10.48047/jocaaa.2024.33.08.125

FUZZY SOFT LOCALLY B OPEN SETS IN FUZZY SOFT BI TOPOLOGICAL SPACES

Sandhya G V[1*], Anil P N[2]
[1]Assistant Professor, Department of Mathematics, Global Academy of Technology, Bengaluru, Karnataka, India.
[2] Professor, Department of Mathematics, Global Academy of Technology, Bengaluru, Karnataka, India.
***Corresponding author email:** sandhyasureshcs@gmail.com

Abstract: The current work introduces the notion of fuzzy soft b locally open set in fuzzy soft bi topological spaces. Fuzzy soft b* and fuzzy soft b** locally open sets in fuzzy soft bi topological spaces are defined. These sets represent variations on the concept of fuzzy soft b locally open sets, offering a more refined classification of open sets in fuzzy soft bi topological spaces. The relationships between fuzzy soft b, b*, and b** locally open sets are explored. Building on these set definitions, the work also initiates optimal definitions for fuzzy soft b locally continuous functions. These functions are designed to preserve the b-local openness of sets under mappings, offering a new perspective on coherence in fuzzy soft bi topological spaces.

Keywords: Fuzzy soft b locally open set, fuzzy soft b* locally open set, fuzzy soft b** locally open set and fuzzy soft b locally continuous functions.

1. INTRODUCTION

Fuzzy set theory provides a powerful mathematical approach for dealing with imprecision and uncertainty, where traditional crisp sets are inadequate. This theory has been further enriched by the development of soft set theory, which offers a flexible and effective tool for handling uncertainties in complex systems. The integration of these two theories into fuzzy soft sets has created a comprehensive mathematical framework that is particularly useful for addressing problems involving ambiguity and incomplete information.

Fuzzy soft [fs] sets have been successfully applied in various domains, including engineering, physics, social sciences, and medical sciences. These applications have demonstrated the rich potential of fs sets in modelling and solving real-world problems where uncertainty is inherent. L.A Zadeh, (1965) introduced fuzzy sets to handle uncertainty, which was further extended into topological structures by Chang, (1968) to address challenges in fuzzy set theory, Molodtsov, (1999) introduced soft sets, applicable in various fields like game theory and probability. P K Maji etal, (2001) later combined fuzzy and soft sets into fuzzy soft sets. Topological structure of these sets was developed by Tanay and Kandemir, (2011) and further explored by Varol, Aygun, (2012) and Roy and Samanta, (2011). Anil P. N, (2016) introduced a new form of fuzzy soft set known as fuzzy b open soft set and also defined b closure, b interior of fs set. Further Anil P N and Sandhya G V, (2019) initiated fuzzy soft locally open sets in fuzzy soft topological spaces.

Fuzzy soft bi topological spaces are an advanced concept that combines elements from fuzzy set theory, soft set theory, and bi topological spaces. These are spaces endowed with two topologies, which can be used to study different types of convergence and continuity within a single space. Mukherjee and Park [6] initiated fuzzy soft bi topological spaces, defining notions such as $\tau_1\tau_2$-fuzzy soft open and closed sets, as well as $\tau_1\tau_2$- interior and closure operators. They also explored some fundamental properties of these concepts. Additionally, by describing a novel class of fuzzy soft sets

10.48047/jocaaa.2024.33.08.125

and introducing a few separation axioms in fuzzy soft bi topological spaces, A. F. Sayed (2017) and (2018) carried on this trend. Sayed A. F, (2021) presented (1, 2)∗-fuzzy soft b-continuous maps, fuzzy soft b-irresolute maps, and their correlation with other weaker forms of fuzzy soft continuous maps in fuzzy soft bi topological spaces. Additionally, the fundamental characteristics of (1, 2)∗-fuzzy soft b-open (closed) and (1, 2)∗-fuzzy soft b-irresolute maps are examined.For fuzzy soft points, three concepts of property in fuzzy soft bitopological spaces in the sense of quasi-coincidence have been presented and examined. These concepts satisfy projective, productive, and hereditary qualities. Saikh Shahjahan Miah,Ruhul Amin,Raihanul Islam,Muhammad Shahjalal,Rezaul Karim (2022). Ray S, Das B etal, (2024) introduced the concept (j,k)γ∗–operation in fuzzy bi topological spaces. The same operation was applied to study (j,k)∗-fuzzy open sets in a given fuzzy bitopological space. Finally, results on locally finiteness of a given fuzzy bitopological space were established via operation approach.

2. PRELIMINARIES

Definition 2.1: Let U be the initial universe and E be the parameter set. P(U) be the set of all subsets on U & $\lambda : E \to P(U)$ is a mapping. Then (λ, E) is called fuzzy soft (fs) set or delicate fuzzy set over U. It is represented as λ_E. For every $e \in E, \lambda_E : U \to P(U)$ is a fuzzy set on U.

Definition 2.2: Let τ be the collection of all fs sets over a universe U with a set of parameters K, then (U, τ, K) is called fuzzy soft topological space [FSTS] if it satisfies (i) $\tilde{0}_K, \tilde{1}_K \in \tau$ (ii) $\cup a_i \in \tau, \forall a_i \in \tau$ (iii) $a \cap b \in \tau, \forall a, b \in \tau$. Each member of τ is called fs open set in U and its complement is known as fs closed set.

Definition 2.3: The intersection of all fs sets containing f_K is called fs closure of f_K denoted by,

$$fscl(f_K) = \bigcap \{ h_K, h_K \text{ is fuzzy soft closed set and } f_K \subseteq h_K \}$$

Definition 2.4: The union of all fs open subsets of g_K is called fs interior of g_K denoted by $fs\,\mathrm{int}(g_K) = \bigcup \{ h_K, h_K \text{ is fuzzy soft open set and } h_K \subseteq g_K \}$

Definition 2.5 : A fs set f_K in (U, τ, K) is known as fs b open if it satisfies the constraint, $f_K \le (f\sin tfscl(f_K) \vee fsclf\sin t(f_K))$, and fs b closed if the condition $f_K \ge (f\sin tfscl(f_K) \wedge fscl(f_K) \vee fsclf\sin t(f_K))$ is satisfied.

Definition 2.6: Let f_K be a fs set in a fs topological space (U, τ, K) then fs b-closure of f_K (fsbcl (f_K)) is given as, $fsbcl(f_K) = \bigcap \{ g_K : g_K \text{ is a } fsb-closed \text{ set of } U \ \& \ g_K \ge f_K \}$, - fs b-interior of f_K (fsbint (f_K)) is defined as,

$$fsb\,\mathrm{int}(f_K) = \bigcup \{ h_K : h_K \text{ is a } fsb-open \text{ set of } U \ \& \ h_K \le f_K \}$$

10.48047/jocaaa.2024.33.08.125

Definition 2.7 : Let (U,K,σ_1) and (U,K,σ_2) be two distinct fuzzy soft topologies on (U, K). Then (U,K,σ_1,σ_2) is known as fuzzy soft bi topological space on which no assumptions of separation properties unless explicitly stated. The members of $\sigma_i (i = 1, 2)$ are called fuzzy open soft sets and their complements are called $\sigma_i (i = 1, 2)$ -fuzzy closed soft sets.

3. FUZZY SOFT BI TOPOLOGICAL SPACES

Definition 3.1: A subset f_K of (U,K,σ_1,σ_2) is called $(\sigma_1,\sigma_2)-$fs locally open if there exists σ_1-fs closed set g_K and σ_2 -fs open set h_K such that $f_K = g_K \cup h_K$

Definition 3.2: A subset f_K of (U,K,σ_1,σ_2) is called $(\sigma_1,\sigma_2)-$fs b locally open if there exists σ_1 -fs b closed set g_K and σ_2 -fs b open set h_K such that $f_K = g_K \cup h_K$

Definition 3.3: A subset f_K of (U,K,σ_1,σ_2) is called $(\sigma_1,\sigma_2)-$fs b*-locally open if there exists σ_1 -fs b closed set g_K and σ_2 -fs open set h_K such that $f_K = g_K \cup h_K$

Definition 3.4: A subset f_K of (U,K,σ_1,σ_2) is called $(\sigma_1,\sigma_2)-$fs b**-locally open if there exists σ_1 -fs closed set g_K and σ_2 -fs b open set h_K which satisfies $f_K = g_K \cup h_K$

Theorem 3.5 : In a fs bi topological space (U,K,σ_1,σ_2), every $(\sigma_i,\sigma_j)-$fs locally open set is both (a) $(\sigma_i,\sigma_j)-$fs b*-locally open (b) $(\sigma_i,\sigma_j)-$fs b**-locally open.

Proof: Let f_K be $(\sigma_i,\sigma_j)-$fs locally open set. Then one can find σ_i fs closed set m_K and σ_j fs open set h_K so that $f_K = m_K \cup h_K$

(a) Since m_K is σ_i fs closed, $Fs\operatorname{int} Fscl\,(m_K) \le m_K$ and $FsclFs\operatorname{int}(m_K) \le m_K$. Therefore $Fs\operatorname{int} Fscl(m_K) \wedge FsclFs\operatorname{int}(m_K) \le m_K$. Thus m_K is $\sigma_i -$ fs b closed. Therefore f_K is $(\sigma_i,\sigma_j)-$fs b* locally open.

(b) Since h_K is σ_j fs open, we have $h_K \le Fs\operatorname{int} Fscl(h_K)$ and $h_K \le FsclFs\operatorname{int}(h_K)$. Therefore

$h_K \le Fs\operatorname{int} Fscl(h_K) \wedge FsclFs\operatorname{int}(h_K)$. Hence h_K is σ_j b open. Thus we have $f_K = m_K \cup h_K$, where m_K is $\sigma_i -$ fs closed and h_K is σ_j fs b open. Hence f_K is $(\sigma_i,\sigma_j)-$fs b** locally open. Reverse implication is not true.

10.48047/jocaaa.2024.33.08.125

Example 3.6: Let $U=\{u_1,u_2,u_3\}$ and $K=\{e\}$. Consider the fs subsets $f_{1K}=\left\{\left\{\frac{0.3}{u_1},\frac{0.9}{u_2},\frac{0}{u_3}\right\}\right\}$, $f_{2K}=\left\{\left\{\frac{0.4}{u_1},\frac{0.4}{u_2},\frac{0}{u_3}\right\}\right\}$, $f_{3K}=\left\{\left\{\frac{0.4}{u_1},\frac{0.9}{u_2},\frac{0}{u_3}\right\}\right\}$, $f_{4K}=\left\{\left\{\frac{0.3}{u_1},\frac{0.4}{u_2},\frac{0}{u_3}\right\}\right\}$

(a) For $\sigma_1=\{\tilde{1},\tilde{0},f_{1K},f_{2K},f_{3K},f_{4K}\}$ and $\sigma_2=\{\tilde{1},\tilde{0},f_{1K}\}$ the fs set $g_K=\left\{\left\{\frac{0.4}{u_1},\frac{0.5}{u_2},\frac{0}{u_3}\right\}\right\}$ is σ_1 fs b closed. Then $h_K=g_K\cup f_{1K}$ is (σ_1,σ_2)- fs b*locally open but not (σ_1,σ_2)- fs locally open.

(b)For $\sigma_1=\{\tilde{1},\tilde{0},f_{1K}\}$ and $\sigma_2=\{\tilde{1},\tilde{0},f_{1K},f_{2K},f_{3K},f_{4K}\}$ the fs set $g_K=\left\{\left\{\frac{0.4}{u_1},\frac{0.9}{u_2},\frac{0}{u_3}\right\}\right\}$ is σ_2 – fs b open. Then $h_K=f_{1K}\cup g_K$ is (σ_1,σ_2) – fs b** locally open but not (σ_1,σ_2) – fs locally open.

Theorem 3.7: In a fs bi topological space (U,K,σ_1,σ_2), every (σ_i,σ_j) – b* locally fs open set is fs b locally open.

Proof: Suppose f_K is (σ_i,σ_j) – fs b* locally open set, then we can find σ_i – fs b closed f_K and σ_j – fs open sets f_K such that $f_K=g_K\cup h_K$. Since h_K is σ_j – fs open, we have $h_K\le F\sin tFs\,cl(h_K)$ and $h_K\le FsclFs\,\text{int}(h_K)$.

Therefore $h_K\le Fs\,\text{int}\,Fscl(h_K)\wedge FsclFs\,\text{int}(h_K)$. Hence h_K is σ_j – delicate fuzzy b open. Thus we have $f_K=g_K\cup h_K$, where g_K is σ_i – delicate fuzzy b closed & h_K is σ_j – delicate fuzzy b open. Hence f_K is (σ_i,σ_j) – fs b locally open. But not conversely.

Example 3.8: Let $U=\{a,b,c\}$ and $K=\{e\}$. Consider fs subsets. $f_{1K}=\left\{\left\{\frac{0.2}{a},\frac{0}{b},\frac{0.7}{c}\right\}\right\}$, $f_{2K}=\left\{\left\{\frac{0.3}{a},\frac{0}{b},\frac{0.3}{c}\right\}\right\}$, $f_{3K}=\left\{\left\{\frac{0.3}{a},\frac{0}{b},\frac{0.7}{c}\right\}\right\}$, $f_{4K}=\left\{\left\{\frac{0.2}{a},\frac{0}{b},\frac{0.3}{c}\right\}\right\}$ For $\sigma_1=\{\tilde{1},\tilde{0},f_{1K},f_{2K},f_{3K},f_{4K}\}$ and $\sigma_2=\{\tilde{1},\tilde{0},f_{1K},f_{2K},f_{3K},f_{4K}\}$ the fs set $g_{1K}=\left\{\left\{\frac{0.3}{a},\frac{0}{b},\frac{0.4}{c}\right\}\right\}$ is clearly σ_1 - fs b closed and $g_{2K}=\left\{\left\{\frac{0.3}{a},\frac{0}{b},\frac{0.8}{c}\right\}\right\}$ is σ_2 - fs b open. Then $h_K=g_{1K}\cup g_{2K}$ is (σ_1,σ_2) fs b locally open but not fs b*-locally open.

10.48047/jocaaa.2024.33.08.125

Theorem 3.9: In a fs bi topological space $(U, K, \sigma_1, \sigma_2)$, every fs b**- locally open set is fs b locally open in U.

Proof: If (δ, K) is $(\sigma_i, \sigma_j)-$ fs b**-locally open then there are two sets, $\sigma_i -$ fs closed (G, K) and $\sigma_j -$fs b open set (h, K) so that (δ, K) is expressed as union of (G, K) & (h, K). And (δ, K) is $\sigma_i -$ fs closed. We have $F\sin tFs\,cl(G,K) \le (G,K)$ & $FsclFs\,\text{int}(G,K) \le (G,K)$.Therefore $(G,K) \ge FsclFs\,\text{int}(G,K) \wedge F\sin tFscl(G,K)$. Hence (G, K) is $\sigma_i -$fs b closed. Thus (δ, K) is union of $\sigma_i -$ fs b closed and σ_j- fs b open sets. Hence (δ, K) is $(\sigma_i, \sigma_j)-$ fs b locally open. But every $(\sigma_i, \sigma_j)-$ fs b locally open set may not be fs b**- locally open.

Illustration 3.10: Let $U = \{a,b,c\}$ and $K = \{e\}$. Consider the fs subsets, $f_{1K} = \left\{\left\{\frac{0}{a}, \frac{0.3}{b}, \frac{0.8}{c}\right\}\right\}$ $f_{2K} = \left\{\left\{\frac{0}{a}, \frac{0.4}{b}, \frac{0.4}{c}\right\}\right\}$ $f_{3K} = \left\{\left\{\frac{0}{a}, \frac{0.4}{b}, \frac{0.8}{c}\right\}\right\}$ For $\sigma_1 = \left\{\tilde{1}, \tilde{0}, f_{1K}, f_{2K}, f_{3K}, f_{4K}\right\}$ and $\sigma_2 = \left\{\tilde{1}, \tilde{0}, f_{1K}, f_{2K}, f_{3K}, f_{4K}\right\}$ the fs set $g_{1K} = \left\{\left\{\frac{0}{a}, \frac{0.4}{b}, \frac{0.5}{c}\right\}\right\}$ is σ_1 fs b closed and $g_{2K} = \left\{\left\{\frac{0}{a}, \frac{0.4}{b}, \frac{0.9}{c}\right\}\right\}$ is σ_2 fs b open. Then $h_K = g_{1K} \cup g_{2K}$ is $(\sigma_1, \sigma_2)-$ fs b locally open but not $(\sigma_1, \sigma_2)-$ fs b**- locally open.

Theorem 3.11: If $(U, K, \sigma_1, \sigma_2)$ is fs- bi topological space and f_K is fs b locally open and g_K is $\sigma_i -$fs b closed and $\sigma_j -$fs b open in $(U, K, \sigma_1, \sigma_2)$ then $f_K \cap g_K$ is $(\sigma_i, \sigma_j)-$ fs b locally open.

Proof : Consider $(\sigma_i, \sigma_j)-$ delicate fuzzy b locally open set f_K, then $f_K = f_{1K} \cup f_{2K}$ where f_{1K} is $\sigma_i -$ fs b closed and f_{2K} is $\sigma_j -$ fs b open.

Consider $f_K \cap g_K = (f_{1K} \cup f_{2K}) \cap g_K = (f_{1K} \cap g_K) \cup (f_{2K} \cap g_K)$. Since g_K is $\sigma_i -$ fs b closed, $g_K \cap f_{1K}$ is $\sigma_i -$ fs b clopen, $f_{2K} \cap g_K$ is $\sigma_2 -$fs b open. Thus $f_K \cap g_K$ is $(\sigma_i, \sigma_j)-$ fs b locally open in $(U, K, \sigma_1, \sigma_2)$.

Remark 3.12: In fs bi topological space $(U, K, \sigma_1, \sigma_2)$, if f_K is $(\sigma_1, \sigma_2)-$ fs b* locally open and $\sigma_i -$fs closed and $\sigma_j -$fs open set g_K then $f_K \cap g_K$ is $(\sigma_i, \sigma_j)-$ fs b* locally open.

10.48047/jocaaa.2024.33.08.125

Proof : Since f_K is $(\sigma_1,\sigma_2)-$fs b*-locally open, we have $f_K = f_{1K} \cup f_{2K}$ where f_{1K} is σ_i-fs b closed and f_{2K} is σ_j- fs open.

Then $f_K \cap g_K = (f_{1K} \cup f_{2K}) \cap g_K = (f_{1K} \cap g_K) \cup (f_{2K} \cap g_K)$. Since g_K is σ_i-fs closed, $g_K \cap f_{1K}$ is σ_j-fs b closed and since σ_j-fs b open, $f_{2K} \cap g_K$ is σ_j-fs open. Thus $f_K \cap g_K$ is $(\sigma_i,\sigma_j)-$ fs b* locally open in (U,K,σ_1,σ_2).

Theorem 3.13: Let (U,K,σ_1,σ_2) be a fs- bi topological space. If f_K is $(\sigma_i,\sigma_j)-$fs b** locally open and g_K is σ_i-fs closed and σ_j- fs open, then $f_K \cap g_K$ is $(\sigma_i,\sigma_j)-$ fs b** locally open.

Proof : Since f_K is $(\sigma_i,\sigma_j)-$ fs b**- locally open, we have $f_K = f_{1K} \cup f_{2K}$ where f_{1K} is σ_i-fs closed and f_{2K} is σ_j- fs b open.

Then $f_K \cap g_K = (f_{2K} \cup f_{1K}) \cap g_K = (f_{1K} \cap g_K) \cup (f_{2K} \cap g_K)$. Since g_K is σ_i-fs closed, $f_{1K} \cap g_K$ is σ_i- fs closed and since σ_j- fs b open, $f_{2K} \cap g_K$ is σ_j- fs b open. Thus $f_K \cap g_K$ is $(\sigma_i,\sigma_j)-$ fs b**- locally open in (U,K,σ_1,σ_2).

Theorem 3.14: Let f_K be a member of (U,K,σ_1,σ_2). Then f_K is $(\sigma_i,\sigma_j)-$ fs b*-locally open if and only if $f_K = g_K \cup \tau_2 - f\sin t(f_K)$, for some σ_i- fs b-closed set g_K.

Proof : Let f_K is $(\sigma_i,\sigma_j)-$fs b*- locally open. We have $f_K = g_K \cup h_K$ where g_K is σ_i- fs b closed and h_K is σ_j-fs open. Clearly $g_K \le f_K$ and $\sigma_j - Fs\operatorname{int}(f_K) \le f_K$, we have $(g_K \bigcup \sigma_j - Fs\operatorname{int}(f_K)) \le f_K$. And also $h_K \le \sigma_j - Fs\operatorname{int}(f_K)$ and hence $g_K \bigcup h_K \le g_K \cup \sigma_2 - Fs\ \operatorname{int}(f_K))$.That is $f_K \le g_K \bigcup \sigma_j - Fs\operatorname{int}(f_K)$ (2). From (1) and (2) $f_K = g_K \cup \sigma_j - Fs\operatorname{int}(f_K)$.

Conversely, given $f_K = g_K \cup \sigma_j - Fs\operatorname{int}(f_K)$ where g_K is σ_i- fs b closed and $\sigma_j - Fs\operatorname{int}(f_K)$ is σ_j- fs open, we have f_K is $(\sigma_i,\sigma_j)-$fs b*- locally open.

Theorem 3.15: Let f_K and g_K be any two fs- subsets of a bi topological space (U,K,σ_1,σ_2). If f_K is $(\sigma_i,\sigma_j)-$ fs b-locally open and g_K is either σ_i- fs b closed or σ_j- fs b open then $f_K \cup g_K$ is $(\sigma_i,\sigma_j)-$fs b locally open.

10.48047/jocaaa.2024.33.08.125

Proof : Let f_K be $(\sigma_i,\sigma_j)-$fs b locally open, then $f_K = f_{1K} \cup f_{2K}$ where f_{1K} is $\sigma_i -$ fs b closed and f_{2K} is $\sigma_j -$fs b open. If g_K is $\sigma_i -$ fs b closed, then $f_{1K} \cup g_K$ is $\sigma_i -$ fs b closed and hence $f_K \cup g_K = (f_{1K} \cup f_{2K}) \cup g_K = (f_{1K} \cup g_K) \cup f_{2K}$ is $(\sigma_i,\sigma_j)-$fs b locally open. Conversely if g_K is $\sigma_2 -$ fs b open, then $f_{2K} \cup g_K$ is $\sigma_j -$ fs b open and hence $f_K \cup g_K = (f_{1K} \cup f_{2K}) \cup g_K = f_{1K} \cup (f_{2K} \cup g_K)$ is $(\sigma_i,\sigma_j)-$fs b locally open.

Theorem 3.16: Let f_K and g_K be any two fs subsets of a bi topological space (U,K,σ_1,σ_2). If f_K is $(\sigma_i,\sigma_j)-$fs b*- locally open and g_K is either $\sigma_i -$ fs closed or $\sigma_j -$ fs open then $f_K \cup g_K$ is $(\sigma_i,\sigma_j)-$fs b*- locally open.

Proof: Let f_K is $(\sigma_i,\sigma_j)-$fs b* locally open, then $f_K = f_{1K} \cup f_{2K}$ where f_{1K} is $\sigma_j -$ fs b closed and f_{2K} is $\sigma_j -$fs open. If g_K is $\sigma_i -$ fs closed, then $f_{1K} \cup g_K$ is $\sigma_j -$ fs b closed and $f_K \cup g_K = (f_{1K} \cup f_{2K}) \cup g_K = (f_{1K} \cup g_K) \cup f_{2K}$ is $(\sigma_i,\sigma_j)-$fs b*- locally open. If g_K is $\sigma_j -$ fs open, then $f_{2K} \cup g_K$ is $\sigma_j -$ fs open and hence $f_K \cup g_K = (f_{1K} \cup f_{2K}) \cup g_K = f_{1K} \cup (f_{2K} \cup g_K)$ is $(\sigma_i,\sigma_j)-$fs b*- locally open.

Theorem 3.17: Let f_K and g_K be members of fs bi topological space (U,K,σ_1,σ_2). If f_K is $(\sigma_i,\sigma_j)-$fs b**- locally open and A_K is either $\sigma_j -$ fs closed or $\sigma_j -$ fs open then $f_K \cup A_K$ is $(\sigma_i,\sigma_j)-$fs b**- locally open.

Proof: Let f_K is $(\sigma_i,\sigma_j)-$fs b**- locally open, then $f_K = f_{1K} \cup f_{2K}$ where f_{1K} is $\sigma_i -$ fs closed and f_{2K} is $\sigma_j -$fs b open. If A_K is $\sigma_i -$ fs closed, then $f_{1K} \cup A_K$ is $\sigma_i -$ fs closed.

And hence $f_K \cup A_K = (f_{1K} \cup f_{2K}) \cup A_K = (f_{1K} \cup A_K) \cup f_{2K}$ is $(\sigma_1,\sigma_2)-$fs b**- locally open. Alternatively, if A_K is $\sigma_j -$ fs b open, then $f_{2K} \cup A_K$ is $\sigma_j -$ fs b open. And hence $f_K \cup A_K = (f_{1K} \cup f_{2K}) \cup A_K = f_{1K} \cup (f_{2K} \cup A_K)$ is $(\sigma_i,\sigma_j)-$fs b**- locally open.

Theorem 3.18: Let f_K and g_K be any two fs- subsets of a bi topological space (U,K,σ_1,σ_2). If f_K and g_K are $(\sigma_i,\sigma_j)-$fs b locally open then $f_K \cup g_K$ is $(\sigma_i,\sigma_j)-$fs b locally open.

10.48047/jocaaa.2024.33.08.125

Proof : If f_K and g_K are $(\sigma_i,\sigma_j)-$fs b locally open then $f_K = f_{1K} \cup f_{2K}$ and $g_K = g_{1K} \cup g_{2K}$ where f_{1K}, g_{1K} are σ_i-fs b closed and f_{2K}, g_{2K} are σ_j-fs b open. $f_K \cup g_K = (f_{1K} \cup f_{2K}) \cup (g_{1K} \cup g_{2K})$ and it is equal to $(f_{1K} \cup g_{1K}) \cup (f_{2K} \cup g_{2K})$ is $(\sigma_1,\sigma_2)-$fs b locally open as $f_{1K} \cup g_{1K}$ is σ_i-fs b closed and $f_{2K} \cup g_{2K}$ is σ_j- fs b open.

Theorem 3.19: Let a_K and b_K be any two fs- subsets of a bi topological space (U,K,σ_1,σ_2). If a_K and b_K are $(\sigma_1,\sigma_2)-$fs b* locally open then $a_K \cup b_K$ is $(\sigma_i,\sigma_j)-$fs b* locally open.

Proof: If a_K and b_K are $(\sigma_i,\sigma_j)-$fs b*- locally open then $a_K = a_{1K} \cup a_{2K}$ and $b_K = b_{1K} \cup b_{2K}$ where a_{1K}, b_{1K} are σ_i-fs b closed and a_{2K}, a_{2K} are σ_j-fs open.

$a_K \cup b_K = (a_{1K} \cup a_{2K}) \cup (b_{1K} \cup b_{2K}) = (a_{1K} \cup b_{1K}) \cup (a_{2K} \cup b_{2K})$ is $(\sigma_i,\sigma_j)-$fs b*- locally open as $a_{1K} \cup b_{1K}$ is σ_i-fs b-closed and $a_{2K} \cup b_{2K}$ is σ_j-fs open.

Theorem 3.20: Let f_K and g_K belongs to fs bi topological space (U,K,σ_1,σ_2). If f_K and g_K are $(\sigma_i,\sigma_j)-$fs b** locally open then $f_K \cup g_K$ is $(\sigma_i,\sigma_j)-$fs b**- locally open.

Proof: Suppose f_K and g_K are $(\sigma_i,\sigma_j)-$fs b**-locally open then $g_K = g_{1K} \cup g_{2K}$ where f_{1K}, g_{1K} are σ_i-fs closed and f_{2K}, g_{2K} are σ_j-fs b open. Then $f_K \cup g_K = (f_{1K} \cup f_{2K}) \cup (g_{1K} \cup g_{2K}) = (f_{1K} \cup g_{1K}) \cup (f_{2K} \cup g_{2K})$ is $(\sigma_1,\sigma_2)-$fs b** locally open as $f_{1K} \cup g_{1K}$ is σ_i-fs closed and $f_{2K} \cup g_{2K}$ is σ_j-fs b open.

Remark 3.21: A $(\sigma_1,\sigma_2)-$ fs locally (b, b*,b** locally) open set need not be $(\sigma_2,\sigma_1)-$fs locally (b, b*,b** locally) open.

Example 3.22: Let $U = \{u,v,w\}$ and $K = \{e_1,e_2\}$. Let $\sigma_1 = \{\tilde{1},\tilde{0},f_K\}$ $\sigma_2 = \{\tilde{1},\tilde{0},g_K\}$, $f_K = \left\{\left\{\frac{0.2}{u},\frac{0.3}{v},\frac{0.9}{w}\right\},\left\{\frac{0}{u},\frac{0.4}{v},\frac{0.7}{w}\right\}\right\}$ $g_K = \left\{\left\{\frac{0.4}{u},\frac{0.7}{v},\frac{0.1}{w}\right\},\left\{\frac{0.5}{u},\frac{0.5}{v},\frac{0.6}{w}\right\}\right\}$

Then $h_K = \left\{\left\{\frac{0.8}{u},\frac{0.7}{v},\frac{0.1}{w}\right\},\left\{\frac{1}{u},\frac{0.6}{v},\frac{0.6}{w}\right\}\right\}$ is $(\sigma_1,\sigma_2)-$ fs locally open set but not $(\sigma_2,\sigma_1)-$ fs locally open.

10.48047/jocaaa.2024.33.08.125

Example 3.23: Let $U=\{x,y,z\}$ and $K=\{e_1,e_2\}$. Let $\sigma_1=\{\tilde{1},\tilde{0},f_K\}$ and $\sigma_2=\{\tilde{1},\tilde{0}\}$, $f_K=\left\{\left\{\frac{0.2}{x},\frac{0.3}{y},\frac{0.9}{z}\right\},\left\{\frac{0}{x},\frac{0.4}{y},\frac{0.7}{z}\right\}\right\}$ then $h_K=\left\{\left\{\frac{0.8}{x},\frac{0.7}{y},\frac{0.1}{z}\right\},\left\{\frac{1}{x},\frac{0.6}{y},\frac{0.3}{z}\right\}\right\}$ is $(\sigma_1,\sigma_2)-$ fs b (b*, b**)-locally open but not $(\sigma_2,\sigma_1)-$ fs b (b*,b**)-locally open.

Theorem 3.24: Let (U,K,σ_1,σ_2) be fs bi topological space, then a subset A of U is said to be $(\sigma_1,\sigma_2)-$ fs b open if $A\subseteq\sigma_1-f\sin t(\sigma_2-fscl(A))\cup\sigma 2-fscl(\sigma_1-f\sin t(A))$

(ii) $(\sigma_1,\sigma_2)-$ regular open if $A=\sigma_1-f\sin t(\sigma_2-fscl(A))$

(iii) $(\sigma_1,\sigma_2)-$ regular closed if $A=\sigma_1-fscl(\sigma_2-f\sin t(A))$

The complement of $(\sigma_1,\sigma_2)-$ fs b open set is said to be $(\sigma_1,\sigma_2)-$ fs b closed.

Theorem 3.25: Let (U,K,σ_1,σ_2) be fs bi topological space and $A\subseteq U$. Then,

(i) $(\sigma_1,\sigma_2)-$ fs b closure of A denoted by $(\sigma_1,\sigma_2)-$ fsbcl (A), is defined as the intersection of all $(\sigma_1,\sigma_2)-$ fs b closed sets containing A.

(ii) $(\sigma_1,\sigma_2)-$ fs b interior of A denoted by $(\sigma_1,\sigma_2)-$ fsbint (A), is defined as the union of all $(\sigma_1,\sigma_2)-$ fs b open sets contained in A.

Lemma 3.26: Let (U,K,σ_1,σ_2) be fs bi topological space and $A\subseteq U$. Then,

(i) $(\sigma_1,\sigma_2)-$ fsbint (A) is $(\sigma_1,\sigma_2)-$ fs b open.

(ii) $(\sigma_1,\sigma_2)-$ fsbcl (A) is $(\sigma_1,\sigma_2)-$ fs b closed.

(iii)A is $(\sigma_1,\sigma_2)-$ fs b open if and only if A= $(\sigma_1,\sigma_2)-fsb\,\mathrm{int}(A)$

(iv)A is $(\sigma_1,\sigma_2)-$ fs b closed if and only if A= $(\sigma_1,\sigma_2)-fsb\,cl(A)$

Lemma 3.27 : Let (U,K,σ_1,σ_2) be fs bi topological space and $A\subseteq U$. Then,

(i) $U\setminus(\sigma_1,\sigma_2)-fsbcl(A)=(\sigma_1,\sigma_2)-fsb\,\mathrm{int}(U\setminus A)$

(ii) $U\setminus(\sigma_1,\sigma_2)-fsb\,\mathrm{int}(A)=(\sigma_1,\sigma_2)-fsbcl(U\setminus A)$

Lemma 3.28: Let (U,K,σ_1,σ_2) be fs bi topological space and $A\subseteq U$. Then $f_K\in(i,j)-fsbcl(A)$ if and only if for every (i, j) –fs b open set B containing f_K such that $B\cap A$ is not null.

10.48047/jocaaa.2024.33.08.125

Definition 3.29: Let (U,K,σ_1,σ_2) and (V,K,μ_1,μ_2) be fs topological spaces. Any fs function from U to V is said to be.

(i) $(\sigma_i,\sigma_j)-$ fs locally continuous if inverse of each $(\mu_i,\mu_j)-$ fs locally open set g_K in V, is $(\sigma_i,\sigma_j)-$ fs locally open in U.**(ii)** $(\sigma_i,\sigma_j)-$ fs b locally continuous if inverse of each $(\mu_i,\mu_j)-$ fs locally open set g_K in V is $(\sigma_i,\sigma_j)-$ fs b locally open in U. **(iii)** $(\sigma_i,\sigma_j)-$ fs b* locally continuous if inverse of every $(\mu_i,\mu_j)-$ fs locally open set g_K in V is $(\sigma_i,\sigma_j)-$ fs b* locally open in U. **(iv)** $(\sigma_i,\sigma_j)-$ fs b** locally continuous if inverse image of every $(\mu_i,\mu_j)-$ fs locally open set g_K in V is $(\sigma_i,\sigma_j)-$ fs b** locally open in U.**(v)** $(\sigma_i,\sigma_j)-$ fs b locally irresolute if for each $(\mu_i,\mu_j)-$ fs b locally open set g_K in V, inverse is $(\sigma_i,\sigma_j)-$ fs b locally open set in U, for $i,j\in\{1,2\}$.

Definition 3.30 : A function λ from (U,K,σ_1,σ_2) to (V,K,μ_1,μ_2) is said to be

(i) $(\sigma_i,\sigma_j)-$ fs b continuous if inverse of each σ_i-fs open set is $(\sigma_i,\sigma_j)-$ fs b open.

(ii) $(\sigma_i,\sigma_j)-$ fs weakly b continuous if for each $f_K\in U$ and each σ_i-fs open set $g_K\in V$ containing $\lambda(f_K)$, there exists $(\sigma_i,\sigma_j)-$fs b open set containing f_K such that $\lambda(f_K)\subseteq\sigma_j-fscl(g_K)$.

(iii) $(\sigma_i,\sigma_j)-$ fs almost b continuous if for each $f_K\in U$ and each σ_i-fs open set $g_K\in V$ containing $\lambda(f_K)$, there exists $(\sigma_i,\sigma_j)-$fs b open set containing f_K such that $\lambda(f_K)\subseteq\sigma_i-f\sin t(j-fscl(g_K))$.

Definition 3.31 : Any function $\lambda:(U,K,\sigma_1,\sigma_2)\to(V,K,\mu_1,\mu_2)$ is called fs continuous if inverse of every $(\mu_i,\mu_j)-$fs open set in V is $(\sigma_i,\sigma_j)-$fs open in U.

Definition 3.32: Any function $\lambda:(U,K,\sigma_1,\sigma_2)\to(V,K,\mu_1,\mu_2)$ is known as fs b continuous if inverse of every $(\mu_i,\mu_j)-$fs open set in V is $(\sigma_i,\sigma_j)-$fs b open in U.

Theorem 3.33: Every fs continuous function in fs bi topological spaces is fs b continuous.

Proof: If $\eta:(U_1,K,\sigma_1,\sigma_2)\to(V_1,K,\mu_1,\mu_2)$ is fs continuous then inverse of every $(\mu_i,\mu_j)-$fs open set in V is $(\sigma_i,\sigma_j)-$fs open in U. And hence $(\sigma_i,\sigma_j)-$fs b open in U.

Therefore η is fs b continuous. But reverse implication is not valid.

10.48047/jocaaa.2024.33.08.125

Example 3.34 : Let $\eta:(U_1,K,\sigma_1,\sigma_2)\to(V_1,K,\mu_1,\mu_2)$ be fs identity mapping. $U=\{u,v\}$ and $V=\{w,z\}$, $\sigma_1=\{\tilde{1},\tilde{0},A_K\}$, Where $A_K=\left\{\left\{\frac{0.3}{u},\frac{0.5}{v}\right\},\left\{\frac{0.2}{u},\frac{0.4}{v}\right\}\right\}$, $\sigma_2=\{\tilde{1},\tilde{0},B_K\}$ and

$B_K=\left\{\left\{\frac{0.4}{u},\frac{0.5}{v}\right\},\left\{\frac{0.4}{u},\frac{0.4}{v}\right\}\right\}$. Fs open sets of U are $\left\{\tilde{0},\tilde{1},\left\{\frac{0.4}{u},\frac{0.5}{v}\right\},\left\{\frac{0.4}{u},\frac{0.4}{v}\right\}\right\}$,

$\mu_1=\{\tilde{1},\tilde{0},C_K\}$, & $C_K=\left\{\left\{\frac{0.3}{w},\frac{0.2}{z}\right\},\left\{\frac{0.1}{w},\frac{0.3}{z}\right\}\right\}$, $\mu_2=\{\tilde{1},\tilde{0},D_K\}$, &

$D_K=\left\{\left\{\frac{0.3}{w},\frac{0.4}{z}\right\},\left\{\frac{0.4}{w},\frac{0.4}{z}\right\}\right\}$. Fs open sets of V are $\left\{\tilde{0},\tilde{1},\left\{\frac{0.3}{w},\frac{0.4}{z}\right\},\left\{\frac{0.4}{w},\frac{0.4}{z}\right\}\right\}$

It is verified that inverse of $(\mu_i,\mu_j)-$ fs open set in V is $(\sigma_i,\sigma_j)-$ fs b open, but not fs open in U. Therefore η is not fs continuous but fs b continuous

Thus, in fs bi topological spaces, fs continuous functions $\Rightarrow$ fs b continuous functions

But every fs b continuous functions $\neq>$ fs continuous functions.

4. CONCLUSION

This work contributes to the growing body of research in fuzzy soft topology by introducing new concepts and establishing foundational relationships between them. By defining and exploring fuzzy soft b locally open sets and their variations, as well as initiating the research of fuzzy soft b locally continuous functions and fs b continuous functions, this research enhances our understanding of how continuity and openness can be conceptualized and applied in the more complex setting of fuzzy soft bi topological spaces.

5. AKNOWLEDGEMENT

The authors are grateful to the editors and reviewers for their valuable comments and suggestions.

6. REFERENCES

[1] Anil P. N. (2016). Fuzzy soft B-open sets in fuzzy soft topological spaces: *International Journal of Science Technology & Engineering*, *3*(2), 277–279.

[2] Chang, C. L. (1968). Fuzzy topological spaces: *Journal of Mathematical Analysis and Applications*, *24*(1), 182–190.

[3] Maji, P. K., Biswas, R., & Roy, A. R. (2001). Fuzzy soft sets: *Journal of Fuzzy Mathematics*, *9*(3), 589–602.

[4] Miah, S. S., Amin, R., Islam, R., Shahjalal, M., & Karim, R. (2022). New concepts on R1 fuzzy soft bitopological space in quasi-coincidence sense: *Journal of Mechanics of Continua and Mathematical Sciences*, *17*(4), 51–59.

[5] Molodtsov, D. (1999). Soft set theory—First results: Computers and Mathematics with Applications, 37(4-5), 19–31

10.48047/jocaaa.2024.33.08.125

[6] Mukherjee, P., & Park, C. (2015). Fuzzy soft bi topological spaces: *Mathematics and Computer Science Journal*, *10*, 1–8.

[7] Pazar Varol, B., & Aygun, H. (2012). Fuzzy soft topology: *Hacettepe Journal of Mathematics and Statistics*, *41*(3), 407–419.

[8] Roy, S., & Samanta, T. K. (2011). A note on fuzzy soft topological spaces: *Annals of Fuzzy Mathematics and Informatics*, *3*(2), 305–311.

[9] Ray, S., Das, B., Bhattacharya, B.,et al. (2024). Application of operation approach in fuzzy bitopological spaces: *Soft Computing*, *28*, 9225–9232

[10] Sandhya, G. V., & Anil, P. N. (2019). Fuzzy soft locally closed sets in fuzzy soft topological spaces: *Journal of New Theory*, *26*, 84–89.

[11] Sayed, A. F. (2017). Characterizations of some types of fuzzy soft sets in fuzzy soft bi topological spaces: *Journal of Advances in Mathematics and Computer Science*, *24*, 1–12.

[12] Sayed, A. F. (2018). Some separation axioms in fuzzy soft bi topological spaces: *Journal of Mathematics and Computer Science*, *8*, 28–45.

[13] Sayed, A. F. (2021). (1,2) * fuzzy soft b-continuity in fuzzy soft bitopological spaces: *Fuzzy Systems and Their Mathematics*, *25*, 8643–8652.

[14] Tanay, B., & Kandemir, M. B. (2011). Topological structure of fuzzy soft sets: *Computers and Mathematics with Applications*, *61*(10), 2952–2957.

[15] Zadeh, L. A. (1965). Fuzzy sets: *Information and Control*, *8*(3), 338–353.