	152.6283	0.100	0.97	6.71	152.6283
4	0.87	6.71	157.0717	0.125	0.67	6.73	146.9939

Table-17: **The sensitivity analysis for sub case 8 when** $M = 4.7\, and\ N = 5.3$

$b=0.5, c=3, \theta=0.1$				$a=140, c=3, \theta=0.1$			
a	t_1	T	$[TAP]_8$	b	t_1	T	$[TAP]_8$
135	0.02	5.34	220.3333	0.4	0.01	5.34	238.4481
140	0.04	5.33	235.5543	0.5	0.04	5.33	235.5543
145	0.01	5.33	251.9112	0.6	0.01	5.33	233.6006
$a=140, b=0.5, \theta=0.1$				$a=140, b=0.5, c=3$			
c	t_1	T	$[TAP]_8$	θ	t_1	T	$[TAP]_8$
2	0.01	5.33	233.1638	0.075	0.01	5.34	245.5956
3	0.04	5.33	235.5543	0.100	0.04	5.33	235.5543
4	0.01	5.34	238.8888	0.125	0.01	5.33	226.7946

Table-18: **The sensitivity analysis for sub case 9 when** $M = 2.5\, and\ N = 2.7$

$b=0.5, c=3, \theta=0.1$				$a=140, c=3, \theta=0.1$			
a	t_1	T	$[TAP]_8$	b	t_1	T	$[TAP]_8$
135	0.01	1.20	357.8813	0.4	0.01	1.29	363.6771
140	0.01	1.28	362.5652	0.5	0.01	1.28	362.5652
145	0.01	1.38	366.7634	0.6	0.01	1.25	362.0126
$a=140, b=0.5, \theta=0.1$				$a=140, b=0.5, c=3$			
c	t_1	T	$[TAP]_8$	θ	t_1	T	$[TAP]_8$
2	0.01	1.25	361.8789	0.075	0.01	1.28	362.5658
3	0.01	1.28	362.5652	0.100	0.01	1.28	362.5652
4	0.01	1.29	363.9752	0.125	0.01	1.28	362.5644

7. Conclusion and future scope :

This study presents a production inventory model for deteriorating items with two level trade credit policy under price and green sensitive demand and fully backlogged shortages over an infinite time horizon. The model is solved numerically by Genetic Algorithm (GA) and then compared. Sensitivity analyses are also performed for different parameters to study the effect of the decision variables. Finally, a future study will incorporate more realistic assumptions in the proposed model, such as variable deterioration rate, stochastic nature of demand parameters and production rate with two warehouse.

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